THE STRESSES

IN

FRAMED STRUCTURES.

INCLUDING THE

STRENGTH OF MATERIALS AND THEORY OF FLEXURE.

ALSO THE

DETERMINATION OF DIMENSIONS AND DESIGNING OF DETAILS. SPECIFICATIONS.
COMPLETE DESIGNS AND WORKING DRAWINGS.

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PREFACE TO THE TENTH EDITION.

THE present edition is essentially a new work. It appears not only in a new form, greatly reduced in size and weight, but with additions and changes so extensive and important that it is practically rewritten and reset.

In the change of title from "Strains" to *Stresses*, we have at last conformed to the established usage of all physicists and the large and increasing majority of engineers. In changing the signs for compression, tension, and rotation we conform to the universal notation of analytical mechanics. These minor changes would have been made by the author long since had it not been for the expense, necessitating as they do an almost entire resetting of the type. The present work is thus in harmony as regards notation and nomenclature with modern practice.

As to more important changes, in Part I, Section I, page 37, we have replaced the articles upon "Graphic Representation of Moments for any Number of Forces" by new and correct matter. These articles have until now appeared in successive editions, erroneous in statement and demonstration. Later writers have not called the attention of the author to this blemish. Indeed, some have done him the favor of giving the same erroneous presentation without calling attention to its source. It is, however, hoped that in their future editions they may give him the credit of its correction.

Examination of the rest of this section will reveal many other minor changes, too numerous for mention here.

In Part I, Section II, many of the changes are of a more important character.

On page 97 *et seq.* we give a revised treatment of the "Method of Calculation by Concentrated Load Systems;" also by "equivalent uniform load," by "one locomotive excess and equivalent uniform load," and by "two locomotive excesses and actual uniform train load." We also give a comparison of the results of these various methods, and point out the reasons why the last method is now and always has been preferred by the author, and used in all the illustrative examples. At the same time the matter is so presented that any method may be used, the illustrations of the application of the statical principles being so full and complete that the reader can have no difficulty in using any system of loading he may prefer.

In Chapter VI, page 148, we have given the principle of least work, and illustrated its application to redundant members and to the deflection of a framed girder. We have there given our reasons for considering such applications of little practical value, although in recent works much stress has been laid upon them. This principle of least work is, however, capable of applications of great value in directions where it has not hitherto been applied, and we believe that in the present work several such applications will be found made for the first time, and the way opened for others.

Thus in Chapter VII, page 155, we have applied it to Swing Bridges, and have obtained new and general methods and formulas which apply to varying depth and chord section. We have given an example fully worked out, for a centre-bearing pivot-span, also a comparison of results with those obtained by the formulas hitherto in use. It is also shown that these formulas are but special cases of our more general method, when depth and chord-sections are constant. They do not therefore apply to most practical cases, although heretofore they have been the best in use. The new formulas of this chapter are deduced by themselves at the end. They follow from the principle of least work so directly and simply, that the value of this principle is well brought out.

Again, in Chapter IX, page 190, we have applied this principle to the Braced Arch, with the same results. Here also we obtain new and general methods and formulas which apply to varying depth and chord sections and to all forms, flat or full centre. Again we find that the formulas heretofore in use are but special cases for a flat parabolic arch of constant depth. They do not therefore apply to many practical cases. Here again the formulas of the chapter are deduced at the end, and those familiar with the usual mathematical treatment of the braced arch will appreciate the directness and simplicity of that afforded by the principle of least work.

Again, in Chapter X, page 217, this principle gives a new solution for the Suspension System. The new formulas thus obtained are recapitulated on page 228 for ready reference, in shape for practical application, and an example is worked out and results given both by the old and new methods. The old method assumes that the cable carries the entire load, dead and live, and that the truss acts merely to distribute a partial loading. The error of such an assumption is pointed out, and by the application of the principle of least work we determine the share of the loading, dead and live, which each system must carry. The new formulas obtained are simple and easy of application.

In the appendix to Part I, Chapter II, page 270, the theory of Flexure and Mechanics of Materials is rewritten and given in greater fulness, clearness, and with improved notation. Here again the principle of least work is applied, giving a direct and simpler determination of the reactions in the cases of beam fixed at one end and supported at the other, and fixed at both ends. On page 289 the fundamental formulas are recapitulated for reference, and these formulas are derived anew from the principle of work. We have also treated the topics of combined stresses and secondary stresses.

With a view to complete treatment, we have given in Chapter III, page 328, the application of the Theory of Flexure to Torsion, although the subject finds no application in Framed Structures. Thus the student need not go outside of the present work for his course on Mechanics of Materials.

In Chapter IV., page 333, we have given a new treatment of the "ideal column" and of long struts. The new formula of Mr. Prichard, M. Am. Soc. C. E., is here given for the first time. This formula is probably the most important on this subject since the derivation of Euler's well-known formula. We have also given the practical column formulas in general use. For the reasons stated in this chapter, we consider the formula of Prof. Merriman as the best practical formula thus far proposed.

As to Part II., which deals with practical designing and structural principles, the changes and additions are numerous, mainly relating to improved method of presentation or to present practice. This part of the subject is in process of change and development. The present work was one of the first to give due prominence to it, and the first to give anything like a complete and practical presentation. In subsequent editions this portion has been greatly enlarged. A point has been reached, however, where it would be impossible to try to keep pace with changing details and minutiæ of practice. Nor is such an attempt desirable in a work of this character. We have therefore sought in the present edition to give clearly and logically those permanent principles and practices, never out of date, upon which design must always rest, in such a manner as to indicate their application to constructions in general, whether framed structures or machines, together with such illustrations, special details, and applications to framed structures in especial, as may serve to give the student ability to use and apply those principles intelligently to other cases. The teacher must always supplement such an endeavor by more detailed instruction and by as great a variety of examples as the time allotted to the subject may permit; while the student and designer must seek in other works than this, in specifications, and in the records of the most recent practice for those changing rules and details which constitute the latest usage.

With the kind permission of Theodore Cooper, C.E., we give his new and revised Specifications for 1896 for Steel Railroad Bridges.

John Sterling Deans, C.E., the Chief Engineer of the Phœnix Bridge Co., has kindly revised his chapter on Erection, bringing it up to date for this edition.

The work closes with a chapter upon High-Building Construction, by Wm. W. Crehore, M. Am. Soc. C. E.

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PART I.

SECTION I.

DIFFERENT METHODS OF CALCULATION.

I. GENERAL PRINCIPLES.

INTRODUCTORY.

DEFINITION OF FRAMED STRUCTURES.—A framed structure, or "*truss*," is a collection of straight "*members*" so joined together by pins or rivets as to form a rigid framework.

The office of such a structure may be either to transmit or transform motion or work, in which case it may form part or whole of a mechanism or machine; or to resist the action of external forces tending to cause motion, in which case it is a structure of stability, or a statical construction. The principles which govern the discussion of the first case are therefore dynamical, and belong to the science of kinetics; while in the second they are those of statical equilibrium, and belong to the science of statics. The latter class of structures alone is discussed in this work.

The simplest kind of truss is a triangle, because that is the only figure whose shape cannot alter without changing the length of its sides. The triangle is thus the truss element, and all framed structures, no matter how complicated, which contain no superfluous members, may be considered as assemblages of triangles.

The members are always straight, because if a member is curved its axis does not coincide with the force at each end which it is designed to resist. The consequence is a tendency to deformation.

EXTERNAL FORCES.—Every structure which we shall consider is acted upon by external forces, such as the loads applied at various points, the reactions of the supports, the weight of the structure itself, the force of the wind, the weight of snow, shocks, etc. These external forces act to distort the structure and the various members of which it is composed. As we shall see later, they can all be resolved into forces applied at the ends of each member. These forces we may distinguish by the effect they produce in the member.

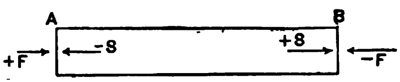
We thus distinguish:

Force of tension, or tensile force, which acts to elongate a member in the direction of its length.

Force of compression, or compressive force, which acts to compress a member in the direction of its length.

Force of shear, or shearing force, which acts upon a member at right angles to its length.

STRESS.—Let a member be acted upon at its ends by two equal and opposite external forces in the direction of its length, so that it is compressed or extended. Then, if equilibrium exists, it follows that at any imaginary section through the member there must exist two internal forces equal and opposite to the external forces at each end. These internal forces are called *stresses*. Thus, if a member AB is compressed by the two equal and opposite external forces $+F$, $-F$, we have



acting at each end, A and B, an internal force or *stress* $+ S$, $- S$ equal and opposite to the external force at that end. We may distinguish the stress according to the character of the external force it balances.

We thus distinguish:

Stress of tension, or tensile stress, due to attraction between the particles, which resists a tensile force.

Stress of compression, or compressive stress, due to repulsion between the particles, which resists a compressive force.

Stress of shearing, or shearing stress, which resists a shearing force.

Stress, then, is always internal. We speak of the force *on* a member, and the resulting stress *in* the member.

STRAIN.—When a member is acted upon by two equal and opposite forces in the direction of its length it is compressed or elongated. This compression or elongation in opposition to existing stress is called *strain.*

If the resisting stress is compressive, the strain is a compression.

If the resisting stress is tensile, the strain is an extension.

If the resisting stress is shearing stress, the strain is a shearing strain.

Stress, then, is measured in units of force, as, for instance, in pounds; while strain is measured in units of length, as, for instance, in inches, and it must always be opposite in direction to coexisting stress.

STRUT, TIE, BRACE, COUNTERBRACE, ETC.—The word "member," which we have used already so many times, signifies a body whose length is generally great in comparison to its other dimensions. It is always straight. By the union of such members the structure is formed, and the whole combination is termed a *framework.* The member has different names according to the stress it is designed to resist. When it resists a compressive stress in general it is called a *Strut*, and when the strut is vertical it becomes a *Post.* When the stress is tensile the member is called a *Tie.* The term *Brace* is used to denote both struts and ties. When a brace is rendered capable of acting either as a strut or as a tie indifferently it is said to be *counterbraced.*

BEAM, GIRDER.—In the case of a bending stress the member is called a *Beam.* When the beam is of considerable length and subjected to *transverse stresses only* it is called a *Girder*, and may be either *solid* or *flanged.* The cross-section of a solid girder is either rectangular, triangular, or round, or some modification of these forms. The flanged girder consists of one or two flanges of any desirable cross-section united to a thin vertical *web.** The office of the flanges is to resist the compressive and tensile stresses. That of the web is to resist the shearing stress. The web may be continuous, as in *plate girders*,* or open-work as in *framed girders.* It is with the latter only that we have to do in this work. The intersection of a brace with a flange is called an *Apex.** That portion of a flange between two adjacent apices is called a *Bay* or *Panel.*

FUNDAMENTAL PRINCIPLES.—All the various methods of investigating the conditions of stability of framed structures are based upon one of two principles—the so-called "principles of statical equilibrium.

The first of these is as follows:

If any number of forces, all in the same plane, and acting at a common point of application, or at different points of the same rigid body, are in equilibrium, the algebraic sum of all their components in any given direction is zero. That is, the sum of all the components tending to cause

* For illustration of a flange cross-section with web, see Fig. 280, Plate 20. For illustration of a plate girder, see Fig. 280, Plate 20, and for a framed girder, see Plate 11(a). For illustration of panel and apex, see Fig. 7, page 11.

motion in any one given direction is exactly equal to the sum of all those tending to cause motion in the precisely opposite direction.

This we shall call the "principle of the resolution of forces."

The second principle is as follows:

If any number of forces, all in the same plane, and acting at a common point of application, or at different points of the same rigid body, are in equilibrium, the algebraic sum of the moments of these forces, taken with reference to any point whatever in the plane of the forces, is zero. That is, the sum of the moments tending to cause rotation in one direction is balanced by the sum of the moments tending to cause rotation in the other direction.

This we shall call the "principle of the equality of moments."

DEFINITION OF "MOMENT."—The "moment" of a force is the product of the force into its "*lever arm.*" The lever arm of a force with respect to any point, which is called the "centre of moments," is the shortest distance of that point from the direction of the force, that is, it is the length of the perpendicular let fall from the point upon the force, prolonged in direction if necessary.*

UNNECESSARY MEMBERS.—*If any framed structure be conceived as cut entirely through so as to divide it into two parts, it is evident that if it held the outer forces in equilibrium before it was cut, the stresses in the cut members must have formerly held in equilibrium all the outer forces acting upon each of the parts into which the structure is divided.*

This principle is evident and does not need demonstration. Now we may resolve each of the outer forces, whatever their direction, and also the stresses in the cut members, into vertical and horizontal components respectively.

We then have, according to our fundamental principles:

1st. The algebraic sum of all the vertical components is zero.

2d. The algebraic sum of all the horizontal components is zero.

3d. The algebraic sum of the moments with reference to any point in the plane of the forces is zero.

Here, then, are three conditions, which furnish us in general with three equations between the acting forces. If only three of these forces are unknown, they can therefore be determined. But if more than three are unknown, they cannot be determined, because there are more forces to be found than there are equations of condition. Now in general all the outer forces acting upon a framed structure are known. It follows, therefore, *that if it is impossible to divide the structure in any direction without cutting more than three members, the stresses in which are necessarily unknown, the problem is indeterminate, and the structure has unnecessary or superfluous members.*

The frame should therefore be altered so as to dispense with one or more of these members, when the problem becomes determinate.

We can easily deduce a criterion for determining whether any frame has superfluous members. Assume, in general, the position of one side, thus fixing the position of two apices. Now, from these two apices we can locate another by two new sides. Then we can locate another by two sides from two previously located, and so on. If, then, m is the number of sides necessary for stability, and n is the number of apices, we have $m = 2(n - 2) + 1$, or $m = 2n - 3$, for the number of necessary sides. If the number of sides in any case exceeds $2n - 3$, the extra number are unnecessary for rigidity. If the number of sides in any case is less than $2n - 3$, the frame can change its shape without changing the length of its sides, and is therefore not rigid.

METHODS OF CALCULATION.—The two fundamental principles already given, give rise to two methods of calculation: the method by "resolution of forces," and the "method of

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* See page 23, Fig. 11, for illustration.

sections," or, as it is often called, the "method of moments." Each of these may be applied graphically or analytically. We may therefore draw up the following scheme, which includes all the methods of solution of framed structures of equilibrium:

I. Resolution of forces { (*a*) Graphic method of solution.
(*b*) Algebraic " " "

II. Method of moments { (*c*) Algebraic method of solution.
(*d*) Graphic " " "

Any one of these methods may be used in the solution of any given case, but in general there will be one, the employment of which in any special case will be found preferable in point of ease and simplicity to the others. Or, it may be, a combination of two or more of these methods furnishes a readier solution. It is therefore desirable that the engineer should be familiar with the principles and application of all, in order to proceed in the best manner in any special case.

The presentation and illustration of these four methods, in the order named, will therefore constitute the first Section of this work.

POSTULATES.—There are certain postulates which we require shall be understood and agreed to, before we can proceed to the application of our fundamental principles.

As the structures which we are to discuss are all of them structures of stability, that is, must oppose the action of outer forces and hold these forces in a state of rest, we assert:

1st. That all the forces which act upon any apex of a framed structure must constitute a system of forces in equilibrium, for which, therefore, the fundamental principles of equilibrium just stated hold good.

If, therefore, the outer forces at any apex are not in equilibrium already, they must be held in equilibrium by the stresses which they cause in the members meeting at that apex.

2d. If the entire structure or frame-work is required to remain at rest, it follows *that all the outer forces acting upon it must also constitute a system of forces in equilibrium.*

3d. A uniformly distributed load may, without sensible error, be assumed to be grouped into weights resting upon the apices, each apex supporting a weight equal to the load resting upon the adjoining half panels.

This is evidently correct in the case of pin joints, and in the case of riveted joints the influence of continuity can be disregarded. In practice, moreover, cross-girders * occur generally only at the apices, so that no panel is subject to transverse stress except from its own weight.

4th. The stress in each panel or brace is uniform throughout its length, and acts in the direction of the length only.

This must evidently be the case for any assemblage of straight members connected by pin joints. In riveted structures there may be a slight wrenching at the joints if the members are not accurately in the direction of the lines of stress, which can be neglected.

5th. A brace cannot undergo tension and compression simultaneously.

6th. The effect of several stresses acting at once upon any brace is the same as the algebraic sum of the effects produced by each stress when acting separately.

Thus, if the stresses are all tensile or all compressive, the combined effect is equal to the sum of the effects produced by each. If some are tensile and some compressive, the difference between the sum of the tensile and the sum of the compressive will be the resultant stress.

* For illustration of cross-girder, see Fig. 280, Plate 20.

UNIT-STRESS—INCH-STRESS.—The stress in any member per unit of area of its cross-section is called the *Unit-stress.* If this unit is the square inch, then the stress per square inch of cross-section is the inch-stress. The entire stress upon a member is then equal to its area of cross-section multiplied by its unit-stress.

SIGNS FOR TENSION AND COMPRESSION.—In mechanics generally a force acting away from the origin is always positive, acting towards the origin, negative. If, then, we take any apex as origin, a tensile stress in any member meeting at that apex will act away from the apex, and a compressive stress towards the apex. We therefore denote a tensile stress by a plus (+) sign and a compressive stress by a minus (−) sign.

The student may be aided in memorizing this by noting that the word "compression" contains the letter "m," and "m" stands for "minus."

CHAPTER I.

GRAPHIC RESOLUTION OF FORCES.

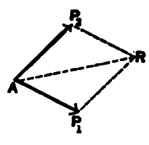
A. GENERAL PRINCIPLES. FORCES IN THE SAME PLANE, COMMON POINT OF APPLICATION.

GRAPHIC REPRESENTATION OF A FORCE.—Three things are necessary to be known in order that a force may be completely given—its point of application, its direction, and its magnitude. All three may be at once represented by a straight line. Thus the length of the line to any convenient scale, may represent the magnitude of the force; one end of this line then gives the point of application, and the direction of the line from this point gives the direction in which the force acts.

All forces of which we shall have occasion to speak will be considered as lying and acting in the same plane.

TWO FORCES—COMMON POINT OF APPLICATION.—If two forces P_1 and P_2, given in direction and magnitude by AP_1 and AP_2 have a common point of application A, Fig. 1, we may find the resultant R according to well known principles, by completing the parallelogram, as indicated by the dotted lines and drawing the diagonal AR. AR is the resultant in direction and intensity. If then we apply to the point A, a force AR it will have the same effect upon the point as the two forces P_1 and P_2 had when acting together, that is, it will *replace* P_1 and P_2. If, however, the resultant R acts in the direction RA, it will produce a precisely opposite effect from P_1 and P_2 acting together. If, therefore, we let P_1, P_2, and R all act upon the point A simultaneously, and suppose R to act in the direction from R to A, then these three forces *will be in equilibrium.*

Fig. 1



Now we wish to call attention to the fact that it is unnecessary to complete the parallelogram fully. Thus it would have been sufficient to have drawn a line as P_1R parallel and equal to AP_2, or a line P_2R parallel and equal to AP_1. In either case we should have found the point R, and would have found, therefore, the magnitude of the resultant.

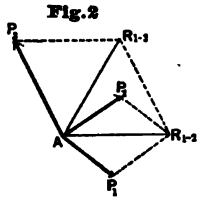
Next, as to the direction of the resultant, notice that if it acts in the direction from R to A it holds the forces in equilibrium. If it should act in the direction AR it would replace the forces.

If then the resultant is supposed to act in the direction obtained by following round either triangle AP_1R or AP_2R, *in the direction of the forces*, as from A to P_1 and P_1 to R and R to A, or from A to P_2 and P_2 to R and R to A, the direction RA thus obtained is the direction for equilibrium. The opposite direction is that in which the resultant must act when it *replaces* the forces.

THREE FORCES—COMMON POINT OF APPLICATION.—Suppose we have three forces

acting at A, as in Fig. 2. Then from the preceding, $R_{1\text{-}2}$ is the resultant of the forces P_1 and P_2. If we suppose it to act in the direction from A to $R_{1\text{-}2}$, it will replace P_1 and P_2 completely. We have then only to find the resultant of $R_{1\text{-}2}$ and P_3, by completing the parallelogram upon these forces, and we find $R_{1\text{-}3}$ the resultant of the forces P_1, P_2 and P_3.

Fig. 2



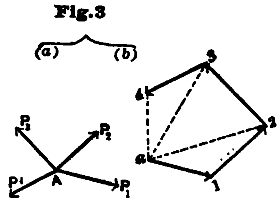
Again we see it is unnecessary to complete all the parallelograms. It would have been sufficient to draw $P_1R_{1\text{-}2}$ parallel and equal to P_2, and then $R_{1\text{-}2}R_{1\text{-}3}$ parallel and equal to P_3, and we should have found the resultant $R_{1\text{-}3}$.

Again, if we go around *in the direction of the forces*, from A to P_1 and P_1 to $R_{1\text{-}2}$, then to $R_{1\text{-}3}$, and then back to A, the direction thus obtained is, as before, *the direction of the resultant for equilibrium.*

It is not necessary, or even desirable, to go through the construction upon the diagram of the forces. It is better to keep the two constructions separate.

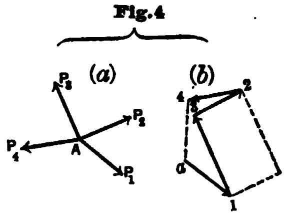
FOUR FORCES.—Let us apply these remarks to four forces P_1, P_2, P_3, P_4, acting at the point A, Fig. 3. The diagram (a) we call the *force diagram.* Now parallel to every force in the force diagram, we draw a line equal by scale to the magnitude of the force to which it is parallel. We thus obtain the polygon a 1 2 3 4. Thus $a1$ is parallel to P_1, and equal by scale to the magnitude of P_1. Then from the end of $a1$, we draw 1 2 parallel and equal to P_2, then 2 3 parallel and equal to P_3, then 3 4 parallel and equal to P_4. The polygon we thus obtain is called the *force polygon.* As we see, it is precisely the outline we should have obtained had we completed all the parallelograms directly upon the diagram (a) as in the last case, Fig. 2. Thus the diagonal $a2$ is the resultant of 1 and 2, $a3$ is the resultant of 1, 2 and 3, and $a4$ is the resultant of 1, 2, 3 and 4.

Fig. 3



ORDER OF FORCES IMMATERIAL.—The order in which the forces are laid off in the force polygon is immaterial. Thus in Fig. 4, it is evidently a matter of indifference whether we lay off the forces in the order 1, 3, 2, 4, or in the order 1, 2, 3, 4. In both cases we obtain the same resultant $a4$, and the same direction and magnitude, for the resultant. But by the same change of two and two we can produce any order we please.

Fig. 4



GENERAL PRINCIPLE.—We see, in Figs. 3 and 4, that the direction obtained for the resultant by following around the force polygon *in the direction of the forces* as laid off, is the direction *necessary for equilibrium.* The opposite direction is that which *replaces* the forces. Thus $a2$, Fig. 3 (b), is the resultant of forces P_1 and P_2, just as in Fig. 1, and if it is conceived as acting at A in the force diagram (a) in the direction given by $a2$, it will replace forces P_1 and P_2. We have then $a3$ as the resultant of $a2$ and 3 or of the forces 1, 2, and 3, and acting at the common point of application A in the direction from a to 3 it will replace forces 1, 2, and 3. Finally then, $a4$ is the resultant of forces 1, 2, 3, and 4, and acting in the direction from a to 4 will replace these forces, or will have the same effect upon the point of application A, as all the forces when acting together. Of course the opposite direction, or the direction from 4 to a, obtained by following round the force polygon in the direction of the forces, is the direction necessary for equilibrium. If then we conceive a force applied at A in the force diagram (a) equal and parallel to $4a$ and acting in the direction from 4 to a, as given by the force polygon (b), we should have a system of five forces all acting at the same point, *in equilibrium.* We have then the following general principle:

If any number of forces in the same plane having a common point of application are in

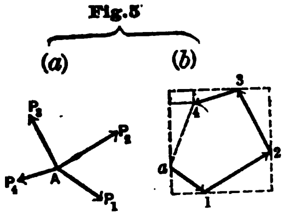
equilibrium, the force polygon closes. If the force polygon does not close, the line necessary to close it is the resultant. If this resultant acts upon the point of application in the direction obtained by following around the force polygon with the forces, it will hold the forces in equilibrium. If taken as acting in the opposite direction, it will replace the forces.

We see also that any diagonal of the force polygon, as shown by the dotted lines in Fig. 3 (*b*), is the resultant of the forces on each side, and replaces those upon one side, and holds in equilibrium those upon the other, or *vice versa*, according to the direction in which we let it act.

Thus, in Fig. 3 (*b*), we have 5 forces in equilibrium, because the polygon is closed by $4a$. If these are in equilibrium, then any two, as $a1$, $1\ 2$, must hold the others in equilibrium, but the resultant of $a1$ and $1\ 2$ is $a2$, and acting in the direction from a to 2, replaces these two forces. It would therefore hold the other forces in equilibrium if acting in this direction.

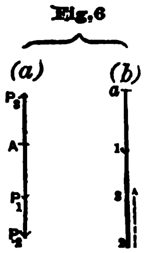
First Fundamental Principle of Equilibrium.—The general principle just enunciated is nothing more than a statement in other words of our first fundamental principle of equilibrium given on page 4. For if we resolve each force represented by a line of the polygon, into a horizontal and vertical component, for instance, as shown in Fig. 5 (*b*), it is evident, that if the algebraic sum of all the vertical components is zero, and the algebraic sum of all the horizontal components is zero, the polygon must be closed. Hence when the force polygon closes, the forces must be in equilibrium. Thus, starting from the point a, we see that three of the forces give downward vertical components, viz. 1, 4, and the resultant $4a$, and the sum of these, since the polygon closes, must be equal to the upward vertical components. So also for the horizontal components, 1 and 2 give components acting from left to right. Their sum is the horizontal distance from a to 2. But 3, 4 and the resultant give horizontal components acting from right to left, and if the polygon closes, their sum must be equal to the horizontal distance from 2 to a.

Fig. 5



Forces all Parallel.—If the forces are all parallel, the force diagram will be a straight line as in Fig. 6 (*a*), where we have three forces P_1, P_2, P_3, all vertical and acting at the same point A.

Fig. 6



If we lay off these forces in the order given we have the force polygon (*b*), which in this case is also a straight line. Thus $a1$ is laid off downwards, equal to P_1, then $1\ 2$ equal to P_2, then $2\ 3$ upwards equal to P_3. The line $3a$ then closes the polygon, and hence the resultant is the algebraic sum of the forces, or $P_1+P_2-P_3$. The line $a\ 1\ 2\ 3$ in Fig. 6 (*b*) should be regarded still as a polygon or double line. Thus following round in the direction of the forces we go from a to 1, 1 to 2, 2 to 3, and hence the resultant $3a$, which closes, must act upwards for equilibrium.

B. ILLUSTRATION OF GENERAL PRINCIPLES.

The foregoing principles, simple as they are, furnish us with the means of finding the stresses in any framed structure, however complicated, which the civil engineer can legitimately be called upon to erect, *provided only all the external forces are known.* They will be applied in detail to many different kinds of structures hereafter (see p. 66), so that the reader may obtain complete mastery of the method. We shall content ourselves here with a single example, merely to illustrate the method of application. For this purpose we select a very simple structure.

APPLICATION TO A ROOF TRUSS.

Dimensions of Truss.—Frame Diagram.—The truss shown in Fig. 7 is 50 feet span, and 12.5 feet high. Each rafter is divided into four equal panels, and the lower horizontal tie is divided into six equal panels. The bracing is as shown in the Figure. Each half of the frame is perfectly symmetrical with the other half. The Fig. 7 we call the *frame diagram*. It may be drawn to any convenient scale, *the larger the better*.

Loading of the Truss.—According to our postulate 3, page 6, we suppose all that portion of the weight of roof covering which extends from the centre of one panel to the

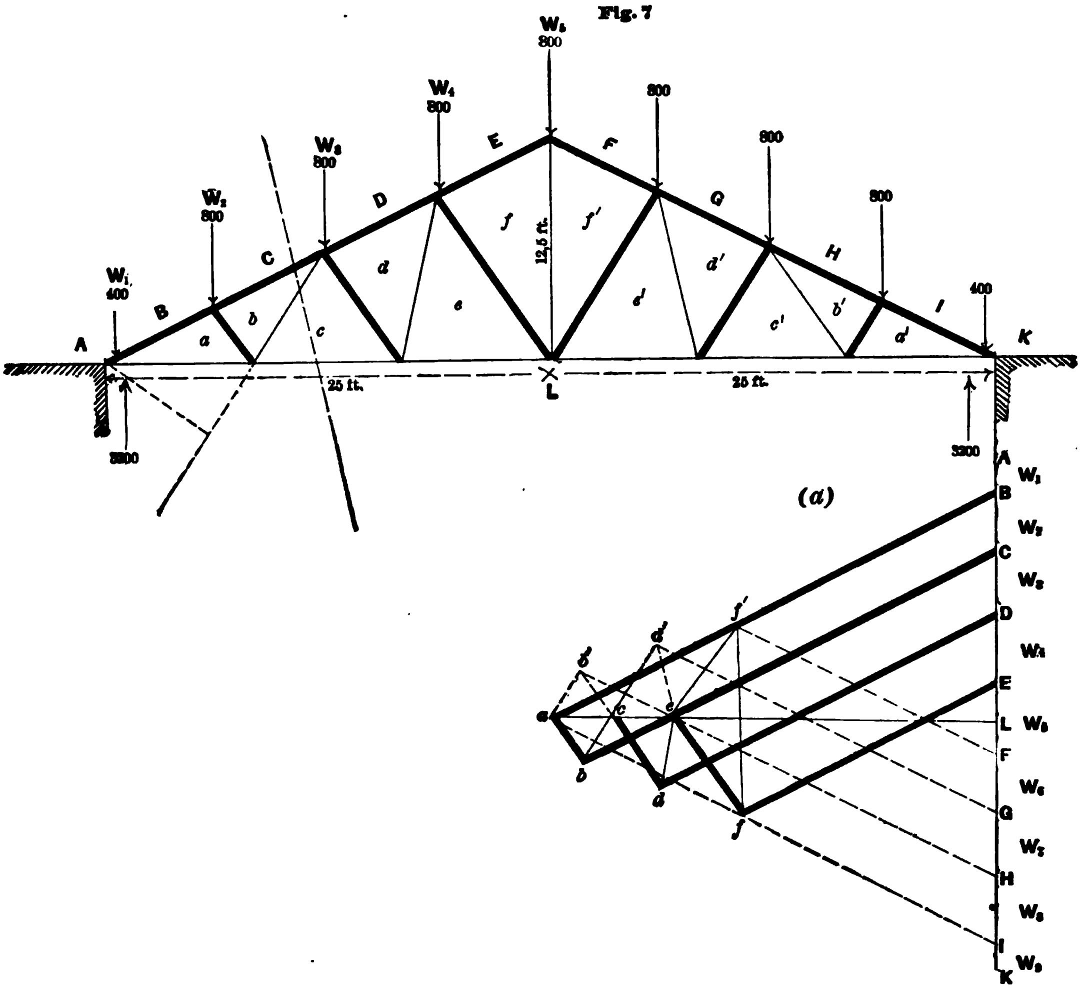


Fig. 7

centre of the next, including weight of cross-pieces, planking, shingles, etc., to be concentrated at each apex. Let us assume that we thus have a weight of 800 lbs. acting at each upper apex, except the two end ones, where the weight is one-half of this, or 400 lbs. Since the truss is symmetrical, with respect to the centre, and symmetrically loaded, the upward reaction or pressure upon the wall at each end will be one half the sum of all the weights, or 3,200 lbs. at each end. These constitute all the external forces which act upon the frame-work.

Notation.—The notation which we adopt in order to conveniently designate any member or stress is as follows. We letter the triangular spaces into which the truss is divided

by the braces, also the spaces between the forces. The letter L refers to all the space below the truss. Thus the panels into which the rafter is divided are Ba, Cb, Dd, Ef, etc. The panels into which the lower horizontal tie is divided are La, Lc, Le, etc. In general any member is denoted by the letters upon each side of it. Thus ab is the first brace, bc the next, and so on. In like manner AB is the first weight, BC the second, etc.

FORCE POLYGON.—We can now proceed to form the "*force polygon.*" Thus in (*a*), Fig. 7, we lay off the weights to any convenient scale, in regular order one after the other, and thus obtain the line $A, B, C, D, \ldots K$. Then the two reactions are laid off upwards from K to L and L to A, thus closing the polygon, as should be the case, since, if the truss is not to move bodily, the stresses must form a system in equilibrium. This is in accordance with our postulate 2, page 6. The force polygon in this case is therefore a straight line, or rather a double line, from A to K and K back to A again. This is evidently because all the external forces are parallel. [Fig. 6, p. 10.]

STRESS DIAGRAM.—We may now proceed to form the "*stress diagram,*" or find the stress in each member caused by these forces. According to our postulate 1, page 6, the stresses in all the members which meet at any apex, together with all the forces at that apex, must form a system of forces in equilibrium. Hence the polygon obtained by drawing lines parallel to these forces, and equal by scale to their magnitude, must close. Wherever, then, in general we know all the forces acting at any apex except two, we can easily find these two, if their directions are given, by drawing lines parallel to these given directions, and prolonging them until they close the incomplete polygon formed by the known forces. These remarks will be evident from the construction. Thus at the left end, Fig. 7, we have two known forces, viz., the half weight (400 lbs.) acting down, and the reaction (3,200 lbs.) acting up. We have also the unknown stresses in the members Ba and La, and these four forces are all which act at the apex A. If equilibrium exists they must therefore form a closed polygon.

But the reaction LA and weight AB are already laid off in order in the force polygon (*a*), the one up, the other down. We have therefore only to unite the points B and L by lines parallel to Ba and La, and we shall have the stresses in these members respectively, *to the same scale as that chosen for the force polygon.*

Now that we know the stress in the member Ba, we can pass to the next upper apex. Because of the four forces acting there, we know already Ba and the weight BC, and hence there are only two unknown, viz., the stresses in ab and Cb. But in the stress diagram (*a*) now commenced, we have already Ba and BC laid off, and we have therefore only to join the points C and a by lines parallel respectively to ab and Cb above.

We next proceed to the first lower apex, where La and ab are known, and bc and Lc are to be found. We therefore join L and b in the stress diagram by lines parallel to Lc and bc above, and we obtain the stresses in these members. Thus the polygon $LabcL$ is made to close.

We then proceed to the next upper apex, where we have Cb, bc, and the weight CD now known. Hence we join D and c below by lines parallel to these pieces, and thus complete the polygon $DCbcdD$.

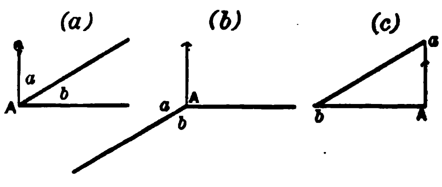
It is unnecessary to follow out the method of procedure further. The reader, however, ought to do it for himself carefully and thoroughly.

THE SYMMETRY OF THE FIGURE A CHECK UPON THE ACCURACY OF THE WORK.—Proceeding in the method indicated, we have found the stresses in every member of the frame. The broken lines give the stresses in the right-hand half. It will be at once seen that they should be precisely equal to the stresses in the corresponding pieces of the left half. This affords several excellent checks upon the accuracy of our work. Thus the two

halves of the Figure should be perfectly symmetrical, and the broken half should unite with the full half exactly at the points e, c and a.

CHARACTER OF THE STRESSES.—The determination of the character of the stresses is second only in importance to the determination of the stresses themselves. Suppose we have a force Aa acting at any point as A, upwards, as shown by the arrow in Fig. 8, and that this force is held in equilibrium by the stresses in the two members ab and Ab, which also act upon the same point A. Then, as we know, these forces must make a closed polygon as given at (c). Now follow round this polygon in the direction given by Aa, and we find that for equilibrium the stress in ab must act upon the point A in the direction from a to b given in Fig. (c). As this force can only act upon the point A *by means of the member ab* which conveys it there, if ab is on the right of the point A, as in Fig. (a), the piece ab must be in *compression.* If ab is on the left of A, the stress in it must be tension. So for the member Ab. We find from Fig. (c) its equilibrium direction from b to A, or from left to right. Transferring this direction to the Figs. (a) and (b), we see that in (a) the stress in Ab must be tension and in (b) tension also. This is sufficient to furnish us with a general rule for finding the character of the stress in any member, as well as to illustrate the reason of the rule.

Fig. 8



If we take any apex of the frame and consider the forces acting upon that apex as a system of forces in equilibrium, the rule is:

Follow round the polygon formed by these forces, in the direction indicated by those forces which are already known in direction, and transfer the directions thus obtained for the forces to the apex under consideration. If the stress in any member is thus found acting away from the apex, the corresponding member is in tension; if towards the apex, it is in compression.

An application of this to Fig. 7 will make it plain. Thus take the first apex. Here we have the reaction known to act up, and the weight AB acting down, in equilibrium with Ba and La. Following round the polygon in (a), therefore, we go *up* from L to A, then *down* from A to B, then, continuing round, we go in order from B to a, and then from a to L. We thus find the direction for the stresses in Ba and aL, viz., Ba from right to left, and aL from left to right. Referring now to the frame itself, and transferring these directions to the corresponding members, we see that the direction for Ba gives us the stress in that member acting towards the apex; it is therefore in compression. In like manner we have the stress in aL acting away from the apex, or tension.

Once more: take the next apex. Here the weight BC acts down. We follow round the polygon in (a), then, from B to C, then to b, then to a, and then back to B. We thus find the direction for aB from a to B. Referring back to the frame, we find that this gives us the stress in this member acting towards the apex we are now considering, and therefore compressive, just as we have already found it.* The direction Cb gives us the stress in Cb acting towards the apex, hence compression. The direction ba gives us the stress in ba acting towards the apex, and therefore compression.

Again: take the first lower apex. Here we have already found La to be in tension, hence the stress in that member must act away from the apex we are now considering. With this to guide us we refer to Fig. (a) and follow round from L to a, then from a to b, b to c, and c back to L. We thus find ab acting towards the apex, and therefore compression, just as we have already found it. Also bc acting away from apex, or tension, and cL away, and therefore tension also.

* Observe that by changing the apex we have the stress in Ba opposite in direction to what it was before, but *in each case it is towards the apex considered,* and therefore in each case compression. When a member is in compression the stresses in it act *towards* the apices *at each end;* when in tension *away* from the apices *at each end.* See page 4.

This is enough to indicate the application of our rule. The reader will do well to apply it carefully to every apex until thoroughly familiar with it. We have denoted compression in Fig. 7 (*a*) by heavy lines and tension by light lines.

It is well, when solving any problem, to avoid confusion in following round the various polygons, to determine the character of the stresses by our rule ***as we go along, and not to wait until the stress polygon* (*a*) *is completed.***

REMARKS UPON THE METHOD.—The truth of the principle enunciated upon page 5, viz., that if the truss be cut entirely in two at any point, the stresses in the members cut will hold the outer forces in equilibrium, is also evident from Fig. 7.

Thus suppose a section cutting Dd, de and Le, then the stresses in these members ought to be in equilibrium with the algebraic sum of the weights and reaction. We see from the stress diagram below that this is the case, because the stresses Dd, de and eL make a closed polygon with $LD = LA - AB - BC - CD =$ algebraic sum of weights and reaction.

The Figure also shows other relations not evident from any principles and peculiar to the frame of the truss. Thus we see that the stress in ab will be the least possible when it is perpendicular to the rafter. We can see, also, at a glance how the stresses would be affected by altering the inclination of any member.

Finally, the application of the method is equally simple and easy of execution, no matter how irregular the frame-work of the truss.

CHOICE OF SCALES.—In general the larger the frame is drawn the better, as it gives us more accurately the direction of the members composing it. The force polygon should be taken to as small a scale as possible consistently with reading off the forces conveniently to as great a degree of accuracy as is required—so as to avoid the intersection of very long lines, where a slight deviation from true direction multiplies the error. When the stress polygon is completely finished, the stresses may be read off according to scale, and written down upon the frame if required. Thus a good scale, dividers, triangle, straight-edge, and hard fine-pointed pencil are all the tools required. The work should be done with care, all lines drawn light with a hard pencil, and points of intersection carefully located, and lettered properly to correspond with the frame. Care should be exercised to secure perfect parallelism in the lines of the stress and frame diagrams. Thus in Fig. 7, since the member ab is very short, its direction is better given by the member ef, which is parallel to it and longer. ALWAYS OBSERVE THE NOTATION GIVEN.

The student will find in SECTION II. many examples for practice, and details of construction for various cases. He would do well to refer now to the examples there given. Some practice is necessary in order to obtain always reliable results. It should be remembered finally, that careful habits of intelligent manipulation, while they tend to give constantly increased skill and more accurate results, affect very slightly the rapidity and ease with which these results are obtained.

NUMERICAL DETERMINATION OF STRESSES.—In the case of Fig. 7, we have drawn the frame to a scale of 12 feet to an inch, and taken as our scale of force, 3,200 lbs. to an inch. Scaling off the stresses in (*a*), we have, calling tension plus (+) and compression minus (−), the stresses in the various members as follows:

For the rafters,

$$Ba = -6280, \quad Cb = -5816, \quad Dd = -4700, \quad Ef = -3580 \text{ lbs.}$$

For the lower panels,

$$La = +5624, \quad Lc = +4832, \quad Le = +4024 \text{ lbs.}$$

For the diagonals,

$ab = -720$, $bc = +720$, $cd = -1060$, $de = +928$, $ef = -1452$, $ff' = +2410$ lbs.

The checking of both halves of the Figure gives assurance of the substantial correctness of the result. The scale actually adopted for the frame by which the above results were found was 10 feet to an inch, and for the forces 800 lbs. to an inch. As the error of the author working rapidly does not exceed $\frac{3}{100}$ths of an inch, the stresses may be depended upon within about 24 or 25 pounds.

CHAPTER II.

ANALYTIC RESOLUTION OF FORCES.

A. GENERAL PRINCIPLES. FORCES IN THE SAME PLANE, COMMON POINT OF APPLICATION.

FUNDAMENTAL PRINCIPLE.—The principle upon which the method of solution by means of the analytic resolution of forces depends, is the same as that upon which the graphic method of the preceding chapter is based, viz.:

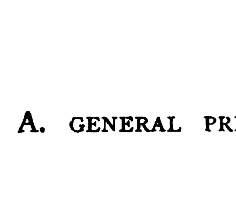
If any number of forces, in the same plane and acting upon the same point, are in equilibrium, the algebraic sums of their vertical and horizontal components must be respectively zero.

The two methods are therefore identical, and the present is only the algebraic solution of the preceding graphical construction.

If then, at any apex of a framed structure, which is the point of application for a system of forces in equilibrium, we know the directions of all the acting forces and the magnitude of all but two, we can at once write down two equations of condition between these two unknown forces, by means of which their magnitude may be determined.

NOTATION.—We always measure the angle of inclination of any member *from the vertical* through the apex. This angle we denote in general by θ, and denote by subscripts the member to which it refers. Thus, Fig. 9, let *ab*, *ac*, *ad* and *ae* be four members meeting at the apex *a*. Then the angles of inclination of these members are measured from the vertical line aV_1 through the apex. Thus θ_{ab} is numerically the angle baV_1. θ_{ac} is numerically the angle caV_1. θ_{ad} is numerically the angle daV_1. θ_{ae} is numerically the angle eaV_1.

Fig. 9



It is, however, necessary that we should always introduce the sines and cosines of these angles with their proper signs in the expression for the algebraic sum of the vertical and horizontal components. For this purpose we adopt the following conventions:

A compressive stress in a member is always minus, a tensile stress plus. This convention we have already introduced in the preceding chapter.

Any force acting vertically *upwards*, as, for instance, a reaction, is plus; when it acts *downwards*, as, for instance, a weight, it is minus.

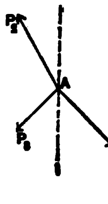
The cosine of θ is plus when the member in question lies *above* the horizontal through the apex. Thus, Fig. 9, cos θ_{ab} is plus and cos θ_{ac} is plus. Similarly cos θ_{ad} and cos θ_{ae} are minus.

The sine of θ is plus when the member lies to the *right* of the vertical through the apex. Thus, Fig. 9, sin θ_{ae} and sin θ_{ab} are plus, while sin θ_{ad} and sin θ_{ac} are minus.

The reader will observe that these are the ordinary conventions of analytical mechanics. That is, upward acting forces are positive, downward acting forces negative. Also

$h_1 a V_1$ is the first quadrant, for which sine and cosine are both positive. The second quadrant is $V_1 a h_1$, for which sine is negative and cosine positive. The third quadrant is $h_1 a V_2$, for which cosine is negative and sine negative. The fourth quadrant is $V_2 a h_2$, for which cosine is negative and sine positive. Hence our rule just given. If we adhere strictly to this notation we shall always be able to write down the various terms in the algebraic sum of the vertical and horizontal components with their proper signs. If, then, we find any stress plus, it will denote tension; if minus, compression.

Fig. 10



GENERAL FORMULAS.—Suppose we have three forces, P_1, P_2, P_3, acting at the point A, Fig. 10, in equilibrium. Then if we resolve each of these forces into a vertical and horizontal component, the algebraic sum of the vertical components must be zero, and the algebraic sum of the horizontal components must be zero. Adhering to the notation just described, the signs of these components in any particular case will take care of themselves, and we can write down the general equations:

For the vertical components,

$$P_1 \cos \theta_1 + P_2 \cos \theta_2 + P_3 \cos \theta_3 + \text{etc.} = 0.$$

For the horizontal components,

$$P_1 \sin \theta_1 + P_2 \sin \theta_2 + P_3 \sin \theta_3 + \text{etc.} = 0.$$

If now P_1 is known, we have two equations containing two unknown quantities, P_2 and P_3, and hence these forces can be easily found.

It is evident, then, that the method is applicable to any apex of any framed structure, where all the acting forces at that apex are known, *except two only*.

B. ILLUSTRATION OF GENERAL PRINCIPLES.

Let us apply the foregoing principles to the same example, as in the preceding chapter, and thus check the results there obtained by the graphic method of resolution of forces.

APPLICATION TO A ROOF TRUSS.

DIMENSIONS.—We take the same dimensions as before, page 11, and refer to Fig. 7, p. 11. The angle, then, which the upper panels make with the vertical through any apex is about 63° 26′. The angle for any panel of the horizontal tie is 90°. The angle for all the parallel braces *ab*, *cd*, *ef*, Fig. 7, is 33° 41′. The angle for the brace *bc* is also 33° 41′. The angle for the brace *de* is 12° 31′.

For the apex *BC*, for instance, we have the panel *Cb*, $\theta_{Cb} = 63° 26'$, and according to our convention, $\cos \theta_{Cb}$ is plus, because the member *Cb* lies in the first quadrant, and $\sin \theta_{Cb}$ is plus for the same reason.

CALCULATION.—Remembering, then, always to take the sines and cosines with their proper signs in the general formulas for the algebraic sum of the vertical and horizontal components, and also recollecting that upward forces are positive and downward forces negative, we can proceed to the calculation.

The numerical values of the sines and cosines are easily found to be as follows:

For the upper panels, $\begin{cases} \cos \theta = 0.44724 \\ \sin \theta = 0.89441 \end{cases}$ lower panels, $\begin{cases} \cos \theta = 0 \\ \sin \theta = 1 \end{cases}$

braces parallel to *ab*, $\begin{cases} \cos \theta = 0.83212 \\ \sin \theta = 0.55460 \end{cases}$

for $bc\left\{\begin{array}{l}\cos\theta = 0.83212\\ \sin\theta = 0.55460\end{array}\right.$ $\qquad de\left\{\begin{array}{l}\cos\theta = 0.97623\\ \sin\theta = 0.21672\end{array}\right.$

We are now ready to apply our principles.

Take the left hand apex, Fig. 7. Here we have the reaction R, the weight W_1, and the stresses in Ba and La, forming a system of forces in equilibrium. We have, then, for the algebraic sum of the vertical forces,

$$R + W_1 + Ba \cos\theta_{Ba} + La \cos\theta_{La} = 0 \quad \ldots \quad (a)$$

and for the algebraic sum of the horizontal components,

$$La \sin\theta_{La} + Ba \sin\theta_{Ba} = 0 \quad \ldots \quad (b)$$

From (a) we find, since La is horizontal, and hence $\cos\theta_{La} = 0$,

$$\text{stress in } Ba = \frac{-R - W_1}{\cos\theta_{Ba}} \quad \ldots \quad (1)$$

From (b) we find

$$\text{stress in } La = \frac{-Ba \sin\theta_{Ba}}{\sin\theta_{La}} \quad \ldots \quad (2)$$

Inserting numerical values, and observing our notation and conventions, we have, since $R += 3200$, $W_1 = -400$, $\cos\theta_{Ba} = +0.44724$, and $\sin\theta_{La} = 1$,

$$\text{stress in } Ba = \frac{-3200 + 400}{+0.44724} = -6260 \text{ lbs.}$$

Hence Ba is in compression.

$$\text{stress in } La = \frac{+6260 \times 0.89414}{1} = +5600 \text{ lbs.}$$

Hence La is in tension.

Let us pass to the next apex. Here we have for the algebraic sum of the vertical components,

$$W_2 + Ba \cos\theta_{Ba} + Cb \cos\theta_{Cb} + ab \cos\theta_{ab} = 0 \quad \ldots \quad (c)$$

and for the horizontal components,

$$Ba \sin\theta_{Ba} + Cb \sin\theta_{Cb} + ab \sin\theta_{ab} = 0 \quad \ldots \quad (d)$$

Inserting in equation (c) the value of $Ba \cos\theta_{Ba}$, as found from equation (a), we have, after substituting value of Cb from (d) and reducing,

$$\text{stress in } ab = \frac{W_2 \sin\theta_{Cb}}{\sin\theta_{ab}\cos\theta_{Cb} - \cos\theta_{ab}\sin\theta_{Cb}} = \frac{W_2 \sin\theta_{Cb}}{\sin(\theta_{ab} - \theta_{Cb})} \quad \ldots \quad (3)$$

In the same way we find from equation (d),

$$\text{stress in } Cb = -\frac{Ba \sin\theta_{Ba}}{\sin\theta_{Cb}} - \frac{W_2 \sin\theta_{ab}}{\sin(\theta_{ab} - \theta_{Cb})} \quad \ldots \quad (4)$$

Inserting numerical values, we have

$$\text{stress in } ab = \frac{-800 \times 0.89441}{\sin(146^\circ\ 19' - 63^\circ\ 26')} = \frac{-715.528}{+0.99730} = -720 \text{ lbs.}$$

Hence ab is in compression. (Observe that the angles θ_{ab} and θ_{Cb} are reckoned as shown in Fig. 9, page 16 Thus $\theta_{ab} = 146^\circ\ 19'$.)

We have in like manner

$$\text{stress in } Cb = 6260 - \frac{-800 \times 0.55460}{+0.99230} = -6260 + 447 = -5813 \text{ lbs.}$$

Hence Cb is in compression.

At the first lower apex, Fig. 7, we have for the vertical components,

$$ab \cos \theta_{ab} + bc \cos \theta_{bc} = 0 \quad \ldots \quad (e)$$

and for the horizontal components,

$$La \sin \theta_{La} + ab \sin \theta_{ab} + bc \sin \theta_{bc} + Lc \sin \theta_{Lc} = 0 \quad \ldots \quad (f)$$

From the first of these equations we obtain

$$\text{stress in } bc = -\frac{ab \cos \theta_{ab}}{\cos \theta_{bc}} \quad \ldots \quad (5)$$

and for the second,

$$\text{stress in } Lc = \frac{-La \sin \theta_{La} - ab \sin \theta_{ab} - bc \sin \theta_{bc}}{\sin \theta_{Lc}} \quad \ldots \quad (6)$$

Inserting numerical values, we have

$$\text{stress in } bc = -\frac{-720 \times +0.83212}{+0.83212} = +720 \text{ lbs.}$$

Hence bc is in tension.

Also,

$$\text{stress in } Lc = \frac{-5600 \times -1 + 720 \times -0.55460 - 720 \times 0.55460}{+1}$$

$$= +5600 - 399 - 399 = +4802 \text{ lbs.}$$

Hence Lc is in tension.

At the apex CD, Fig. 7, we have for the vertical components,

$$W_8 + Cb \cos \theta_{Cb} + bc \cos \theta_{bc} + cd \cos \theta_{cd} + Dd \cos \theta_{Dd} = 0 \quad \ldots \quad (g)$$

and for the horizontal components,

$$Cb \sin \theta_{Cb} + bc \sin \theta_{bc} + cd \sin \theta_{cd} + Dd \sin \theta_{Dd} = 0 \quad \ldots \quad (h)$$

From these two equations we have, after reduction,

$$\text{stress in } cd = \frac{W_8 \sin \theta_{Dd} + bc \sin (\theta_{Dd} - \theta_{bc})}{-\sin (\theta_{Dd} - \theta_{cd})} \quad \ldots \quad (7)$$

Also,

$$\text{stress in } Dd = \frac{Cb \sin \theta_{Cb} + bc \sin \theta_{bc} + cd \sin \theta_{cd}}{-\sin \theta_{Dd}} \quad \ldots \quad (8)$$

Inserting numerical values, we have

$$\text{stress in } cd = \frac{-800 \times 0.89441 + 720 \sin (63°26' - 213°41')}{-\sin (63°26' - 146°19')}$$

$$= \frac{-716 - 360}{+0.99230} = -1081$$

Hence cd is in compression.

Also,

$$\text{stress in } Dd = \frac{-5813 \times -0.89441 + 720 \times -0.55460 - 1081 \times 0.55460}{-0.89441}$$

$$= \frac{+5199.205 - 399 - 599}{-0.89441} = -4696 \text{ lbs}$$

Hence Dd is in compression.

By comparison with the formulas already found, we can now write down the formulas for the remaining members at once, without first writing down the equations of condition. Thus, at the second lower apex, Fig. 7, we have at once, by simply referring to the formulas already found for bc and making the proper changes in the subscripts,

$$\text{stress in } de = -\frac{cd \cos \theta_{cd}}{\cos \theta_{de}} \quad \ldots\ldots \quad (9)$$

In like manner, referring to equation (6),

$$\text{stress in } Le = \frac{-Lc \sin \theta_{Lc} - cd \sin \theta_{cd} - de \sin \theta_{de}}{\sin \theta_{Le}} \quad \ldots\ldots \quad (10)$$

Inserting numerical values, we have

$$\text{stress in } de = \frac{+1081 \times +0.83212}{+0.97623} = +920 \text{ lbs.}$$

$$\text{stress in } Le = \frac{+4802 + 1081 \times -0.55460 - 924 \times 0.21672}{+1}$$

$$= +4802 - 599 - 200 = +4003 \text{ lbs.}$$

In similar manner, for the apex DE, Fig. 7, referring to equations (7) and (8), we can write down at once,

$$\text{stress in } ef = \frac{W_4 \sin \theta_{Ef} + de \sin (\theta_{Ef} - \theta_{de})}{-\sin (\theta_{Ef} - \theta_{ef}}, \quad \ldots\ldots \quad (11)$$

$$\text{stress in } Ef = \frac{Dd \sin \theta_{Dd} + de \sin \theta_{de} + ef \sin \theta_{ef}}{-\sin \theta_{Ef}} \quad \ldots\ldots \quad (12)$$

Inserting numerical values, we have

$$\text{stress in } ef = \frac{-800 \times 0.89441 + 924 \sin (63° 26' - 192° 31')}{-\sin (63° 26' - 146° 19')}$$

$$= \frac{-715 - 717}{+0.99230} = -1443 \text{ lbs.}$$

$$\text{stress in } Ef = \frac{-4696 \times -0.89441 + 924 \times -0.21672 - 1443 \times 0.55460}{-0.89441}$$

$$= \frac{+4200 - 200 - 800}{-0.89441} = -3577 \text{ lbs.}$$

At the centre apex in the lower chord, Fig. 7, we have, since by reason of the symmetry of the frame and the symmetrical loading the stresses in all the members of the right half are equal to those in the left,

$$2fe \cos \theta_{fe} + ff' \cos \theta_{ff'} = 0.$$

Or

$$\text{stress in } ff' = \frac{-2fe \cos \theta_{fe}}{\cos \theta_{ff'}} \quad \ldots\ldots \quad (13)$$

Inserting numerical values,

$$\text{stress in } ff' = \frac{+2 \times 1443 \times +0.83212}{+1} = +2401 \text{ lbs.}$$

A comparison with the stresses found for the same case in Chapter I. shows a satisfactory agreement in the results of the two methods. Thus:

	Ba	*Cb*	*Dd*	*Ef*	*La*	*Lc*	*Le*	*ab*	*bc*	*cd*	*de*	*ef*
Method of Chapter I.........	−6280	−5816	−4700	−3580	+5624	+4832	+4024	−720	+720	−1060	+928	−1452
Method of Chapter II	−6260	−5813	−4696	−3577	+5600	+4802	+4003	−720	+720	−1081	+924	−1443

COMPARISON WITH PRECEDING METHOD.—We see that the application of the present method to the case chosen is much more difficult than the graphic method of Chap. I., in that it involves much calculation and requires very careful attention to avoid errors. The present method, therefore, does not adapt itself readily to cases where the various members have different inclinations, although, as we shall see hereafter in the applications of Section II., page 103, there are many cases of frequent occurrence in practice where the application of the method is quick and easy. When the calculations are performed with proper care, the results are more accurate than by the graphic method. This latter, however, by the proper choice of scales, gives results practically correct.

One important point of difference we may note here, however, which holds good for all analytic methods as compared with graphic—that is, the graphic method gives indeed a *general method* of solution, but, in any case, only *particular results*, while the analytic method gives general results or formulas which hold good *for all similar cases*. Thus the formulas we have just obtained hold good for *all* trusses of the pattern of Fig. 7, no matter what their dimensions. We have, in any case, only to insert the special numerical values, and the formulas give us at once the stresses for that case.

In solving, then, any particular case, we solve at the same time all others like it, while the graphic method must be applied anew for every fresh case. This is generally true of all graphic methods.

If it were required to compute a large number of trusses, therefore, of different dimensions but same type, the present method would possess perhaps practical advantages superior to the graphic. Each method has thus its particular advantages, and the engineer should be able to choose in any case, that which leads most directly and easily to the required results. Illustrations of the use of this method will occur in Section II., wherever it is advantageous to make use of it.

THE METHOD IDENTICAL WITH THE METHOD OF SECTIONS.—We have stated at page 5 the principle that if the truss is conceived as cut in two at any point, the stresses in the cut members are in equilibrium with the outer forces acting upon each portion into which the truss is divided. We can therefore write down two equations of condition for the cut members, expressing the condition that the sums of all the horizontal and vertical components are zero, and thus if only the stresses in two cut members are unknown, we can find them. The formulas thus obtained would be precisely identical with those already found, and we can therefore, if we choose, call the present method the analytic method of sections, instead of the analytic method of resolution of forces.

Thus by the application of our principle we have for the apex *BC*, Fig. 7, the equation (*c*), page 18, viz.:

$$W_2 + Ba \cos \theta_{Ba} + Cb \cos \theta_{Cb} + ab \cos \theta_{ab} = 0.$$

But we have already found for the preceding apex,

$$R + W_1 + Ba \cos \theta_{Ba} + La \cos \theta_{La} = 0.$$

If we find the value of $Ba \cos \theta_{Ba}$ from this, and insert in the first equation, we have, since for the second apex $\cos \theta_{Ba}$ is minus,

$$W_2 + R + W_1 + La \cos \theta_{La} + Cb \cos \theta_{Cb} + ab \cos \theta_{ab} = 0,$$

which is precisely the same equation as we should obtain by the method of sections, and expresses the condition that the vertical components of the cut members La, Cb, and ab are in equilibrium with the outer forces. The two methods are therefore identical, and whether we had proceeded from the principle that all the forces at any apex are in equilibrium or from the principle just stated of sections, we would have obtained in either case precisely the same results and equations.

ALGEBRAIC REPRESENTATION OF THE STRESS DIAGRAM.—We can write down all the formulas obtained for the various members directly from the stress diagram Fig. 7 (*a*), without stating the equations of condition at all. Since the present method and the graphic method of Chapter I. are both based upon precisely the same principle, Fig. 7 (*a*) is simply the graphic interpretation of our algebraic work. The simple trigonometrical solution of the various lines in the stress diagram Fig. 7 (*a*) will therefore give us at once the formulas of this chapter. Thus a little inspection of the stress diagram will suffice to make evident that

$$ab \sin (\theta_{ab} - \theta_{Cb}) = W \sin \theta_{Cb}.$$

This is the same expression as equation (3), page 18. So for the other members.

Any one therefore familiar with the graphic method of Chapter I. can readily deduce from the stress diagram itself the trigonometrical formulas for the stresses in the various members.

CHAPTER III.

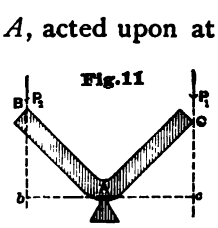
METHOD OF MOMENTS—ALGEBRAIC SOLUTION.

A. GENERAL PRINCIPLES. FORCES IN THE SAME PLANE, DIFFERENT POINTS OF APPLICATION.

MOMENT, LEVER ARM, CENTRE OF MOMENTS.—The "*moment*" of a force with reference to any point is the product of the force into its "*lever arm.*" The point with reference to which the moment is taken is called the "*centre of moments.*" The lever arm of a force is the length of the perpendicular let fall from the centre of moments upon the direction of the force. For this purpose the force must be considered as prolonged in direction if necessary.

Thus in Fig. 11, if we have a bent lever BAC, with its fulcrum at A, acted upon at C by the force P_1 and at B by the force P_2, the lever arm of P_1 with reference to A is Ac, the perpendicular to the direction of P_1 prolonged, and the moment of P_1 with reference to A is $P_1 \times Ac$. In like manner the lever arm of P_2 is Ab and its moment $P_2 \times Ab$.

Fig. 11



FUNDAMENTAL PRINCIPLE.—The methods of solution of the two preceding chapters are based upon the first fundamental principle of equilibrium, viz.: that if any number of forces acting upon a rigid body are in equilibrium, the algebraic sum of the vertical components must be zero, and the algebraic sum of the horizontal components must be zero. That is, all the forces tending to raise the body vertically must be balanced by those tending to move the body downwards, and all those tending to move it horizontally in one direction must be balanced by all those tending to move it horizontally in the other.

The method of solution of the present chapter is based upon the *second* fundamental principle of equilibrium, viz.:

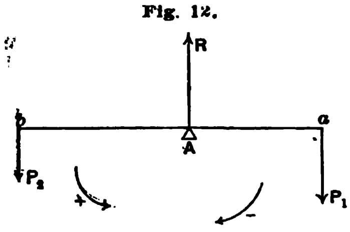
If any number of forces, in the same plane and acting upon the same point or at different points of the same rigid body, are in equilibrium, the algebraic sum of the moments with reference to any point in the plane must be zero.

We may therefore call the present method the "method of moments." As the solution is algebraic, it is the "algebraic method of moments."

SIGN FOR MOMENTS.—As in analytical mechanics generally, we take rotation counter clock-wise as positive and clock-wise as negative. If then any force tends to cause rotation about the centre of moments in a counter clock-wise direction, or from right to left, we take its moment as positive. The opposite direction is negative. If we adhere strictly to this notation, we shall always be able to write down the various terms in the algebraic sum of the moments of any number of forces about any assumed centre of moments, with their proper signs.

Thus, in Fig. 12, suppose we have a lever ab resting upon a fulcrum at A and acted upon by the forces P_1 and P_2 and the reaction of the fulcrum R. If there is equilibrium, we must have the algebraic sum of the moments of the forces *with reference to any point in the plane* of the forces equal to zero.

Fig. 12.



Taking the centre of moments at A, and adding the moments of P_1 and P_2 algebraically as directed, we have

$$+P_2 \times bA - P_1 \times Aa = 0 \text{ or, } P_2 \times bA = P_1 \times Aa; \quad \therefore \frac{P_2}{P_1} = \frac{Aa}{bA}.$$

This is the well-known "law of the lever," viz., the forces are to each other inversely as their lever arms.

It makes no difference where we take the centre of moments. The algebraic sum of the moments must always be zero for equilibrium. Thus when we took the centre of moments at A, the moment of R is zero and does not appear. If, however, we take the centre of moments at b we should have, since now the moment of P_2 disappears,

$$R \times bA - P_1 \times ba = 0, \text{ or } P_1 = R\frac{bA}{ba}.$$

Again, taking the centre of moments at a, we have, since the moment of P_1 now disappears,

$$P_2 \times ba - R \times Aa = 0, \text{ or } P_2 = R\frac{Aa}{ba}.$$

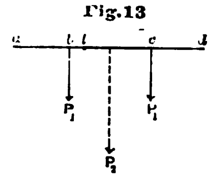
Adding these last values of P_1 and P_2, we obtain

$$P_1 + P_2 = R\frac{bA + Aa}{ba} = R, \text{ or } R - P_1 - P_2 = 0.$$

That is, the algebraic sum of the vertical forces is zero, or the reaction R is equal and opposite to the sum of the weights, as should be for equilibrium.

PAIR.—Two forces having different points of application, but in the same plane, equal in magnitude and parallel, and having the same direction, are called a *pair*. Thus in Fig. 13, the two equal and parallel forces, P_1, P_1, are called a pair. Suppose we take any point to the left of b, as for instance a, as a centre of moments, then we shall have for the combined moment,

Fig. 13



$$-P_1 \times ac - P_1 \times ab = -P_1(ac + ab) = -P_1(ab + bc + ab)$$
$$= -P_1(2ab + bc) = -2P_1\left(ab + \frac{bc}{2}\right).$$

If we take any point as d, to the right, as the centre of moments, we have for the combined moment

$$+P_1(cd + bd) = +P_1(2cd + bc) = +2P_1\left(cd + \frac{bc}{2}\right).$$

If we take any point as l between the forces as the centre of moments, we have for the resultant moment

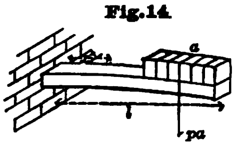
$$-P_1 \times cl + P_1 \times bl = -P_1(cl - bl) = -P_1(bc - bl - bl)$$
$$= -P_1(bc - 2bl) = -2P_1\left(\frac{bc}{2} - bl\right).$$

We see, therefore, that wherever the centre of moments is taken, the moment of a pair is equal to the moment of the sum of the forces $2P_1 = P_2$ acting at a point midway betweeen them. A pair can therefore be replaced by a single force, P_2, equal to the sum of the two forces and parallel to them, acting at a point midway between them.

UNIFORM LOAD.—Any uniformly distributed load can be regarded as a system of pairs, symmetrically placed with reference to the centre of the load.

Thus let Fig. 14 represent a beam fixed horizontally in the wall at the left, whose length is l, and let a load of p pounds per unit of length be distributed over a distance of a units from the right end. This load is then composed of a number a of unit loads, each of which is equal to p. Consider the two extreme ones, right and left. These form a pair, and can therefore be replaced by a weight of $2p$ acting at the centre of the loaded portion. The same holds true for the next pair right and left, and so on. *The whole load can then always be replaced by the sum of all the unit loads, or the whole load, pa, applied at the centre of the loaded portion.* The moment of this force with reference to any point *not covered by the load* is the same as the moment of the load itself. Thus the moment with reference to a point distant x from the left end is, from the Fig., if the point is not covered by the load,

Fig. 14.



$$-pa \times \left(l - x - \frac{a}{2}\right).$$

If the point is at the left end of the load, we have $x = l - a$, and the moment is

$$-pa \times \frac{a}{2} = \frac{pa^2}{2}.$$

If the point is covered by the load, or $x > l - a$, we have the loaded portion on the right equal to $l - x$, and hence the load on the right of the point equal to $p\,(l - x)$. Its lever arm is $\frac{l - x}{2}$, hence the moment of all the right hand unit weights is

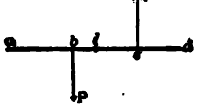
$$-\frac{p\,(l - x)^2}{2}.$$

In any case, then, wherever the centre of moments, the moment of any system of uniform loads is equal to the moment of the sum of these loads when concentrated at the centre of the system.

COUPLE.—Two forces in the same plane, having different points of application, parallel and equal in magnitude, but having opposite directions, are called a couple.

Thus the two forces P, P, in Fig. 15, form a couple. If we take any point to the left, as a, as a centre of moments, we have for the resultant moment

Fig. 15



$$-P \times ab + P \times ac = +P(ac - ab) = +P \times bc.$$

If we take any point to the right, as d, as a centre of moments, we have

$$-P \times cd + P \times bd = +P(bd - cd) = +P \times bc.$$

If we take any point between the forces, as l, we have

$$+P \times cl + P \times bl = +P(cl + bl) = +P \times bc.$$

The moment, therefore, of a couple is constant, wherever the centre of moments is chosen, and equal to the product of either force into the distance between the forces.

A couple, then, can only be replaced or balanced by another couple in the same plane. The forces of the new couple may have any magnitude, provided the distance between them is so chosen that the product of either force into this distance is constant and equal to the moment of the first couple.

METHOD OF APPLICATION OF PRINCIPLES.—We have already seen (page 5) that if a truss is properly braced and has no superfluous members, it is always possible to divide it at some point in some direction, such that not more than three members whose stresses are necessarily unknown shall be cut. Also that the stresses in the members cut must hold in equilibrium the outer forces acting upon either portion of the truss. According to our principle, then, the algebraic sum of the moments of the stresses in the members and of the outer forces must be zero. Now, in any case, the outer forces are always given, or they must first be found before we can attempt to determine the stresses. There are, then, at most, only three unknown stresses to be determined, viz., the stresses in the members cut by the section. Now, as we can take the centre of moments anywhere we please, we have only to take it at the intersection of two of the members, and we shall have at once the moment of the stress in the other, balanced by the sum of the moments of the outer forces, because the lever arms, and therefore the moments of the other two cut members, will be zero.

We have thus the following rule:

Conceive at any point a section completely through the truss, cutting not more than three members the stresses in which are unknown. In order to find the moment of the stress in any one of these members, take the centre of moments at the intersection of the other two.

For equilibrium, the algebraic sum of the moment of the stress in this member and of the moments of the outer forces acting upon either portion of the truss must be equal to zero. If, then, we know the moments of these outer forces and the lever arm for the member, we can find the stress in the member.

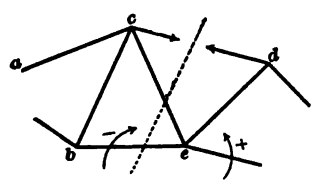
It is evident that the section may cut more than three members whose stresses are unknown, in fact any number, *provided all but that one in which the stress is required meet at a common point.* We have only to take this point as the centre of moments.

NOTATION.—We denote the lever arm for any member in general by the letter l, with subscripts denoting the member. Rotation counter clock-wise is plus, clock-wise minus. A tensile stress is plus, a compressive stress minus.

Since now the outer forces are all known, both in direction, magnitude, and points of application, we can easily write down their moments in any case, for any assumed centre of moments, each with its proper sign, according to the direction of rotation which each force severally tends to cause about that centre of moments. It remains only to give a rule for determining the proper sign to give to the moment of the member the stress in which is required, in order that a plus sign for the stress in the result may indicate tension, and a minus sign compression.

PROPER SIGN FOR MOMENT OF THE MEMBER.—Let Fig. 16 represent a portion of any truss subjected to the action of known outer forces, not shown in the figure. Suppose we wish the stress in the member *cd*. Taking a section through *cd*, *ce*, and *be*, we have the centre of moments for *cd* at *e*, the intersection of the other cut members. Now, in order to always write the moment of the stress with its proper sign in the algebraic sum, we have the following rule:

Fig. 16.



Place arrows upon the cut member at the section, pointing away from each end, as shown in Fig. 16. The moment of the stress is to be taken with the same sign as the rotation indicated by these arrows.

Thus in Fig. 16, if we consider the *left-hand* portion into which the truss is divided by the section, we take the arrow belonging to this *left-hand* portion. If, then, we denote the stress in *cd* by *cd*, and its lever arm by l_{cd}, we have, considering the left-hand portion, the rotation indicated by the arrow belonging to that portion, with reference to the centre of moments at *e*, clock-wise or negative in the figure. We have then for equilibrium

$$-cd \times l_{cd} + \left\{ \begin{array}{l} \text{algebraic sum of moments of all outer forces} \\ \text{acting upon the left-hand portion} \end{array} \right\} = 0.$$

If we were to consider the right-hand portion, we should have from the figure

$$cd \times l_{cd} + \left\{ \begin{array}{l} \text{algebraic sum of moments of all outer forces} \\ \text{acting upon the right-hand portion} \end{array} \right\} = 0.$$

In the first case, if the moment sum for the left-hand portion is negative, we should evidently have compression in *cd*, and by our rule we should have

$$-cd \times l_{cd} - M = 0, \quad \text{or} \quad cd = -\frac{M}{l_{cd}};$$

that is, the stress in *cd* is minus or compression.

In the second case, the moment sum for the right-hand portion would be positive and we should have *cd* negative as before. In all cases the lever arm *is taken without sign.*

If we observe the above rule and notation, the signs of the stresses will denote the character of the stress according to the convention we have adopted of plus for tension and minus for compression.

Unless otherwise stated, *we shall always consider the left-hand portion* into which the truss is divided by the section and write down the algebraic sum of the moments for all the outer forces acting upon this *left-hand portion.*

B. ILLUSTRATION OF GENERAL PRINCIPLES.

Let us choose as an example to illustrate these points the same truss which we have already become familiar with in the preceding chapters, represented in Fig. 7.*

APPLICATION TO A ROOF TRUSS.

LEVER ARMS.—It is necessary first to find the lever arms of the various members. This in any case is a simple question of trigonometry. The lever arms for the upper panels, Fig. 7, are evidently the perpendiculars drawn to those panels from each opposite lower apex. For the lower panels we have the perpendiculars let fall upon these panels from each opposite upper apex. For each brace, the lever arm is the perpendicular to the direction of the brace drawn through the left end *A*, where rafter and tie intersect. This will be evident by considering sections through the truss and applying our rule.

Thus suppose a section cutting *Cb*, *bc* and *Lc*, as indicated by the broken line, or *Dd*, *dc* and *Lc*. Then, by our rule, the point of moments for *Lc* is the apex *CD*, the point of intersection of the other two members. For *Cb* it is the second lower apex. For *bc* it is the apex *AB*, or the left end of the truss. The panels *Ba* and *Cb* have evidently the same lever arm.

If we pass a section through *Ef*, *ff'*, *f'e'* and *Le'* it cuts, to be sure, more than three braces. The stress in *f'e'* can, however, be easily found, since it is equal to *ef* by reason

* The student will find the method of moments of this chapter applied in detail to Bridges and Roofs of various kinds in "*Dach und Brücken-Constructionen,*" by *A. Ritter*, Hanover, 1873, a translation of which, under the title of "*Elementary Theory and Calculation of Iron Bridges and Roofs,*" by *H. R. Sankey*, has been published by E. & F. N. Spon.

of the symmetry of the frame and loading. If this were not the case we could easily find it by working toward it from the right end. The intersection of the unknown members Ef and Le' is at the left end, and this is therefore the centre of moments for ff'.

We can easily find, then, the lever arms for the various pieces by simple trigonometrical computation.

It is unnecessary to explain this work in detail. The lever arms thus computed are as follows:

For the lower panels,

	La	Lc	Le
lever arms =	3.125	6.25	9.375 ft.

For the upper panels,

	Ba	Cb	Dd	Ef
lever arms =	3.727	3.727	7.454	11.181 ft.

For the braces,

	ab	bc	cd	de	ef	ff'
lever arms =	6.934	6.934	13.869	16.27	20.803	25 ft.

Length of each lower panel = $8\frac{1}{3}$ ft.

Horizontal projection of each upper panel = 6.25 ft.

CALCULATION.—Let us first calculate the lower panels. Conceive La cut.* The centre of moments is then at the apex BC, Fig. 7. Let R be the reaction at the left end, and let us always consider the left-hand portion of the truss.

Then,

$$R \times l_R + La \times l_{La} = 0. \quad \ldots\ldots \quad (1)$$

Inserting numerical values, and having regard to our notation and rule for sign of moments, we have, since the rotation due to R is negative according to our rule, because the arrow for La gives positive rotation for the assumed centre of moments,

$$-2800 \times 6.25 + La \times 3.125 = 0.$$

Hence,

$$La = +\frac{2800 \times 6.25}{3.125} = +5600 \text{ lbs.}$$

La is therefore in tension.

For Lc we have by our rule, page 27, the centre of moments at the apex CD, whether we pass a section cutting Cb, bc and Lc, or Dd, cd and Lc.

We have for the general equation of equilibrium,

$$R \times l_R + W_2 \times l_{W_2} + Lc \times l_{Lc} = 0. \quad \ldots\ldots \quad (2)$$

As the centre of moments is on the right of Lc, according to our rule, page 27, the moment for Lc is plus.

Inserting numerical values, and having regard to the signs for positive and negative rotation, we have

$$-2800 \times 12.5 + 800 \times 6.25 + Lc \times 6.25 = 0.$$

Hence,

$$Lc = \frac{+2800 \times 12.5 - 800 \times 6.25}{6.25} = +4800 \text{ lbs.}$$

Lc is therefore also in tension.

---

* Let the section cut La, ab and Cb. Of these three pieces the two not desired meet at the apex BC. This, therefore, is our centre of moments for La. For Ba, in like manner, take a section through Ba, ab, bc and Lc.

For Le we have, in like manner,

$$R \times l_R + W_2 \times l_{W_2} + W_3 \times l_{W_3} + Le \times l_{Le} = 0. \quad \ldots \ldots \quad (3)$$

Inserting numerical values,

$$-2800 \times 18.75 \times 800 \times 12.5 + 800 \times 6.25 + Le \times 9.375 = 0.$$

Hence,

$$Le = \frac{+2800 \times 18.75 - 800 \times 12.5 - 800 \times 6.25}{9.375} = +4000 \text{ lbs.}$$

Let us now calculate the upper panels.

For the panel Ba, the centre of moments is at the first lower apex. The general equation is

$$R \times l_R + Ba \times l_{Ba} = 0. \quad \ldots \ldots \quad (4)$$

According to our rule, the moment for Ba is minus, because the arrow for Ba gives negative rotation for the assumed centre of moments.

Inserting numerical values,

$$-2800 \times 8.33 - Ba \times 3.727 = 0.$$

Hence,

$$Ba = \frac{-2800 \times 8.33}{3.727} = -6260 \text{ lbs.}$$

Ba is therefore in compression.

For the panel Cb we have the same point of moments; but when we pass a section through Cb, ab and La, the weight W_2 acts upon the left-hand portion also, as well as R.

Hence,

$$R \times l_R + W_2 \times l_{W_2} + Cb \times l_{Cb} = 0. \quad \ldots \ldots \quad (5)$$

Inserting numerical values, we have, since R causes negative rotation and W_2 positive, and since the arrow for Cb gives negative rotation,

$$-2800 \times 8.33 + 800 \times 2.08 - Cb \times 3.727 = 0.$$

Hence,

$$Cb = \frac{-2800 \times 8.33 + 800 \times 2.08}{3.727} = -5813 \text{ lbs.}$$

For the panel Dd, the centre of moments is at the second lower apex. The moment, according to our rule, is minus. The general equation is

$$R \times l_R + W_2 \times l_{W_2} + W_3 \times l_{W_3} + Dd \times l_{Dd} = 0. \quad \ldots \ldots \quad (6)$$

Inserting numerical values,

$$-2800 \times 16.66 + 800 \times 10.416 + 800 \times 4.166 - Dd \times 7.454 = 0.$$

Hence,

$$Dd = \frac{-2800 \times 16.66 + 800 \times 10.416 \times 800 + 4.166}{7.454} = -4695 \text{ lbs.}$$

For the panel Ef we have the centre of moments at the centre of the lower tie. The moment is minus according to rule. We have, then,

$$R \times l_R + W_2 \times l_{W_2} + W_3 \times l_{W_3} + W_4 \times l_{W_4} + Ef \times l_{Ef} = 0. \quad \ldots \ldots \quad (7)$$

Inserting numerical values,

$$-2800 \times 25 + 800 \times 18.75 + 800 \times 12.5 + 800 \times 6.25 - Ef \times 11.181 = 0.$$

Hence,

$$Ef = \frac{-2800 \times 25 + 800 \times 18.75 + 800 \times 12.5 + 800 \times 6.25}{11.181} = -3590.$$

Let us now calculate the stresses in the braces. For the brace *ab*, and indeed for all the braces, the centre of moments, according to our rule, is at the left end. The moment for *ab* is minus according to rule. The general formula is

$$W_2 \times l_{W_2} + ab \times l_{ab} = 0. \quad (8)$$

Inserting numerical values, we have, since W_2 tends to cause negative rotation, and the moment for *ab* is minus,

$$-800 \times 6.25 - ab \times 6.934 = 0.$$

Hence,

$$ab = \frac{-800 \times 6.25}{6.934} = -721 \text{ lbs.}$$

For the brace *bc* we have, according to rule, the moment plus, because the arrow for *bc* gives positive rotation for the assumed centre of moments. We have, for the general formula,

$$W_2 \times l_{W_2} + bc \times l_{bc} = 0. \quad (9)$$

Inserting numerical values,

$$-800 \times 6.25 + bc \times 6.934 = 0.$$

Hence,

$$bc = \frac{+800 \times 6.25}{6.934} = +721 \text{ lbs.}$$

For the brace *cd* the moment is minus, and we have,

$$W_2 \times l_{W_2} + W_3 \times l_{W_3} + cd \times l_{cd} = 0. \quad (10)$$

Inserting numerical values,

$$-800 \times 6.25 - 800 \times 12.5 - cd \times 13.869 = 0.$$

Hence,

$$cd = \frac{-800 \times 6.25 - 800 \times 12.5}{13.869} = -1081 \text{ lbs.}$$

For the brace *de*, in like manner, the moment is plus. We have then,

$$W_2 \times l_{W_2} + W_3 \times l_{W_3} + de \times l_{de} = 0. \quad (11)$$

Inserting numerical values,

$$-800 \times 6.25 - 800 \times 12.5 + de \times 16.27 = 0.$$

Hence,

$$de = \frac{+800 \times 6.25 + 800 \times 12.5}{16.27} = +926 \text{ lbs.}$$

For the brace *ef* the moment, according to rule, is minus. We have,

$$W_2 \times l_{W_2} + W_3 \times l_{W_3} + W_4 \times l_{W_4} + ef \times l_{ef} = 0. \quad (12)$$

Inserting numerical values,

$$-800 \times 6.25 - 800 \times 12.5 - 800 \times 18.75 - ef \times 20.803 = 0.$$

Hence,

$$ef = \frac{-800 \times 6.25 - 800 \times 12.5 - 800 \times 18.75}{20.803} = -1442.$$

For the brace ff' we pass a section cutting Ef, ff', $f'e'$, and Le'. Since the point of moments is on the left, according to our rule, the moment for ff' is plus. The stress in $f'e'$ is, by reason of the symmetry of frame and loading, equal to that already found for ef. We have then

$$W_2 \times l_{W_2} + W_3 \times l_{W_3} + W_4 \times l_{W_4} + f'e' \times l_{f'e'} + ff' \times l_{ff'} = 0. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Inserting numerical values,

$$-800 \times 6.25 - 800 \times 12.5 - 800 \times 18.75 - 1442 \times 20.803 + ff' \times 25 = 0.$$

Hence,

$$ff' = +\frac{800 \times 6.25 + 800 \times 12.5 + 800 \times 18.75 + 1442 \times 20.803}{25} = +2400 \text{ lbs.}$$

REMARKS.—These results compare favorably with those found for the same case in the two preceding chapters. The student will do well to select another example and compute it thoroughly, according to our method, paying special attention to the rules for determining the centres of moments and the signs for the moments, and checking his results by the method of Chapter I. Only in such way can he obtain mastery of the method. He would do well also to remember that time cannot be better spent than in getting familiar with the *principles* in these first four chapters. When we pass to applications in the second section, he will then find no difficulty in following the text, and will not be confused by the special details peculiar to different structures.

COMPARISON OF METHODS.—Much use will be made of the present method in this work. We shall call it hereafter the "*method of sections.*" We see that it is general in its application to all properly braced structures—that is, all framed structures which have no superfluous members. As compared with the analytic method by resolution of forces, of the preceding chapter, it will be seen that its application in the case chosen is much simpler and involves much less calculation. Still, for trusses in which the members have various inclinations, all different, the computation of the lever arm is tedious, and the graphic method of Chapter I commends itself as specially adapted to such cases. Indeed it is the special advantage of the graphic method, that it is entirely unaffected by irregularities of form and loading which necessitate much calculation by the other methods.

The present method can, however, in all cases, be used as a *check* upon the accuracy of the results obtained by the graphic method, to great advantage, inasmuch as it gives the stress in any member without reference to any others of the frame.

Thus in the example Fig. 7, after having found all the stresses by the graphic method, as shown in Fig. 7 (*a*), we can compute the stresses *in the last member* of that Figure, *Le*, by the present method of moments. If this is found to agree with the stress given by the graphic method, we may have confidence in the accuracy of all the others, because any error would have been carried along from member to member, and would have shown itself in the last.

CHAPTER IV.

METHOD OF MOMENTS.—GRAPHIC SOLUTION.

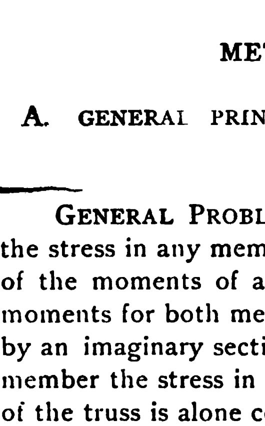
A. GENERAL PRINCIPLES. FORCES IN THE SAME PLANE, DIFFERENT POINTS OF APPLICATION.

General Problem.—We have seen in the preceding chapter that in order to find the stress in any member of a framed structure we have simply to divide the algebraic sum of the moments of all the outer forces by the lever arm for this member. The centre of moments for both member and outer forces is at the intersection of the other members cut by an imaginary section which completely divides the truss into two portions and cuts the member the stress in which is required. The outer forces acting upon *the left-hand portion* of the truss is alone considered.

We see, then, that in any case the problem to be solved is—What are the moments of these outer forces? If the algebraic sum of these is once found, we have only to divide by the lever arm of the member in order to find its stress. The object of the present chapter, therefore, is to deduce a *graphic method for finding the algebraic sum of the moments of the outer forces.*

Position of Resultant.—Suppose we have any number of forces, Fig. 17, given in direction and magnitude, and acting at different points of application in the same plane.

Fig. 17



If we lay these forces off to scale, the one after the other, and thus form the force polygon (a), the line necessary to close this polygon will be, as in Chapter I, the resultant to scale, and given in direction. But we do not know whereabouts in the plane of the forces, in Fig. 17, this resultant should act.

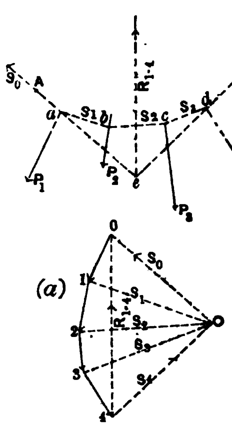
In the present case a ready method suggests itself at once. Thus we can consider P_1 as acting at any point in its line of direction, and so also for P_2. The resultant of P_1 and P_2, then, we can consider as acting at the intersection a of P_1 and P_2, prolonged if necessary. But the resultant of P_1 and P_2 is given in the force polygon (a) in direction and magnitude by the diagonal o2, because that diagonal closes the polygon commenced by the forces P_1 and P_2. At a then, parallel to o2 below, we can draw a line representing the direction of the resultant of P_1 and P_2, and produce it till it meets P_3, prolonged if necessary, at b. At b we can consider the resultant of $R_{1\cdot 2}$ and P_3 acting, or the resultant of P_1, P_2 and P_3. But o3 in the force polygon (a) below gives this resultant in direction and magnitude. Parallel to o3 then draw a line through b till it meets P_4, prolonged if necessary, at c.

Thus c is a point in the plane of the forces through which the direction of the resultant passes. In the force polygon (a), o4 is this resultant in direction and magnitude,

Parallel to o4 draw a line through c, and it will represent the resultant in proper position and direction. This resultant, taken as acting in the direction obtained by following around the force polygon in the direction of the forces, or in the direction from 4 to o, as shown by the arrow, will hold the forces in equilibrium.

If in the opposite direction, it will replace the forces (see page 10).

Fig. 18



THE PRECEDING METHOD NOT GENERAL.—This method of finding the position of the resultant, though sufficiently obvious, is evidently not general in its application. Thus, suppose the forces were all parallel or inclined so slightly as not to intersect within the limits of the drawing. In such case the method would fail. It is necessary, therefore to find some method which shall avoid this difficulty.

GENERAL METHOD FOR FINDING POSITION OF RESULTANT.—In Fig. 18 we have four forces given: required to find the resultant in direction, magnitude and position.

We shall evidently find the first two from the force polygon as before. Thus lay off the forces to scale in Fig. 18 (a), one after the other, and we shall have the resultant given in magnitude and direction by the closing line o4. If this resultant acts in the direction from 4 to o, obtained by following around the polygon in the direction of the forces, it will hold the forces in equilibrium. In the opposite direction it will replace the forces.

It remains to determine the *position* of the resultant in the plane of the forces.

For this purpose we choose a point O at any convenient point, and draw the lines Oo and $O4$. This point thus chosen we shall hereafter call a "*pole.*" Now, since every line in the force polygon represents a force, by thus choosing a pole and drawing lines to the extremities of the resultant, *we have resolved the resultant into* the *two forces*, Oo *and* $O4$. This is evident from the fact that these two lines close the polygon, and hence, taken as acting from 4 to O and O to o, as shown by the arrows, hold the forces P_1, P_2, P_3, P_4 in equilibrium. But these same lines make a closed polygon with o4, and taken in the direction shown by the arrows, *replace* the resultant when acting in the direction necessary for equilibrium. As the pole O can be taken anywhere, we can thus resolve the resultant into any two directions we wish.

Let us then consider the resultant as *replaced* by the two forces Oo and $O4$. Anywhere in the plane of the forces above, Fig. 18, draw a line S_0 parallel to Oo, and produce it till it meets P_1, produced if necessary, at a. The resultant of S_0 and P_1 will pass through a and be parallel to S_1, in the force polygon, since S_1 in the force polygon is the resultant of P_1 and S_0, given in direction and magnitude. Through a then draw a line parallel to S_1 and produce to intersection b with P_2. The line S_2 in the force polygon is the resultant of S_0, P_1, and P_2. Parallel to this line draw S_2 through b above, and produce to intersection c with P_3. The point c will be the point where the resultant of S_0, P_1, P_2, and P_3 should act. The force polygon gives the direction of this resultant as well as its magnitude. It is S_3. Parallel to this draw S_3 above, and produce to intersection d with P_4. Finally through d, draw a line S_4 parallel with S_4 in the force polygon.

Proceeding in this manner, we thus find for any assumed position of S_0 in the plane of the forces, the proper corresponding position for S_4. Since now, S_0 and S_4 are components of the resultant, and each may be considered as acting at any point in its line of direction, we have only to prolong them and *their intersection gives a point through which the resultant must act.* Through the point e, therefore, draw a line parallel to the

line 40 in the force polygon, and it will represent the resultant in proper direction and position. Acting as shown by the arrow it causes equilibrium. The magnitude of the resultant is given to scale in the force polygon.

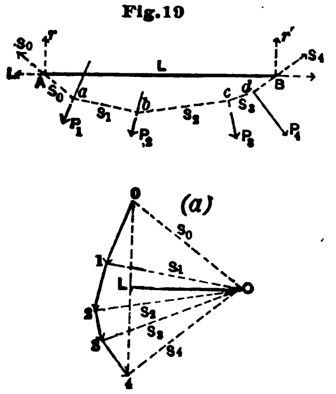
POSITION OF POLE AND OF S_0 INDIFFERENT.—A little inspection will make it apparent that our method is general, no matter where in the plane of the forces we take S_0 as acting, that is no matter where the point a is taken. Thus if a had any other position upon the direction of P_1, if everything else remained unchanged, we should evidently obtain a polygon every side of which would be parallel to that shown in Fig. 18. The new S_4 would then be parallel to that line in the present Figure as also S_0. Their intersection would, therefore, lie in a point upon the direction of the resultant as drawn.

Also any other position of pole would give different directions for the lines S_0, S_1, S_2, etc, but the intersection e of the end lines would still lie in the same line.

POLE, EQUILIBRIUM POLYGON, CLOSING LINE, RAYS.—The point O we call the "*pole*" in the force polygon. It may be taken where we please. The polygon *abcd* above we call the "*equilibrium polygon*" and *ab, bc, cd,* etc., are its *segments*. In the present case it is evidently the shape a string would take if suspended at any two points, as A and B on S_0 and S_4 respectively. The stresses in the segments would be tensile. We denote these stresses Or, $O1$, $O2$, etc., by S_0, S_1, S_2, etc., and call them "*rays*." In general, forces may act up as well as down, in which case some of the rays might represent compressive stresses, and our polygon above would contain struts as well as ties.

Let us suppose, in Fig. 19, that we take any two points, as A and B, upon the end segments S_0 and S_4, and suppose them to be made fixed. The force S_0 acting at A we shall then have to *replace* by two forces, one parallel to the resultant, and one through AB. So also for S_4. The sum of the two components parallel to the resultant must be equal and opposite to the resultant, and the component in the direction AB must be resisted in the present case by a strut or compressive member AB. This resolution we can make at once, by drawing through O in the force polygon a line OL parallel to AB. The line AB we call the "*closing line*." Thus we see from Fig. 19 (a) that the sum of the components $4L$ and Lo equals the resultant 04.

Fig. 19



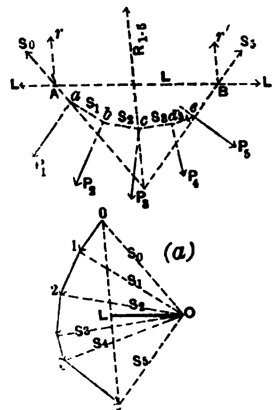
In any case then we can fix any two points of the polygon, as A, B, by drawing the closing line AB. A line OL through O parallel to this in the force polygon gives the components into which S_0 and S_4 are resolved. We must consider, then, the entire polygon $AabcdB$, with its closing line AB, as a *frame in equilibrium*, and can apply to it the principles of Chapter I. Thus take the apex A. Here we have the force r in equilibrium with the stresses in AB and Aa. Following round in the force polygon from L to 0 and so around, we find by our rule, page 13, Aa in tension and L in compression. So also for the other end B, we find Bd in tension and L in compression. The components r and r' act opposed to the resultant which replaces the forces, and the forces at A and B parallel to L are equal and opposite, hence there is no motion of the entire frame in any direction.

RECAPITULATION; FORCE AND EQUILIBRIUM POLYGON FOR ANY NUMBER OF FORCES IN A PLANE.—Suppose then we have any number of forces, as $P_1 \ldots P_6$, Fig. 20. Our method is as follows:

1st. Form the force polygon, Fig. 20(a), by laying off the forces to scale, one after the other in any order. The line 05 which closes the polygon is the resultant in magnitude

and direction. When it acts in the direction from 5 to 0, obtained by following round in the direction of the forces, it will cause equilibrium. In the opposite direction it will replace the forces.

Fig.20



2d. Choose a pole O at any convenient point, and draw the rays S_0, S_1 . . . S_5.

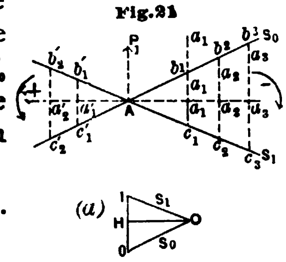
3d. Form the equilibrium polygon by drawing anywhere in the plane of the forces a line parallel to S_0 until it meets P_1, prolonged if necessary, at a. From a a line parallel to S_1 till it meets P_2 at b. From b a line parallel to S_2 till it meets P_3 at c. From c a line parallel to S_3 till it meets P_4 at d. From d a line parallel to S_4 till it meets P_5 at e. From e a line parallel to S_5. The first and last segments of this polygon intersect at a point upon the resultant. Moreover, *any two segments, as ab and cd, intersect at a point upon the resultant for the forces P_2 and P_3 acting between these segments.* The intersection of ab and de gives thus a point upon the resultant of P_2, P_3 and P_4. These resultants may be found in magnitude and direction from the force polygon (a).

4th. Fix any two points in the end segments of the equilibrium polygon by drawing the closing line AB. Resolve S_0 and S_5 into forces r and L, and r' and L, respectively parallel to the direction of the resultant and closing line. This is at once done by drawing the line OL in the force polygon parallel to AB. Then OL is the force to scale, acting at each end of the closing line AB, and Lo is the component r, and $5L$ the component r'. If these forces are to *replace* S_0 and S_5, they must act as shown by the arrows, in directions opposite to those obtained by following round in the direction of the forces in the force polygon. Thus S_0 acts from O to 0 for equilibrium. Following round, we obtain, then, r acting from L to 0, and L acting from O to L, as the directions necessary to *replace* S_0. In the same way we find, since S_5 acts from 5 to O for equilibrium, $5L$ and LO as the directions for r' and L at the right end of the closing line.

5th. Conceive now the forces S_0 and S_5 removed, and *replace* them at the points A and B, by r, L, and r' and L, and we have a frame-work, $AabcdeB$, which, acted upon by the forces P_1 . . . P_5 and r, L, r' and L, is in equilibrium. Applying the principles of Chapter I. to the apex A, where we have r, L, and the stress in Aa in equilibrium, we find the stress in AB in this case compression. So also for the apex B. The stresses in all the segments are tensile in this case. The magnitude of these stresses can be found to scale from the force polygon (a).

CULMANN'S PRINCIPLE.—Suppose we have a single force P_1, Fig. 21. The force polygon (a) becomes a straight line equal by scale to P_1. Let us choose a pole O anywhere, and draw the rays S_0 and S_1. This is the same thing as resolving the force P_1 into two components parallel to S_0 and S_1. These components are given in direction and magnitude in the force polygon (a). Parallel to them draw lines S_0, S_1, through the point of application A of the force P_1.

Fig.21



Now draw from the pole O a line OH perpendicular to 01. This distance OH we call the "*pole distance.*"

In the plane of the forces take any point whatever having any position, as a_1 or a_2, and draw through this point the ordinates b_1c_1, b_2c_2, etc. Now the moment of P_1 with reference to any point, as a_1, is $P_1 \times Aa_1$. But referring to the force polygon (a), we have by similar triangles,

$$P_1 : H :: b_1c_1 : Aa_1.$$

Hence,
$$P_1 \times Aa_1 = H \times b_1c_1.$$

That is, *the moment of the force P_1, with reference to any point, is equal to the ordinate through this point, parallel to P_1, included by the two components into which P_1 is resolved, multiplied by the pole distance in the force polygon.*

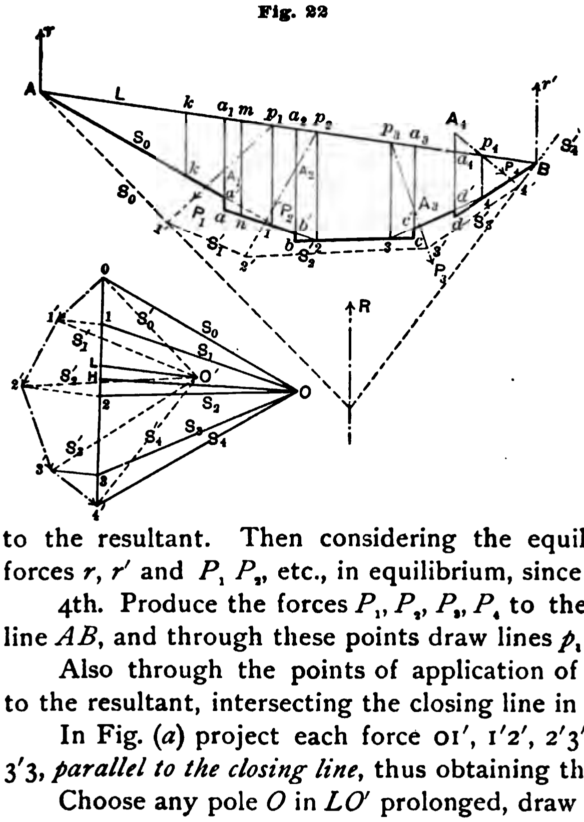
This principle we call Culmann's principle. It has, as we shall see, direct application to the equilibrium polygon. For any point situated to the left of P_1, the moment is by convention plus. For any point to the right it is minus, provided the force P_1 acts upward as represented. If it acted downward, the moments to the right would be plus, and those to the left minus.

This principle can also be proved as follows: The moment of a force is equal to the algebraic sum of the moments of its components. In Fig. 21, P_1 is resolved into components S_0 and S_1. The moment of P_1 then about any point as a_1 is equal to the sum of the moments of S_0 and S_1 about that point. But S_0 can be resolved at b_1 into a vertical force passing through a_1 and the horizontal force H. So also S_1 at c_1 can be resolved into a vertical force passing through a_1 and the horizontal force H. The vertical forces passing through a_1 have no moment. The sum of the moments of the horizontal forces is $H \times a_1b_1 + H \times a_1c_1 = H \times b_1c_1$.

GRAPHIC REPRESENTATION OF MOMENTS FOR ANY NUMBER OF FORCES IN EQUILIBRIUM.—SIGNIFICANCE OF EQUILIBRIUM POLYGON.*—We are now able to solve the problem proposed at the beginning of this chapter, and find graphically the moments of any number of forces in equilibrium.

Thus suppose we have any number of forces P_1, P_2, P_3, P_4, etc., acting at the points of application A_1, A_2, A_3, A_4, Fig. 22.

Fig. 22



1st. Construct the force polygon (a); choose any pole O' and draw the rays S_0', S_1', S_2', S_3', S_4' (broken lines).

2d. Construct the corresponding equilibrium polygon (broken lines) $A1'2'3'4'B$. Produce the two end segments. The resultant R passes through their point of intersection and is parallel and equal by scale to $4'0$ in (a).

Draw a closing line L anywhere, as AB.

3d. Parallel to AB draw $O'L$ in (a). It will be to scale the stress in the closing line AB, and the segments into which it divides the resultant, viz. $L0$ and $4'L$, will be the forces r and r' at A and B, parallel to the resultant. Then considering the equilibrium polygon as a frame, we have the forces r, r' and P_1 P_2, etc., in equilibrium, since they make a closed polygon in (a).

4th. Produce the forces P_1, P_2, P_3, P_4 to their intersections p_1, p_2, p_3, p_4 with the closing line AB, and through these points draw lines $p_1 1$, $p_2 2$, $p_3 3$, $p_4 4$, parallel to the resultant.

Also through the points of application of the forces A_1, A_2, A_3, A_4 draw lines parallel to the resultant, intersecting the closing line in a_1, a_2, a_3, a_4.

In Fig. (a) project each force $01'$, $1'2'$, $2'3'$, $3'4'$, on the resultant $4'0$, by lines $1'1$, $2'2$, $3'3$, *parallel to the closing line*, thus obtaining the points 1, 2, 3, on $4'0$.

Choose any pole O in LO' prolonged, draw the pole distance $H = OH$ *perpendicular to* $4'0$, and the rays S_0, S_1, S_2, S_3, S_4 (full lines).

Form the corresponding equilibrium polygon $A1234B$. Since P_1 passes through p_1, its moment about p_1 is zero. The moment of r with reference to p_1 is then, by Culmann's principle, preceding, equal to the pole distance OH, multiplied by the ordinate $p_1 1$.

* The student may omit here to "Application to Parallel Forces," page 38, if he finds what follows to be at first difficult of comprehension

If we suppose P_1 acting at p_1 to be resolved into a component parallel to the resultant (01 in (*a*)) and along the closing line (1′1 in (*a*)), the latter component will have no moment for any point in the closing line AB. The first component, by Culmann's principle, has a moment at a_1 equal to $a'a^*$ multiplied by $OH = H$. The moment at a_1, then, of r and P_1 is equal to $a_1a' \times H$ for r, and $a'a \times H$ for P_1, or a total of $a_1a \times H$.

For any point k in the closing line AB, then, from A to a_1, the moment of all the forces on left (or right) is equal to the ordinate kk (parallel to the resultant) $\times H$. Beyond a_1, for any point m between a_1 and a_2, we have the moment equal to $mn \times H$.

5th. We have then simply to prolong the sides 12, 23, 34 of the equilibrium polygon $A1234B$ already drawn, to intersections a, b, c, d, with the resultant parallels through A_1, A_2, A_3, A_4. We thus have the polygon $Aa'ab'bcc'd'dB$. *The moment at any point m of the closing line AB, of all the forces left* (*or right*) *of this point, is equal to the ordinate mn of this polygon, parallel to the resultant, multiplied by the pole distance H.*

POLE DISTANCE A MATTER OF INDIFFERENCE.—If in Fig. 22 the pole were in the same line LO, parallel to AB, but at any other distance, as, for instance, twice as far, all our ordinates would be decreased in the same proportion, or be half as much. The product of the ordinates by H would, therefore, be unchanged. We can therefore take O at any convenient distance in the line LO.

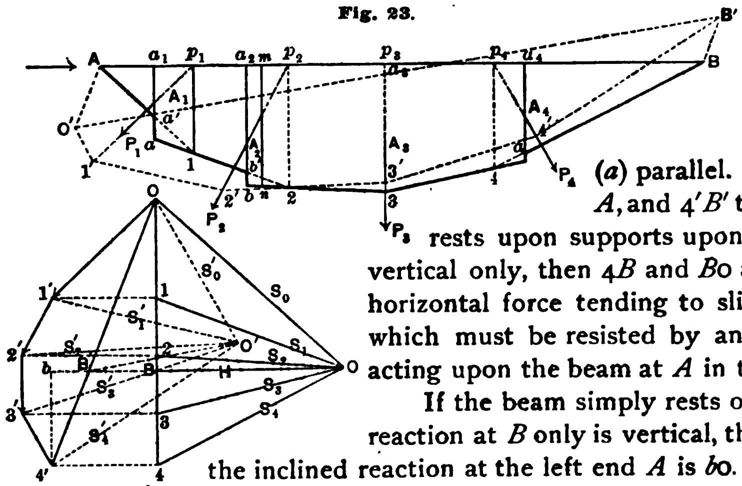
APPLICATION TO A BEAM.—In the case of a beam resting upon supports, the method is the same, except that we must have the closing line AB coinciding with the beam.

Thus let AB, Fig. 23, be the axis of a beam, supported at A and B and acted upon by the forces P_1, P_2, P_3, P_4, applied at the points A_1, A_2, A_3, A_4; *these points being rigidly connected with the beam.* Required to find the reactions at the supports A and B and the combined moment of all the forces left (or right) of any point of the axis with reference to that point.

1st. Construct the force polygon 01′2′3′4′0, Fig. 23 (*a*). Choose any pole O' and draw the rays S_0', S_1', S_2', S_3', S_4' (broken lines).

2d. Construct the corresponding equilibrium polygon (Fig. 23, broken lines) $O'1'2'3'4'B'$ and draw a closing line $O'B'$ through the points O', B' at the ends of the beam in the lines AO', BB', *parallel to the resultant* 4′0. Draw $O'B'$ in (*a*) parallel. Then B'0 is the reaction at A, and $4'B'$ the reaction at B. If the beam rests upon supports upon both of which the pressure is vertical only, then $4B$ and B0 are these reactions and 44′ is a horizontal force tending to slide the beam off its supports, which must be resisted by an equal horizontal outer force acting upon the beam at A in the opposite direction.

FIG. 23.



If the beam simply rests on its right support, so that the reaction at B only is vertical, that reaction is equal to $4'b$ and the inclined reaction at the left end A is b0. If the beam simply rests on its left support, so that the reaction at A only is vertical, that reaction is equal to B0, and the inclined reaction at the right end B is $4'B$.

3d. In (*a*) choose a new pole O anywhere *in the horizontal through B'*. Then the closing line for our new polygon will be horizontal also. We construct this new polygon as in the preceding article.

Thus, produce P_1, P_2, etc., to intersections p_1, p_2, etc., with AB, draw verticals through a_1, a_2, a_3, etc., and also through A_1, A_2, A_3, etc. In (*a*) project 01′, 1′2′, etc., vertically upon 04, thus obtaining 1, 2, 3, 4. Draw the rays S_0, S_1, S_2, S_3, S_4 (full lines) and construct the corresponding equilibrium polygon $A1234B$ (full lines).

Produce 12 to *a*, 23 to *b*, 34 to *d*, and we have, as before, the polygon $Aa'ab'b3dd'B$.

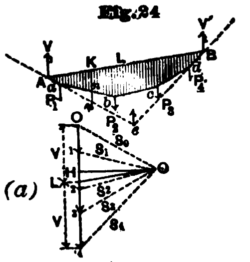
At any point *m* of AB, the vertical ordinate *mn* to this polygon, multiplied by the pole distance $OB = H$, will give the moment of all the forces left (or right) of *m*.*

COROLLARY.—It is evident that if the points of application A_1, A_2, etc., coincide with p_1, p_2, etc., we have simply the polygon $A1234B$, and the sudden changes $a'a$, $b'b$, etc., disappear.

APPLICATION TO PARALLEL FORCES.—The outer forces acting upon framed structures are generally weights and the reactions of supports due to these weights, and therefore in a majority of practical cases, it is required to investigate a system of parallel forces.

Suppose we have a number of parallel forces, $P_1 \, . \, . \, P_4$, Fig. 24.

Fig. 24



1st. Form the force polygon (*a*). This becomes in this case a straight line 04.

2d. Choose a pole O and draw S_0, $S_1 \, . \, . \, S_4$.

3d. Form the equilibrium polygon *abcd*.

4th. Fix any two points, as A and B, by drawing the closing line AB. Parallel to AB in the force polygon draw the line L. Then Lo and $4L$ are the upward reactions which must be applied at A and B to produce equilibrium. Their sum is equal to the resultant, as should be.

Now the resultant will act at *e*, and be parallel to the forces and equal to their sum. The pole distance is the perpendicular from the pole O upon the direction of the forces. The projection of each of the rays S_0, S_1, etc., in this direction is constant and equal to the pole distance OH.

From Culmann's principle, the moment at any point K of the reaction V, is therefore the ordinate Km measured parallel to the resultant, multiplied by the pole distance H. But according to the same principle, the moment of the force P_1 is equal to the ordinate $nm \times H$. Hence, the combined moment, since V acts up and P_1 down, is $H\,(Km - nm) = H \times Kn$.

For parallel forces then, any *ordinate of the equilibrium polygon parallel to the resultant is directly proportional to the algebraic sum of the moments of the forces on one side of the section with reference to any point in that ordinate, whether on the line AB or not.*

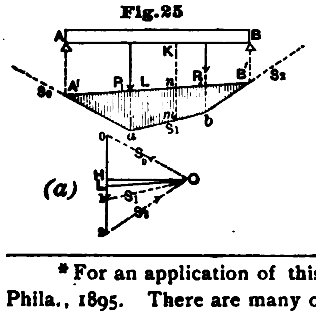
The moment itself is equal to the ordinate to the scale of length, multiplied by the pole distance to the scale of force.

We see, also, that *the ordinate included between any two segments of the equilibrium polygon prolonged, gives in the same way the moment of the force at their intersection, with reference to any point on that ordinate.* Thus *mn* multiplied by H, gives the moment of P_1 with reference to any point on Km, as K, or *n*, or *m*.

EXAMPLE I.—A few examples will make the above principles clear and show their application to practical problems.

Let AB, Fig. 25, be a beam subjected to two unequal weights P_1 and P_2 applied at any two points. Required the reaction at the supports A and B, also the moment at any point of all the forces right or left of that point, when equilibrium exists.

Fig. 25



1st. Form the force polygon (*a*).

2d. Choose a pole O, and draw the rays S_0, S_1 and S_2, and the pole distance H.

3d. Construct the equilibrium polygon by drawing a line parallel to S_0 till it meets P_1, produced if necessary, at *a*. From *a*

* For an application of this method to axles, see *Rouleaux*—"*The Constructor*." Translation by H. Supler. Phila., 1895. There are many other interesting and important properties of the equilibrium polygon, which may be found in "Elements of Graphical Statics," DuBois—Wiley & Sons. Upon these properties the entire science of graphic statics is based. The above are, however, all of which we shall need to make use in this work.

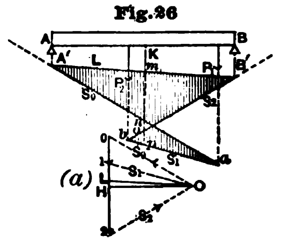
a line parallel to S_1 till it meets P_2 at b. From b draw a parallel to S_2, and prolong it indefinitely. Drop verticals from the ends A and B of the beam, and draw the closing line $A'B'$. Parallel to $A'B'$ draw OL in the force polygon.

Then Lo and $2L$ are the reactions at the ends A and B, and acting upwards they hold the weights in equilibrium. The supports should, therefore, be below the beam at each end.

The moment at any point K of the beam is equal to the ordinate nm, multiplied by the pole distance H.

EXAMPLE 2.—It is well to observe that the order in which the forces are taken, makes no difference as to the results, although the Figure obtained may be very different.

Fig. 26



Thus let us take the same example as before, but number the forces in inverse order.

We form the force polygon as before, choose a pole and draw S_0, S_1 and S_2. Now parallel to S_0 we must draw a line *till it meets* P_1 at a. [Note that S_0 must *always* be prolonged to intersection with P_1.] Then from a a parallel to S_1 till it meets P_2 at b. Then from b a parallel to S_2. Draw the closing line $A'B'$. A parallel to it in (*a*) gives the reactions Lo and $2L$ as before. Since S_0 acts from O to o for equilibrium, Lo must act up to *replace* it. Hence the support at A is below. So also for $2L$. Since S_2 acts from 2 towards O for equilibrium, $2L$ must act up to *replace* it, and the support at B should be below also.

In general, always take the directions of the reactions *opposed* to the directions of S_0 and S_n for equilibrium, obtained by following round the force polygon.

As to the moments, we see that the moment of the left reaction with reference to any point, as K, is $mn \times H$. But the moment of P_2 with reference to the same point, is $op \times H$. The difference then of mn and op, gives us the same ordinate as in the first example. The lower ordinates subtracted from the upper, will give us the same Figure as before.

We see, therefore, that whenever we obtain a double Figure, as in the present case, it shows simply that we have taken the forces in inconvenient order. We have only to change the order, to obtain the moments directly from the polygon.

In Fig. 25, we have a comprehensive picture of the way in which the moments change for every point of the beam from end to end.

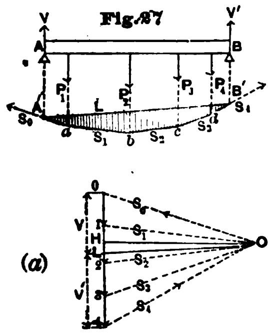
CLOSING LINE PARALLEL TO BEAM ; CHOICE OF POLE DISTANCE.—It makes no difference what inclination the closing line may have, because, as we have seen, the ordinate in the equilibrium polygon parallel to the resultant, multiplied by the pole distance, gives the combined moment *with reference to any point on that ordinate*, of all the forces right or left.

We can, however, if we wish, always render the closing line parallel to the beam itself, and this it is sometimes desirable to do. We have only first to find by preliminary construction, the reactions, or the point L where the parallel to the closing line in the preliminary force polygon intersects the force line (Figs. 25 and 26). If then we take a new pole anywhere upon a line through this point, *parallel to the beam*, the closing line will be parallel to the beam.

As to choice of pole distance, we have only to so choose the position of the pole as to give good intersections for the polygon. The multiplication may be directly performed by properly changing the scale in the equilibrium polygon. The ordinate to the new scale will then give the moment at once. Thus if our scale of length in Fig. 25 is five feet to an inch, and the pole distance in the force polygon measured to the scale adopted

for forces is, say, ten pounds, we have only to take fifty moment units to the inch as the scale for the ordinates, and they will then give the moments directly.

EXAMPLE 3.—BEAM WITH ANY NUMBER OF WEIGHTS.—Suppose we have any number of weights as $P_1 \ldots P_4$, Fig. 27.



The method of procedure is as follows:

1st. Construct the force polygon (*a*). Choose a pole O and draw the rays $S_0 \ldots S_4$.

2d. Construct the equilibrium polygon.

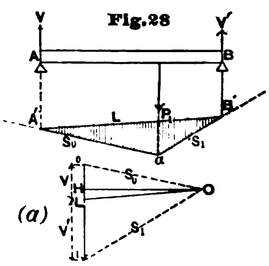
3d. Draw the closing line through the points A', B', *vertically beneath the supports.*

A parallel in the force polygon gives the reactions at the end, Lo and $4L$. These reactions must always act so as to *replace* the stresses in those lines of the equilibrium polygon which meet at A and B. Thus at A', Lo must replace the stresses in $A'B'$ and $A'a$. In the force polygon below, we see that $A'a$ or S_0, for equilibrium, acts from O to o. Hence Lo must act opposed, or up. In same way, stress in dB' or S_4 acts from 4 to O for equilibrium, hence $4L$ must act opposed, or up also. The supports at A and B must then be below.

Knowing the reactions, we can now make the closing line parallel to the beam if we choose, by simply taking a new pole anywhere upon a line through L parallel to the beam, and making a new equilibrium polygon. No advantage would be gained by such construction in this case.

The moment at any point is given by the ordinate, in the equilibrium polygon, parallel to the resultant or to the forces, multiplied by the pole distance H.

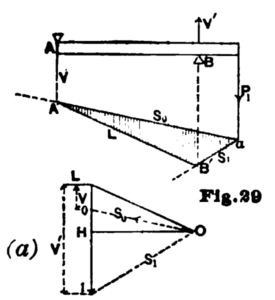
EXAMPLE 4.—BEAM WITH A SINGLE WEIGHT.—Let the weight P_1, Fig. 28, act at any point of the beam AB. Then the equilibrium polygon is $A'aB'$. The vertical reaction at the ends of the beam are Lo and $1L$, both acting up, and hence the supports must be below the beam.



We see at once that the moment is greatest at the weight, and decreases both ways to zero at the ends.

EXAMPLE 5.—BEAM WITH WEIGHT BEYOND BOTH SUPPORTS.—Observe in the construction of the equilibrium polygon that S_0 is *always* prolonged *till it meets* P_1. Also that the closing line $A'B'$ always unites the two points vertically below the supports. The equilibrium polygon $A'aB'$ is then easily drawn.

The reactions require special notice. Thus, the reaction at B is the resultant of the stresses in the lines S_1 and L, which meet at B'. This, as shown by the force polygon, is $1L$. Since S_1 has the direction from 1 to O for equilibrium, the reaction $1L$ to *replace* S_1 and L must act up. The support at B is therefore below the beam. Again, the reaction at A is the resultant of the stresses in S_0 and L which meet at A'. This is given in the force polygon by Lo. But S_0 acts from O to o for equilibrium. The reaction, then, in order to *replace* S_0 and L, must act opposed to this direction, or down. Hence the support at A must be above the beam. The reaction at B is then greater than the weight P_1 by the amount of the reaction V at A, just as should be the case.



The moment at any point is given, as always, by multiplying the ordinate in the equilibrium polygon into the pole distance H.

EXAMPLE 6.—BEAM WITH ONE DOWNWARD AND ONE UPWARD FORCE BETWEEN

THE SUPPORTS.—Here we need only call special attention to the fact that as P_2 acts up and is less than P_1, S_2 in the force polygon lies between S_0 and S_1. The reactions are Lo and $2L$, and obtaining the directions of S_0 and S_2 for equilibrium, we see that one of the reactions must act up in this case and the other down.

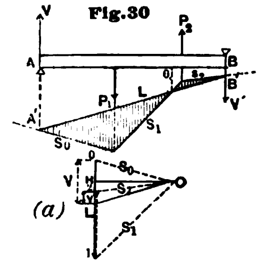


Fig.30

We see also that if P_2 should be taken less, so that 2 falls below L in the force polygon, the reaction at B would be upward also, and the support there would have to be below the beam. The student would do well to sketch the construction for P_2 greater than P_1.

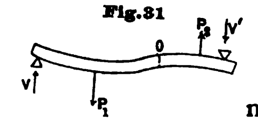


Fig.31

At the point o we see that the moment is zero. At this point the moment of V is equal and opposite to the moment of P_1. At o, then, we would have a "point of inflection," or the beam would be concave upward as far as o, and from o on convex upward, as shown in Fig. 31. At o the two curves would have a common tangent.

EXAMPLE 7.—BEAM SAME AS BEFORE, BOTH FORCES EQUAL.—Laying off the force polygon, the first force extends from o to 1, Fig. 32 (*a*), and the second from 1 back to o again. Choosing, therefore, a pole O and drawing S_0, S_1, S_2, we find that S_0 and S_2 fall together. The directions of S_0 and S_2 for equilibrium are shown by the arrows.

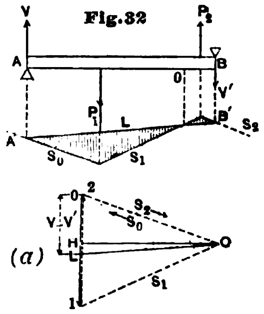


Fig.32

Constructing the equilibrium polygon, and drawing the closing line $A'B'$ and its parallel L in the force polygon, we see that the reaction at A, or the resultant of S_0 and L is Lo, and the reaction at B, or the resultant of S_2 and L, is also Lo. The reactions V and V' are therefore equal. This is in accordance with our principle, page 26, that a couple can only be held in equilibrium by another couple. As to the direction of these reactions, taking S_0 as acting as shown by the arrow for equilibrium, V, in order to replace, must act up. In like manner V' must act down. The support at A should be below and B above. At o we have the moment zero. Here then is a point of inflection, and the beam has the deflected shape shown in Fig. 31.

The moments at any point are, as always, given by the ordinates multiplied by the pole distance H. We see that the moment is greatest at each force, and zero at o and the two ends.

EXAMPLE 8.—BEAM WITH TWO EQUAL WEIGHTS BEYOND THE SUPPORTS.—Fig. 33 needs no explanation, except to call attention to the reactions.

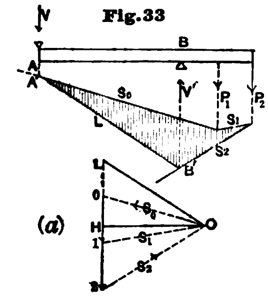


Fig.33

Thus the reaction at A is oL acting down. At B, it is the resultant of S_2 and L, or $2L$, acting up.

We see from the ordinates in the equilibrium polygon, how the moments vary from point to point.

We repeat here, that the order in which the forces are taken in all these examples, is indifferent, as also the position of the pole. The student will do well to work out cases to scale and satisfy himself that this is so.

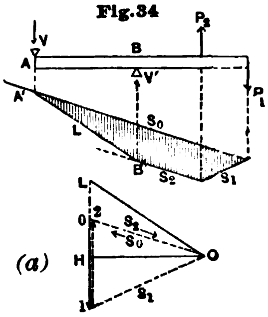


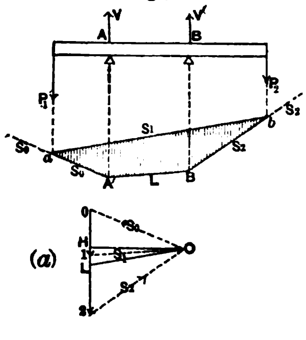
Fig.34

EXAMPLE 9.—BEAM WITH A COUPLE BEYOND THE SUPPORTS.—Observe that S_0, Fig. 34, is produced till it intersects P_1 in the equilibrium polygon. Then S_1 to P_2, then S_2 parallel to S_0. The closing line $A'B'$ is then drawn. A parallel to it in the force polygon (*a*) gives Lo acting down as the reaction at A and oL acting up as the reaction at B. Between B and P_2 we see that the moment is constant, because S_0 and S_2 are parallel. This is the graphical interpretation of our principle, page 26, that the moment of a couple is constant

for any point in the plane. We see, also, that since S_0 and S_2 intersect at an infinite distance and the resultant of S_0 and S_2 is zero in the force polygon, that *the resultant of a couple is an infinitely small force situated at an infinite distance.*

EXAMPLE 10.—BEAM WITH A VERTICAL WEIGHT BEYOND EACH SUPPORT.—Here we would call attention, Fig 35, to the construction of the equilibrium polygon. We draw a parallel to S_0 in the force polygon till it meets P_1 at a. From a a parallel to S_1 till it meets P_2 at b. Through b a parallel to S_2. Prolong S_2 and S_0 till they meet the verticals through B and A at B' and A'. $B'A'$ is the closing line.

Fig. 35



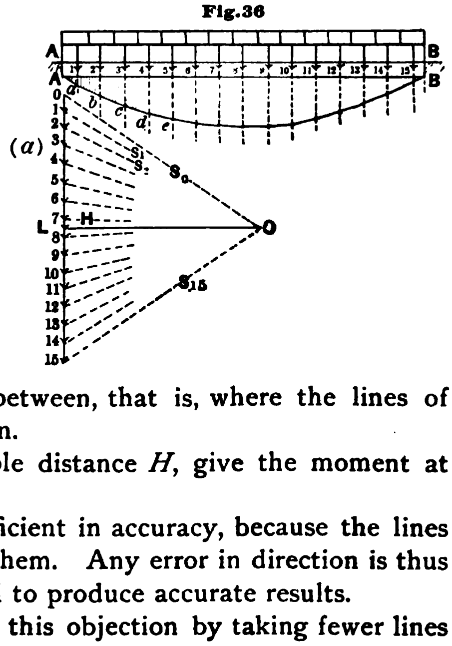
This gives us in (*a*) Lo acting up for reaction V at A, and $2L$ acting up for reaction V' at B.

If we had taken the forces in inverse order, we should have got a double figure, as in Fig. 26.

EXAMPLE 11.—UNIFORM LOADING.—A uniform load may be considered as a system of equal and equidistant weights very close together.

Thus, in Fig. 36, the load area, which is a rectangle whose height is the load per unit of length, and whose length is the length of the beam, may be divided into any number of equal parts. The area of each of these parts we may consider as the weight which acts at its centre of gravity, and lay it off to any assumed scale in the force polygon. Since the reaction at A and B must be equal, we take the pole in a horizontal through the centre of the force line. The closing line will then be parallel to the beam, and $OH = OL$.

Fig. 36

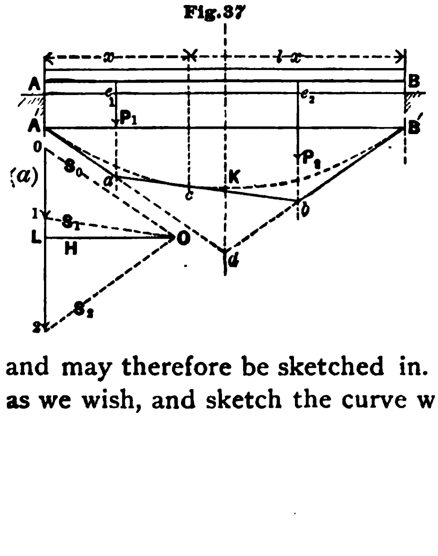


Now draw the rays $S_0\ S_1 \ldots S_{15}$, and then construct the equilibrium polygon. It is evident that the points *abcde*, etc., of the polygon will enclose a curve tangent to *ab*, *bc*, *cd*, *de*, etc., at the points midway between, that is, where the lines of division of the load area meet the sides of the polygon.

The ordinates to this curve, multiplied by the pole distance H, give the moment at any point of the beam.

It will be seen, however, that this method is deficient in accuracy, because the lines *ab*, *bc*, etc., are so short, and there are so many of them. Any error in direction is thus carried on, and even the most extreme care would fail to produce accurate results.

Fig. 37



We may avoid this objection by taking fewer lines of division.

Thus, suppose, Fig. 37, we divide the load into only two portions, x and $l-x$. The entire weight over the portion x can be considered as acting at the centre e_1 of the load-area x (page 25). The same holds good for the load over $l-x$. We thus have two forces, P_1 and P_2. Taking the pole as before, so that the closing line shall be parallel to the beam, construct the equilibrium polygon $A'abB'$. The curve of moments required will be tangent at A', c and B', as shown by the dotted curve, and may therefore be sketched in. We may thus determine as many points of tangency as we wish, and sketch the curve with considerable accuracy.

We may, however, devise a still better method. It is evident that the curve required is symmetrical with respect to the vertical through the centre of the beam. If we can determine what this curve is, it may be possible to construct it directly without using the force polygon at all.

Now we see that no matter where the load area is supposed to be divided, we shall have always, Fig. 37,

$$e_1e_2 = \frac{1}{2}x + \frac{1}{2}(l\text{-}x) = \frac{1}{2}l.$$

That is, no matter where the line of division, the horizontal projection of the line ab of the equilibrium polygon is constant and equal to $\frac{1}{2}l$. But the line ab is a tangent to the curve required. But if from any point on the line $A'd$ we draw a line ab, limited by the line $B'd$, in such a way that its horizontal projection is constant, the line ab *will envelop a parabola.*

This may easily be proved analytically as follows :—Let the load per unit of length be p. Then the entire load is pl and the reaction at each end is $\frac{pl}{2}$.

The moment at any point distant x, Fig. 37, from the left end, is then

$$y = -\frac{pl}{2}x + P_1\frac{x}{2},$$

but since P' is always equal to px,

$$y = -\frac{pl}{2}x + \frac{px^2}{2} = -\frac{p}{2}.\,x\,(l - x).$$

That is, *the moment at any point in a beam subjected to a uniform load is equal to one half the unit load multiplied by the product of the two segments of the beam.*

This is the equation of the curve of moments when the origin is at A'. For the centre of the beam, $x = \frac{l}{2}$, and we have, therefore, the ordinate to the curve at the centre, $-\frac{pl^2}{8}$.

If we take the origin at K, we have,

$$x = \frac{l}{2} + x', \qquad y = -\frac{pl^2}{8} + y',$$

hence,

$$-\frac{pl^2}{8} + y' = -\frac{pl}{2}\left(\frac{l}{2} + x'\right) + \frac{p}{2}\left(\frac{l}{2} + x'\right)^2,$$

or

$$x'^2 = \frac{2}{p}y',$$

which is the equation of a parabola referred to its vertex.

We have, therefore, the following construction :

In Fig. 38, lay off a perpendicular eK to the beam at its centre, and make it equal by scale to $\frac{pl^2}{8}$. Through A, B, and K, construct a parabola, having its vertex at K. The ordinate to this parabola at any point of the beam will give the moment at that point, to the same scale as that by which eK was laid off. The distance Kd is equal also to $\frac{pl^2}{8}$, because

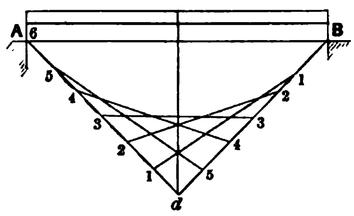
Fig. 38

ed, or the moment of the reaction is $\frac{pl}{2} \times \frac{l}{2} = \frac{pl^2}{4}$, and $ed - eK = Kd = \frac{pl^2}{4} - \frac{pl^2}{8} = \frac{pl^2}{8}$.

The distance ed then is equal to $\frac{pl^2}{4}$.

HOW TO DRAW A PARABOLA.—Since in any case we know, then, the distance *ed*, we can always draw the lines *Ad* and *Bd*, Fig. 39. If then we divide *Ad* into any number of equal parts, as say, six, and *Bd* into the same number of equal parts, and number these parts in the one case away from *d*, and in the other case towards *d*, we have only to draw lines joining any two points having the same number, and these lines will all have the same horizontal projection equal to $\frac{l}{2}$. They will, therefore, enclose the parabola required. Tangent to these lines we may sketch the curve.

Fig. 39

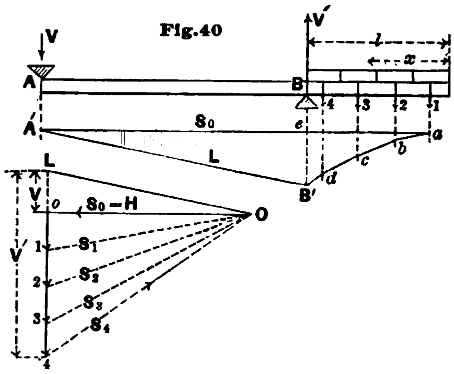


A better method, because more accurate, is to plot the ordinates to the curve, from its equation,

$$y = \frac{pl}{2}x - \frac{px^2}{2}$$

by inserting for x, measured from the left end different values, as $\frac{1}{10}l$, $\frac{2}{10}l$, $\frac{3}{10}l$, etc., and finding the corresponding values for the ordinate y.

EXAMPLE 12.—BEAM LOADED UNIFORMLY BEYOND THE SUPPORTS.—Let the beam, Fig. 40, be loaded uniformly beyond the support B. If we divide the total load into say four equal parts, and consider each weight acting at the point midway between the points of division, we may form the force polygon and then draw the equilibrium polygon as shown.



Take the pole in a horizontal through o. Then S_0 will be parallel to the beam and be equal to H. We obtain the equilibrium polygon $A'abcdB'$, and $A'B'$ is the closing line. This gives us Lo for the reaction at A, acting down, and $4L$ for the reaction at B, acting up. The ordinate at any point, multiplied by H, gives the moment.

Again, we see that the polygon $abcdB'$ is properly a curve, tangent to ab, be, etc, at the points where the lines of division prolonged meet these sides. The polygon approaches this curve more nearly, the greater the number of parts into which we divide the load.

It is better, to insure accuracy, to plot this curve, which is, as before, a parabola, by points.

Thus, if the loaded portion is l, we have for the moment at any point distant x from the right end the moment $y = \frac{px^2}{2}$.

Inserting different values for x, we can find the corresponding moment and lay it off to scale. The moment at B is then $\frac{pl^2}{2}$. Laying this off from e to B' by scale, we can join $B'A'$, and thus obtain the equilibrium polygon. The ordinate at any point taken to the scale adopted gives then at once the moment at that point, *i. e.*, the pole distance is unity.

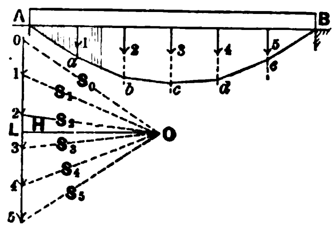
EXAMPLE 13.—BEAM LOADED WITH CONCENTRATED EQUAL WEIGHTS, EQUI-DISTANT.—Let the distance from the ends to the nearest weight be equal to the distance between the weights.

Take the pole as before, Fig. 38, so that the closing line shall be horizontal. We can then construct the polygon *abcde*, Fig. 41, the ordinates to which, multiplied by the pole

distance, give the moments. As the weights are concentrated, we have not in this case a curve, but a true polygon. We meet, however, the same practical difficulties of construction as in Fig. 36.

These difficulties may be overcome, as in that case, by constructing the parabola for an equal uniform load, and then remembering that the polygon required *is inscribed in this parabola*, that is, has its angles upon the curve.

Fig. 41



We can then construct the parabola for an equal uniform load, as directed, page 44, and where the weights intersect the curve, we have the points *abcd*, etc. The polygon can then be drawn. The moment for the point of application of any weight then is given by the ordinate to the curve. The moment for a point between any weight *is given by the ordinate to the polygon*, and not to the curve.

To construct the parabola, we divide one of the equal weights by the distance between two weights. This gives us the equivalent uniform load per unit of length p. We can then plot the parabola by points from its equation

$$y = \frac{pl}{2}x - \frac{px^2}{2}$$

by inserting for x the distance of each weight from the left end in terms of l. The moment is then given directly by y. We can lay off the values for y thus found, to scale, and the force polygon is unnecessary, since the pole distance is thus assumed as unity.

The above is sufficient to enable any careful student to thoroughly master the method. We see that in any case we can easily find, by a graphical construction, the moment of all the outer forces acting upon any rigid body, right or left of any point, and this was the problem proposed for solution at the beginning of this chapter. The student should at first draw all the examples with parallel ruler. Afterwards he can sketch merely by eye for purposes of elucidation only.

B. ILLUSTRATION OF GENERAL PRINCIPLES.

Let us choose as an example to illustrate the application of these principles the same truss as that already discussed in the preceding chapter.

APPLICATION TO A ROOF TRUSS.

Let Fig. 42 represent the truss. The end weights can evidently be disregarded in the force polygon, since they act directly upon the supports. This is also shown in Fig. 7 (*a*), where the end weights have no effect upon the stresses, and the Figure is the same as though they were left out, provided the reaction is taken at 2,800 lbs. instead of 3,200 lbs. In the methods of Chapter II. and Chapter III. also, the same is the case. In general, a weight upon the support has no effect upon the truss, and can be disregarded.

Numbering the weights then as in Fig. 42, we can construct a force polygon (*a*), and then the equilibrium polygon, as shown. This, however, is not advisable, for reasons already given. It will be more accurate to assume the pole distance as unity, thus discarding the force polygon altogether, and construct the parabola from its equation

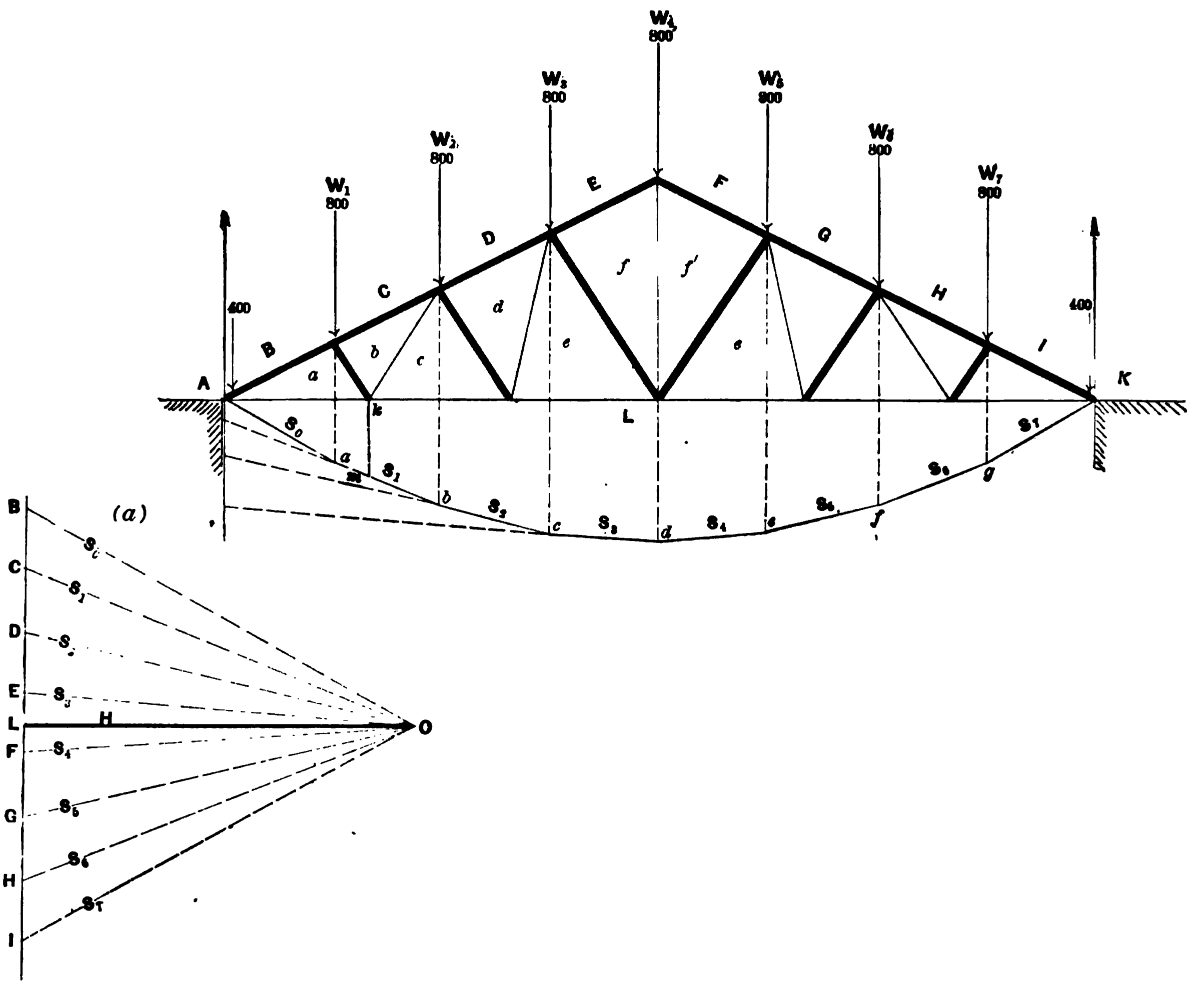
$$y = \frac{pl}{2}x - \frac{px^2}{2},$$

as directed in Example 13.

Putting then $x = \frac{1}{8}l, \frac{2}{8}l, \frac{3}{8}l, \frac{4}{8}l$, etc., we obtain for the moments at the points of appli cation of the weights, and, therefore, for the apices a, b, c, d, etc., upon the curve, the ordinates,

$$y = \frac{7}{128}pl^2, \frac{12}{128}pl^2, \frac{15}{128}pl^2, \frac{16}{128}pl^2, \text{ etc.}$$

Fig. 42



The total weight acting upon the truss in the present case, *including the end weights*, is 6,400 lbs.* The length of the span $l = 50$ feet. Hence $p = \frac{6400}{50} = 128$ lbs.

We have, then, for

$$x = \frac{1}{8}l \qquad \frac{2}{8}l \qquad \frac{3}{8}l \qquad \frac{4}{8}l$$

$$y = 17500 \qquad 30000 \qquad 37500 \qquad 40000 \text{ moment units.}$$

Laying these off to any convenient scale, we determine very accurately the points *abcd*, Fig. 42. The other half of the polygon is precisely similar.

* Observe that in calculating p, or the equivalent load per unit of length, *all* the weights must be taken, or else we must divide one of the equal weights by the distance between. Thus $\frac{800}{6.25} = 128$ lbs. per ft.

The ordinates to this polygon will give, to the scale adopted for moment units, the moment for any point of the truss, of the outer forces left or right of this *point*. Thus the moment with reference to k, of all the forces right or left, is km, Fig. 42. We find by scale $km = 21666\frac{2}{3}$ moment units. In the same way, the moment for the next lower apex is 35000 moment units to scale. The moment at the next lower apex, or for the centre of the span, is 40000 moment units, since it is vertically beneath the weight W_4.

Now our rule is, as before, Chapter III., page 27, for any member,

$$\text{Stress} \times \text{lever arm} + \Sigma \text{ moments of outer forces} = 0.$$

The second term is given by our ordinates to the polygon to scale. We have then only to divide these by the lever arm for any member in order to obtain the stress.

As regards the centre of moments for any member, we must observe the rule, Chapter III., page 26, *viz.:* Cut the truss entirely through by a section cutting only three members, the stresses in which are unknown. For any one of these members take the point of moments at the intersection of the other two cut.

For the proper sign of the stress moment, we have, as before, the rule of Chapter III., page 26, *viz.:* Considering only the left-hand portion of the truss thus divided in two, imagine an arrow at the section pointing away from the left end of the cut member. Take the stress moment with the same sign as the rotation indicated by this arrow.

The lever arms for this case have been calculated for each member, and are given in Chapter III., page 28.

As always, a moment causing rotation in the direction of the hands of a watch is negative, in the reverse direction positive.

Observing these conventions, a plus sign in the result will indicate tension in a member, a minus sign compression.

Let us first find the stress in the lower panels. For La, the centre of moments is at the first upper apex BC, according to rule. The moment for this point is given by the ordinate na, or is 17500 moment units. Considering always the left portion, this moment is negative, because the reaction—the only force acting on that portion—acts up. The stress moment is, according to the rule, plus, because the arrow for La would give positive rotation. The lever arm has been found to be 3.125 feet (page 28).

We have, then,

$$La \times 3.125 - 17500 = 0,$$

or,

$$La = + \frac{17500}{3.125} = + 5600 \text{ lbs.}$$

In similar manner we have

$$Lc \times 6.25 - 30000 = 0,$$

or,

$$Lc = + \frac{30000}{6.25} = + 4800 \text{ lbs.}$$

For Le, we have

$$Le \times 9.375 - 37500 = 0,$$

or,

$$Le = + \frac{37500}{9.375} = + 4000 \text{ lbs.}$$

Let us now find the stresses in the upper panels. For the panel Ba, the centre of moments is at k. Since, when we cut Ba and La, the only force acting on the left-hand portion of the truss is the reaction, the moment at k is the moment of this reaction. That is, it is the ordinate from k to the line *Aa of the polygon produced.* It is therefore negative and larger than km, which gives the combined moment of the reaction and first weight. We find it by scale to be $23333\frac{1}{3}$ moment units. The stress moment of Ba is negative, because the arrow for Ba would indicate negative rotation.

We have then

$$-Ba \times 3.727 - 23333\tfrac{1}{3} = 0,$$

or,

$$Ba = -\frac{23333}{3.727} = -6260 \text{ lbs.}$$

In like manner, for Cb we have the moment $km = 21666\frac{2}{3}$. Hence,

$$-Cb \times 3.727 - 21666\tfrac{2}{3} = 0,$$

or,

$$Cb = -\frac{21666}{3.737} = -5813 \text{ lbs.}$$

In the same way we have

$$-Dd \times 7.454 - 35000 = 0,$$

or,

$$Dd = -\frac{35000}{7.454} = -4691 \text{ lbs.}$$

Also,

$$-Ef \times 11.151 - 40000 = 0,$$

or,

$$Ef = -\frac{40000}{11.151} = -3587 \text{ lbs.}$$

For the braces, the point of moments is at A. Taking a section through Cb, ab and La, we have acting on the left-hand portion only the weight at BC, which causes a moment about A. But the moment of this weight with reference to A is, by our principles, the ordinate *through A which meets S_1 produced.* This moment is negative. We take it off to scale $= 5000$ moment units. The lever arm for ab is given on page **28.** The stress moment for ab is negative by our rule. We have, then,

$$-ab \times 6.934 - 5000 = 0,$$

or,

$$ab = -\frac{5000}{6.934} = -721.$$

In like manner, for bc the stress moment is positive and the same as for ab.
We have, then,

$$bc = +721 \text{ lbs.}$$

Again, for the brace cd, the moment is the sum of the moments of the weights at BC and CD with reference to A, because when we cut Dd, cd, and Lc, both of these weights act upon the left-hand portion. This moment is given to scale by the ordinate through A which meets the line S_2 in the equilibrium polygon produced. It is to scale 15000 and is negative.

We have, then, since the stress moment is minus,

$$-cd \times 13869 - 15000 = 0,$$

or,

$$cd = -\frac{15000}{13869} = -1081 \text{ lbs.}$$

For the brace *de* we have the same moment, because only the same weights act upon the left-hand portion, but the stress moment is positive.

We have, then,

$$de \times 16.2 - 15000 = 0,$$

or,

$$de = +\frac{15000}{16.2} = +926 \text{ lbs.}$$

For the brace *ef*, in like manner, the moment is positive and equal to the ordinate through A, limited by the line *cd* of the equilibrium polygon, produced. This ordinate to scale is 30000 moment units, and is negative. We have then

$$-ef \times 20.803 - 30000 = 0,$$

or,

$$ef = -\frac{30000}{20.803} = -1442 \text{ lbs.}$$

For the brace ff' we have the same moment, but the stress moment is positive. But the piece $f'e'$, which is also cut, has also a moment with respect to A, which must be taken into account. Since, by reason of the symmetry of frame and loading, the stress in $f'e'$ is the same as that already found for *ef*, and its lever arm is the same, its moment is also -30000.

We have, then,

$$ff' \times 25 - 60000 = 0,$$

or,

$$ff' = +2400 \text{ lbs.}$$

These values are precisely the same as those already found for the roof truss in the preceding chapters.

REMARKS UPON THE METHOD.—The present method is convenient for finding the stresses in the upper and lower panels, *but it should never be used for the braces.* We see from Fig. 42 that in prolonging the sides *ab*, *bc*, etc., of the equilibrium polygon till they meet the vertical through A, which is necessary in order to find the moments for the braces, a little variation in direction will make considerable difference. As the sides *ab*, *bc*, etc., are short, they do not give direction accurately enough.

In fact, of all our four methods, none are so well adapted to the case of Fig. 42 as the method of Chapter I., checked in one or two of the last pieces by the method of Chapter III. The more irregular the frame the more advantageous is the graphic method of Chapter I. For girders with parallel flanges, like most bridge trusses, however, the method of the present chapter is very extensively used for the upper and lower flanges, and is in such cases very easy of application.

TEXT-BOOKS ON GRAPHIC STATICS.

The student will find the graphical method of Chapters I. and IV., as well as many other applications and principles, explained and treated in the following works :

Culmann, K.—"Die Graphische Statik." With Atlas of 36 Plates. Zürich, Meyer & Zeller, 1866. [I. Part, 1864 ; Elements and Graphical Investigations of Structures. Also a second edition,

first volume, 1875, with 17 Plates. General Principles, second volume, to follow shortly. This is the pioneer work on the subject, and also the most complete.]

Bauschinger.—" Elemente der Graphischen Statik." With Atlas of 20 Plates. München, 1871. [A more popular presentation of the subject, requiring less mathematical preparation to read.]

Ott, K. Von.—" Die Grundzüge des Graphischen Rechnens und der Graphischen Statik." Prag, 1872. [English translation by G. S. Clarke. A small elementary treatise.]

Favaro, Antonio.—" Lezioni di Statica Grafica." Padua, 1877. Pp. 650.

Levy.—" La Statique Graphique et ses Applications." Paris, 1874. With Atlas of 24 Plates. [Principles and numerous applications.]

Du Bois, A. J.—" The Elements of Graphic Statics, and their Application to Framed Structures." Pp. 400. With Atlas of 32 Plates. New York, John Wiley & Sons.

Clarke, G. S.—" The Principles of Graphic Statics." Pp. 138. With Atlas of 11 Plates and numerous illustrations in Text. E. & F. N. Spon, London.

Greene, Charles E.—" Trusses and Arches Analyzed and Discussed by Graphical Methods." John Wiley & Sons, New York.

Bow, R. H.—" Economics of Construction in Relation to Framed Structures." E. & F. N. Spon, London.

The literature of graphic statics is now quite extensive. Our space forbids mention of monographs and papers. A more complete list may be found in the author's treatise above, which was the first systematic presentation in English. The preceding list comprises all the text-books upon the subject proper known to the author.

PART I.

SECTION II.

PRACTICAL APPLICATIONS.

SECTION II.

PRACTICAL APPLICATION OF PRECEDING METHODS TO VARIOUS STRUCTURES.

INTRODUCTORY.—CLASSIFICATION OF STRUCTURES.

PLAN OF THIS SECTION.—The preceding section includes all the methods used in the solution of framed structures. They are, as we have seen, four in number—two graphic and two algebraic. Special cases lead sometimes to modifications of these general methods, which we shall point out in their proper place. In the present section we shall discuss more in detail the various forms of framed structures most frequently met with, and, in doing so, shall sufficiently indicate the application of our principles to enable the reader to easily solve any other case not specially treated. We shall choose for each form that method or that combination of methods which in each case seems most advantageous. The student familiar with the principles of the preceding section can easily apply any other combination which seems to him to offer superior advantages as to accuracy or facility.

The choice of any method for any special case is in some measure a matter of individual preference. While, therefore, we shall adopt those methods which seem to us the best suited to the case in hand, or which are most generally in use, the student will understand clearly that he is by no means confined to such method unless it commends itself to him as, on the whole, the best.

CLASSIFICATION OF STRUCTURES.—We may divide all those structures of which we shall treat into two classes: those which sustain the action of a permanent load, or unvarying forces, and those which are subject to forces of variable magnitude. To the first class belong *Roof Trusses*, *Cranes*, *Cantilevers*, and in general all those structures which have to sustain a "dead load," such as their own weight and exterior forces of constant magnitude, such as the weight of roofing, snow, etc. To the second class belong *bridges*, which have to sustain, besides a "dead load" proper, consisting of their own weight and outside forces of constant magnitude, also the action of a "live load," such as that of moving cars or vehicles, cattle, and men.

ROOF TRUSSES.—Roof trusses are of almost innumerable forms. It will be unnecessary to discuss each form. The principles which apply to one apply to all. The selection of a few well-chosen cases will suffice for all. Such cases will be found in the next chapter.

TRUSS ELEMENT.—The truss element is in all cases a triangle. No rigid framework can be made which does not consist of a repetition of the triangle. Any frame of three sides is rigid. Its shape cannot be altered without altering the lengths of its sides. Any framework of more than three members can thus alter its shape, unless divided into triangles by diagonals which constitute the bracing.

SUPERFLUOUS MEMBERS.—The conditions of equilibrium are three, viz.: 1st. The

algebraic sum of the vertical components must be zero; 2d. The algebraic sum of the horizontal components must be zero; 3d. The algebraic sum of the moments of the forces must be zero. As in any framed structure we know, or must first independently determine, all the outer forces, it follows that these outer forces must be held in equilibrium at any point of the frame by the stresses in the members cut by a section through the frame at that point (p. 5). If there are only three such members the stresses in which are necessarily unknown, we can always write down three equations of condition between the stresses in these members, and therefore determine them. If there are more, the problem is indeterminate; there are more unknown quantities than there are equations of condition between them.

At any point, therefore, of any properly framed structure it should be possible to make, in some direction, a section, cutting the structure entirely in two, which shall not cut more than three members, the stresses in which are *necessarily* unknown. Of course it may cut any number of members, provided it is possible to find independently the stresses in all but three. Any framed structure which violates this rule is improperly framed, and has superfluous members.

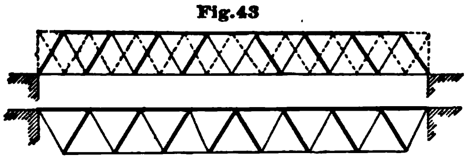
BRIDGE TRUSSES.—We may divide all bridge trusses into two classes, those in which the upper and lower members, or "chords," are horizontal or parallel, and those in which the chords are not parallel, and modifications of these.

I. GIRDERS WITH PARALLEL CHORDS.

In the first class there are two pure types which admit of many varieties. These are the *triangular* and the *quadrilateral* types, so called from the character of the bracing.

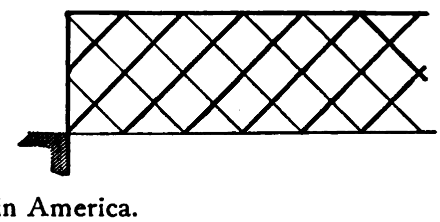
WARREN GIRDER.—The "*Warren*" girder, Fig. 43, is an example of the pure triangular type. Its bracing consists always of *equilateral triangles.* When the triangles are not equilateral, but isosceles, or have, indeed, any other shape, the truss is simply a "triangular truss." A common form is to make the height, or distance from centre to centre of chords, half the length of panel, in which case the angles of the braces with the chords are 45°. This truss is of more frequent occurrence in England than in this country.

Fig.43



DOUBLE TRIANGULAR-LATTICE TRUSS.—The triangular is the simplest form of truss, consisting simply of repetitions of the single truss element, or triangle. When, owing to the great length of panels, we have two or more systems of triangulation, as shown in Fig. 43, by the dotted lines, the truss becomes the "*double triangular*," or "*triple triangular*," as the case may be.

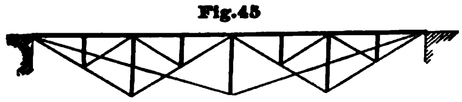
Fig.44



When there are in general more than three systems, and the braces are riveted to each other at their intersections, we have what is known as the "*lattice*" girder or truss, Fig. 44. A few lattice girders executed in wood are still to be found. With these exceptions, this style of truss may be said to be almost unknown in America.

FINK TRUSS.—This is essentially a triangular truss with the lower chord left out, Fig. 45. The span is trussed or supported at the centre by a strut and ties from each end. Then the half spans, if sufficiently long to need it, are trussed as shown in the Figure. Again, the quarter spans may be trussed in similar manner, and so on. A number of these trusses are to be found in this country, but it is not now generally regarded with favor by bridge builders.

Fig.45



These are, in general, all the modifications of the simple triangular type as applied to bridges.

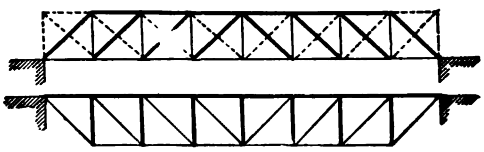
QUADRILATERAL TYPE.—It is evident that if such a structure as Fig. 43 is subjected to the action of a live load, some of the braces may be sometimes extended and sometimes compressed, according to the position of the moving load. It is not advisable, from a practical point of view, to subject the same member to alternating stresses of different character. Such action tends to deteriorate the material of which the piece is made, and shorten its life in the structure.

This has given rise to various constructions, in which each member is required to sustain a stress of only one character, although this stress may indeed vary considerably in amount. In Fig. 43, the difficulty might be met by having each brace, when necessary, double, consisting of a hollow cylindrical member for compression, enclosing a tie rod to take the tension.

Such considerations have led to the quadrilateral type of truss, in which each member takes only stress of a certain character.

QUADRILATERAL TRUSS—HOWE, PRATT, MURPHY-WHIPPLE.—A very common form is shown in Fig. 46. We may call it a single quadrilateral because it has but one system of bracing, and the panels are rectangular in form.

Fig. 46



When the vertical members sustain only compression, and the inclined members tension, it is known as the "*Pratt*" or "*Murphy-Whipple*" system. In this shape it is often constructed of iron, and is then an advantageous form, because the shortest braces are compressed. As a long piece in compression always requires extra material to stiffen it and prevent it from doubling up or "buckling," this tends to save material. When there are two systems it is called the Whipple or double-intersection truss.

When the vertical members are in tension and the inclined braces in compression, the form is known as the "*Howe*" system. This is still often executed in wood and iron combined. The long braces are made of wood and the verticals of iron rods. This is again an advantageous use of material, as wood is comparatively cheap and best used in compression, while wrought iron is dearer and better adapted for tension.

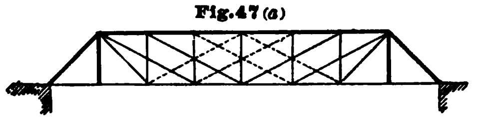
COUNTER-BRACES.—Where in any quadrilateral system the action of the live load tends to cause in any inclined brace a stress opposite in character to that which it is designed to take, a brace in the direction of the other diagonal is inserted, as shown by the dotted braces in Fig. 46. Thus a load which tends to shorten one brace or diagonal cannot do so without elongating the other. If, for instance, the braces in full lines in Fig. 46 will take only tension, and buckle up under the action of a compressive stress, the dotted braces will be called into action. Such braces are called "*counter-braces.*" The stress in a counter-brace is, therefore, due entirely to the action of the live load. The dead load causes no stresses in it whatever. The main braces, therefore, in any case are those braces which are called into action by the dead load; the counter-braces, those which are called into action by the live load only.

SCREWING UP COUNTER-BRACES.—By properly screwing up the counters of such a truss as Fig. 46, the girder may be held down to that deflection which would be caused by the live load when it covers the whole span, and the girder thus rendered very rigid. The live load as it comes on would then act simply to relieve the stresses in the counters without adding anything to those existing in the braces themselves. Under such circumstances all the members sustain always a steady stress, except the counter-braces, and in these the stress, though fluctuating in amount, is always the same in character.

DOUBLE QUADRILATERAL—WHIPPLE TRUSS.—When the panels in Fig. 46 become

very long we may divide them up, and thus obtain the double quadrangular system of Fig. 47(*a*), or, as it is called sometimes, the "*Whipple*" or double-intersection Pratt truss. In the same way we may obtain triple quadrilateral, etc. All such systems as Figs. 44 and 47(*a*) may be called ***multiple systems.*** The pure types are the triangular and the quadrilateral, from which they are derived by multiplication of the system of bracing.

Fig. 47 (*a*)



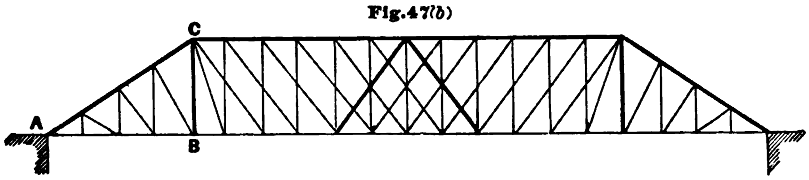
This is still a common form of truss, though the best practice avoids all multiple systems. A modification of it of European origin is shown in Fig. 47(*b*).

If h represents the height and l the length of span, the best length a of the portion AB is given by

$$a = 0.006\, l + 1.08\, h.$$

The object of the variation is of course to effect a saving of material, but it may be doubted whether the design would compare favorably with the double-intersection Pratt truss as executed in America with pin connections. Two such bridges are in existence in Vienna, over the Danube, each about 200 feet span.

Fig. 47(*b*)



Another modification, known in Germany as the *Schwedler* truss, consists in curving the ends AC, Fig. 47(*b*). In this truss the length of the portion AB is given by the formula

$$a = l\frac{w}{w'}\left[\sqrt{1 + \frac{w}{w'}} - 1\right],$$

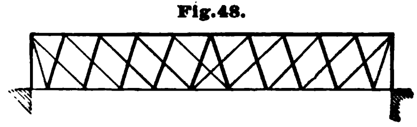
where l is the span, w the dead weight, and w' the live load per unit of length. The height h at any distance x from the end of the curved portion is given by

$$h = \frac{h_1 w}{l}\left[\sqrt{1 + \frac{w'}{w}} + 1\right]^2 \frac{x(l - x)}{wl + w'x},$$

where h_1 is the height of the straight portion. All the members are, of course, straight, and only the apices of the portion AC lie in the curve given by the above equation. The height is so regulated by these equations that no counter-braces are required in the portions AB.

Post Truss.—A well-known form of double quadrilateral is that known as the "*Post*" truss, Fig. 48. In this truss the ties are made to slope at an angle of 45°, and the struts at an angle of 18° 26′ with the vertical. The dimensions, therefore, are taken so that, the height being equal to one panel and a half, the ties extend across one panel and a half and the struts across one-half a panel. The apices in one chord are midway between those of the other.

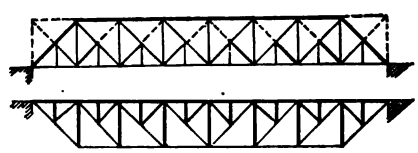
Fig. 48.



Baltimore Bridge Co.'s Truss.—A modification of the single quadrangular system is shown in Fig. 49. It is known as the *Baltimore Bridge Co.'s Truss*, also as the Petit Truss, or more commonly as the "sub-Pratt." It is used in modern practice in preference to multiple systems, which are generally avoided.

Its peculiarity consists in the way in which a large panel is divided into two smaller ones by inserting half-braces and suspending ties.

Fig. 49



KELLOGG TRUSS.—This is another modification of the simple quadrangular. The object, as in all modifications, is to diminish the length of panel in a long span with the least material. The construction is shown in Fig. 50. For this purpose additional ties are run from the top of each post to the centre of the bay or panel. The counter-braces are shown by dotted lines. This like other multiple systems is avoided by modern practice.

BOLLMAN TRUSS.—A compound system consisting of a suspension system combined

Fig. 50

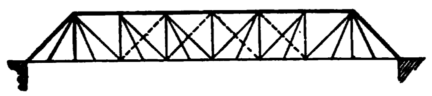
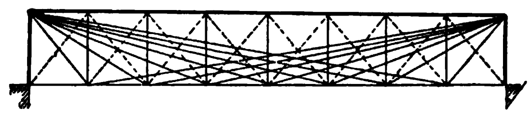


Fig. 51

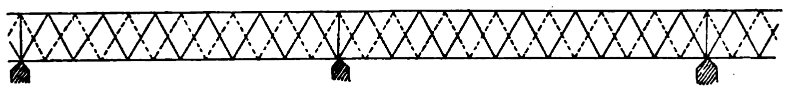


with a stiffening truss of the simple quadrangular type, is known as the Bollman truss, Fig. 51. A tie is run from each end directly to each loaded apex, thus forming a suspension system, which is stiffened by a quadrangular truss. This truss is no longer built.

The above comprise all the best known varieties of quadrangular truss, as applied to bridges.

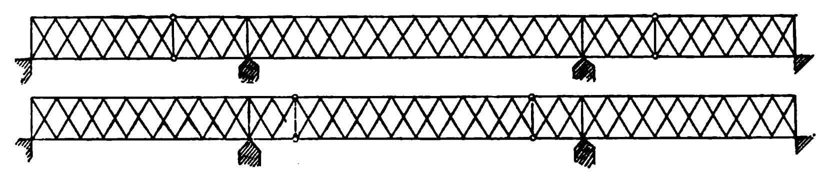
CONTINUOUS GIRDER.—When a girder with parallel chords is extended over more than two supports it is called a continuous girder, Fig. 52 (*a*).

Fig. 52 (*a*)



The bracing may be of any character, either triangular or quadrangular, single or multiple, properly arranged so that each system shall transfer pressure directly to the supports. Thus if a double system is adopted in Fig. 52 (*a*), as shown by the dotted lines, we must introduce verticals over each support. A system shown in Fig. 52 (*b*), has been patented by Gerber, in Germany, in which the girder is continuous over the supports and *hinged* beyond the supports. The system is claimed to have all the advantages for long spans of the continuous girder, so far as saving in material is concerned, without the disadvantages of the latter system. It shows on stress sheet considerable gain over the simple girder in the parallel chords, amounting to over 25 per cent., and is equally simple and certain in its calculation and construction. The distance of the hinges from the centre supports should be, for long spans of over 200 feet, about 0.2 of the centre span. This it will be seen is the forerunner of the "cantilever."

Fig. 52 (*b*)



In the case of a succession of long spans the system is worthy of more attention than it has heretofore received, as it offers some advantages over the discontinuous girder.

DECK AND THROUGH BRIDGE—LATERAL BRACING.—In all these forms, and in bridge trusses generally, the system may be so arranged as to allow the live load to traverse either the upper or the lower chord. A truss in which the live load traverses the lower or tension chord, is called a "*through*" truss. If the truss in this case is not high enough to admit of cross-bracing over head, it is called a "*pony*" truss. Such trusses are necessarily short. If over 100 feet in length they are apt to be deficient in lateral stability. If

the live load traverses the upper or compression chord, it is called a "deck bridge." A bridge consists essentially of two or more trusses placed side by side over the interval to be spanned, and connected together at either top or bottom, or both, by horizontal or lateral trussing, usually of the quadrilateral type. The object of this bracing is to support the trusses and stiffen the structure against the action of the wind. The same principles apply to it as to the main trusses, and it is calculated in similar manner. From apex of one truss to apex of the other, floor beams are laid across, upon which the flooring is put.

II. GIRDERS WITH INCLINED CHORDS.

Girders whose chords are not parallel are named according to the general shape of truss, rather than the character of bracing adopted. They are used in general where, owing to the length of the span, the height of a girder with horizontal chords would be excessive.

Fig. 53

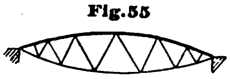
Fig. 53 represents a girder with a curved upper chord and straight lower chord. The bracing is usually of the quadrilateral type. The well known Kuilenberg bridge in Holland is of this class. For long spans there is a saving of material over the girder with parallel chords. The curve of the upper chord is usually that of a parabola.

BOWSTRING GIRDER.—The bowstring girder, Fig. 54, consists of a curved upper chord, usually parabolic or circular, and straight horizontal lower chord. The bracing may be of any character, generally quadrilateral. It is a common form, and well adapted to bridges of long span. It may sometimes be inverted, so that the bottom chord is arched, in which case we may call it the inverted bowstring.

Fig. 54

DOUBLE BOW OR LENTICULAR.—The double bowstring, or bowstring suspension, or lenticular truss, Fig. 55, consists of two arched chords, so arranged that the thrust of the one outwards is balanced by the pull of the other inwards. The bracing may be of any sort. The roadway may pass through the centre or be above or below the truss. Of this class are the famous Saltash bridge, and the bridge over the Rhine at Mayence. In Germany, this shape is known as the *Pauli* truss.

Fig. 55



The form of the *Pauli* truss is so arranged that the maximum stresses in the chords shall be constant. For this purpose the depth at any point distant x from the end is

$$h = 4h_0 \frac{x}{l}\left(1 - \frac{x}{l}\right)\left[1 + 2\frac{h_0^2}{l^2}\left(1 - 2\frac{x}{l}\right)^2\right],$$

where l is the span and h_0 the depth at centre.

This truss has all the advantages of the double bowstring, and is said to be from 4 to 17 per cent. more economical of material. It is often found in Germany, the most noteworthy example being the Mayence bridge, which consists of four spans of about 345 feet each, and 24 smaller ones of from 52 to 87 and 115 feet.

Fig. 56

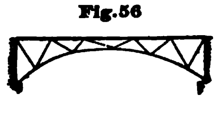


Fig. 57

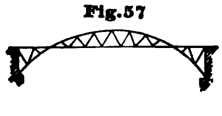


Fig. 58

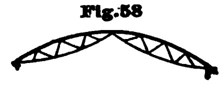
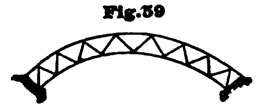


Fig. 59



BRACED ARCH.—The braced arch, as its name implies, consists, Fig. 56, of an arched chord stiffened so as to resist the action of the live load by bracing in various ways. The

bracing may be of either the triangular or quadrilateral types. The system admits of many modifications, as shown in Figs. 57, 58 and 59.

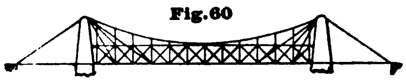
In Fig. 59 we have two parallel arches, braced together. This is the system of the braced arch over the Mississippi at St. Louis.

Many other modifications may be devised. The braced arch may be divided into three kinds in which the distribution of stresses are entirely different.

Thus we may have, 1st, the arch hinged or free to turn at the crown and at both ends; 2d, the arch hinged at ends only; 3d, the arch without hinges. The St. Louis arch is of the latter kind.

SUSPENSION SYSTEM.—A common form of suspension bridge is that shown in Fig. 60. A cable is stretched from towers at either end, over which it passes to anchorages where it is made fast. The office of this cable is to sustain the total load. The system is stiffened by a horizontal truss of ordinary form, and by stays extending out from each tower.

Fig. 60



This is the system of the suspension bridge at Niagara, of the East River bridge at New York, and of others erected by Roebling. This system also admits of many modifications. Thus Figs. 56, 57, 58, 59, all become suspension systems when inverted.

DOUBLE SYSTEMS—LONG SPANS.—In general, all double systems of bracing are now avoided by good practice, owing to the indeterminate character of the stresses, and the difficulty of ensuring that each system shall carry its own share, and no more or less. The Lattice Truss, Fig. 44; Fink Truss, Fig. 45; Post Truss, Fig. 48; Kellogg Truss, Fig. 50; Bollman Truss, Fig. 51, are also antiquated. No more will probably be built in America.

Of the forms remaining, only one system of bracing should be used. The tendency of modern practice is towards long panels, much longer than formerly. The Pratt Truss, Fig. 46, has thus become the standard form for horizontal chords. The Warren is less often used.

When, owing to length of span, the panels would become excessively long, the "sub-Pratt" or Baltimore Truss, Fig. 49, or some modification of it, either with or without inclined chords, is used.

Thus Fig. 60 (a) is a sketch of one of the spans of the Cincinnati and Covington Bridge, span 484.5 feet; centre depth, 75 feet; depth at ends, 52 feet. It will be observed that the bracing is that of the "sub-Pratt"; the chords are inclined, and the long compression panels in the top chord are supported at the centre. The length of panel is 27 feet $9\frac{1}{2}$ inches at ends, and 26 feet 8 inches for the others. This is a good illustration of recent practice, long length of panel, large centre depth, inclined chords, and single-system bracing.

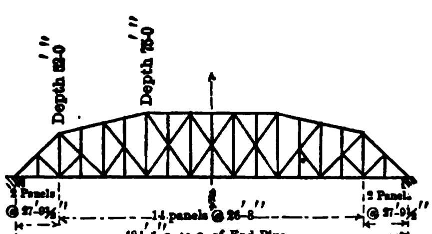


Fig. 60 (a).

CANTILEVER SYSTEM.*—The cantilever system counts the longest spans of the day. The Forth Bridge, in Scotland, the longest existing clear span, is of this type. Its longest span is 1,710 feet, the central girder being 350 feet long, and the cantilever arms extending out 680 feet on each side.

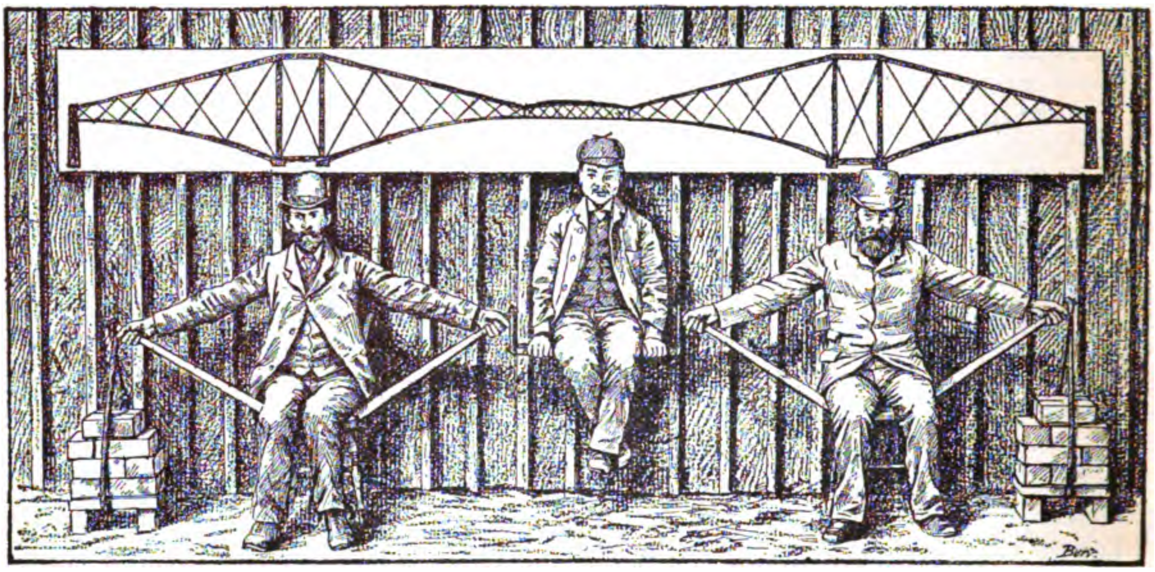
The principle of this system is better illustrated by the subjoined cut than by any lengthy description.

This cut was given in the *Engineering News*, June 11, 1887, from the original photograph furnished by Tho. C. Clarke, C. E., and was used by Mr. Benjamin Baker in a lecture on the Forth Bridge, before the Royal Institution.

* The theory of the cantilever will be found, page 262.

The sketch represents the Forth Bridge.

The four sticks which form the "compression members" simply abut against the chairs and are grasped by the "tension members" at each end. The "central span" is hung from the inner ends. The outer ends are anchored down.



OBJECT OF SECTION II.—It is not the place here to discuss the relative merits of these different forms, nor the conditions which lead to the adoption of one or the other in any special case. These will be alluded to as we discuss in turn each typical form. The object of the present section of this work, therefore, is to so apply the principles of Section I., to selected cases, as to enable the student to readily determine the stresses in any of the preceding structures, or any modification of them. In doing this, we shall have occasion to make such comparisons as shall enable him to appreciate the special advantages of each form.

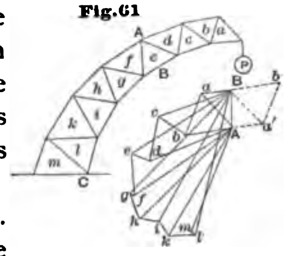
CHAPTER I.

STRUCTURES WHICH SUSTAIN A DEAD LOAD ONLY—ROOF TRUSSES.

THE method which we adopt for all structures of this class is the Graphic method of Section I., Chapter I., checking in every case our results by the calculation of the stresses in one or two members, by the method of moments of Chapter III., Section I. The method is so simple and general in its application, that but little remains to be added to the remarks of Chapter I, Section I.

BENT CRANE.—In Fig. 61 we have given the frame diagram and stress diagram for a bent crane, bearing the load P at the peak. The notation is the same as on page 11. The student should follow out the stress diagram carefully with reference to determining the proper *character* of the stresses in the various members. Thus he will observe that the stresses in the braces alternate in character up to gh; at this point the stresses in gh and hi are of the same character.

Fig. 61

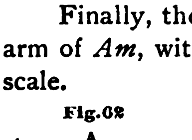


All the lower chords radiate from B and are in compression. All the upper chords radiate from A and are in tension. Observe also that the stress diagram could have been laid off equally well upon the right of the weight line, BA, in which case the letters B and A should be interchanged, and all the upper chords would radiate from the top of the weight line, and the lower chords from the bottom, as shown by the dotted lines. In general the stress diagram may thus be laid off upon either side of the weight line.

Observe, also, that to obtain accurate results in such a case, the frame should be drawn carefully to as large a scale as possible, as the braces Aa and the flanges Ba and Ab, etc., are very short. It may even be desirable to calculate the slope of these members from the given dimensions of the frame, and plot their directions by ordinates, so as to obtain longer lines of direction. The scale for the stress diagrams should, on the other hand, be taken as small as is consistent with reading off the stresses to the desired degree of accuracy.

Finally, the stress in Am may be calculated by moments. For this purpose the lever arm of Am, with reference to C, may be calculated or measured directly from the frame to scale.

Fig. 62

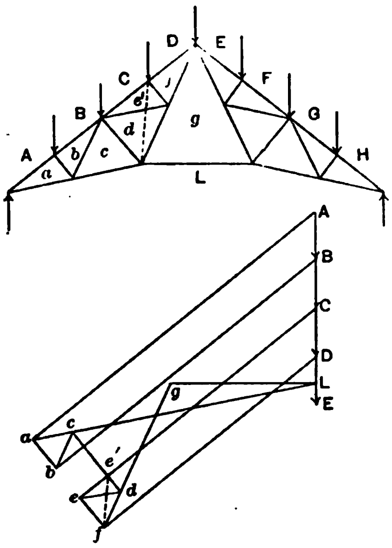


In all cases the student should determine the character of the stresses in each member as he makes the stress diagram, and not wait until it is completed. When it is all completed the stresses may be taken off to scale.

CANTILEVER CRANE.—In Fig. 62 we have represented a cantilever crane. The same remarks apply as in the preceding case. It is given as an example for the student to solve, in accordance with the preceding remarks.

FRENCH ROOF TRUSS.—In Fig. 63 we have represented what is sometimes known as the "French Roof Truss." The bracing is formed by supporting each principal rafter at its middle point by a strut, *cd*, perpendicular to the rafter, and two tie rods, one from each end. If the half rafter is still too long, we insert the perpendicular struts *ab* and *ef* at the quarter points, as shown in the Figure, and join the ends to the centre by ties *bc* and *de*. This system of trussing is repeated as often as may be requisite. The form is a very common one, and we have chosen it as an illustration because there is in it at the apex *BC* an apparent indeterminance and violation of our rule, page 5, which is apt to cause trouble to the beginner.

Fig. 63.



Thus the student will find no difficulty in constructing the stress diagram and finding the stress as given in the Figure as far as apex *BC*. When we arrive at this apex, however, we have the stresses in *Bb* and *bc* already determined, and the known apex weight at *BC* in equilibrium with *three* unknown stresses, viz. *cd*, *de*, and *Ce*. The problem would, therefore, seem to be indeterminate.

If, however, we apply the criterion for superfluous members, page 5, we find that there are no superfluous members and there should be, therefore, no real indeterminance. Thus the number of members m is 27 (we disregard the dotted member). The number of apices n is 15. We have then, applying our criterion, $m = 2n - 3$. There are then no superfluous members.

If now we *remove* the two members *de* and *ef* and replace them by the dotted member $e'f$, where e' takes the place in the new notation of the two letters e and d, we still have a rigid frame with no superfluous members. For the number of members is now $m = 25$ and the number of apices $n = 14$. We have then $m = 2n - 3$.

But this change has evidently not affected the stress in the member *Lg*. We can now carry on the diagram until we find the stress in *Lg*. When the stress in *Lg* is thus found, we can replace the members *de* and *ef*. We can now consider the second lower apex and find the stresses in *cd* and *dg*. We then proceed to the apex *BC* and shall have no difficulty.

We might also find the stress in *Lg* directly by the method of sections, page 26.

Every case of apparent indeterminance can be solved in this manner.

CURVED MEMBERS.—Whenever curved members occur, as they sometimes do in roof trusses, the direction of the stress should be taken in the line connecting the two ends. Such members are subject to a bending stress as well as to direct compression or tension, and must be proportioned accordingly. The method of proportioning the cross-section of members to resist the stresses to which they are subjected will be taken up in Part II. of this work. In all such cases as Figs. 61 and 62, the members in the curved chords are always straight, and are chords of the curve in which the apices are situated.

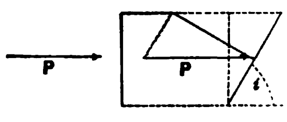
WIND FORCES.—Roof trusses have not only to sustain the weight of roofing, snow, etc., but also the pressure caused by wind. This is often, especially in the case of large trusses, placed at considerable intervals apart, very great. The action of the wind, moreover, may often be to cause in certain members stresses opposite in character to those caused in the same members by the dead load. The calculation of the stresses caused by

wind forces is thus often of considerable importance, and ought never to be left out of account in designing iron roofs of large span.

When a horizontal current of wind strikes against an inclined surface, it is deviated from its original direction and causes a normal pressure upon that surface. This normal pressure, owing to the fluidity of the air, is found to be greater than the normal component of the pressure upon a surface at right angles to the wind.

Thus, Fig. 65, if P is the pressure of the wind in lbs. per sq. ft., upon a surface perpendicular to its direction, the normal component of this pressure upon a surface inclined at the angle i to the horizon, is not $P \sin^2 i$ as it would be by the resolution of forces, but is found by experiment to be given by the experimental formula

Fig. 65



$$P_n = P \sin i^{1.84 \cos i - 1}. *$$

If we take the maximum pressure of the wind against a surface perpendicular to its direction, as 50 lbs. per square foot, we shall probably allow sufficient margin for the heaviest gales in our latitudes. The highest pressures, according to Unwin, do not exceed 55 lbs. per square foot, and the accuracy of these observations is stated by him as "doubtful."

Taking, then, the greatest pressure of wind to be anticipated at 50 lbs. per square foot we have, from our formula, the normal pressure per square foot upon surfaces inclined at various angles to the horizon, as follows:

ANGLE OF ROOF WITH HORIZON.	NORMAL PRESSURE PER LBS.	ANGLE OF ROOF WITH HORIZON.	NORMAL PRESSURE PER LBS.
5°	6.6	45°	44.0
10°	12.1	50°	47.6
15°	17.5	55°	49.5
20°	22.9	60°	50.6
25°	28.1	65°	51.1
30°	33.1	70°	51.2
35°	37.7	80°	50.5
40°	41.5	90°	50.0

From this Table we can find by interpolation the normal pressure upon a roof having any angle of inclination to the horizon, due to a gale of wind which would cause a pressure of 50 lbs. upon a square foot of surface perpendicular to its direction.

Again, in order to find the pressure from the velocity, let v be the velocity of the current in feet per second, and p be the pressure of the current in lbs. per square foot upon a surface perpendicular to its direction. Then, if w is the weight of the fluid in lbs. per cubic foot, we have, according to hydraulic principles,

$$p = 2hw,$$

where h is the "head" due to the velocity, or

$$p = \frac{v^2 w}{g}.$$

Since for ordinary atmospheric air, $w = 0.08$ lbs. approximately,

$$p = \frac{0.08}{32} v^2 = \left(\frac{v}{20}\right)^2.$$

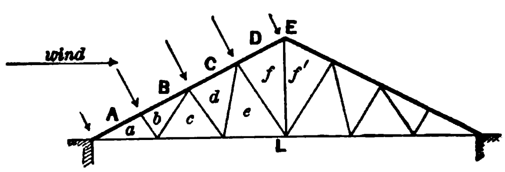
From the formula $p = \left(\frac{v}{20}\right)^2$ we have the following Table:

* This formula is given by Unwin, "*Iron Bridges and Roofs*," London, 1869, and is by him attributed to Hutton.

VELOCITY IN FEET PER SECOND.	VELOCITY IN MILES PER HOUR.	PRESSURE IN LBS. PER SQUARE FOOT.
10	6.8	0.25
20	13.6	1.00
40	27.2	4.00
60	40.8	9.00
70	47.6	12.25
80	54.4	16.00
90	61.2	20.25
100	68.0	25.00
110	74.8	30.25
120	81.6	36.00
130	88.4	42.25
150	102.0	56.25

APPLICATION TO A ROOF TRUSS.—Let Fig. 66 represent a roof truss. Let the span be 50 feet and height 12.5 feet. The length of each rafter is then 27.95 feet. Suppose that the truss supports 8 feet of roofing, that is, that the main trusses, of which Fig. 66 is one, are placed 8 feet apart. Then the area of roof supported by one rafter is $27.95 \times 8 = 223.6$ square feet.

Fig. 66



Now the inclination of the rafter to the horizon is $i = 26° 34'$. From our Table, then, we have the normal wind pressure $= 29.6$ lbs. per square foot. The total normal wind pressure upon one side of the roof due to the wind is, then, $223.6 \times 29.6 = 6{,}619$ lbs. This pressure we divide into four equal parts of 1,655 lbs. each, or say, in round numbers, 1,600 lbs. Let us suppose the wind blowing upon the left side. Then we have a normal pressure of 1,600 lbs. at each of the apices AB, BC and CD, and a normal pressure of 800 lbs. at the left end and top apex. Since all the pressure from the centre of one bay to the centre of the next is supposed to be concentrated at the intermediate apex, we have at the top and bottom apex only half as much pressure as at the other apices.

It remains to determine the reactions. As soon as these are known, we shall know all the outer forces which act upon the truss, and can then proceed to diagram the stresses.

DETERMINATION OF REACTIONS.—The action of the wind, blowing horizontally upon the left, is to slide the entire truss off of its supports. The truss, if it is necessary, should, therefore, be fastened at the ends to the wall. In general, the weight of the truss and its roofing is sufficient to cause friction enough to keep it in place. If not, it can easily be fastened. Large iron trusses may sometimes be put upon friction rollers at one end, so as to allow for changes of temperature, in which case the other end must be fixed.

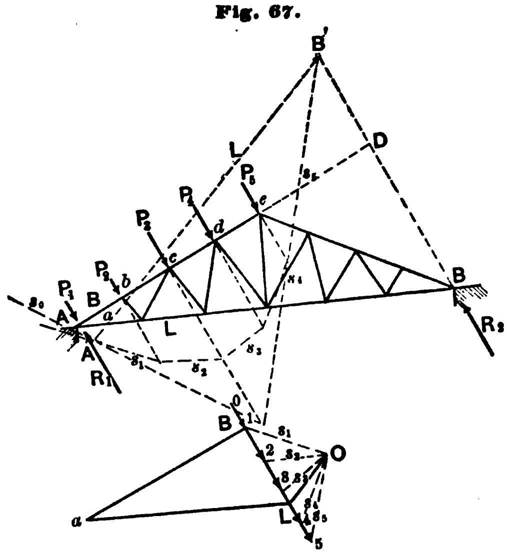
We have then two cases; 1st, when both ends are fixed; 2d, when one end only is fixed and the other is upon rollers.

I. REACTIONS WHEN BOTH ENDS ARE FIXED.

In this case, the two reactions are parallel to the normal wind forces, and can easily be calculated. Thus if we take moments about the left end A, Fig. 67, we have the moment of the reaction R_2 balanced by the sum of the moments of the forces, or

$$R_2 \times AD = P_2 \times Ab + P_3 \times Ac + P_4 \times Ad + P_5 \times Ae.$$

Fig. 67.



Or we may take the resultant of all the forces acting at the apex c, and therefore*

$$R_2 \times AD = (P_1 + P_2 + P_3 + P_4 + P_5) \times Ac.$$

The reaction R_2, at the right end, is thus easily found. Of course, the reaction at the left end is found by subtracting R_2 from the sum of the forces. The reactions being thus known, we now know all the outer forces acting upon the truss. We can, therefore, form the force polygon and then proceed to construct the stress diagram. Thus the force polygon is the line o 1 2 3 4 5, Fig. 67. The reaction R_2 is the distance $5L$, and the reaction R_1 is the distance oL.

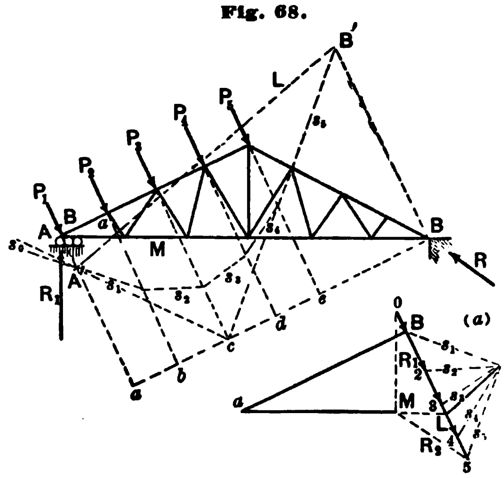
Instead of calculating R_2, we may take a pole o, draw the rays, $s_1, s_2 \ldots s_5$, construct the corresponding equilibrium polygon, draw the closing line $A'B'$, and thus determine the reactions $5L$ and Lo (see page 39). Note that it is only necessary in this case to draw the two sides s_0 and s_5, meeting in P_3.

II. REACTIONS WHEN ONE END IS FIXED AND THE OTHER ON ROLLERS.

Suppose the wind end is on rollers, Fig. 68. Then the reaction at the roller end must be vertical. Since, then, we know its direction, we can easily calculate it by taking moments about the right hand end. Thus

$$R_1 \times AB = P_1 \times Ba + P_2 \times Bb + P_3 \times Bc + P_4 \times Bd + P_5 \times Be.$$

Fig. 68.



The lever arms Ba, Bb, etc., can be easily found from the given dimensions of the Figure. Or we may consider the resultant of all the forces acting at the apex where P_3 acts. Hence

$$R_1 \times AB = (P_1 + P_2 + P_3 + P_4 + P_5) \times Bc.$$

Having thus found R_1, R_2 may be easily found. Thus, if we lay off the forces P_1, P_2, etc., to scale, as shown in Fig. 68(a), and then lay off oM vertically to scale equal to R_1 already found, the line $M5$ necessary to close the polygon is the resultant R_2 in magnitude and direction. The force polygon is then closed and we can proceed to form the stress diagram.

Instead of calculating R_2, we may take a pole o, construct the corresponding equilibrium polygon, draw the closing line $A'B'$, and thus determine the reactions Mo and $5M$ (see page 39). Note that in this case it is only necessary to draw the two sides s_0 and s_5, meeting in P_3 at c, in order to determine the closing line $A'B'$.

But the wind may blow upon the fixed end side, in which case the stresses in the members may be very different. Instead of supposing the wind to blow on the right side, let us suppose the left end fixed and the right on rollers, Fig. 69. Then the reaction R_1 is inclined and R_2 must be vertical. We can, therefore, easily calculate R_2 by taking moments about the left end A. Thus

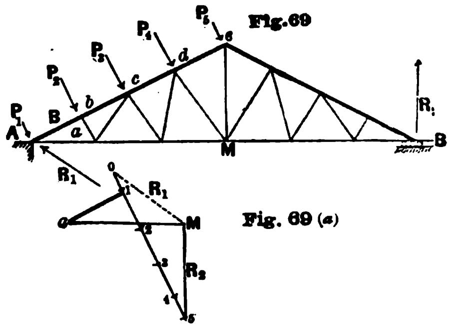
* The lever arms AD, Ab, Ac, etc., are understood, of course, to be the *perpendiculars* from A to R_2, P_2, P_3, etc.

$$R_2 \times BA = P_2 \times Ab + P_3 \times Ac + P_4 \times Ad + P_5 \times Ae,$$

or,

$$R_2 \times BA = (P_1 + P_2 + P_3 + P_4 + P_5) \times Ac.$$

If, then, we lay off the wind forces to scale in Fig. 69(a), and lay off $5M$ vertical and equal to R_2 already calculated, the line Mo necessary to close the polygon is the reaction R_1 in magnitude and direction. We can now proceed to form the stress diagram.



We can construct the reactions in this case also by means of an equilibrium polygon.

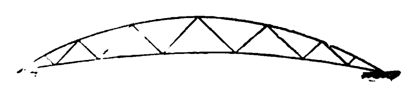
The student can now easily diagram the stresses for the three cases of Figs. 67, 68, and 69. In each case we have given the stresses in the first two members, in order to call attention to the fact that the first half weight P_1 has no influence upon the stresses. Thus in Fig. 69 we have acting at the apex A the reaction R_1 and the force P_1. We have, then, in the force diagram (a), to join 1 and M by lines parallel to Ba and Ma.

We see at once from Figs. 69 and 68 that the stress in Ma, for instance, is much greater when the wind blows on the fixed end side than when it blows on the roller end. This is evident, because when it blows on the roller end it tends to shut up the truss and thus relieve the tension in Ma. If the forces were great enough, we might even have compression in Ma, that is, the intersection a, Fig. 68(a), might fall to the right of L.

COMPLETE CALCULATION OF A ROOF TRUSS.—We see then that the complete calculation of a roof truss consists of two parts. 1st. We must find the stresses due to the greatest dead load or weight of truss, together with roofing, snow load, etc. 2d. We must find the stresses in each member due to wind force, as already detailed. Here again, in case of rollers, we may have in any member two stresses, according as the wind comes on from right or left. If both these stresses are of the same character as the dead load stress, we should add the *greatest* of them to the dead load stress to obtain the greatest stress in the member. If one of these is of the same character as the dead load stress, we add it. As to the other, if it is less in amount than the dead load stress, it will only tend, when the wind blows, to relieve the stress due to dead load by that amount; but if it is *greater* than the dead load stress it will cause a stress of opposite character, and the piece should be counterbraced for the difference. If both the wind stresses are of different character from that caused by the dead load, we need only consider the greater of the two. If this is less than the dead load stress, it will produce no effect, except sometimes to diminish the stress in the member. But if it is greater, it will cause stress of an opposite character, and the member should be counterbraced for the difference. In all cases the wind has great influence upon the stresses, and it should always be taken into account in the designing of large spans.

CURVED ROOFS.—For a curved roof, such as Fig. 70, the inclination of the surface exposed to the wind is different at every apex, and is always to be taken as tangent to the curve at each apex. In such a case as Fig. 70, it may often happen that the wind causes stresses in certain members opposite in character to those caused by the dead load. We give in Plates 1 to 7 a large number of roofs of various kinds, with their stress diagrams.* For the sake of generality, acting forces and reactions are often taken as inclined. The student cannot do better than to

Fig. 70



* These Plates are copied from "*Economics of Construction*," Bow, London, 1873.

follow out the stress diagrams for a number of cases, until he feels himself thoroughly master of the method, and can ***determine the character of the stresses*** in each case.

EXAMPLES.*

1. If the pole of an equilibrium polygon describe a straight line, show that the corresponding sides of the successive equilibrium polygons will intersect in a straight line which is parallel to the locus of the pole.

2. A system of heavy bars, freely articulated, is suspended from two fixed points; determine the magnitudes and directions of the stresses at the joints. If the bars are all of equal weight and length, show that the tangents of the angles which successive bars make with the horizontal are in arithmetic progression.

3. If an even number of bars of equal length and weight rest in equilibrium in the form of an arch, and $a_1, a_2, \ldots a_n$, be the respective angles of inclination to the horizon of the 1st, 2nd, ... nth bars counting from the top, show that $\tan a_{n+1} = \frac{2n+1}{2n-1} \tan a_n$.

4. Four bars of equal weight and length, freely articulated at the extremities, form a square $ABCD$. The system rests in a vertical plane, the joint A being fixed, and the form of the square is preserved by means of a horizontal string connecting the points B and D. If W be the weight of each bar, show, 1st, that the stress at C is horizontal and $= \frac{W}{2}$; 2d. That the stress on BC at B is $W\frac{\sqrt{5}}{2}$ and makes with the vertical an angle $\tan^{-1}\frac{1}{2}$; 3d. That the stress on AB at B is $W\frac{\sqrt{13}}{2}$ and makes with the vertical an angle $\tan^{-1}\frac{3}{2}$. 4th. That the stress upon AB at A is $\frac{5}{2}W$; 5th. That the tension of the string is $2W$.

5. Five bars of equal length and weight, freely articulated at the extremities, form a regular pentagon $ABCDE$. The system rests in a vertical plane, the bar CD being fixed in a horizontal position, and the form of the pentagon being preserved by means of a string connecting the joints B and E. If the weight of each bar be W, show that the tension of the string is $\frac{W}{2}(\tan 54^\circ + 3 \tan 18^\circ)$, and find magnitudes and directions of the stresses at the joints.

6. Six bars of equal length and weight ($= W$), freely articulated at the extremities, form a regular hexagon.

First, if the system hang in a vertical plane, the bar AB being fixed in a horizontal position, and the form of the hexagon being preserved by means of a string connecting the middle points of AB and DE, show that, 1st, the tension of the string is $3W$; 2d, the stress at C is $\frac{W}{2\sqrt{3}}$ and horizontal; 3d, the stress at D is $W\sqrt{\frac{13}{12}}$ and makes with the horizontal an angle $\tan^{-1} 2\sqrt{3}$.

Second, if the system rest in a vertical plane, the bar DE being fixed in a horizontal position, and the form of the hexagon be preserved by means of a string connecting the joints C and F, show that, 1st, the tension of the string is $W\sqrt{3}$; 2d, the stress at C is $W\sqrt{\frac{31}{3}}$ and makes with CB an angle $\sin^{-1}\sqrt{\frac{3}{124}}$; 3d, the stress at B is $W\sqrt{\frac{7}{12}}$ and makes with CD an angle $\sin^{-1}\sqrt{\frac{3}{28}}$.

Third, if the system hang in a vertical plane, the joint A being fixed, and the form of the hexagon be preserved by strings connecting A with the joints E, D and C, show that, 1st, the tension of each of the strings AE and AC is $W\sqrt{3}$; 2d., the tension of the string AD is $2W$, and determine the magnitudes and directions of the stresses at the joints, assuming that the strings are connected with pins distinct from the bars.

7. Show that the stresses at C and F, in the first case of Ex. 6, remain horizontal when the bars AF, FE, BC, CD, are replaced by any others, which are all equally inclined to the horizon.

8. An ordinary jib-crane is required to lift a weight of 10 tons at a horizontal distance of 6 ft. from the axis of the post. The post is a hollow cast-iron cylinder of 10 ins. external diam.; find its thickness, assuming the safe tensile and compressive stress to be 3 tons per sq. in.

*The following Examples are taken from "*Applied Mechanics*," by Prof. Henry T. Bovey, Montreal, 1882. It is believed that the intelligent student can solve them by an independent application of preceding principles.

The hanging part of the chain is in *four* falls; the jib is 15 ft. long, and the top of the post is 12 ft. above ground; find the stresses in the jib and tie when the chain passes, (1)—along the jib; (2)—along the tie.

The post turns round a vertical axis; find the direction and magnitude of the pressure at the tie, which is three feet below the ground.

9. If the post in the preceding question were replaced by a solid cylindrical wrought-iron post, what should its diam. be; the safe inch-stress being 3 tons as before?

10. The horizontal traces of the two back-stays of a derrick-crane are x and y feet in length, and the angle between them is a; show that the stress in the post is a maximum when $\frac{\cos(a-\theta)}{\cos\theta} = \frac{x}{y}$, θ being the angle between the trace x and the plane of the jib and tie.

11. The inner flange of a bent crane (Fig. 61, page 62,) forms a quadrant of a circle of 20 ft. radius, and is divided into *four* equal bays. The *outer* flange forms the segment of a circle of 23 ft. radius. The two flanges are 5 ft. apart at the foot, and are struck from centres in the same horizontal line. The bracing consists of a series of isosceles triangles, of which the bases are the equal bays of the inner flange. The crane is required to lift a weight of 10 tons. Determine the stresses in all the members.

12. A braced semi-arch is 10 ft. deep at the wall, and projects 40 ft. The upper flange is horizontal, is divided into *four* equal bays, and carries a uniformly distributed load of 40 tons. The lower flange forms the segment of a circle of 104 ft. radius. The bracing consists of a series of isosceles triangles, of which the bases are the equal bays of the upper flange. Determine the stresses on all the members.

13. Three bars, freely articulated, form the equilateral triangle ABC. The system rests in a vertical plane upon the supports B and C in the same horizontal line, and a weight, W, is suspended from A. Determine the stress in BC, neglecting the weight of the bars.

14. A triangular truss of white pine consists of two equal rafters, AB, AC, and a tie beam BC; the span of the truss is 30 ft., and its rise is 7½ ft.; the uniformly distributed load upon each rafter is 8,400 lbs., and a weight of 10,000 lbs. is suspended from the centre of the tie-beam. Determine the dimensions of the rafters and tie-beam, assuming the safe tensile and compressive inch stresses to be 3,300 and 2,700 lbs., respectively.

15. A triangular truss consists of two equal rafters, AB, AC, and a tie beam BC, all of white pine; the centre D of the tie-beam is supported from A by a wrought-iron rod AD; the uniformly distributed load upon each rafter is 8,400 lbs., and upon the tie-beam is 36,000 lbs. Determine the dimensions of the different members, BC being 40 ft. and AD 20 ft.

What will be the effect upon the several members if the centre of the tie-beam be supported upon a wall, and if for the rod a post be substituted, against which the heads of the rafters can rest?

16. A triangular truss of white pine consists of a rafter AC, a vertical post AB, and a horizontal tie-beam AC; the load upon the rafter is 300 lbs. per lineal ft.; $AC = 30$ ft., $AB = 6$ ft. Find the resultant pressure at C.

How much strength will be gained if the centre of the rafter be supported by a strut from B?

17. The rafters of a roof are 20 ft. long, and inclined to the vertical at 60°; the feet of the rafters are tied by two rods, which meet under the vertex, and are tied to it by a rod 5 ft. long: the roof is loaded with a weight of 3,500 lbs. at the vertex. Determine the stresses in all the members.

18. The feet of the equal roof rafters AB, AC, are tied by rods BD, CD, which meet under the vertex and are joined to it by a rod AD. If W and W' are the distributed loads in lbs. upon the rafters, and if S is the span of the roof in feet, show that the weight of metal in the ties in lbs. is $\frac{5}{6}\frac{W+W'}{f}S\cot.\beta$, f being the safe inch stress in lbs., and β the angle ABD.

19. A roof truss consists of two equal rafters inclined at 60° to the vertical, of a horizontal tie-beam of length l, of a collar-beam of length $\frac{l}{3}$, and of a queen-post at each end of the collar-beam; the truss is loaded with a weight of 2,600 lbs. at the vertex, a weight of 4,000 lbs. at one collar-beam joint, a weight of 1,200 lbs. at the other, and a weight of 13,500 lbs. at the foot of each queen. Determine the stresses in the members.

20. A frame is composed of a horizontal top-beam 40 ft. long, two vertical struts 3 ft. long, and three tie-rods, of which the middle one is horizontal and 15 ft. long. Find the greatest stress produced in the several members when a single load of 12,000 lbs. passes over the truss.

21. An equilibrium polygon is drawn for a series of parallel loads at given distances. Show that, 1st. By properly drawing the closing line of the polygon a bending moment curve is obtained which corresponds to any position of the series of loads on any given beam; 2d. So long as the closing line lies on the

same two polygon sides, its positions for any given beam form the envelope of a parabola ; 3d. The centre of the beam corresponding to a given closing line bisects the distance between the verticals through the intersection of the polygon sides, and the point where the closing line touches the parabola.

22. Vertical loads of 4, 3, 7, and 2 tons are concentrated upon a horizontal beam of 20 ft. span, at distances of 3, 7, 12, and 15 ft., respectively, from the left support. Prove generally that the vertical ordinate intercepted between a point in the corresponding equilibrium polygon and a closing line whose horizontal projection is the span of the beam. represents on a certain determined scale the bending moment of a section at any point. Find its value by scale measurement for a section at 9 ft. from the left support, using the following scale : For *lengths*, $\frac{1}{4}$ inch = 1 foot ; for forces, $\frac{1}{4}$ inch = 1 ton ; the polar distance = 5 tons. Determine graphically, by means of the same diagram, the greatest bending moment that can be produced on the same section by the same series of loads traveling over the span at the stated distances apart.

PLATE I.

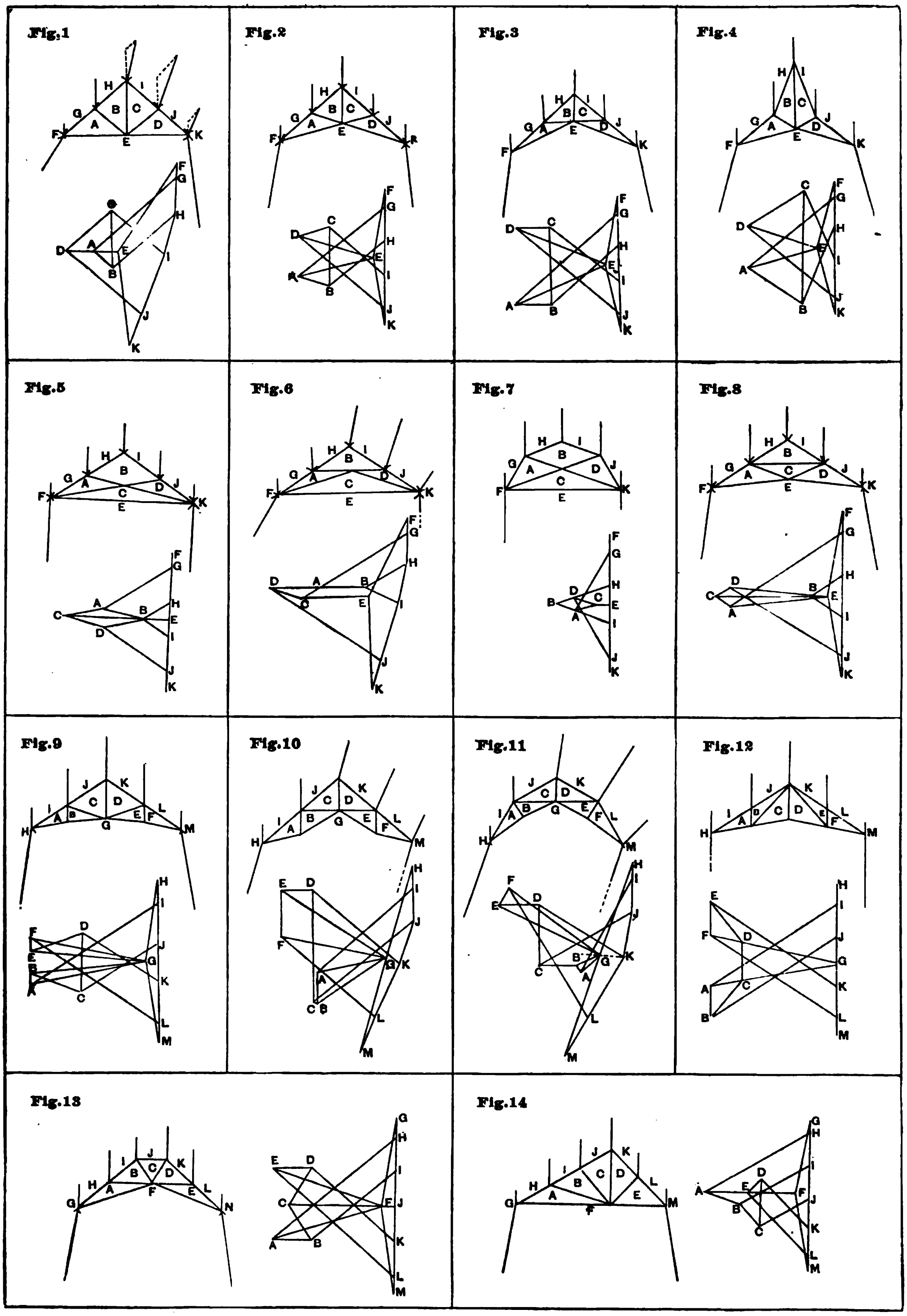


Fig.1
Fig.2
Fig.3
Fig.4
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Fig.10
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Fig.12
Fig.13
Fig.14

PLATE 2.

Fig. 15

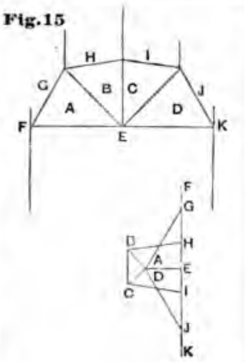


Fig. 16

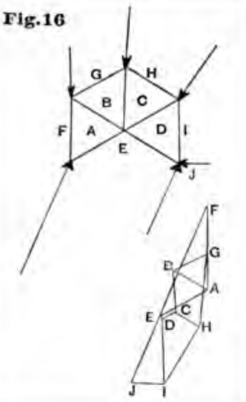


Fig. 17

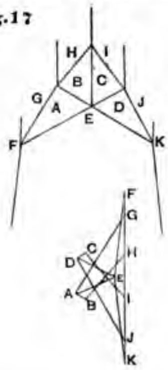


Fig. 18

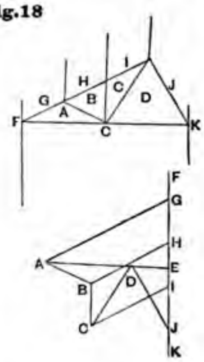


Fig. 19

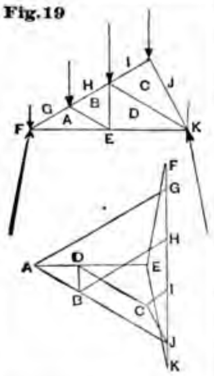


Fig. 20

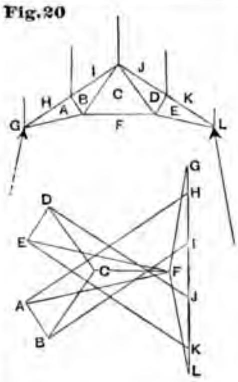


Fig. 21

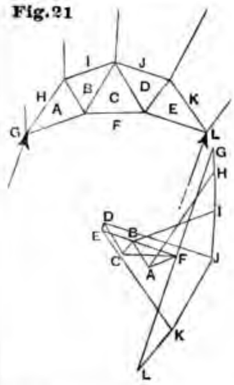


Fig. 22

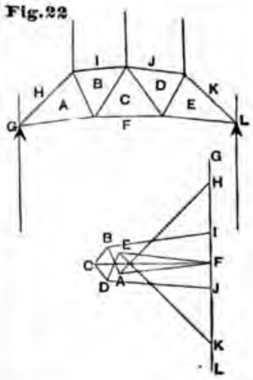


Fig. 23

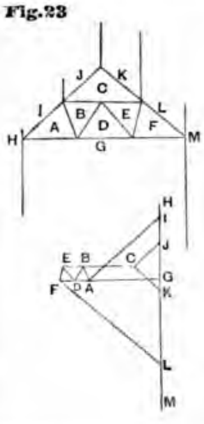


Fig. 24

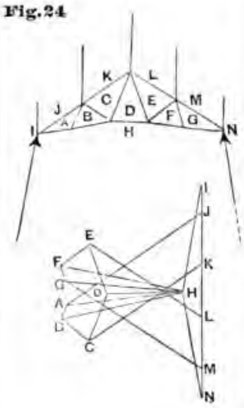


Fig. 25

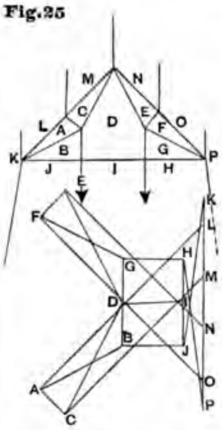


Fig. 26

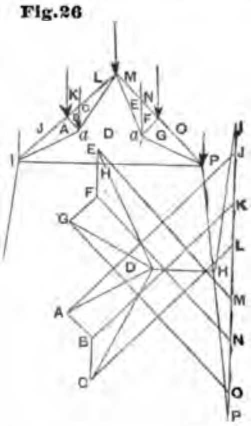


Fig. 27

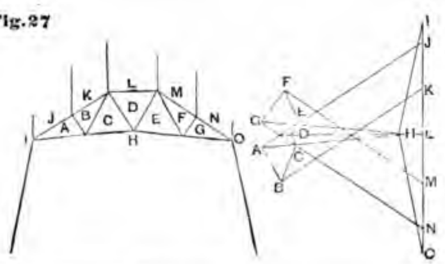


Fig. 28

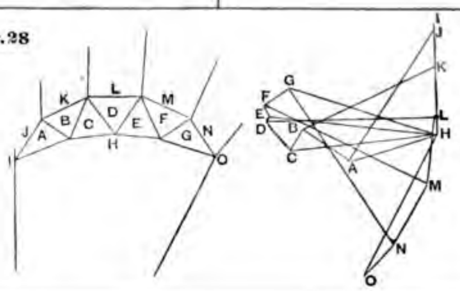


PLATE 3.

Fig.29

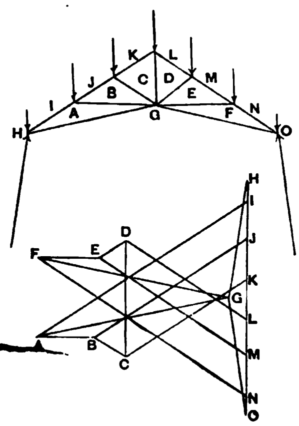


Fig.30

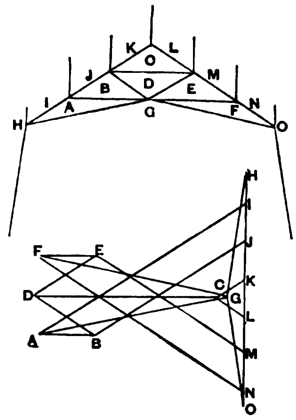


Fig.31

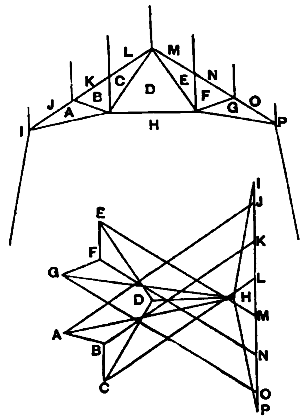


Fig.32

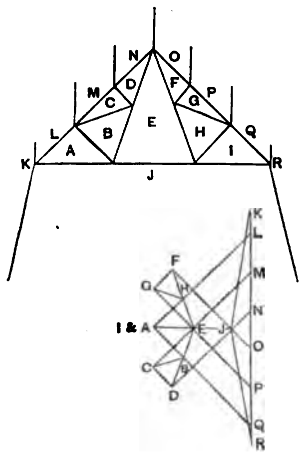


Fig.33

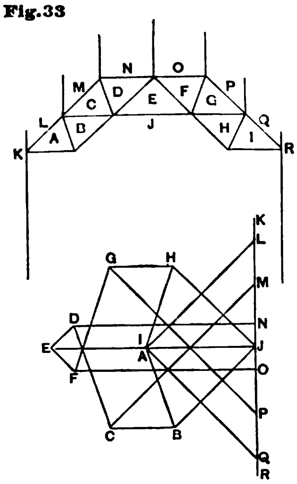


Fig.34

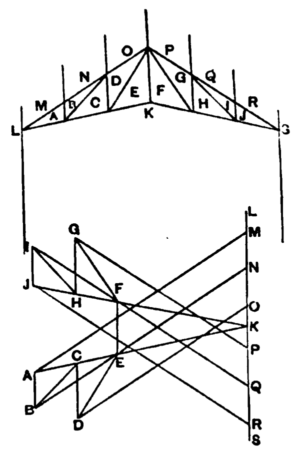


Fig.35

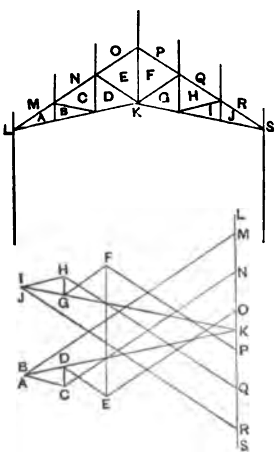


Fig.36

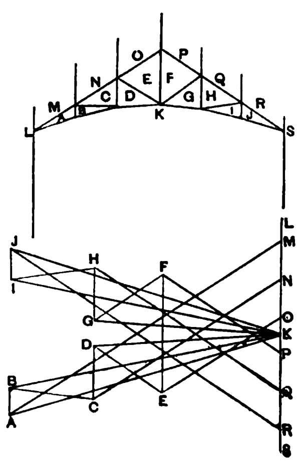


Fig.37

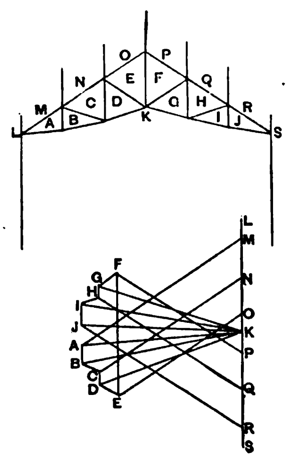


PLATE 4.

Fig.38

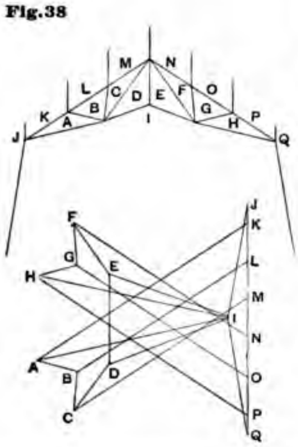


Fig.39

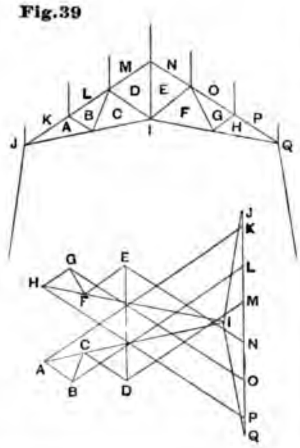


Fig.40

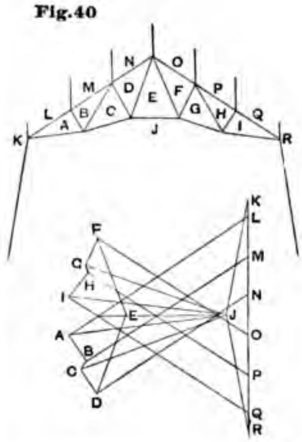


Fig.41

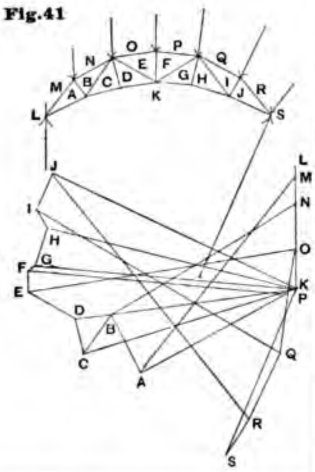


Fig.42

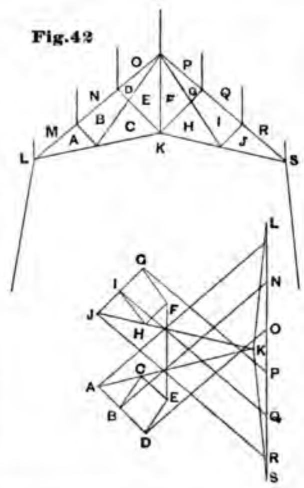


Fig.43

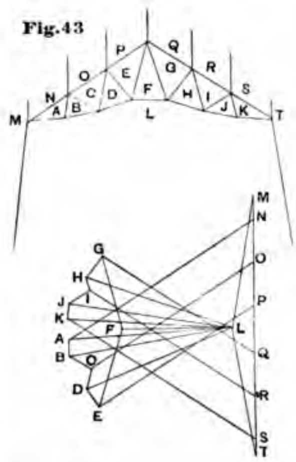


Fig.44

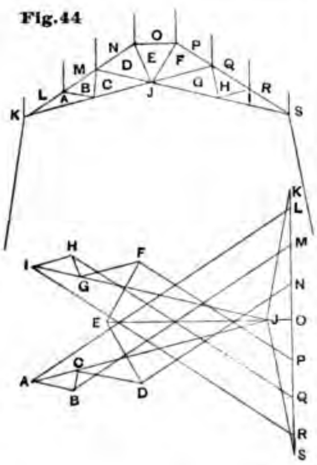


Fig.45

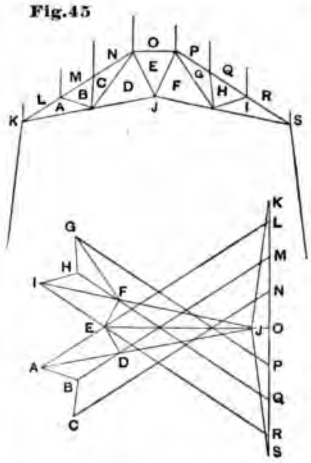


Fig.46

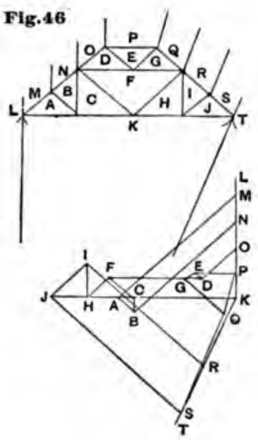


PLATE 5.

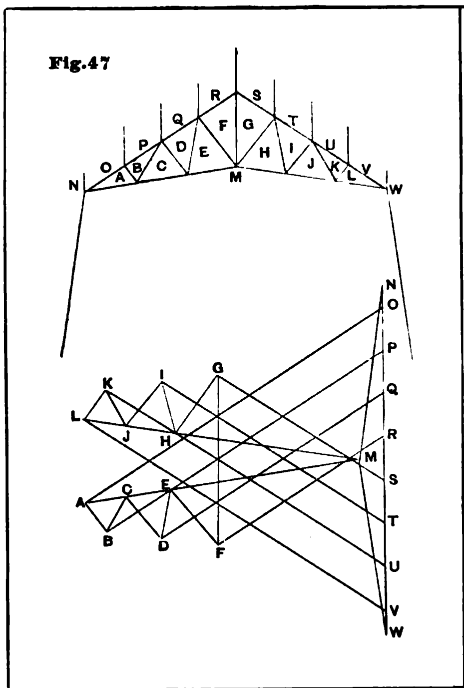


Fig.47

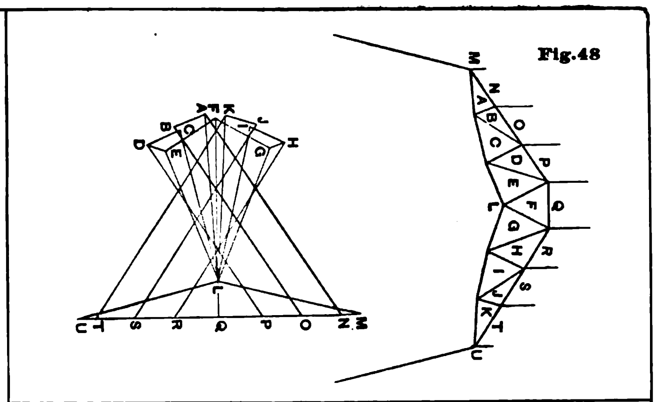


Fig.48

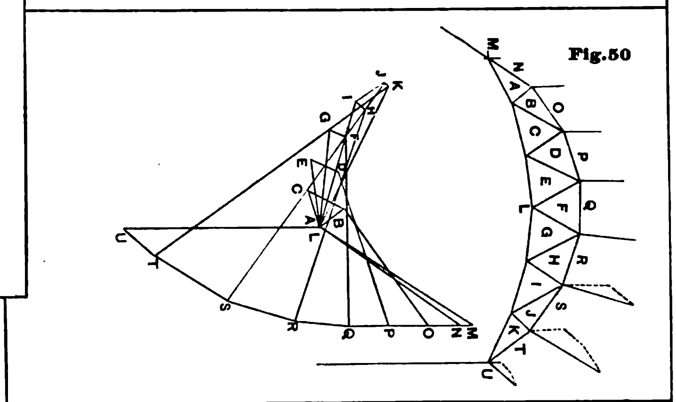


Fig.50

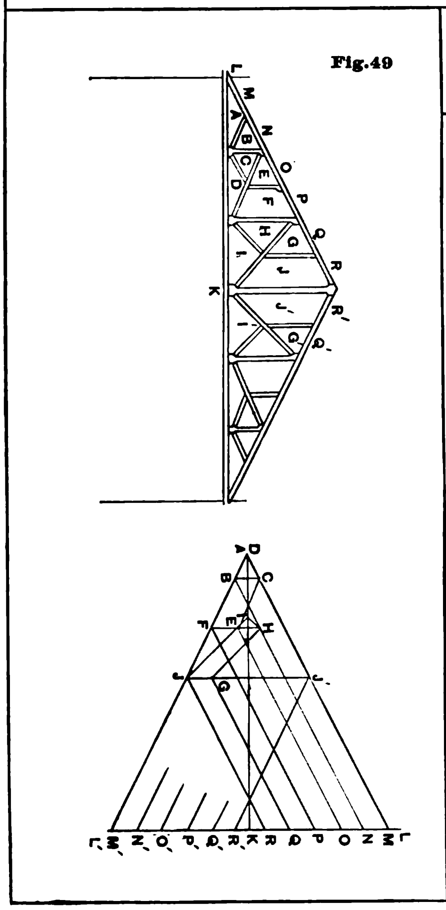


Fig.49

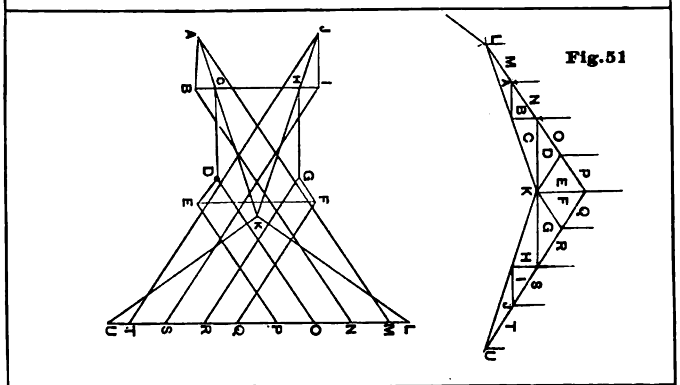


Fig.51

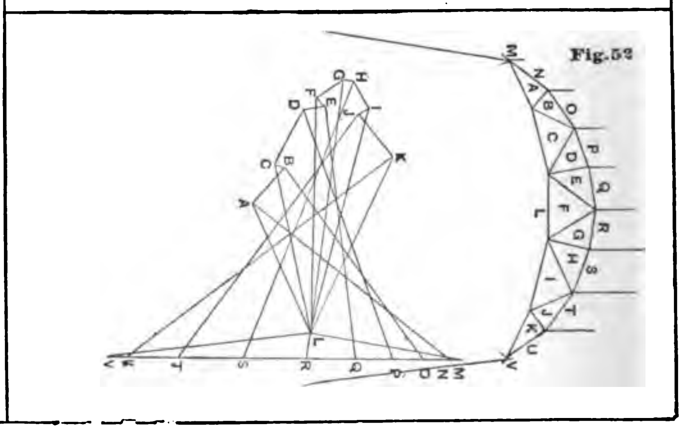


Fig.52

PLATE 6.

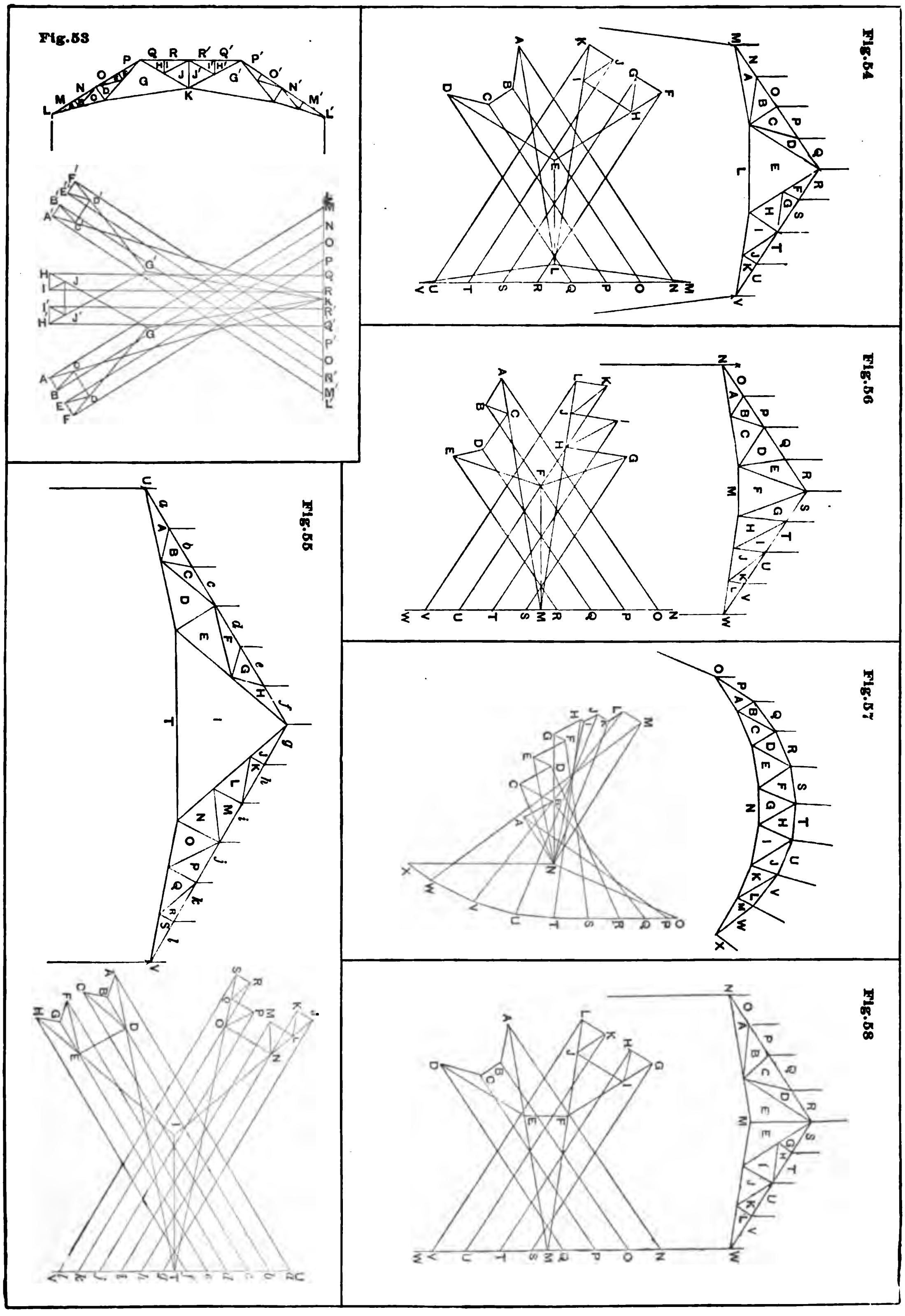


Fig. 53

Fig. 54

Fig. 55

Fig. 56

Fig. 57

Fig. 58

PLATE 7.

Fig.59

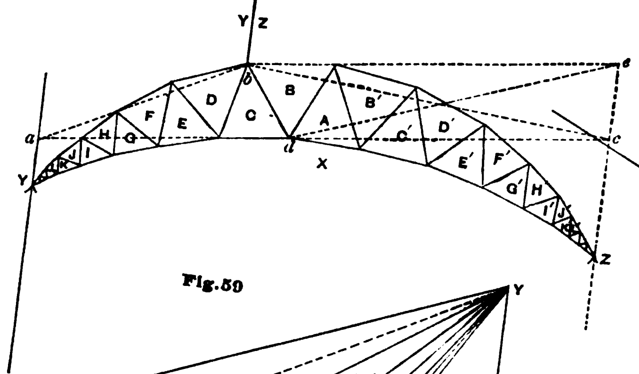


Fig.59

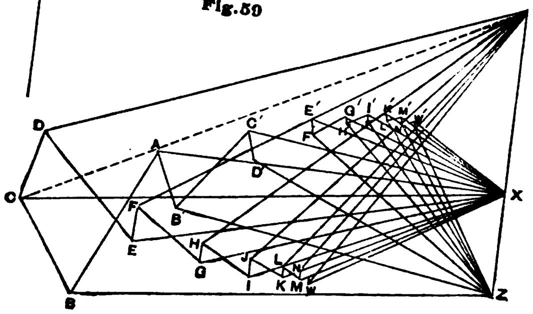


Fig.60

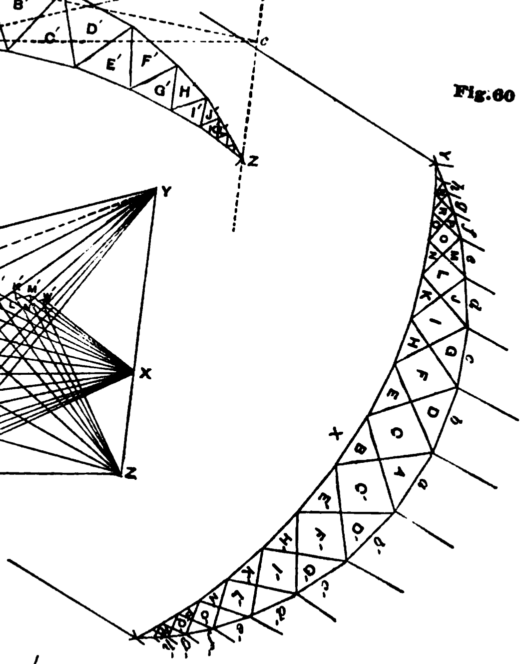


Fig.61

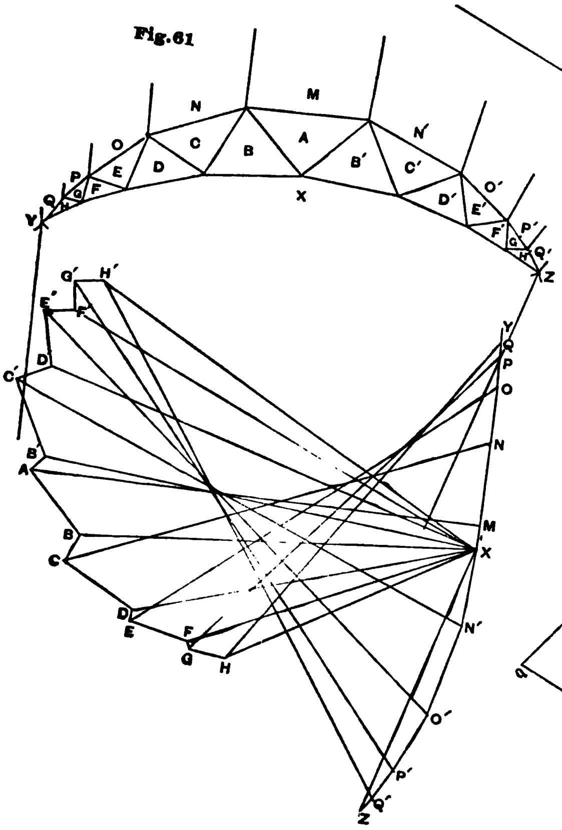
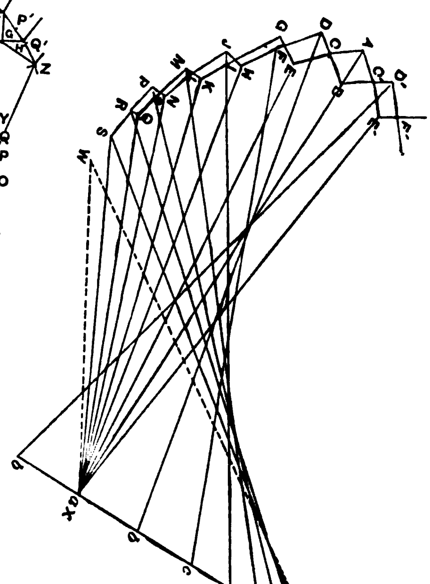


Fig.60



CHAPTER II.

STRUCTURES WHICH SUSTAIN A LIVE AS WELL AS A DEAD LOAD—BRIDGE TRUSSES.

GENERAL PRINCIPLES.

SHEAR—DEFINITION OF.—Let Fig. 71 represent a beam fixed horizontally at one end and sustaining a load P, at the other. Imagine the beam cut completely in two at any point, and then consider what forces are necessary in order that the separated portion may still retain its place and perform its duty. We know that before the section was made all the fibres above the neutral axis were extended, and all below were compressed. We can replace these forces by a link above and a strut or compression piece below, as shown in Fig. 71. But these alone are not sufficient. The link and strut prevent the right hand portion from turning about the section under the action of the weight P, and that is all. In order to prevent the right hand portion from falling vertically we must apply an upward force at the section equal and opposed to P, as shown in Fig. 71. The weight P, we call the "*shearing force*," and the equal and opposite force at the section, the "*resistance to shear*," or "*shearing stress.*"

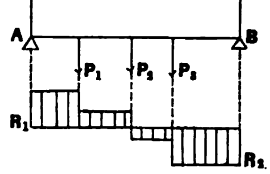
Fig. 71

The shearing force is so called, because its action is to cause one section to slide upon the next, just as if the separation were effected by cutting with a pair of shears.

We may then define shearing force, as ***that force which at any section tends to make that section slide upon the one immediately following.***

Thus, in Fig. 72, which represents a horizontal beam, AB, subjected to the action of outer forces R_1, P_1, P_2, P_3, R_2, the shear at any section between the left end and P_1 is constant and equal to the reaction R_1. R_1 acting up at the left end tends to slide each section upon the next, until we come to P_1. Here we have R_1 still tending to slide the section up, but P_1 tends to slide it down. The difference or algebraic sum is then the shear for any section between P_1 and P_2. Thus the ordinates to the shaded area below, give to scale the shear at any point of the beam AB.

Fig. 72



When, therefore, the section is vertical, and the outer forces all vertical, we may define the shearing force as ***the algebraic sum of all the outer forces acting upon the beam, right or left of the section in question.***

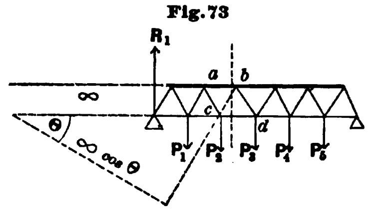
If any of the outer forces are not vertical, we must resolve them into components parallel and perpendicular to the vertical section, and take the former in the algebraic sum.

In general, to find the shear in a section taken in any direction, resolve all the outer

forces into components, perpendicular and parallel to the plane of the section, and the algebraic sum of all the latter, right or left of the section, will be the shear in that section, or the force tending to slide it upon the next consecutive section.

FRAMED GIRDER—HORIZONTAL CHORDS—SHEAR.—If in any framed structure we conceive a section dividing the structure into two portions, it is evident from the above that the stresses in the cut members must hold the shear in equilibrium. This principle we have already proved in Chapter II., page 22. Thus in Fig. 73, conceive a section cutting ab, bc and cd. Then the stresses in these three members must hold the shear in equilibrium.

Fig. 73



The shear in the present case is the algebraic sum of all the outer forces left or right of the section, because the section and forces are all vertical. In taking the algebraic sum we adhere to the conventions of Chapter II., page 16, and take, therefore, upward forces as positive, and downward forces as negative, and consider always only that portion of the truss *on the left of the section.* This must be carefully noted, for if we took the right-hand portion, our conventions should be reversed. The shear, then, in the present case, is $+R_1 - P_1 - P_2$, and the stresses in the cut members ab, bc, and cd must hold this shear in equilibrium.

But if the chords are horizontal, as in Fig. 73, the vertical components of their stress is zero. That is, they cannot take any part in resisting the shear or transverse action of the forces, and simply answer the purpose of the link and strut in Fig. 71. The brace bc must then resist the shear, and hence the vertical component of its stress must be equal and opposed to the shear.

Thus according to the conventions of Chapter II., page 16, that is, taking tension as plus, and compression as minus, and measuring the angle θ made by any brace with the vertical, as shown in Fig. 9, page 16, and considering always the left-hand portion of the truss,

$$\text{stress in } bc \times +\cos\theta_{bc} + R_1 - P_1 - P_2 = 0,$$

or

$$\text{stress in } bc = \frac{(+R_1 - P_1 - P_2)}{-\cos\theta_{bc}} = -\text{ shear} \times \sec\theta_{bc}.$$

That is, for horizontal chords and vertical loads, *the stress in any brace is equal to the shear multiplied by the secant of the angle which the brace makes with the vertical.*

The angle θ should always be measured as directed in Fig. 9, page 16. There may arise some uncertainty as regards the proper sign for this angle θ. Thus, Fig. 73, if we measure the angle θ round from the vertical through c, the sec. of θ is positive, but if we measure from the vertical through b, the sec. of θ is negative. This uncertainty will be removed if we remember that since we are considering only the left-hand portion of the truss, *we must always measure the angle θ for any brace, from the vertical through that end of the brace* BELONGING TO THE LEFT-HAND PORTION.

Thus in the present case, for instance, the sec. of θ for bc is essentially positive, because measured as above it lies in the first quadrant. (See Fig. 9, page 16.) If all the weights are equal and equidistant, R_1 will be greater than $P_1 + P_2$, and the shear will be plus. We shall have then the stress in bc essentially minus, denoting that bc is in compression.

In like manner, for the brace bd, the shear would be the same as for bc, but as the angle θ is in the fourth quadrant, the sec. of θ for bd will be negative.

We have then,

$$-\text{ stress in } bd \times \cos\theta_{bd} + R_1 - P_1 - P_2 = 0,$$

or

$$\text{stress in } bd = + \text{ shear} \times \sec\theta_{bc}.$$

We see at once that the stress in *bd* will be plus or tension, and equal in amount to the compression just found for *bc*.

We can easily deduce the same result directly from the principle of moments, Chapter III., page 23. Thus the chords *ab* and *cd*, Fig. 73, being parallel, their point of intersection is at an infinite distance. Taking this point as a centre of moments, we have for the lever arm of *bc*, $\infty \cos\theta$. The lever arms of R_1, P_1 and P_2 are each ∞. Then according to our rule, Chapter III., page 27,

$$\text{stress in } bc \times \infty \cos\theta_{bc} + R_1 \infty - P_1 \infty - P_2 \infty = 0,$$

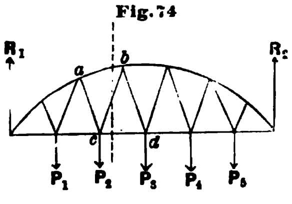
or

$$\text{stress in } bc = \frac{(+R_1 - P_1 - P_2)\infty}{-\infty \cos\theta_{bc}} = -\text{ shear} \times \sec\theta_{bc}.$$

RESIDUAL SHEAR.—If the chords are *not* horizontal, they will themselves take some of the shear, and only what is left is to be resisted by the braces. This remainder we call the "*residual shear.*"

Thus, for instance, Fig. 74, if we take a section cutting *ab*, *bc* and *cd*, the stresses in these members are in equilibrium with the shear.

Fig. 74



The shear is, from the preceding, $R_1 - P_1 - P_2$. But the member *ab* resists a portion of this shear equal to the vertical component of the stress in it. The member *cd*, being in the present case horizontal, has no vertical component. The chord stresses can always be found by moments independently of the braces. Let us suppose *ab* to be thus found, and to be compression. The vertical component of its stress is then,

$$\text{stress in } ab \times \cos\theta_{ab}.$$

The angle θ is to be measured, as already noticed, always from the vertical *through the left end* of the member, as directed in Fig. 9, page 16. We have then for the stress in *cb*,

$$\text{stress in } cb \times + \cos\theta_{cb} + R_1 - P_1 - P_2 + ab \cos\theta_{ab} = 0,$$

or

$$\text{stress in } cb = \frac{(+R_1 - P_1 - P_2 + ab\cos\theta_{ab})}{-\cos\theta_{cb}} = -\text{ residual shear} \times \sec\theta_{cb}.$$

That is, *the stress in any brace is equal to the* RESIDUAL SHEAR *multiplied by the secant of the angle which the brace makes with the vertical.*

ACTION OF LIVE LOAD.—When a structure is designed to resist the action not only of a constant dead load, but also of a moving or live load, which may have various positions, it is evident that the stresses in the members will vary according to the position of the live load. It is of great importance, therefore, to determine what position of the live load gives the greatest stress in any particular member. Comparing, then, this greatest stress due to live load with the stress in the same member due to dead load, if they are both of the same kind, the total maximum stress in the member will be the sum of both. If they are of dif-

ferent kinds, and the live load stress *exceeds* the dead load stress, the members must be counterbraced for the difference. But if the dead load stress is greatest, no counterbracing is necessary, because the action of the live load then is simply to relieve the strained member of a certain amount of its dead load stress.

The live load may also often cause in the same member stresses of different kinds, sometimes compression and sometimes tension, according to its position.

DISTRIBUTION OF UNIFORM LIVE LOAD CAUSING MAXIMUM CHORD STRESSES.—In any properly framed structure, such as we shall discuss hereafter, we can always divide the frame by a section at any point, cutting one brace and two chords. Taking, then, the point of moments at the intersection of the other two members cut, we have the moment of the stress in the chord balanced by the sum of the moments of the outer forces right or left of the section. The stress in any chord will then be greatest when the live load is so disposed as to give the greatest moment possible for that chord. It is required, then, to find that distribution of load which makes the moment at any point a maximum.

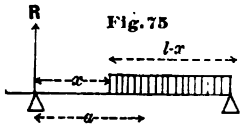


Fig. 75

This is easily found for a uniform load. Thus, in Fig. 75, suppose we have a uniformly distributed moving load of w lbs. per unit of length, coming on from the right. Let it cover the distance $l - x$, the end of the load being at a distance x from the left end. Then, for the reaction R at the left end, we have by moments,

$$-Rl + w(l - x) \times \frac{l - x}{2} = 0,$$

because the weight $w(l - x)$ of the loaded portion can be considered as concentrated at the middle point of the loaded portion (Chapter III., page 25).

The reaction at the left end is, therefore,

$$R = \frac{w(l - x)^2}{2l}.$$

The moment at any point distant from a from the left end, if a is greater than x, is

$$M_a = -Ra + \frac{w(a - x)^2}{2}.$$

Substituting the value of R above,

$$M_a = -\frac{wa(l - x)^2}{2l} + \frac{w(a - x)^2}{2}.$$

If we suppose x constant, and differentiate with respect to a, and put the first differential equal to zero, we have

$$R - w(a - x) = 0.$$

That is, *for any given position of the load, the moment is greatest at that point for which the shear is zero.*

But we can put the preceding equation after easy reduction in the form

$$M_a = -\frac{wa(l - a)}{2} + \frac{wx^2(l - a)}{2l}.$$

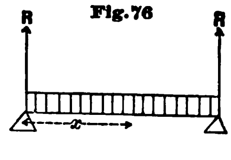
We see at once from this equation that for any given values of a and l, the moment

will be greatest when $x = 0$. That is, *the moment at any point is the greatest possible when the load covers the whole span.*

No special discussion, therefore, is necessary in order to find the methods of loading which give the greatest stresses in the chords for uniform load. We have only to suppose the live load to cover the whole span, just like the dead load. The greatest stresses in the chords will then be found when we suppose the girder fully loaded with both dead and live loads. *This holds good whether the chord are parallel or inclined, provided the girder is a simple girder,* i.e., *not continuous over more than two supports, and whether the girder is framed or is a solid beam.*

GRAPHIC INTERPRETATION OF EQUATION FOR MAXIMUM MOMENTS.—From the preceding principle we can easily find the maximum moment at any point. Thus, let the moving load per unit of length be w and the dead load w'. Then the total load is $(w' + w)\,l$. The reaction at each end is, therefore, $\frac{(w' + w)\,l}{2}$, and the maximum moment at any point distant x from the left end is,

Fig. 76

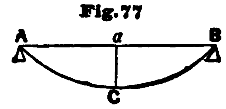


$$M_{Max} = -\frac{(w' + w)\,l}{2}\,x + (w' + w)\frac{x^2}{2} = -\frac{w' + w}{2}\,.\,x\,(l - x).$$

That is, *the moment at any point of a beam for a uniform load, is equal to one half the unit load multiplied by the product of the two segments of the beam.*

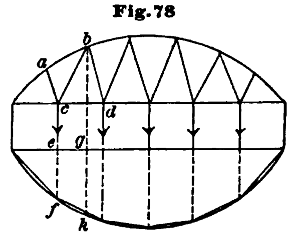
This is the equation of a parabola, Fig. 77, whose middle ordinate at the centre of the span $aC = -(w' + w)\frac{l^2}{8}$, which passes through the ends of the girder A and B, and has its vertex at C. The same result has been already obtained in Chapter IV., page 45. If, therefore, we draw a parabola through A and B, whose middle ordinate aC is by scale $(w' + w)\frac{l^2}{8}$, the ordinates to this parabola will give the maximum moments at any other point of the beam.

Fig. 77



APPLICATION TO A FRAMED GIRDER.—In the case of a framed girder, Fig. 78, the load consists of a succession of concentrated apex loads, and the parabola becomes a polygon whose apices *are at the intersections of the weights with the curve.*

Fig. 78

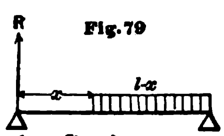


To find the greatest stress in ab, Fig. 78, we first locate the point of moments at c (Chapter III., page 27). Then the ordinate ef gives the moment at c. This moment, divided by the lever arm for ab, gives the stress in ab. In order to obtain the stress with its proper sign, plus for tension and minus for compression, observe the rule for the sign of the stress moment, Chapter III., page 27, and remember that the moments are all negative.

Again, for the stress in the chord cd, the point of moments is at b. From *this* point, then, we drop the ordinate gh to the polygon. The moment is given to scale by gh. Generally, we draw the ordinate *through the point of moments for the chord in question.*

DISTRIBUTION OF UNIFORM LIVE LOAD CAUSING MAXIMUM SHEAR.—The position of a uniform live load, in order to give the greatest shear at any point, is different according as the girder is solid or framed, and in the latter case also varies according as the chords are parallel or inclined.

Fig. 79



1*st. Solid beam without panels.*—Let the load, as before, come on from the right.

Then the left-hand reaction is, as before,

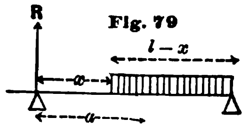


Fig. 79

$$R = \frac{w(l-x)^2}{2l}.$$

This reaction is the shear for any and all points between the left end and the end of the load. For any point distant, a, from the left end, where a is greater than x, the shear is

$$S_a = \frac{w\,(l-x)^2}{2l} - w\,(a-x).$$

We see at once that this is less than the reaction by the amount $w\,(a-x)$, and that the shear will be greatest when $a = x$. That is, *the shear at any point is greatest when the load reaches from that point to the farthest support.* When the load reaches from the point to the nearest support *we have the greatest shear of opposite character.*

The same holds good for residual shear.

Graphic Interpretation.—The equation which gives the greatest shear at any point distant x from the left end, as the load comes on from the right, is then

$$\text{Max. Shear} = \frac{w\,(l-x)^2}{2l}.$$

This is the equation of a parabola, Fig. 80, having its vertex at the right end, B, and the ordinate AC at the left end equal by scale to $\frac{wl}{2}$, or half the live load.

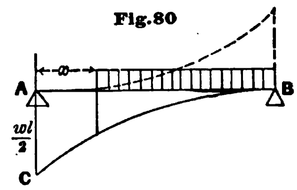


Fig. 80

The ordinates to this parabola at any point give the maximum shear when the load comes on from the right. A similar parabola, indicated by the dotted line, gives the maximum shear for any point when the load comes on from the left.

Shear Caused by Dead Load.—If a beam or girder sustains a uniformly distributed load over its whole extent of w' per unit of length, the total load will be $w'l$, and the reaction at each end $\frac{w'l}{2}$. The shear at any point distant x from the left end, is then

$$\text{Shear} = \frac{w'l}{2} - w'x.$$

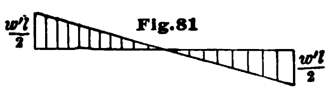


Fig. 81

This is the equation of a straight line, as shown in Fig. 81, the end ordinates being $\frac{w'l}{2}$, and the ordinate at the centre being zero.

2*d. Framed girder, uniform load, maximum shear.*—The stress in any brace is, as we have seen, found by multiplying the shear, or residual shear, by the secant of the angle which the brace makes with the vertical, regard being had to the conventions of positive and negative forces, and the quadrant in which θ is measured, and the definition of shear, pages 79 and 80.

In order to find the maximum stress in any brace, we must then find that position of the loading which gives the greatest shear for that brace and the corresponding shear

This we can easily do for a uniform load. It should be noted that the position is different for parallel and inclined flanges.

Let us first take the case of parallel flanges. It is a common practice to take the load for any brace as extending beyond the brace *to the middle of the panel.* This is not strictly correct. The load reaches into the panel a variable distance, x, as will be seen from the following:

Let $l =$ span, $p =$ panel length, $N =$ number of panels, $m =$ number of panels covered by the load, $w =$ the uniform load per lineal foot, $R =$ the reaction at unloaded end.

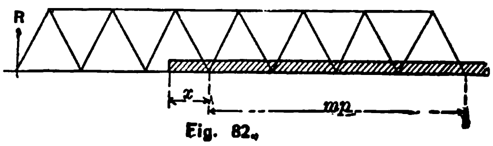


Fig. 82.

Then that portion of the load wx, which takes effect at the panel point beyond the load, is $\frac{wx^2}{2p}$. The shear at the panel point covered by the load is then

$$S = R - \frac{wx^2}{2p}.$$

But we have for the reaction

$$R = \frac{wx\left(\frac{x}{2} + mp\right)}{l} + \frac{w\,(mp)^2}{2l} = \frac{w}{2l}[(mp)^2 + x^2 + 2\,xmp].$$

Substituting this value of R in the expression for the shear, and placing the first differen tial coefficient equal to zero, we have for the condition of maximum shear

$$\frac{w}{2l}(2x + 2mp) - \frac{wx}{p} = 0, \text{ or, since } p = \frac{l}{N},$$

$$\frac{w}{2l}(2x + 2mp - 2Nx) = 0 \qquad \therefore x = \frac{mp}{N - 1}.$$

Substituting this value of x in the expression for the shear, we have, after reduction,

$$\text{Max. Shear} = \frac{wpm^2}{2\,(N - 1)}.$$

For the first panel point from left, $m = N - 1$ and $x = p$, or the whole span is covered, and the shear is $\frac{wp\,(N - 1)}{2}$. These results are independent of the bracing, whether inclined, or vertical and inclined.

EXAMPLE.—Let the span $l = 140$ feet, number of panels $N = 7$, uniform load $w = 4{,}000$ lbs. per lineal foot.

Then, for the maximum shear at the first panel point on left, we have $m = 6$, $N - 1 = 6$, $x = p$, shear $= \frac{6\,wp}{2} = 3\,wp = 3$ full panel loads, or half the effective load $= 3 \times 4{,}000 \times 20 = 240{,}000$ lbs.

At the fourth panel point, $m = 3$, $x = \frac{p}{2}$, or the load reaches just to the middle of the panel, shear $= \frac{3}{4}\,wp$.

At the sixth panel point, $m = 1$, $x = \frac{1}{6}p$, shear $= \frac{1}{12}\,wp$. If we took the panel load wp as concentrated at the panel point, and disregard the portion which goes direct to the right abutment, as is the common practice, we would have shear $= \frac{1}{7}\,wp$.

In general, it will be easily seen that the shear obtained by supposing the panel load wp as concentrated at the panel point, and disregarding the half panel loads at each end, is always somewhat in excess of the strictly correct value. For this reason it is a common and allowable practice to take x as always $\frac{p}{2}$, and suppose all the load from middle to middle of panel as concentrated at the panel point.

For inclined chords the position of the load is different, as the chords themselves take a portion of the shear, and only the rest affects the braces. Let the position of the uniform load giving the maximum stress in the brace T be required, Fig. 83. Let n be the number of panels on the left of the panel point in question, c = the load which takes effect at the forward panel point, and d = the distance from the left support to the intersection of the chords cut by a section through T or P.

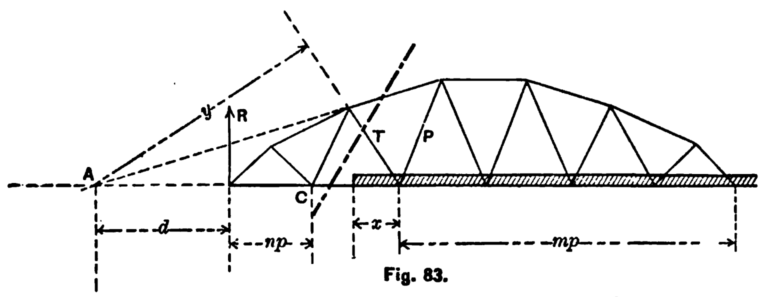


Fig. 83.

Let y = the lever arm of T about the intersection of the chords, A.
Let y' = the lever arm of P about the intersection of the chords, A.

Then we have for the reaction at left support

$$R = \frac{wx\left(\frac{x}{2} + mp\right)}{l} + \frac{w\,(mp)^2}{2l}, \text{ and } c = \frac{wx^2}{2p}.$$

Also, passing a section through T or P, completely severing the truss, and taking moments about A, we have

$$Ty = Rd - c\,(d + np) = \frac{wxd}{l}\left(\frac{x}{2} + mp\right) + \frac{w\,(mp)^2\,d}{2l} - \frac{wx^2d}{2p} - \frac{wx^2n}{2}.$$

Putting the first differential coefficient equal to zero, and $p = \frac{l}{N}$, and reducing, we have, for the condition which makes Ty, and therefore the stress in T or P, a maximum,

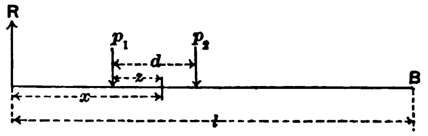
$$x = \frac{mpd}{d\,(N - 1) + nl}.$$

Inserting this value of x, we have

$$\text{Maximum } Ty = \frac{wm^2pd}{2N}\left[\frac{N + \frac{nl}{d}}{(N - 1) + \frac{nl}{d}}\right]$$

If the flanges are parallel, $d = \infty$, $y = \infty \cos\theta$, as on page 80, $\frac{nl}{d} = 0$, and $x = \frac{mp}{N - 1}$, and shear $= T\cos\theta = \frac{wpm^2}{2\,(N - 1)}$, as already found. The character of the bracing, whether vertical and inclined, or inclined only, makes no difference in these results.

MAXIMUM SHEAR AND MOMENT FOR A SYSTEM CONSISTING OF TWO CONCENTRATED LOADS.—The maximum shear and moment at any point of a beam due to a single concentrated load, evidently occurs when the load acts at this point.



For two concentrated loads, the maximum shear and moment at any point on the left of the centre occurs when the first load is at this point.

This is easily found as follows: Let the two loads be p_1 and p_2 at a constant distance d. Let the point be at a distance x from the left end less than the half length $\frac{l}{2}$. Let p_1 be at a distance z from the point. Then p_1 is distant from the right end $(l - x + z)$, and p_2 is distant from the right end $(l - x + z - d)$. We have then for the reaction R at A,

$$R = \frac{p_1(l - x + z) + p_2(l - x + z - d)}{l} = p_1 + p_2 - \frac{p_1}{l}(x - z) - \frac{p_2}{l}(x + d - z).$$

We have then for the shear at the point

$$S = R - p_1 = p_2\left(1 - \frac{d}{l}\right) - \frac{p_1 + p_2}{l}(x - z), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and for the moment at the point

$$M = -Rx + p_1 z.$$

We see then that R and S are greatest for $z = 0$. Hence M is greatest for $z = 0$. That is the shear and moment at any point on left of centre are greatest for p_1 at that point.

METHOD OF CALCULATION BY CONCENTRATED LOAD SYSTEMS.*—It is the present practice of many engineers to calculate all spans below 200 feet, for the system of concentrated loads actually formed by the locomotive and tenders, followed either by a uniform train load, or by a system of concentrated train loads also. Many systems are specified by engineers, and the reader should remember that we seek to illustrate the *method* of procedure, rather than to sanction any special numerical values.

The system of loads which we adopt we believe to represent good practice and to allow margin for future increase. At the same time the tendency is ever toward heavier rolling stock, and our system of loads may shortly be considered too light. Any system, however, may be handled in a precisely similar manner.

In Fig. 84, suppose a series of loads, $p_1, p_2, p_3 \ldots . p_n$, to act upon the girder AB, the distances from load to load being $d_1, d_2, d_3 \ldots . d_{n-1}$, and the distance of the last load from the right end being y.

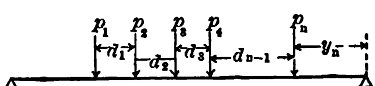


Fig. 84.

* The principles and application of the method here given were worked out independently, but simultaneously, by Mr. Robert Escobar, C. E., of the Union Bridge Company, and by Theodore Cooper, C. E., *Trans. Am. Soc. C. E.*, July, 1889.

Then the total moment at B will be the sum of the moments of each load, or

$$\text{Moment at } B = p_1(d_1 + d_2 + d_3 + \ldots . d_{n-1} + y_n) + p_2(d_2 + d_3 + \ldots . d_{n-1} + y_n) + p_3(d_3 + \ldots . d_{n-1} + y_n) + \ldots . + p_n y_n.$$

Now, let us denote the total moment at the end load p_n, by M_n. We have

$$\text{Moment at } p_n = M_n = p_1(d_1 + d_2 + d_3 + \ldots . d_{n-1}) + p_2(d_2 + d_3 + \ldots . d_{n-1}) + p_3(d_3 + \ldots . d_{n-1}).$$

Comparing this with the value of the moment at the right end of the span, and denoting the sum of all the wheel loads by P_n, we see at once that

$$\text{Moment at } B = M_r = M_n + (p_1 + p_2 + p_3 + \ldots . p_n) y_n = M_n + P_n y_n.$$

This principle holds good for any other point. Thus the moment at *any point* is equal to *the moment at the preceding load on left, plus the sum of all the preceding loads multiplied by the distance from the left preceding load to the point in question.*

If a uniform train load, w per lineal foot, comes on at the right, and covers the distance y_n, then, in order to find the moment at the right end, let M_n stand for the moment *at the head of the train*, instead of the last concentrated load, and we shall have

$$M_r = M_n + P_n y_n + \frac{w y_n^2}{2},$$

which is a general expression for M_r in any case, simply taking for M_n the moment at last wheel, if there is no train load, and at the head of the train if there is; y_n in the first case being the distance from last wheel to right end, and in the second, the distance covered by the train. In both cases P_n is the sum of the wheel loads on the span.

Let us now take, for our system of concentrated loads, that given in the Table which follows. We give in column (1) the wheel loads p_1, p_2, p_3, etc., and in column (2) the distances d_1, d_2, d_3, etc., between the wheels. Any desired system can be tabulated in a similar manner. Then in column (3) we place the distances $d_1, d_1 + d_2, d_1 + d_2 + d_3$, etc., of each wheel from the front wheel, and in column (4) the sum of the loads P_n.

By applying our principle we can find the moment M_n at any load of all preceding loads, as given in column (6).

Thus, for the moment at p_2, we have $16000 \times 8 = 128000$ ft. lbs. At p_3 we have $128000 + 41600 \times 4' \, 3'' = 304800$ ft. lbs. At p_4 we have $304800 + 67200 \times 4' \, 3'' = 590400$ ft. lbs., and so on. Multiplying, then, each value of P_n in column (4) by the corresponding distance in column (2), we obtain the values in column (5), and the successive additions of these give column (6). The sum of all values in (1) should check by giving the last value in (4), and the sum of all in (5) should give the last value in (6).

The Table gives locomotives and tenders, as specified by the Atlantic Coast-Line Railroad. Any desired system of loads can be treated in precisely similar manner. The loads given are the *total loads for one track.*

We suppose these two locomotives and tenders to be followed by a train load of 4000 lbs. per lineal foot,* and the distance from the last wheel load, p_{20}, to the uniform load to be $2' \, 3''$. Then the moment at the beginning of the uniform train load is $22870666 + 448000 \times 2\frac{1}{4} = 23878666$, and this moment is to be taken for M_n in finding M_r = moment at right end, in case there is any train load on the span.

* Since the locomotive and tender concentrates 224000 lbs. on a 54 ft. wheel base, the locomotive excess for this case is $224000 - 54 \times 4000 = 8000$ lbs. The distance between locomotives is then *about* 50 feet.

TABLE FOR TWO 112-TON DECAPOD ENGINES.

ATLANTIC COAST LINE.

(1)	(2)	(3)	(4)	(5)	(6)
LOADS IN POUNDS.	DISTANCES d_1, d_2, ETC., BETWEEN WHEELS.	DISTANCE FROM FIRST WHEEL.	SUMMATION OF LOADS, P_n.	PRODUCT OF P_n BY DISTANCE TO NEXT WHEEL.	MOMENT AT EACH WHEEL, M_n.
p_1 = 16000					
	8′		16000	128000	
p_2 = 25600		8′			128000
	4′ 3″		41600	176800	
p_3 = 25600		12′ 3″			304800
	4′ 3″		67200	285600	
p_4 = 25600		16′ 6″			590400
	4′ 3″		92800	394400	
p_5 = 25600		20′ 9″			984800
	4′ 3″		118400	503200	
p_6 = 25600		25′			1488000
	7′ 6″		144000	1080000	
p_7 = 20000		32′ 6″			2568000
	4′ 8″		164000	765333	
p_8 = 20000		37′ 2″			3333333
	5′ 7″		184000	1027333	
p_9 = 20000		42′ 9″			4360666
	4′ 8″		204000	952000	
p_{10} = 20000		47′ 5″			5312666
	7′ 3″		224000	1624000	
p_{11} = 16000		54′ 8″			6936666
	8′		240000	1920000	
p_{12} = 25600		62′ 8″			8856666
	4′ 3″		265600	1128800	
p_{13} = 25600		66′ 11″			9985466
	4′ 3″		291200	1237600	
p_{14} = 25600		71′ 2″			11223066
	4′ 3″		316800	1346400	
p_{15} = 25600		75′ 5″			12569466
	4′ 3″		342400	1455200	
p_{16} = 25600		79′ 8″			14024066
	7′ 6″		368000	2760000	
p_{17} = 20000		87′ 2″			16784666
	4′ 8″		388000	1810666	
p_{18} = 20000		91′ 10″			18595333
	5′ 7″		408000	2278000	
p_{19} = 20000		97′ 5″			20873333
	4′ 8″		428000	1997333	
p_{20} = 20000		102′ 1″			22870666
			448000	22870666	

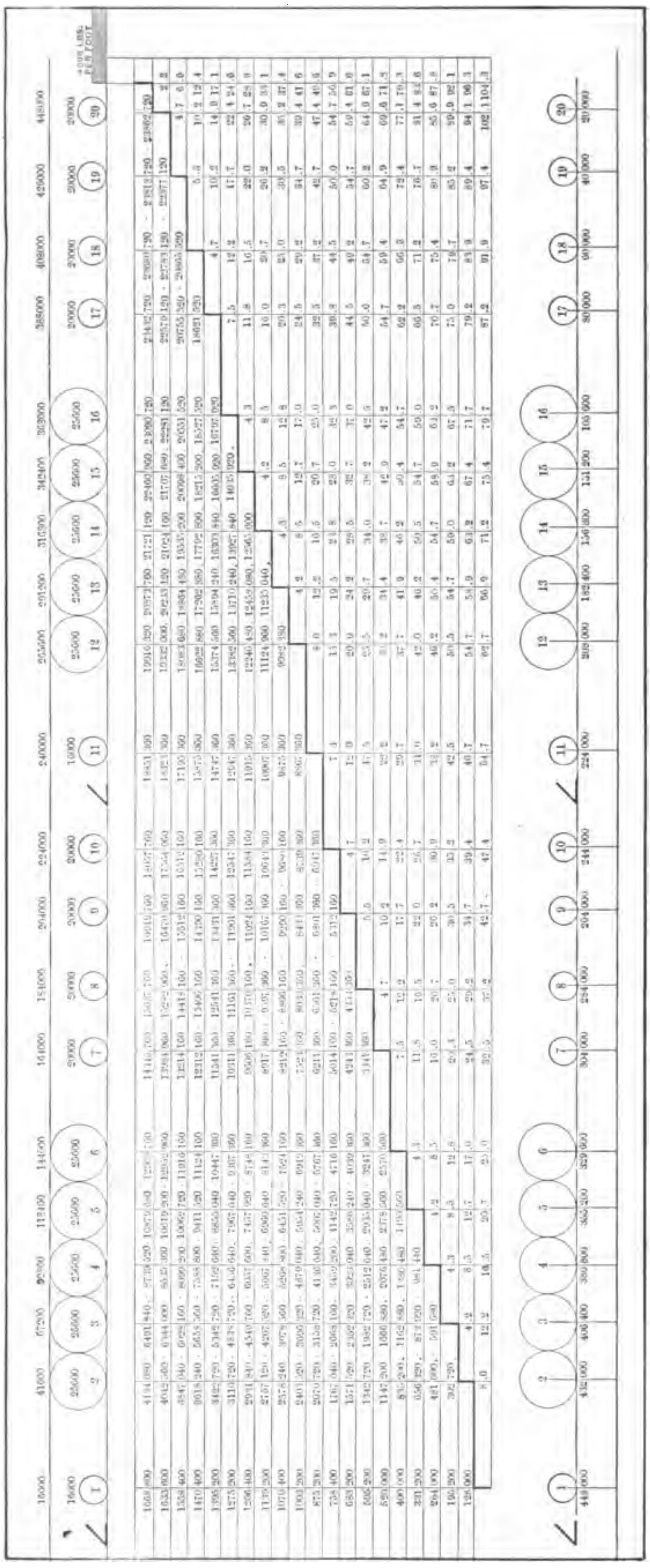
The results of our table can now be embodied in a diagram arranged for ready use. Several forms of diagram are in use, each having its own points of merit and its own advocates. The student who understands one will have no difficulty in understanding any other.

The first horizontal line gives the summation of the loads from the left, and the last line at bottom gives the summation of loads from the right.

The second horizontal line gives the loads themselves.

The numbers in the vertical column under load 1 give the moments of load 1 with reference to the end of the train load and each of the other loads. Thus, 1668800 is the moment in ft.-lbs. of load 1 with reference to the end of the train load; while 1633600 is the moment of load 1 with reference to load 20; 1558400 with reference to load 19; 1470-400 with reference to load 18; and so on, as indicated by the stepped line.

The numbers in the vertical column under load 2 give the moment of loads 1 and 2. Thus, 4134080 is the moment of loads 1 and 2 with reference to the end of the train load;



4042560 is the moment of loads 1 and 2 with reference to load 20; 3847040 is the moment of loads 1 and 2 with reference to load 19, and so on.

The numbers in the vertical column under load 3 give in the same way the moment of loads 1, 2 and 3, and so on.

Hence, 23862720 is the moment of all the loads with reference to the end of the train load, and 22877120 is the moment of loads 1 to 19 inclusive with reference to load 20.

At the upper right hand, below the stepped line, the numbers in the first vertical column give the distances of the loads from the end of the train. Thus, 2.2 ft. is the distance of load 20, 6.9 of load 19, 12.4 of load 18, 17.1 of load 17, etc., all from the end of train.

In the next vertical column on left, we have the distances of the loads from load 20. Thus, 4.7 ft. is the distance of load 19, 10.2 of load 18, 14.9 of load 17, all from load 20, and so on. Hence the distance of load 1 from load 2 is 8 ft.; of load 2 from load 3, 4.2 ft.; of load 3 from load 4, 4.3 ft., and so on.

The diagram gives total loads and moments. For one rail divide by 2.

ILLUSTRATION OF THE USE OF THE DIAGRAM—CRITERION FOR MAXIMUM SHEAR.—Let Fig. 85 represent a girder with a system of concentrated wheel loads, followed by a uniform train load.

It is required to find the position of the system which gives the maximum shear at the point K, whatever the character of the bracing may be, ***the chords being horizontal.***

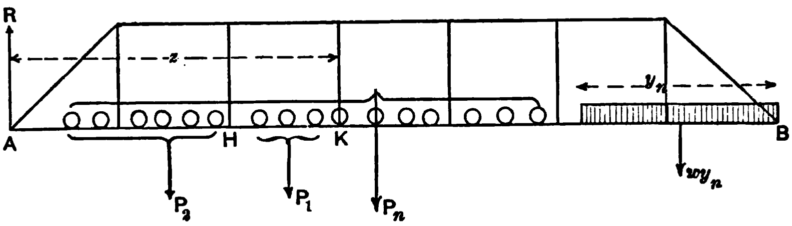


Fig. 85.

Let the sum of all the wheel loads between the ends A and B be $\sum\limits_B^A P = P_n$. Let b denote the distance of any wheel P from the right end B, and k denote the distance of any wheel on the left of K from K. Let the sum of all the wheel weights between A and H be $\sum\limits_H^A P = P_2$, and of all the loads in the panel HK, $\sum\limits_K^H P = P_1$.

Let l = the span of the girder, p the panel length HK, and N the number of panels, when all the panels are equal.

Then, taking moments about B, we have the reaction R at the left end, $R = \frac{1}{l}\sum\limits_B^A Pb$, and the portion of the load in the panel HK, which takes effect at H, is $\frac{1}{p}\sum\limits_K^H Pk$. Hence the shear at K is

$$S = \frac{1}{l}\sum_B^A Pb - \sum_H^A P - \frac{1}{p}\sum_K^H Pk.$$

The last term is increased suddenly whenever a wheel passes K. The shear will therefore be a maximum *when some wheel is at the point K.*

Now, let the system be moved a very small distance, δx, to the left. The shear will be increased by a small amount, δS, and we shall have

$$S + \delta S = \frac{1}{l}\sum_B^A P(b + \delta x) - \sum_H^A P - \frac{1}{p}\sum_K^H P\,(k + \delta x).$$

Subtracting the value of S already found, we have

$$\frac{\delta S}{\delta x} = \frac{1}{l}\sum_B^A P - \frac{1}{p}\sum_K^H P = \frac{P_n}{l} - \frac{P_1}{p}.$$

* Such a large scale diagram will be found at page 215.

The shear S will therefore be a maximum for some wheel at K, which, when it is moved to the left, so as to enter the panel HK, causes the value of $\frac{\delta S}{\delta x}$ to be either zero or to pass from positive to negative. If any of the uniform train load, w per lineal foot, is on the span, and covers the distance y_n, it should be included in the total load from A to B.

We have, therefore, for the criterion for maximum shear, in the case of parallel chords, whatever the character of the bracing,

$$\frac{P_n + wy_n}{l} \overset{=}{>} \frac{P_1}{p}; \text{ or, } \frac{P_n + wy_n}{N} \overset{=}{>} P_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

The second form holds for equal panels, so that $Np = l$. The first form is general, p being the panel length HK. We see that P_2 does not appear in the criterion. In all practical cases, or when the number of panels is not very great, P_2 will be zero. Whether it is or not, it does not affect the criterion.

The shear for horizontal chords, whatever the bracing, is then a maximum at any panel point, *when one of the wheels is at the point and when the average load on the span is equal to or just greater than the average load in the panel in front of the panel point.* *

The value of P_1 does NOT *include the load at the point.* Without that load, P_1 should be equal to or less than the first term of the criterion; and when that load is added to P_1, by reason of a small shift to the left, the sum should be greater than the first term of the criterion.

We can thus easily find, by trial with the diagram, the position of the system giving a maximum. This position being known, we can find the moment M_r at the right end, of the entire load on the span, including the uniform train load, if any. Dividing M_r by the length of span l, we have the reaction at the left end. Subtract from this reaction $P_2 + \frac{1}{p}\sum_K^H Pk$ and we have the shear. In finding the moment M_r at the right end, the moment of the train load $\frac{w}{2}y_n^2$ must be included if the train comes on. If, then, M_n is the moment of all the wheels with reference to the head of the train, the moment at the right end $M_r = M_n + P_n y_n$. If we denote the moment of all the wheels in the panel with reference to K, or $\sum_K^H Pk$ by M_1, we have then, in general,

$$\text{Shear} = \frac{M_r}{l} - P_2 - \frac{M_1}{p} = \frac{M_n + P_n y_n + \frac{w}{2}y_n^2}{l} - P_2 - \frac{M_1}{p}.$$

Usually it will be found that there are no loads beyond the panel on the left of the point, so that P_2 is zero, and M_1 can be taken directly from the diagram. We have in this case

$$\text{Shear} = \frac{M_r}{l} - \frac{M_1}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

where M_r is the moment at the right end of all the loads on the span, including the uniform train load, if any.

EXAMPLE.—Suppose the span $l = 140$ feet, the number of panels $N = 7$, and the maximum shear is required at 20 feet from the left end.

Suppose the first wheel, p_1, is at the point, 20 feet from left end of span. In this position of the system P_1 is zero, the train load covers the distance $y_n = 120 - 104.3 = 15.7$ feet, $P_n = 448000$, and the total load $P_n + wy_n = 448000 + 4000 \times 15.7 = 510800$ lbs. We have, therefore, $\frac{P_n + wy_n}{7}$ greater than P_1, and if p_1 is moved a little to the left of the point, P_1 will become 16000, but the total load is essentially the same, and $\frac{1}{7}$th of this is greater than 16000 also.

* It should be noted that this criterion holds good only for horizontal chords. For inclined chords a modification is necessary (page 245).

We therefore try for p_2 at the point. We have now $P_1 = 16000$, $y_n = 120 + 8 - 104.3 = 23.7$ feet, $P_n = 448000$, total load $= 448000 + 4000 \times 23.7 = 542800$ lbs. Again, $\frac{1}{7}$th of this is greater than $P_1 = 16000$, and if the second wheel is moved a very little to left of the point, P_1 will be 41600, the total load is practically unchanged, and $\frac{1}{7}$th of it is greater than 41600 also.

We therefore try for p_3 at the point. For this position of the system $P_1 = 41600$, $y_n = 120 + 12.2 - 104.3 = 27.9$ feet, $P_n = 448000$, the total load is $448000 + 4000 \times 27.9 = 559600$ lbs., and $\frac{1}{7}$th of this is greater than P_1. If the third wheel is moved a very little to left, P_1 becomes 67200, the total load is unchanged, and $\frac{1}{7}$th of it is greater than 67200 also.

We therefore try for p_4 at the point. For this position $P_1 = 67200$, $y_n = 120 + 16.5 - 104.3 = 32.2$ feet, $P_n = 448000$, total load $= 448000 + 4000 \times 32.2 = 576800$ lbs., and $\frac{1}{7}$th of this is greater than P_1. But if the fourth wheel is moved a little to left, P_1 becomes 92800, the total load is unchanged, and $\frac{1}{7}$th of it is *less than* 92800.

The *fourth wheel at the point* gives, therefore, a maximum shear, since this is the one for which $\frac{P_n}{7}$ is greater than $P_1 = 67200$, and less than 92800; that is, it is the position for which $\frac{P_n + wy_n}{N}$ is *just* greater than P_1.

Assuming this position, we have at once $M_1 = 590400$, and moment at right end of span $M_r = 23878666 + 448000 \times 32.2 + \frac{4000 \times (32.2)^2}{2} = 40377946$. We have, therefore, from (2), ***maximum shear*** $= \frac{40377946 - 7 \times 590400}{140} = 258893$.

Whenever we thus determine the position for maximum shear, if when we place the next load on the point the front wheel goes off the span, we should see whether there is not another maximum which is greater than that already found.

Thus in the present case, for the fifth wheel at the point, p_1 passes off. Hence $P_1 = 92800 - 16000 = 76800$ lbs. The train load covers $y_n = 120 + 20.7 - 104.3 = 36.4$ feet, and total load $= 448000 - 16000 + 4000 \times 36.4 = 577600$ lbs., and $\frac{P_n}{7}$ is greater than P_1. But for p_6 a little to left of the point P_1 becomes 102400, the total load remains the same, and $\frac{P_n}{7}$ is *less* than P_1. Hence p_5 at the point also gives a maximum.

For this position we have $M_1 = 984800 - 16000 \times 20.7 = 653600$, and moment at right end of span $M_r = 23878666 + 448000 \times 36.4 + \frac{4000 \times (36.4)^2}{2} - 16000 \times 140.7 = 40584586$. Hence ***maximum shear*** $= \frac{40584586 - 7 \times 653600}{140} = 257210$ lbs. As this is less than p_4 at the point, that position gives the true maximum.

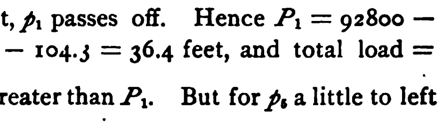
Finally, we should test and see whether the uniform load alone does not give a greater shear.

In the present case the shear due to uniform load is, as found on page 85, 240000 lbs.

We may find in similar manner the maximum shear at any other panel point. The shear thus found is correct for double track and two trusses for each track. We should take *one half* of it for each truss, for single track and two trusses.

CRITERION FOR MAXIMUM SHEAR FOR ANY POINT OF A SOLID BEAM.—Let the wheel p_1, Fig 85 (*a*), be at any point K of a beam AB of length l. Then the reaction at the left end A or the shear at K is given by

Fig. 85 (a).



$$-Rl + \sum_B^A Pb = 0; \quad \text{or,} \quad R = \frac{\sum_B^A Pb}{l}.$$

Now let the system be moved until p_2 is at K. If d_1 is the distance between p_1 and p_2, we have for the reaction,

$$-Rl + \sum_B^A P(b + d_1) = 0; \quad \text{or,} \quad R = \frac{\sum_B^A P(b + d_1)}{l}.$$

The shear then in the second case is

$$\frac{\sum_B^A P(b+d_1)}{l} - p_1.$$

If from this we subtract the shear in the first case we have for the change of shear, if we denote $\sum_B^A P$ by P_n,

$$\frac{P_n d_1}{l} - p_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If this expression is positive, it shows that the shear is increased when p_2 is at K; if negative, the shear is greatest for p_1 at K. Expression (3) then is the criterion for determining which of two consecutive wheels gives the maximum shear at any point K of a beam. If when p_2 is at K additional loads come on the beam, let P_n' be the sum of all loads for p_1 and P_n'' the sum of all loads for p_2 at K. Then the change of shear will be between

$$\frac{P_n' d_1}{l} - p_1 \quad \text{and} \quad \frac{P_n'' d_1}{l} - p_1.$$

If the first expression is negative and the second positive, we must try both positions in order to find which load gives the greatest shear at K. This will be the case only for a short distance, on the left of which both expressions are positive, giving p_2 at the point, and on the right both are negative, giving p_1.

For the point at which both p_1 and p_2 give the same shear we have, if no loads come on or go off,

$$\frac{P_n}{l} = \frac{p_1}{d_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This then is the condition for equal shear.

EXAMPLE.—Let $l = 70$ ft., $p_1 = 16000$ lbs., $d_1 = 8$ ft. and the distance of K from the left end be $AK = z = 20$ ft. Required to find the position of the load system which gives a maximum shear at K.

Take first p_1 at K. Then from our diagram, page 88, we see that p_{10} is the last load on the beam and $P_n' = 224000$. If now p_2 is at K, p_{11} is the last load on, and $P_n'' = 240000$. We have then for the change of shear $\frac{224000 \times 8}{70} - 16000 = +9600$ lbs. We have also $\frac{P_n'' d_1}{l} - p_1$ positive.

The shear at K is then greatest for p_2 at the point.

From (4) we have

$$P_n = \frac{16000 \times 70}{8} = 140000 \text{ lbs.}$$

When P_n then is equal to or *greater* than 140000 lbs. we have p_2 at the point, and for P_n equal to or *less* than 140000 lbs. we have p_1 at the point. That is, from our diagram, page 88, for a distance from the right end up to 20.7 ft., we have p_1 at the point, and for a distance from the right end greater than 25 ft. we have p_2 at the point. For any point between 20.7 ft. and 25 ft. from the right end, both positions must be tested.

CRITERION FOR MAXIMUM MOMENT.—for the moment at the panel point K, Fig. 85, we have, if we call the distance AK, z,

$$M = -Rz + \sum_K^A Pk = -\frac{z}{l}\sum_B^A Pb + \sum_K^A Pk$$

If the system is moved a small distance, δx, to the left, we find, as before,

$$\frac{\delta M}{\delta x} = \frac{z}{l}\sum_B^A P - \sum_K^A P = \frac{z}{l}P_n - P_z,$$

where P_z is the sum of the wheel loads on the segment $AK = z$.

We have, therefore, in general, the criterion

$$\frac{P_n + wy_n}{l} \underset{>}{=} \frac{P_z}{z}; \text{ or, } \frac{(P_n + wy_n)\,z}{l} \underset{>}{=} P_z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

That is, the moment is a maximum *when one of the wheels is at the point, and when the average load upon the span is equal to or just greater than the average load beyoud the panel point.*

It should be remembered, however, that, as we shall see later, this criterion, while it holds good for chords either horizontal or inclined, and also for any point of the loaded chord, whatever the bracing, and for both chords in the case of vertical and diagonal bracing, does *not* hold for points in the *unloaded* chord *when the bracing is triangular.* For this latter case a modification is necessary (page 242).

If we denote the moment of all the wheels between A and K, with reference to K, by M_z, we have for the moment corresponding to the maximum position, as determined by our criterion,

$$M = -\frac{z}{l}M_r + M_z = -\frac{\left(M_n + P_n y_n + \frac{w}{2}y_n^2\right)z}{l} + M_z \quad . \quad . \quad . \quad . \quad (6)$$

where M_r is the moment at the right end of all the loading on the span, including the uniform train load, if any.

EXAMPLE.—*Let* $l = 140$ *feet,* $N = 7$, *and let the maximum moment at* 40 *feet from left end be required.*

Here $z = 40$, $\frac{z}{l} = \frac{2}{7}$, and we proceed as for shear, except that we use criterion (5) and equation (6).

Thus, let us place p_6 at the point. The uniform load covers the distance $y_n = 100 + 25 - 104.3 = 20.7$ feet, total load $= 448000 + 4000 \times 20.7 = 530800$, and $\frac{2}{7}$ths of this $= 151657$. This, we see, is greater than 118400 preceding, and also greater than 144000. There is no maximum for p_6.

We next place p_7 at the point. For this position $y_n = 100 + 32.5 - 104.3 = 28.2$ feet, total load $= 448000 + 4000 \times 28.2 = 560800$, and $\frac{2}{7}$ths of this $= 160228$. This is greater than $P_z = 144000$ and *less* than 164000. There is a maximum for p_7 at the point.

For this maximum we have $M_z = 2568000$, and

$$M_r = 23878666 + 448000 \times 28.2 + \frac{4000\,(28.2)^2}{2} = 38102746.$$

Hence, for p_7, at the point,

$$M = -\frac{2M_r}{7} + M_z = -8318500 \text{ ft. lbs.}$$

It by no means follows, however, that this is the only maximum.

Thus, if we place p_8 at the point, $y_n = 100 + 37.2 - 104.3 = 32.9$ feet; total load $= 448000 + 4000 \times 32.9 = 579600$, and $\frac{2}{7}$ths of this $= 165600$. This is greater than $P_s = 164000$, and *less* than 184000. There is, therefore, a maximum for p_8 at the point.

For this maximum we have $M_s = 3333333$, and

$$M_r = 23878666 + 448000 \times 32.9 + \frac{4000\ (32.9)^2}{2} = 40782686.$$

Hence, for p_8, at the point,

$$M = -\frac{2M_r}{7} + M_s = -8318863 \text{ ft. lbs.}$$

This maximum is, therefore, greater than for p_7.

If we continue to test we shall find no maximum until p_{11} is placed at the point.

For this position $y_n = 100 + 54.7 - 104.3 = 50.4$. But since $p_1 - p_3$ have passed off the span, total load $= 448000 + 4000 \times 50.4 - 67200 = 582420$ lbs., and $\frac{2}{7}$ths of this $= 166400$. Also, for P_s we have $224000 - 67200 = 156800$, and for next value of P_s, $240000 - 67200 = 172800$. We see that 166400 is greater than the first and *less* than the second. There is, therefore, a maximum for p_{11}.

For this maximum we have

$$M_s = 6936666 - 304800 - 67200 \times 42.5 = 3775866, \text{ and}$$

$$M_r = 23878666 + 448000 \times 50.4 + \frac{4000\ (50.4)^2}{2} - 304800 - 67200 \times 142.5 = 41657386.$$

Hence, for p_{11}, at the point,

$$M = -\frac{2M_r}{7} + M_s = -8126244.$$

This is less than for p_8.

If we continue to test we find no maximum until we come to p_{14}.

For this position, we have $y_n = 100 + 71.2 - 104.3 = 66.9$ feet; $p_1 - p_6$ have passed off; total load $= 448000 + 4000 \times 66.9 - 144000 = 571600$.

$P_s = 291200 - 144000 = 147200$, and the next value is $316800 - 144000 = 172800$. Since $\frac{2}{7}$ths of total load $= 163314$ is greater than the first and less than the second, p_{14} gives a maximum.

For this maximum, we have

$$M_s = 11223066 - 1488000 - 144000 \times 46.2 = 3082266, \text{ and}$$

$$M_r = 23878666 + 448000 \times 66.9 + \frac{4000(66.9)^2}{2} - 1488000 - 144000 \times 146.2 = 40260286.$$

Hence, for p_{14}, at the point,

$$M = -\frac{2M_r}{7} + M_s = -8420673.$$

This is greater than for p_8.

For p_{15} at the point, $y_n = 100 + 75.4 - 104.3 = 71.1$ feet; $p_1 - p_7$ are off; total load $= 448000 + 4000 \times 71.1 - 164000 = 568400$, and $\frac{2}{7}$ths of this $= 162400$ lbs.

$P_s = 316800 - 164000 = 152800$, and the next value is $342400 - 164000 = 178400$. Since $\frac{2}{7}$ths of total load is greater than the first and less than the second, we have a maximum for p_{15}.

For this maximum $M_s = 12569466 - 2568000 - 164000 \times 42.9 = 2965866$; and

$$M_r = 23878666 + 448000 \times 71.1 + \frac{4000\ (71.1)^2}{2} - 2568000 - 164000 \times 142.9 = 39838286.$$

Hence, for p_{15}, at the point,

$$M = -\frac{2M_r}{7} + M_s = -8416500 \text{ ft. lbs.}$$

For p_{16}, at the point, we find, in similar manner, $M = -8407196$ ft. lbs.

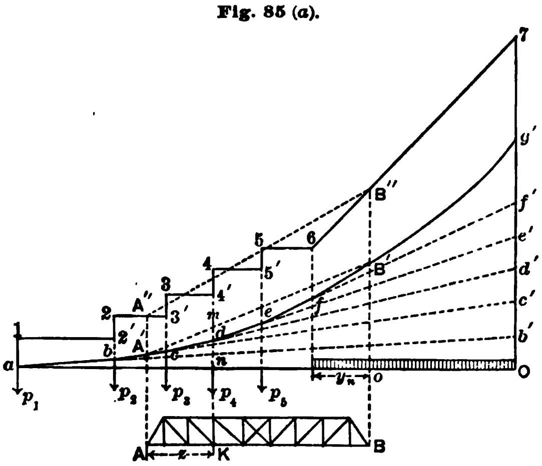
We see, then, that the greatest maximum is for p_4 at the point, and it is 8420670 ft. lbs.

For uniform train load over the whole span the moment is 8000000 ft. lbs. The maximum required is therefore 8420670 ft. lbs.

CONCENTRATED LOAD SYSTEM—GRAPHICAL SOLUTION.—The preceding method of calculation is tedious. For this reason the following graphical construction will often be found to be preferable.* We shall first illustrate the principle of the graphical construction and then give the construction itself.

1st. MOMENTS.—Let p_1, p_2 . . . p_6, Fig. 85(*a*), be a number of concentrated wheel loads, followed by a uniform train load, as shown in the Figure. Lay off each load to scale. We thus obtain the "*load line*" *a* 1 2′ 2 3′ 3 4′ 4 5′ 5 6 7. If now we lay off upon a vertical at o the moments *ob′*, *b′c′*, etc., with reference to o, of the loads p_1, p_2, etc., and draw the lines *ab′*, *bc′*, etc., we obtain the equilibrium polygon *a b c d e f*. From *f* to *g′* we have a parabola which may be easily drawn by laying off the ordinates to a number of its points. Any ordinate as *dn* will then give the moment at *n* of all loads on the left.

Fig. 85 (*a*).



Now suppose $AB = l$ is the length of a truss and we wish the maximum moment at some panel point K distant $AK = z$ from the left end A.

The maximum moment at K will be for some wheel load at K. We therefore try for one after another as follows:

Place the span AB so that p_4 acts at K. Now the criterion for maximum moment is, *for vertical and diagonal bracing* (page 93),

$$\frac{P_n + wy_n}{l} \overset{=}{>} \frac{P_z}{z}, \quad \text{or} \quad \frac{(P_n + wy_n)z}{l} \overset{=}{>} P_z.$$

That is, the moment is a maximum for vertical and diagonal bracing, for one of the wheels at the panel point, and when the average load upon the span is equal to or just greater than the average load beyond the panel point.

If then we project the ends A and B upon the load line at A'' and B'', the line $A''B''$ makes an angle with the horizontal whose tangent is given by $\frac{P_n + wy_n}{l}$. If also we draw the lines $A''4$ and $A''4'$, the line $A''4'$ makes an angle with the horizontal whose tangent is given by $\frac{p_3}{z}$, and the line $A''\ 4$ makes an angle with the horizontal whose tangent is given by $\frac{p_3 + p_4}{z}$. We have then, if the line $A''B''$ cuts the load line between 4 and 4′, $\frac{P_n + wy}{l} > \frac{p_3}{z}$ and $< \frac{p_3 + p_4}{z}$. Hence p_4 at the panel point gives a maximum. If $A''B''$ passes above 4, we should shift the span AB to the left and try for p_3 at the panel point K.

If $A''B''$ passes below 4′, we should shift the span AB to the right and try for p_5 at the panel point K. If there are no loads off the bridge on the left, the point A'' is in the line *ao* vertically over the end A of the bridge. When loads at the left are off, as in the Figure, the

*See paper by Prof. H. T. Eddy, *Trans. Am. Soc. C. E.* Vol. XXII, 1890. Also Prof. Ward Baldwin, Eng. News, Sep. 28, 1889. Also "Graphical Statics," Du Bois, Wiley & Sons, New York, 1875.

point A'' is on the load line vertically over the end A of the bridge. The point B'' is always on the load line vertically over the end B of the bridge. The criterion, it should be noted, holds only for vertical and diagonal bracing. For triangular bracing we have a different criterion and construction (page 243).

We can thus find by trial the position of the load system which gives a maximum moment at the panel point K. In the Figure, p_4 at K gives a maximum. We can now draw the closing line $A'B'$ and the maximum moment is given to scale by the ordinate *md*.

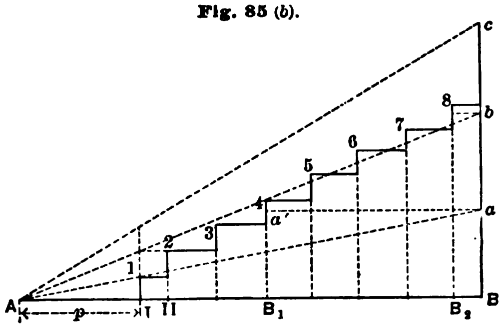
2d. SHEAR—FRAMED GIRDER.—The criterion for maximum shear, *for horizontal chords*, is

$$\frac{P_n + wy_n}{l} \overset{=}{>} \frac{P_1}{p}.$$

That is, the shear at any point is a maximum, for horizontal chords, when one of the wheels is at the point and when the average load on the span is equal to or just greater than the average load in the panel in front of the panel point.

Let 1 2 3, etc., Fig. 85(b), be the load line as before. Place the span AB so that the *first panel point I* is at p_1, and draw Aa, Ab, Ac, so that the ordinates at I are p_1, p_1+p_2, $p_1+p_2+p_3$, etc. Then we see at once from the Figure that Aa makes an angle with the horizontal whose tangent $\frac{p_1}{p}$ is greater than $\frac{p_1+p_2+p_3}{l}$ and less than $\frac{p_1+p_2+p_3+p_4}{l}$. The

Fig. 85 (b).



first wheel load p_1 will then give a maximum shear until p_4 comes on, or for a distance B_1I, from the right end. Mark off on a strip of paper the panel points and place it in the position AB with the first panel point at I. Then if we lay off from B the distance B_1I, we have the distance from the right end for which p_1 gives the maximum shear.

Wheel p_2 gives the maximum shear at every point it passes, when the load system is moved from the position with p_4 at B to the position with p_n at B. If we mark off then on the strip from the right end B the distances B_1II and B_2II, we have the space within which p_2 gives the maximum shear. This space evidently overlaps the first by the distance I, II, within which we must test for both p_1 and p_2. If the ordinate Bc is greater than P_n when p_2 is at I, the maximum shear is given by p_2 for the rest of the truss.

We can thus find the position of the load system for the maximum shear at any panel point. The shear itself is then easily found. Thus in Fig. 85(a) for moments, the reaction at the left end is equal to the ordinate to the equilibrium polygon at the right end, divided by the length of the truss. This gives us $\frac{M_r}{l}$, page 90. From this we have to subtract the loads P_2 beyond the panel, if any, and $\frac{M_1}{p}$, where M_1 is the moment at the right end of the panel of all loads in the panel. This can also be easily taken off the diagram.

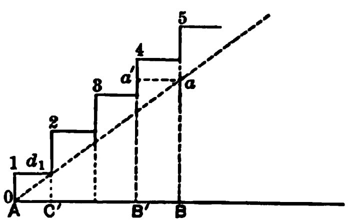
3d. SHEAR FOR SOLID BEAM.—The criterion for equal shear for p_1 and p_2 is (page 92)

$$\frac{p_1}{d_1} = \frac{P_n}{l}.$$

Let 0 1 2 3, etc., be the load line, Fig 85(c), as before. We draw the *line of equal shears Aa*, making the angle with the horizontal whose tangent is given by $\frac{p_1}{d_1}$. If then we

lay off the length of the beam AB from o, and draw the ordinate Ba, we have $\frac{Ba}{l} = \frac{p_1}{d_1}$. In order that Ba may be the total load P_n, we see from the Figure that wheel 4 must be at the right end. If then we lay off the beam from B' to the left for any point between B' and C', the maximum shear is given by p_1 at the point, and for any point between A and the left end the maximum moment is given by p_2 at the point. For any point between A and C' we must try for both p_1 and p_2.

Fig. 85 (c).



APPLICATION OF PRECEDING PRINCIPLES TO CONSTRUCTION OF A DIAGRAM.—We may now construct a diagram as follows: Take a sheet of cross-section paper and indicate the wheels to a scale of say 8 feet to an inch, as shown in the following diagram. Then lay off the load line to a scale of say 24,000 lbs. to an inch. Above the uuiform train load we have a straight line with a slope of 2000 lbs. per foot. Note that we take only the loads *for one rail or one half the loads given on page* 112. We now set off on the right the moments at the end of the train load for wheels 1, 2, 3, etc., and draw the moment lines numbered 1, 2, 3, etc., on the right. We take for the scale of moments 2,000,000 ft. lbs. per inch. We thus construct the equilibrium polygon as shown in the diagram following. The part of the equilibrium polygon above the moment line 19 is a portion of a parabola which can be constructed by computing the moments for different points, and laying off these moments above the bottom line.

The use of the diagram thus prepared has already been explained in connection with Figs. 85(a), (b), and (c). In Fig. 85(a) the lines $A''B''$ and $A'B'$ need not be actually drawn on the diagram. It is sufficient to stretch a thread from A'' to B''. So also in Fig. 85(b). None of the construction lines need to be actually drawn. We thus avoid marking up the diagram.

METHOD OF CALCULATION BY EQUIVALENT UNIFORM LOAD.—For spans under 100 feet the method of calculation by concentrated wheel loads given in the preceding pages is always used. For spans over 100 feet the method of calculation by equivalent uniform load is preferred by many engineers as less tedious and sufficiently accurate.

An "equivalent uniform load" is one which will give the same stress in any member as would be caused by the concentrated wheel loads. Evidently no single uniform load will give the same stresses in all the members as the wheel loads. That uniform load is therefore taken which gives *at the quarter point* of the span the same moment as the wheel loads.

If then l is the length of span, and u the equivalent uniform load per foot, we have, if $M_{\frac{1}{4}}$ is the moment at the quarter point due to the wheel loads as found by calculation or diagram already described,

$$\frac{3}{32} ul^2 = M_{\frac{1}{4}}.$$

Hence we have for the equivalent uniform load per foot u which, when it covers the whole span, will give the same moment at the quarter point as the wheel loads,

$$u = \frac{32}{3} \frac{M_{\frac{1}{4}}}{l^2}.$$

We have proved, page 81, that the moment at any point is greatest when the uniform load covers the whole span. We therefore compute the chord stresses for the uniform load u per foot covering the entire span.

For beams we have proved, page 82, that the shear is greatest at any point of a beam when the uniform load extends from the right end to that point.

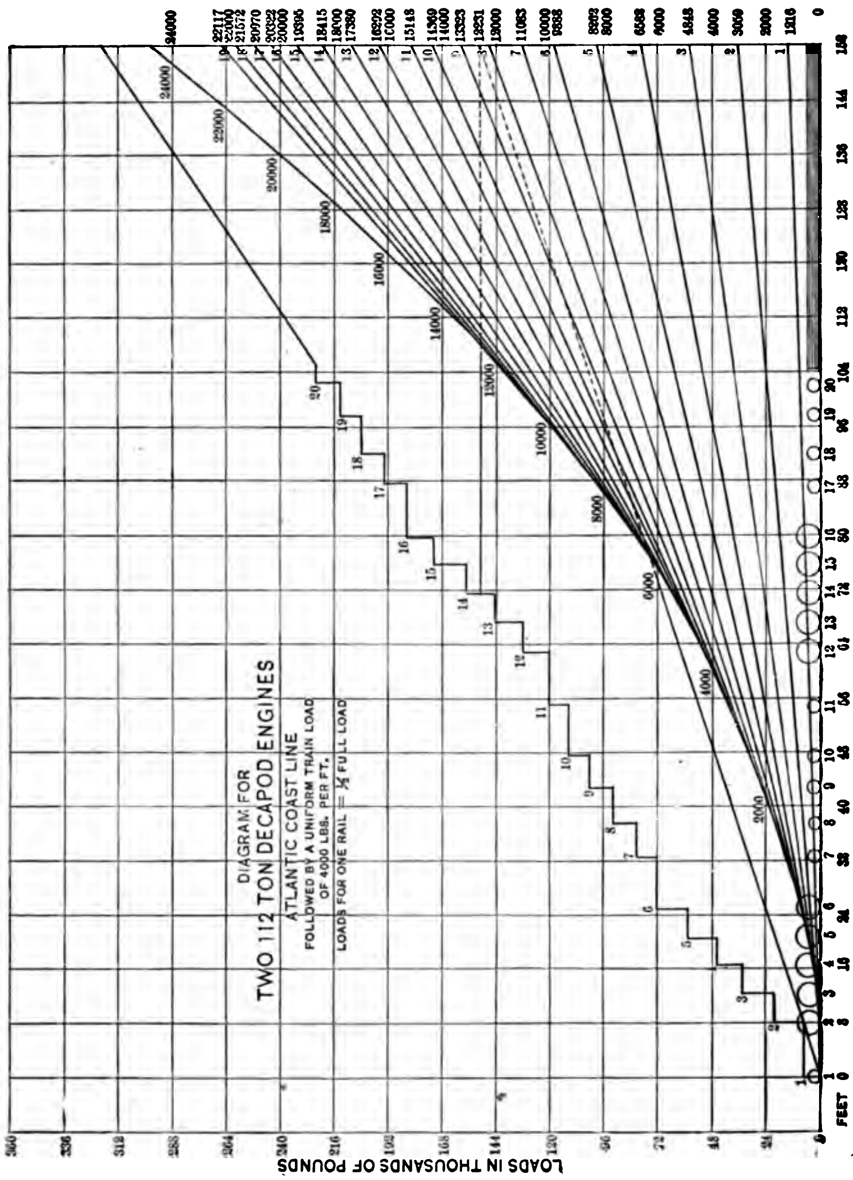


DIAGRAM FOR
TWO 112 TON DECAPOD ENGINES
ATLANTIC COAST LINE
FOLLOWED BY A UNIFORM TRAIN LOAD
OF 4000 LBS. PER FT.
LOADS FOR ONE RAIL = ½ FULL LOAD
LOADS IN THOUSANDS OF POUNDS
FEET

For framed girders, the point to which the uniform load must extend for maximum shear in any panel has been found on pages 83 and 84, Figs. 82 and 83. It is, however, in general sufficiently accurate to consider the uniform load as extending up to the middle of the panel for which the shear is required.

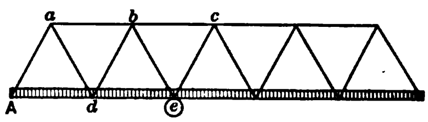
In the table on page 101 we give a comparison of the stresses found by this method with those given by the wheel loads.

METHOD OF CALCULATION BY ONE LOCMOTIVE EXCESS AND EQUIVALENT UNIFORM TRAIN LOAD.—Many engineers use this method instead of the preceding, or the method by wheel loads, for spans over 100 feet.

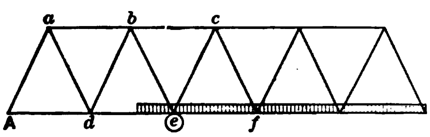
The equivalent train load is placed as before, for maximum moment and shear.

For a beam, for maximum moment and shear at any point, the locomotive excess is placed at that point.

For a framed girder for any chord, the locomotive excess is placed at the point of moments for that chord, or as near to the point of moments as possible without passing to the left of it. Thus, in the Figure for the stress in *bc*, we have the equivalent train over the whole span and the locomotive excess at *e*, the point of moments for *bc*. For *de* the point of moments is *b*, but for through truss the excess cannot be placed there. We therefore put it at *e*, which is as near *b* as it can be placed without passing to the left. In the same way for *ab* we place the excess at *d*, and for *Ad* at *d*. The equivalent train load is always over the whole span.



For maximum shear for *be* we have the equivalent train load as shown in the Figure, reaching from the right end to a point between *d* and *e* as given, pages 83 and 84, Figs. 82 and 83. It is, however, in general sufficiently accurate to take it extending to half-way between *d* and *e*. The locomotive excess is placed at *e*. For *cf*, we should then take the equivalent train load reaching to half-way between *e* and *f*, and place the excess at *f*.



It remains to determine the locomotive excess and the equivalent train load. Let w_t be the actual train load per foot, W the weight of the locomotives and tenders, and b the wheel base of the locomotives and tenders. Then the locomotive excess E is the excess of the weight of the locomotives and tenders over a corresponding length of train. We have then

$$E = W - w_t b.$$

Thus in the system of wheel loads given on page 88, the weight of the locomotives and tenders is $W = 448000$ lbs. The wheel base is $b = 104.3$ ft., and the train load is $w_t = 4000$ lbs. per ft. We have then for the locomotive excess in the same case

$$E = 448000 - 104.3 \times 4000 = 30800 \text{ lbs.}$$

The locomotive excess for any other wheel system is found in the same way.

Let w_e be the equivalent train load, l the length of span, and $M_{\frac{1}{4}}$ the moment at the quarter point due to the wheel loads. Then we have

$$\frac{3}{32} w_e l^2 + \frac{3}{16} El = M_{\frac{1}{4}}$$

Hence we have for the equivalent uniform train load

$$w_e = \frac{32}{3} \frac{M_{\frac{1}{4}}}{l^2} - \frac{2E}{l}.$$

In the table on page 101 we give a comparison of the stresses found by this method with those given by the wheel loads.

METHOD OF CALCULATION BY TWO LOCOMOTIVE EXCESSES AND ACTUAL UNIFORM TRAIN LOAD.—By this method for spans over 100 feet, the actual uniform train load w_t per foot is used instead of the equivalent uniform train load w_e of the preceding method, and placed the same as before for maximum moment and shear.

The actual locomotive excess is computed for each locomotive. The forward locomotive excess is placed as in the preceding method for maximum moment and shear. The next locomotive excess follows at the distance between the two locomotives. If there is but one locomotive, we have of course but one locomotive excess. In such case the placing of this excess would be precisely the same as in the preceding method.

For the system of wheel loads given on page 88 we have then two locomotive excesses of 15400 lbs. instead of one of 30800 lbs. as found on page 99.

COMPARISON OF RESULTS OF DIFFERENT METHODS OF CALCULATION.—We have then the method by wheel loads; the method by equivalent uniform load u; the method by one locomotive excess and equivalent uniform train load w_e; the method by two locomotive excesses and actual uniform train load w_t.

We give in the following table a comparison of the results of these methods for a number of trusses from 100 to 300 ft. span, for the system of wheel loads given on page 88, and also for the system known as "Cooper's Class A, Extra Heavy." This latter has a lighter train and hence the locomotive excess is larger. Computed as on page 99, it is $E = 53500$ lbs. for single excess and 26750 lbs. for two excesses.

The table gives the wheel load stresses in each case for the various members, for one truss, in thousands of pounds. The other results are given by their ratio to the wheel load stresses, so that it may be seen at once how they compare with the wheel load stresses taken as the standard. All results to the nearest decimal.

Thus for the member *ab* for the first span given, the stress due to the wheel loads for the system given on page 88 is 71000 lbs. By the method of equivalent uniform load u the stress is 98.9 per cent of the wheel load stress, and by the method of one excess and equivalent uniform train load, w_e, the same, while for two excesses and actual uniform train load w_t, the stress is 99.7 per cent of the wheel load stress.

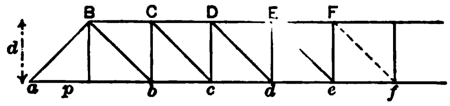
It appears from this table that the methods by equivalent uniform load, and by one excess and equivalent uniform train load, give results fairly close to the more tedious method by wheel loads. The method by one excess and equivalent uniform train load gives on the whole the closest results. The method by two excesses and actual uniform train load gives also fairly close results, but these results are all more or less in excess.

WHY THE METHOD BY TWO EXCESSES AND ACTUAL UNIFORM TRAIN LOAD IS TO BE PREFERRED.—It will be seen from the following table that the method by one excess and equivalent uniform train load gives results on the whole closer to the method by wheel loads than the other methods. We should prefer this method, then, if we regard the method by wheel loads as the standard. This method is accordingly preferred by many engineers.

The student who reads the following chapters of this work will of course be able to use any method which may be desired. But in all our illustrations and numerical examples to follow we shall uniformly give the preference to the last method, viz., by two excesses and actual uniform train load, for the following reasons.

The wheel load system given on page 88, or any other which may be specified, does not represent any actual engines and train, but an imaginary or "typical" system, which is expected to allow for the greatest stresses which may actually occur, with a surplus to cover future increase in the weight of engines and rolling stock.

COMPARISON OF RESULTS OF DIFFERENT METHODS OF CALCULATION.



		Atlantic Coast Line.				Cooper's "Extra Heavy A."			
Span depth, Panel length.	Members.	Wheel Load Stresses in thousands of pounds.	Equivalent Uniform Load u. Ratio to wheel load stresses, per cent.	One Excess Load and w_e. Ratio to wheel load stresses, per cent.	Two Excess Loads and w_t. Ratio to wheel load stresses, per cent.	Wheel Load Stresses in thousands of pounds.	Equivalent Uniform Load u. Ratio to wheel load stresses, per cent.	One Excess Load and w_e. Ratio to wheel load stresses, per cent.	Two Excess Loads and w_t. Ratio to wheel load stresses, per cent.
l = 100 ft.	ab	71.0	98.9	98.9	99.7	67.5	99.1	99.1	106.0
d = 25 "	$BC - bc$	103.0	102.2	102.1	101.5	94.8	105.8	105.8	102.2
p = 20 "	CD	102.0	103.3	103.3	102.6	97.0	103.4	103.4	105.1
	aB	112.2	100.0	100.0	100.9	117.9	90.6	90.3	106.1
	Bb	64.2	104.9	108.6	106.4	62.2	103.0	112.1	112.6
	Cc	31.2	108.0	118.1	111.1	29.2	110.3	135.3	119.0
l = 150 ft.	ab	118.6	95.1	95.1	101.8	110.0	98.6	98.3	105.0
d = 28 "	$BC - bc$	185.9	97.1	97.1	103.5	169.5	102.4	102.0	107.2
p = 25 "	CD	211.4	96.0	96.0	101.4	185.9	105.0	104.6	106.7
	aB	178.4	95.7	94.9	101.6	166.5	97.5	97.3	104.1
	Bb	116.5	96.9	98.8	104.7	110.8	97.6	101.6	107.9
	Cc	67.0	101.2	106.3	110.4	62.6	103.7	114.7	118.4
	Dd	26.5	127.9	140.9	139.6	31.2	104.2	125.9	118.8
l = 200 ft.	ab	135.6	96.8	96.7	101.1	122.7	95.8	95.8	102.7
d = 28 "	$BC - bc$	236.9	98.5	98.5	102.8	211.5	98.9	98.9	105.4
p = 20 "	$CD - cd$	310.0	98.8	98.8	102.9	272.3	100.9	100.8	106.8
	$DE - de$	356.2	98.3	98.3	102.1	312.7	100.3	100.3	105.5
	EF	368.1	99.1	98.9	102.6	322.6	101.3	101.3	105.2
	aB	234.3	96.4	96.4	100.7	209.9	96.5	96.4	103.3
	Bb	186.5	96.8	97.7	101.7	170.5	95.0	97.0	103.9
	Cc	144.0	97.5	99.4	103.1	133.8	94.2	99.3	105.4
	Dd	107.2	98.3	101.5	104.7	100.8	93.7	101.4	107.8
	Ee	75.3	100.0	105.0	107.3	72.0	93.7	105.7	111.0
	Ff	48.6	103.2	111.0	111.8	47.2	95.3	113.5	116.4
l = 200 ft.	ab	150.6	93.4	93.4	96.8	129.7	96.6	96.4	105.2
d = 32 "	$BC - bc$	241.2	100.0	100.0	103.4	214.3	100.2	100.0	106.4
p = 25 "	$CD - cd$	304.0	99.1	99.1	102.3	266.7	100.7	100.5	105.9
	DE	322.0	99.8	99.8	102.6	282.2	101.5	101.3	105.3
	aB	233.7	97.8	97.8	101.3	210.7	96.5	96.4	103.2
	Bb	173.6	98.7	99.7	102.9	159.2	95.8	98.0	105.1
	Cc	121.6	100.6	103.1	105.8	114.7	95.1	100.3	107.4
	Dd	79.6	102.5	107.8	108.8	76.3	95.1	104.7	111.3
	Ee	45.6	107.36	115.4	115.0	43.6	100.0	117.4	120.8
l = 250 ft.	ab	181.7	98.1	98.1	102.0	161.3	96.0	95.9	102.4
d = 32 "	$BC - bc$	320.2	99.0	99.0	102.9	277.4	99.2	99.1	105.6
p = 25 "	$CD - cd$	422.5	98.5	98.4	102.2	361.3	100.0	100.0	106.0
	$DE - de$	480.1	99.0	99.0	102.6	409.2	100.9	100.8	106.3
	EF	495.4	100.0	100.0	103.4	416.4	103.3	103.2	108.0
	aB	266.2	108.8	108.1	109.3	261.4	96.0	96.1	102.7
	Bb	223.5	103.6	104.3	104.4	210.9	95.2	96.8	103.8
	Cc	183.3	98.1	99.8	102.1	165.7	94.3	97.7	104.8
	Dd	128.4	109.1	108.0	106.4	125.1	93.7	99.4	107.0
	Ee	95.8	100.7	104.8	102.2	89.6	93.4	102.2	110.0
	Ff	62.2	103.5	109.8	105.4	59.3	94.1	107.4	114.6
l = 300 ft.	ab	219.2	97.8	97.8	101.7	189.2	97.0	97.1	102.7
d = 38 "	$BC - bc$	385.1	99.0	99.0	102.9	326.6	99.9	99.9	105.5
p = 30 "	$CD - cd$	508.8	98.4	98.4	101.4	426.0	100.5	100.6	105.9
	$DE - de$	578.1	98.9	98.9	102.6	476.8	102.7	102.7	107.7
	EF	600.2	99.3	99.3	102.9	488.4	104.4	104.5	108.9
	aB	354.8	97.5	97.5	101.4	305.2	96.7	97.2	102.8
	Bb	282.5	98.0	98.6	102.4	246.2	95.9	97.5	103.7
	Cc	217.9	98.8	100.1	103.8	192.9	95.2	98.2	105.0
	Dd	161.9	99.7	101.9	105.4	145.9	94.4	99.1	106.7
	Ee	113.7	101.5	104.9	108.0	105.0	93.7	100.7	108.9
	Ff	74.1	103.8	109.1	111.6	69.7	94.1	104.3	113.2

The train is always taken as uniformly distributed in all the methods.

Now the method by wheel loads gives the *static* stresses for the system adopted, whatever it may be, and the methods by equivalent uniform load, and by one excess and equivalent uniform train load, are designed to reproduce, as near as may be, these *static* stresses.

By *static* stresses we mean those due to the system when at rest.

But it is well known, and easily demonstrated by mechanics, that a load instantaneously imposed without impact will produce an effect twice as great as the same load at rest. This result *is not due to impact.* Let, for instance, a load just touch the span and hang by a cord. If that cord is instantaneously cut there is no impact, but the deflection will be twice as great as that due to the same load at rest on the span.

Now the loads we have to deal with are *moving* loads. They are not instantaneously imposed at any point of the span, but they are *suddenly* applied. Their effect therefore must be greater than for the same loads at rest. What allowance is to be made for this action we are not yet in a position to say. Experiments upon this point are very desirable. We do know, however, that the static stresses given by the wheel load system should all be increased.

Reference to the preceding table will show that the stresses obtained by the method of two excesses and actual uniform train load are somewhat greater than those due to the wheel load system.

We therefore prefer this method for spans over 100 feet—

1st, because it is easily and quickly adapted to any given wheel load system, with one or two or more engines and given train load.

2d, because the stresses are easily and quickly found.

3d, because we believe the stresses thus found are superior in accuracy to those found by the wheel load method.

We shall use this method, then, in all our illustrations to follow. For the sake of illustration and for numerical convenience only, we shall assume two excesses of 66,000 lbs. or 33 tons each, 50 feet apart, and a train load of 2000 lbs. or 1 ton per foot.

The student will understand, however, that for any prescribed wheel load system the corresponding excesses must be taken for that system.

Thus for the system of page 88 we have, as computed on page 99, two excesses at 50 feet apart, each one 154,000 lbs., and train load of 4000 lbs. per foot. For Cooper's "Class A, Extra Heavy," we should have two excesses at about 50 feet apart, of 26,750 lbs. each, and train load of 3000 lbs. per foot.

CHAPTER III.

BRIDGE GIRDERS WITH PARALLEL CHORDS—TRIANGULAR GIRDER.

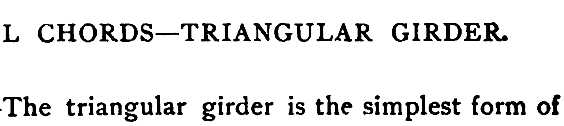
DIFFERENT METHODS OF SOLUTION.—The triangular girder is the simplest form of girder, and we choose it, therefore, as our first example of the application of preceding principles. These principles have given rise to various methods of solution for girders with horizontal chords, some of which are advantageous in some forms of girders, and some in others. We shall give in the present chapter *all* these methods as applied to the same example, and shall then in future chapters, which discuss other forms of girder, choose in each case that method alone which seems best adapted to the case in hand.

We may distinguish four different methods, based upon the principles of the four Chapters of Section I., Part I., viz., the method by graphic resolution of forces, by algebraic resolution of forces, by algebraic method of moments, and by graphic method of moments. The special form which the last two take in the case of parallel chords has been noticed in the preceding Chapter. The application of the first two will be apparent as we proceed.

EXAMPLE FOR SOLUTION.—We shall choose, for convenience merely, a short girder, which will serve to illustrate the methods quite as well as if it were longer.

Let the girder, Fig. 88, be 10 feet high and 80 feet long, having 8 equal panels in the lower chord and 7 in the upper. The live load passes over the lower chord, and the bridge is, therefore, a "through bridge." The bracing consists of isosceles triangles, and hence the angle made by each brace with the vertical is $26° 34'$. Let the dead load be supposed to be known and equal to one half a ton per running foot, and let the live load be taken at one ton per foot.* Our data, then, are as follows:

Fig. 88



$$l = 80, \qquad d = 10, \qquad \theta = 26° 34', \qquad p = 0.5 \text{ ton}, \qquad m = 1.0 \text{ ton};$$

where d = depth of girder, p is dead or permanent load, and m moving load per foot.

Since the length of each panel is 10 feet, we have an apex load of 5 tons for the dead load and 10 tons for the live load. The notation for the various pieces is as represented in Fig. 88.

---

* We do not, therefore, at present take account of the action of locomotive excesses. We shall do that hereafter, page 111. The above loads, it will be noted, are for the entire structure. If there are two trusses, the stresses will be one-half of those found. In this and all following examples the dead load and dimensions are assumed for convenience of calculation and illustration only, and are *not* to be considered as examples of practical cases. We shall see how to estimate dead load and choose best dimensions hereafter. For spans *less than* 100 *feet the method of this chapter should not be used,* but the method by concentrated wheel loads. For spans over 100 feet the method of thischapter, or the method by equivalent uniform load, page 97, or by one excess and equivalent uniform train load, page 99, may be used.

FIRST METHOD—BY GRAPHIC RESOLUTION OF FORCES.

MAXIMUM STRESSES IN THE CHORDS.—According to the principles of the preceding Chapter the chord stresses will be greatest when the girder is fully loaded with both dead and live loads. When this is the case we have at each lower apex a load of $5 + 10 = 15$ tons. The reaction at each end is then 52.5 tons.

We lay off the weights AB, BC, CD, etc., Fig. 89, then the reactions HL and LA, and

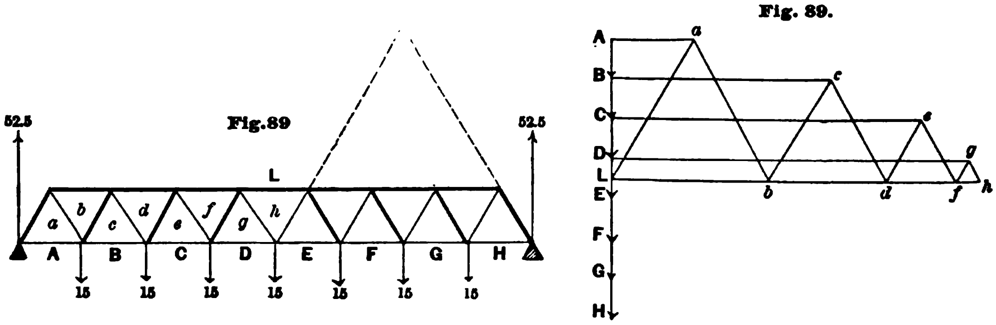


Fig. 89.

then form the stress diagram according to the principles of Chapter I., Section I. The stresses in the chords thus obtained are the greatest which can ever occur. Making the construction, we find

	Lb	Ld	Lf	Lh	Aa	Bc	Ce	Dg
stress	-52.5	-90	-112.5	-120	$+26.25$	$+71.25$	$+101.25$	$+116.25$

It only remains to notice that since the braces are very short they will not give direction very accurately in the stress diagrams. Hence it is well to lay off carefully the directions of the diagonals to a much larger scale, as shown by the dotted lines in Fig. 89, and use these directions in forming the stress diagram.

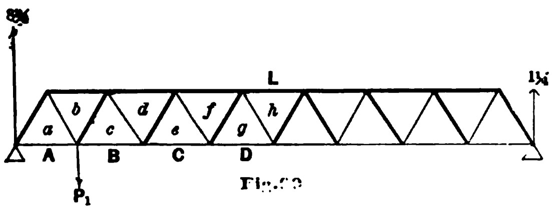


Fig. 90

MAXIMUM STRESSES IN THE BRACES.—In order to find the stresses in the braces we may find the stresses caused by each live load apex weight separately. Tabulating these stresses, we can easily find the dead load stresses and finally the maximum stresses in each brace. Thus, Fig. 90, suppose only the first apex live load of 10 tons to act. The reaction at the left end is then $\frac{7}{8}\ 10 = 8\frac{3}{4}$, and at the right end $\frac{1}{8}\ 10 = 1\frac{1}{4}$. Lay off then AB equal to the weight $P = 10$, and BL and LA equal to the reactions, and form the stress diagram. Scaling off the stresses in the braces, we can enter them in a table as follows:

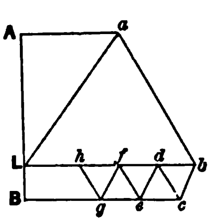


TABLE FOR STRESSES IN THE BRACES.

	La	*ab*	*bc*	*cd*	*de*	*ef*	*fg*	*gh*
P_1	− 9.8	+ 9.8	+ 1.4	− 1.4	+ 1.4	− 1.4	+ 1.4	− 1.4
P_2	− 8.4	+ 8.4	− 8.4	+ 8.4	+ 2.8	− 2.8	+ 2.8	− 2.8
P_3	− 7.0	+ 7.0	− 7.0	+ 7.0	− 7.0	+ 7.0	+ 4.2	− 4.2
P_4	− 5.6	+ 5.6	− 5.6	+ 5.6	− 5.6	+ 5.6	− 5.6	+ 5.6
P_5	− 4.2	+ 4.2	− 4.2	+ 4.2	− 4.2	+ 4.2	− 4.2	+ 4.2
P_6	− 2.8	+ 2.8	− 2.8	+ 2.8	− 2.8	+ 2.8	− 2.8	+ 2.8
P_7	− 1.4	+ 1.4	− 1.4	+ 1.4	− 1.4	+ 1.4	− 1.4	+ 1.4
Live load Comp. −	− 39.2	......	− 29.4	− 1.4	− 21.0	− 4.2	− 14.0	− 8.4
Live load Tens. +	......	+ 39.2	+ 1.4	+ 29.4	+ 4.2	+ 21.0	+ 8.4	+ 14.0
Dead load.	− 19.6	+ 19.6	− 14.0	+ 14.0	− 8.4	+ 8.4	− 2.8	+ 2.8
Max. com. −	− 58.8	.....	− 43.4	......	− 29.4	......	− 16.8	− 5.6
Max. tens. +	......	+ 58.8	......	+ 43.4	......	+ 29.4	+ 5.6	+ 16.8

Thus the first line in the table gives the stresses in all the braces due to the first apex load P_1.

In a similar way we may find and tabulate the stresses due to each of the other apex loads acting separately. This, however, need not involve a separate diagram for each apex load. We can fill up the table directly. Thus, suppose the second weight P_2 to act. It will cause at the right end a reaction twice as great as P_1 caused at that end. The stresses, then, in all the braces to the right of P_2 will be twice as great as they were for P_1. As to their signs, we have only to remember that the two members which meet at the loaded apex *BC*, viz. *cd* and *de*, are both tension (if the load were on the top flange both compression), and the stresses alternate in sign both ways. Thus *de* would be tension and equal to $2 \times 1.4 = +2.8$, *ef* would be -2.8, *fg* $= +2.8$, *gh* -2.8, etc. In similar manner the left hand reaction for P_2 would be $\frac{6}{8}\,10 = 7\frac{1}{2}$, instead of $\frac{7}{8}\,10$ or $8\frac{3}{4}$. The stresses in all the braces to the left of P_2, therefore, are $\frac{6}{7}$ of the stresses caused by P_1 in the braces to the left of it. As to the signs, the same rule is to be observed. Thus the stress in *cd* due to P_2 is tension and equal to $\frac{6}{7} \times 9.8 = +8.4$; for *bc* we have then -8.4, etc. We can, therefore, fill out the table for P_2.

In similar manner for P_3 we have *fg* tension and equal to $3 \times 1.4 = +4.2$. Also *ef* tension and equal to $\frac{5}{7} \times 9.8 = +7$.

For P_4 we have *gh* tension and equal to $\frac{4}{7}$ths of the stress caused by P_1 in the left hand braces, or $\frac{4}{7} \times 9.8 = +5.6$. We can therefore fill out the line for P_4 in the table.

For P_5 we have $\frac{3}{7} \times 9.8 = 4.2$, and by reference to Fig. 90 we see that, starting from the weight and remembering that the braces are alternately tension and compression, *gh* is in tension. We thus fill out the line for P_5 in the table.

In similar manner we fill out the lines for P_6 and P_7.

Our table now contains the stresses in the braces caused by each apex live load, acting separately.

The next two lines give the compression and tension in each member due to the live load. Thus we see at once that all the live loads cause compression in *La*. The live load compression is then 39.2 tons. In the same way we see that the greatest tension on *fg* occurs when only P_1, P_2, and P_3 act, and the greatest compression when P_4, P_5, P_6, and P_7 act. This agrees with our principle in the preceding Chapter, that the stress in any brace is greatest when the live load reaches from the end of the girder to half a bay past the brace.

Having now filled out the lines for live load compression and tension, we can easily find the dead load stresses. The dead load acts at every apex simultaneously, and since it is in the present case half the live load, we have only to take the algebraic sum of all the live load stresses and divide by 2 to obtain the dead load stresses.* We thus fill out the line for dead load stresses at once.

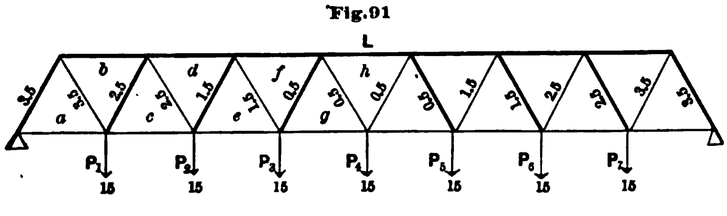
Finally we can find the maximum stresses. Thus for *La* the dead load causes $-$ 19.6 and the live load $-$ 39.2 tons. The maximum is therefore $-$ 58.8 tons. In same way $ab = +58.8$. For *bc* the greatest compression is $-29.4 - 14 = -43.4$. The live load tends to cause a tension of $+$ 1.4 in *bc*, but as this is less than the constant dead load stress of compression it produces no effect. The same holds good for *cd*, *de*, and *fe*. In *fg* the dead load causes compression of 2.8. This, together with the live load compression of 14, gives $-$ 16.8. But the live load may also cause a tension of $+$ 8.4. As this is greater than $-$ 2.8 due to dead load, we must counterbrace *fg* for the difference. Hence the effective tension in *fg* is $+ 8.4 - 2.8 = + 5.6$ tons. We find thus that *fg* and *gh* are the only members which need to be counterbraced, because they are the only braces in which the stress due to dead load is exceeded by the stress of opposite kind due to live load.

We have thus found the maximum stresses in every member of the girder.

SECOND METHOD—BY ALGEBRAIC RESOLUTION OF FORCES.

MAXIMUM STRESSES IN THE CHORDS.—The loading which gives the maximum stresses in the chords is when both dead and live loads cover the whole span, that is, when we have 15 tons at each apex. When this is the case the weight P_4, Fig. 91, being at the centre, we know that it causes a reaction of $\frac{1}{2} P_4$ at each end. That is, the shear, or portion which goes each way through the braces, is 0.5 P_4. This shear multiplied by the secant θ gives the stress in the braces due to P_4 alone, tension for *gh* and alternating toward the left.

Fig. 91



If P_3 acts alone it would cause at the left end a reaction of $\frac{5}{8} P_3$ and at the right end a reaction of $\frac{3}{8} P_3$. But if P_5 acts at the same time, it will cause at the left as much as P_3 causes at the right. Hence when P_3 and P_5 act simultaneously, we can consider that the whole of P_3 goes toward the left end through the braces, and the whole of P_5 toward the right end.

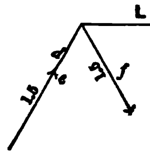
While, then, the stress in *gh* and *gf* would be 0.5 P_4 sec θ, the stresses in *de* and *ef* would be given by 1.5 P_3 sec θ. In the same way for *bc* and *cd* we have 2.5 P_2 sec θ, and for *La* and *ab*, 3.5 P_1 sec θ.

We have accordingly placed upon each brace, in Fig. 91, the coefficients of *P*, which,

* This is on the supposition that the dead load takes effect only at the loaded apices.

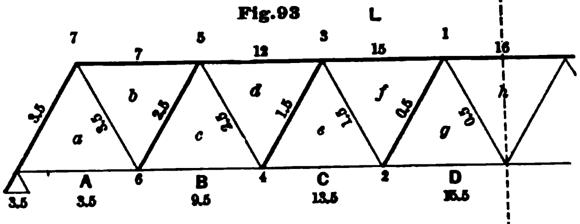
multiplied by P or 15, gives the shear for full load. This shear, multiplied by the sec θ, gives the stress in a brace, multiplied by the tangent θ it gives the stress in the chord. Thus, Fig. 92, we have the stress in ef tension and equal to $1.5\,P \sec\,\theta$. The horizontal component of this stress causes stress of compression in the chord Lf. This horizontal component is $1.5\,P \sec\,\theta \times \sin\,\theta = 1.5\,P \tan\,\theta$. But if ef is tension, de is compression and equal to ef. Hence, de causes also compression in Lf equal to $1.5\,P \tan\,\theta$. The total compression in Lf then is $1.5\,P \tan\,\theta + 1.5\,P \tan\,\theta = 3\,P \tan\,\theta$.

Fig. 92



In general, if we add together the coefficients in Fig. 91, for any two braces which meet at an apex, we shall have the coefficients which multiplied by $P \tan\,\theta$ will give the stress which these braces cause in the chord to the right of them. Thus, Fig. 93, we obtain at the upper apices the coefficients 1, 3, 5, 7, and at the lower apices 2, 4, 6 and 3.5.

Fig. 93



The stress, then, in Lb is $7\,P \tan\,\theta = 7 \times 15 \times 0.5 = -52.5$. In Ld, we have $5\,P \tan\,\theta$ due to the braces bc and cd. But the stress in Lb also acts upon Ld. The total stress in Ld is then $7\,P \tan\,\theta + 5\,P \tan\,\theta = 12\,P \tan\,\theta = 12 \times 15 \times 0.5 = -90$.

If, therefore, commencing at the end, *we add the apex coefficients, and place the results over each chord panel, the coefficients thus determined give the stresses in the chord panels.*

Thus

$$
\begin{aligned}
Lf &= 15\ \ P \tan\,\theta = 15\ \ \times 15 \times 0.5 = -112.5\\
Lh &= 16\ \ P \tan\,\theta = 16\ \ \times 15 \times 0.5 = -120.\\
Aa &= 3.5\,P \tan\,\theta = 3.5 \times 15 \times 0.5 = +\ 26.25\\
Bc &= 9.5\,P \tan\,\theta = 9.5 \times 15 \times 0.5 = +\ 71.25\\
Ce &= 13.5\,P \tan\,\theta = 13.5 \times 15 \times 0.5 = +101.25\\
Dg &= 15.5\,P \tan\,\theta = 15.5 \times 15 \times 0.5 = +116.25
\end{aligned}
$$

These are precisely the same results as those obtained by the preceding method of diagram. The method in the present case is very simple, and involves but little work.

MAXIMUM STRESSES IN THE BRACES.—In order to find the maximum stresses in the braces, we might take each apex live load separately and find the shear which it sends toward each abutment. These shears multiplied by sec θ would give the stresses in braces right and left of the load. We could thus easily form a Table precisely similar to the one on page 105, two simple multiplications only being necessary in order to fill out each line.

Thus let P_1 act alone, Fig. 90. The portion which goes toward the left is $\frac{7}{8}P_1$ and toward the right $\frac{1}{8}P_1$. We have then tension in both ab and bc. For ab we have $+\frac{7}{8}P_1 \sec\,\theta = +\frac{7}{8}\,10 \times 1.117 = +9.8$. For bc we have $\frac{1}{8}P_1 \sec\,\theta = \frac{1}{8}\,10 \times 1.117 = +1.4$. This is enough to fill out the first line in our Table, page 105, for P_1. Other lines can be filled out in similar manner. We have only to remember that the braces which meet at the weight have both the same sign, plus when the weight is below, and minus when it is at the upper apex, and that the signs alternate both ways from the loaded apex.

The Table, page 105, was rendered necessary in order to avoid making a separate diagram for each weight.

In the present case, however, it is unnecessary to draw up a Table at all. We can find the maximum stresses in each diagonal directly.

a. DEAD LOAD STRESSES.—Thus let us first find the dead load stresses. The apex load is 5 tons. We have, then, from our coefficients, Fig. 93,

$$La = 3.5\,P \sec\theta = 3.5 \times 5 \times 1.117 = -\ 19.54.$$

We have La compression because the reaction at its foot is upward. For ab then we have $+19.54$. As the signs are alternately plus and minus,

$$bc = -\ 2.5\,P \sec\theta = -\ 2.5 \times 5 \times 1.117 = -\ 13.96,$$

and $cd = +\ 13.96$.

For de, we have,

$$de = -\ 1.5\,P \sec\theta = -\ 1.5 \times 5 \times 1.117 = -\ 8.38,$$

and $ef = +\ 8.38$.

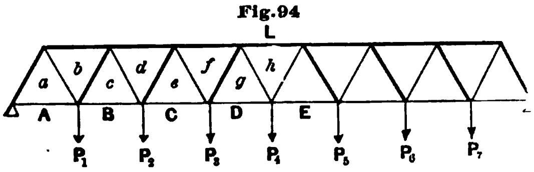
For fg,

$$fg = -\ 0.5\,P \sec\theta = -\ 0.5 \times 5 \times 1.117 = -\ 2.792,$$

and $gh = +\ 2.792$.

These stresses are very closely what we have found for the dead load stresses in our Table, page 105.

Fig. 94



b. LIVE LOAD STRESSES.—The apex live load is 10 tons.

The greatest positive stress for the brace gh will occur when P_4, P_5, P_6, and P_7 act, the other apices being unloaded, Fig. 94. The greatest negative shear for gh will occur when only P_1, P_2, and P_3 act. We have, then, for the positive shear for gh, $(\frac{1}{8}+\frac{2}{8}+\frac{3}{8}+\frac{4}{8})10 = +\ 12\frac{1}{2}$, and for the negative shear $-(\frac{1}{8}+\frac{2}{8}+\frac{3}{8})10 = -\ 7\frac{1}{2}$.

We have, then,

$$-gh\cos\theta + 12\tfrac{1}{2} = 0 \quad \text{or} \quad gh = +\ 12\tfrac{1}{2}\sec\theta = +\ 13.96,$$
$$-gh\cos\theta - 7\tfrac{1}{2} = 0 \quad \text{or} \quad gh = -\ 7\tfrac{1}{2}\sec\theta = -\ 8.38.$$

These are the greatest stresses of each kind the live load can cause in gh.

For fg we have the same stresses only of opposite character, hence $fg = -\ 13.96$, and $+\ 8.38$.

For the brace ef the greatest positive shear is when P_3, P_4, P_5, P_6 and P_7 act, and the greatest negative shear when P_1, P_2 act. We thus find the shears $+\ 18\frac{3}{4}$ and $-\ 3\frac{3}{4}$.

Hence,

$$-ef\cos\theta + 18.75 = 0, \quad \text{or} \quad ef = +\ 20.94,$$
$$-ef\cos\theta - 3.75 = 0, \quad \text{or} \quad ef = -\ 4.19.$$

For de, then, we have $de = -\ 20.94$ or $+\ 4.19$.

For the brace cd the greatest positive shear will be when all the weights except P_1 act. We have then for the shears, $+\ 26\frac{1}{4}$ and $-\ 1\frac{1}{4}$.

Hence,

$$-cd\cos\theta + 26\tfrac{1}{4} = 0, \quad \text{or} \quad cd = +\ 29.32.$$

$$-cd\cos\theta - 1\tfrac{1}{4} = 0, \quad \text{or} \quad cd = -\ 1.4.$$

For bc we have $bc = -29.32$ and $+1.4$.

For the brace ab the greatest shear is positive, and occurs when all the loads act. There is no negative shear. When all the loads act the shear is $+35$.

Hence,

$$-ab\cos\theta + 35 = 0, \text{ or } ab = +39.1.$$

We have, then, $La = -39.1$.

Collecting the above results together, we have the following Table:

TABLE OF STRESSES IN THE BRACES.

		La	ab	bc	cd	de	ef	fg	gh
Dead load.		− 19.54	+ 19.54	− 13.96	+ 13 96	− 8.38	+ 8.38	− 2.8	+ 2.8
Live load.	Comp. −	− 39.1	....	− 29.32	− 1.4	− 20.94	− 4.19	− 13.96	− 8.38
	Tens. +	....	+ 39.1	+ 1.4	+ 29 32	+ 4.19	+ 20.94	+ 8.38	+ 13.96
Max. comp.		− 58.64	....	− 43.28	....	− 29.32	....	− 16.76	− 5.58
Max. tens.		....	+ 58.64	....	+ 43.28	....	+ 29.32	+ 5.58	+ 16.76

The values in this Table agree well with the Table on page 105. The first three lines give the dead load stresses and the live load compression and tension. From these three lines, the last two, which give the maximum stresses, are easily found, just as before.

THIRD METHOD—BY ALGEBRAIC METHOD OF MOMENTS.

MAXIMUM STRESSES IN THE CHORDS.—The point of moments for any chord is at the opposite apex. We take, as before, a full load, or 15 tons per apex. This gives $52\frac{1}{2}$ tons for each reaction. Then, since the depth of truss is 10 feet, and the length of panel 10 feet, we can write down the following equations (see Fig. 94):

For the upper chord panels,

$$-Lb \times 10 - 52.5 \times 10 = 0, \text{ or } Lb \times 10 = -52.5 \times 10,$$

hence $Lb = -52.5$.

In similar manner,

$$-Ld \times 10 - 52.5 \times 20 + 15 \times 10 = 0, \text{ or } Ld = -90,$$

$$-Lf \times 10 - 52.5 \times 30 + 15 \times 20 + 15 \times 10 = 0, \text{ or } Lf = -112.5,$$

$$-Lh \times 10 - 52.5 \times 40 + 15 \times 30 + 15 \times 20 + 15 \times 10 = 0, \text{ or } Lh = -120.$$

For the lower chord panels,

$$Aa \times 10 - 52.5 \times 5 = 0, \text{ or } Aa \times 10 = +52.5 \times 5, \text{ hence } Aa = +26.25.$$

In similar manner,

$$Bc \times 10 - 52.5 \times 15 + 15 \times 5 = 0, \text{ or } Bc = +71.25,$$
$$Ce \times 10 - 52.5 \times 25 + 15 \times 15 + 15 \times 5 = 0, \text{ or } Ce = +101.25,$$
$$Dg \times 10 - 52.5 \times 35 + 15 \times 25 + 15 \times 15 + 15 \times 5 = 0, \text{ or } Dg = +116.25.$$

These values agree perfectly with those already found.

MAXIMUM STRESSES IN THE BRACES.—We have already seen, Fig. 73, page 78, that the application of the method of moments to the braces, gives us for the stress in any brace, as bc, Fig. 73,

$$bc = \text{Shear} \times \sec\theta.$$

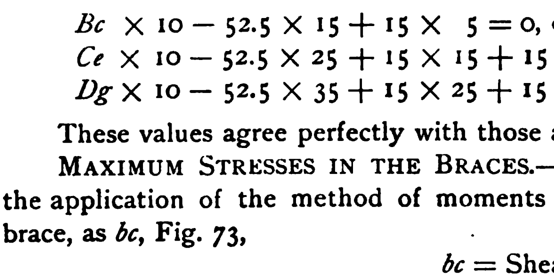
The method for the braces, then, is identical with that of the preceding method, when the chords are horizontal.

FOURTH METHOD—GRAPHIC METHOD OF MOMENTS.

MAXIMUM STRESSES IN THE CHORDS.—The principles of this method are given on pages 44 and 45.

The dead load per unit of length is $p = 0.5$ and the live load $m = 1$. The middle ordinate of the parabola, Fig. 95, is therefore $(p + m)\frac{l^2}{8} = 1.5\frac{80^2}{8} = 1200$. We lay off then, Fig. 95, to any convenient scale $EF = 1200$ and draw the parabola AFB. Drop verticals from the *loaded apices*, and where they intersect the curve, we shall have the apices of the moment polygon. Then to find the moment for any chord panel, as Bc, drop a vertical *from the point of moments for that chord panel*. Thus, in the Figure km, the ordinate by scale from AB to the polygon (*not* to the curve), gives the moment for the chord panel Bc. In like manner the ordinate nO gives the moment for Ld. These moments divided by the depth of truss give the stresses. The division can be at once effected by properly changing the scale of moments. Thus if we lay off $EF = 1200$ to a scale of 600 to an inch, and if we are to divide all the moments by 10, then the ordinates measured to a scale of 60 *tons to an inch* will give the stresses directly.

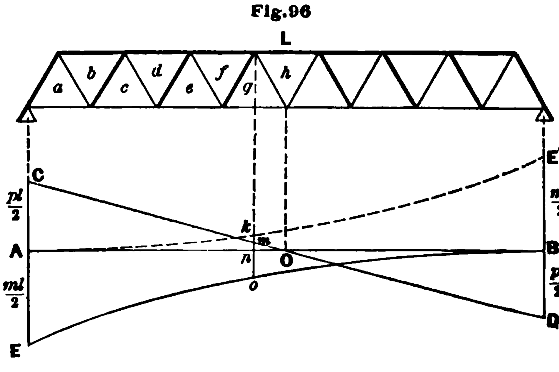
Fig. 95



If the student will make the construction carefully, he will find precisely the same values for the chords as those already obtained by the preceding methods.

MAXIMUM STRESSES IN THE BRACES.—According to the principles of the preceding Chapter, page 82, we draw the shear diagram for dead load, Fig. 96, by laying off $\frac{pl}{2} = \frac{0.5 \times 80}{2} = 20$ at each end of the span and drawing the straight line COD. For the maximum live load shear, we lay off $AE = \frac{ml}{2} = \frac{1 \times 80}{2} = 40$ and draw the parabola EB. This parabola gives the positive shear for load coming on from the right. For load coming on from the left we have the dotted parabola AE', which gives the negative shear for any brace in the left half of truss.

Fig. 96



We have now only to remember that the shear for any brace is given by the ordinate let fall from the *middle* of the panel belonging to that brace. Thus, for gh the greatest positive shear is equal by scale to mo, where mn is the shear due to dead load, and no that due to live load. By our rule

$$gh \cos\theta + \text{shear} = o \text{ or } gh = - \text{Shear} \sec\theta.$$

For gh, as the sec θ is negative and since the shear is positive, $- mo \sec\theta$ will give tension in gh. The sec θ in this case is 1.117. If, then, we divide the scale to which $\frac{pl}{2}$ and $\frac{ml}{2}$ are laid off by 1.117, mo to this new scale will give at once the stress in gh.

In the same way the greatest negative shear due to the live load is kn. But the positive shear due to dead load is mn. The difference, or km, is the effective shear which causes compression in gh. We see, therefore, that only fg and gh, and the corresponding braces on the other side of the centre, require counterbracing. For all the others the dead load positive shear exceeds the maximum negative shear due to live load.

Thus, Fig. 96, we obtain by scale $mn = 2.5$ and $no = 12.65$ or $mo = 15.15$. This multiplied by $\sec\theta = 1.117$ gives 16.9 tension in gh, which agrees with the value already found by the preceding methods. In the same way we find $kn = 7.65$ and $mn = 2.5$, hence $km = 5.15$. This multiplied by $\sec\theta = 1.117$ gives 5.7 tons for compression in gh, which agrees well with the values already found.

STRESSES DUE TO LOCOMOTIVE EXCESS.—In all that precedes we have supposed a uniformly distributed live load of 1 ton per foot. But as we have seen, page 97, we must for spans greater than 100 feet, also take into account the stresses due to the *locomotive excess*. Whatever method, therefore, we adopt, of those just given, the solution is not complete until we have found and properly added the locomotive excess stresses to those already found for uniform live load.

These stresses we now proceed to find.

UPPER CHORD.*—For chord Lb, Fig. 95, we should have a concentrated load equal to the locomotive excess over 1 ton per foot, or 33 tons (page 102) acting at the 1st lower apex, Fig. 95, and another equal load acting at the 6th lower apex or 50 feet from the 1st (page 100). These two loads being conceived to act at these places, we find the left hand reaction easily from

$$- R \times 80 + 33 \times 70 + 33 \times 20 = 0, \text{ or } R = + 37.125.$$

Hence for the stress in Lb, we have

$$- Lb \times 10 - 37.125 \times 10 = 0, \text{ or } Lb = - 37.125.$$

In the same way we have for Ld,

$$- R \times 80 + 33 \times 60 + 33 \times 10 = 0, \text{ or } R = + 28.875,$$

and $$- Ld \times 10 - 28.875 \times 20 = 0, \text{ hence } Ld = - 57.75.$$

$$- R \times 80 + 33 \times 50 = 0, \text{ or } R = + 20.625,$$

and $$- Lf \times 10 - 20.625 \times 30 = 0, \text{ hence } Lf = - 61.875 \text{ tons.}$$

$$- Lh \times 10 - 16.5 \times 40 = 0, \text{ hence } Lh = - 66 \text{ tons.}$$

LOWER CHORD.—For chord Aa, Fig. 95, we have 33 tons at the 1st apex and at the 6th. Hence,

$$Aa \times 10 - 37.125 \times 5 = 0, \text{ or } Aa = + 18.56.$$

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* The method by equivalent uniform load, page 97, may be used instead of the method of this chapter, for spans over 100 feet, or the method by one locomotive excess and equivalent uniform train load, page 99.

For Bc we have,

$$Bc \times 10 - 28.875 \times 15 = 0, \text{ or } Bc = +43.312,$$

$$Ce \times 10 - 20.625 \times 25 = 0, \text{ or } Ce = +51.56,$$

$$Dg \times 10 - 16.5 \times 35 = 0, \text{ or } Dg = +57.75.$$

These stresses must be *added* to those already found for the uniform live load of 2000 lbs. per foot. We see at once how much the chord stresses may be increased owing to the very heavy concentrated loads of the locomotive.

BRACES.—For the braces La and ab we have, according to page 108, the stresses

$$La = -37.125 \times \sec\theta = -37.125 \times 1.117 = -41.47,$$

and

$$ab = +41.47.$$

In similar manner we have,

$$bc = -28.875 \times 1.117 = -32.25 \qquad cd = +32.25,$$

and

$$bc = +4.125 \times 1.117 = +4.6 \qquad cd = -4.6.$$

$$de = -20.625 \times 1.117 = -23.04 \qquad ef = +23.04,$$

and

$$de = +8.25 \times 1.117 = +9.21 \qquad ef = -9.21.$$

$$fg = -16.5 \times 1.117 = -18.4 \qquad gh = +18.4,$$

and

$$fg = +12.375 \times 1.117 = +13.82 \qquad gh = -13.82.$$

These values must be added to the corresponding values found for uniform live load. The actual maximum stresses then are given by the following Table, where the dead load stresses are as before, page 108.

TABLE FOR MAXIMUM STRESSES IN THE BRACES.

		La	ab	bc	cd	de	ef	fg	gh
Live load.	Comp. −	39.1 41.47	....	29.32 32.25	1.4 4.6	20.94 23.04	4.19 9.21	13.82 18.4	8.38 13.82
	Tension +	....	39.1 41.47	1.4 4.6	29.32 32.25	4.19 9.21	20.94 23.04	8.38 13.82	13.82 18.4
Dead load.		− 19.54	+ 19.54	− 13.96	+ 13.96	− 8.38	+ 8.38	− 2.8	+ 2.8
Max. comp.		100.11	....	75.53	....	52.36	5.02	35.02	19.4
Max. tens.		....	100.11	....	75.53	5.02	52.36	19.4	35.02

Comparing these stresses with those found and tabulated on page 109, we see how great an influence the locomotive excess has. We see that de and ef must now also be counterbraced as well as fg and gh. All the stresses are very much increased.

If there are two trusses in the bridge, the stresses in each will be one half of those just found. In general we find the stresses in a truss as if it alone supported the entire load, and then divide these stresses among as many trusses as may compose the bridge.

TABLES UNNECESSARY.—Reviewing all the methods, we see that in the present case the method of calculation by moments (page 109) is decidedly the simplest and best. The Table, page 105, was rendered necessary in order to avoid the necessity of making a separate diagram for each brace. With this exception, the other Tables are unnecessary, and are only given in order to show the relative influence of the dead and train loads and locomotive excess. In practice, we can and should find the maximum stress of either kind upon any member directly by a single equation of moments. We close this Chapter, therefore, by calculating the case in hand in the manner which we recommend for all such cases.

Thus, referring to Fig. 94, let us find once more the maximum stresses. Let the dead load of 5 tons at each apex be x, the train load 10 tons $= y$, and the locomotive excess 33 tons $= z$.

(*a*) STRESSES IN THE CHORDS.

The stresses due to dead load alone are easily found if they are required, as on page 109, for full live load.

For the maximum stresses, we proceed as follows:

At every apex of the lower chord, Fig. 94, let the dead load x and train load y act. We have, then, $x + y = 5 + 10 = 15$ tons at each apex, and the reaction at each end is $\frac{7(x+y)}{2} = 52.5$ tons.

For the panel *Aa*, Fig. 94, we have in addition the locomotive excess of $z = 33$ tons at P_1 and at P_6, 50 feet from P_1. These loads cause a reaction at the left end of $\frac{7}{8}z + \frac{2}{8}z = \frac{9}{8}z = 37.125$ tons. For the panel *Aa*, then, the left reaction is $\frac{7(x+y)}{2} + \frac{9}{8}z = 89.625$ tons, and hence, by moments,

$$Aa \times 10 - \left[\frac{7(x+y)}{2} + \frac{9}{8}z\right] \times 5 = 0, \text{ or } Aa = +44.81.$$

For *Bc* we have z tons at P_2 and at P_7. The left reaction is, therefore, $\left[\frac{7(x+y)}{2} + \frac{7}{8}z\right] = 81.375$ tons. Hence

$$Bc \times 10 - \left[\frac{7(x+y)}{2} + \frac{7}{8}z\right] \times 15 + (x+y)\,5 = 0 \qquad Bc = +114.56.$$

In similar manner,

$$Ce \times 10 - \left[\frac{7(x+y)}{2} + \frac{5}{8}z\right] \times 25 + (x+y)(15+5) = 0 \qquad Ce = +152.81.$$

$$Dg \times 10 - \left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 35 + (x+y)(25+15+5) = 0 \qquad Dg = +174.$$

$$-Lb \times 10 - \left[\frac{7(x+y)}{2} + \frac{9}{8}z\right] \times 10 = 0 \qquad Lb = -89.625.$$

$$-Ld \times 10 - \left[\frac{7(x+y)}{2} + \frac{7}{8}z\right] \times 20 + (x+y)\,10 = 0 \qquad Ld = -147.75.$$

$$-Lf \times 10 - \left[\frac{7(x+y)}{2} + \frac{5}{8}z\right] \times 30 + (x+y)(20+10) = 0 \qquad Lf = -174.375.$$

$$-Lh \times 10 - \left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 40 + (x+y)(30+20+10) = 0 \quad Lh = -186.$$

These are the maximum stresses which can ever occur in the chords.

(*b*) STRESSES IN THE BRACES.

For the greatest tension in *gh*, Fig 94, we have at every lower apex 5 tons $= x$, due to the dead load; also 10 tons $= y$ at all the right hand apices due to train load, and finally, 33 tons $= z$ at P_4 due to the locomotive excess. The left reaction is then $\frac{7x}{2} + \left(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8}\right)y + \frac{z}{2} = \frac{7}{2}x + \frac{10}{8}y + \frac{4z}{8} = 17.5 + 12.5 + 16.5 = 46.5$ tons. The shear for *gh* is, then, the reaction minus the three dead loads at P_1, P_2, and P_3, or $\frac{7}{2}x + \frac{10}{8}y + \frac{z}{2} - 3x = 46.5 - 15 = +31.5$ tons. Therefore,

$$gh = +\left(\frac{7}{2}x + \frac{10}{8}y + \frac{4z}{8} - 3x\right) \sec\theta = +31.5 \times 1.117 = +35.18 \text{ tons.}$$

The greatest compression in *gh* will be when, in addition to the dead load at every lower apex, we have 10 tons $= y$ at P_1, P_2 and P_3 and 33 tons $= z$ and P_3. The reaction at the *right* end is then

$$\tfrac{7}{2}x + \left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8}\right)y + \tfrac{3}{8}z = \tfrac{7}{2}x + \tfrac{6}{8}y + \tfrac{3}{8}z = 17.5 + 7.5 + 12.375 = 37.375.$$

The negative shear for *gh* is then $-37.375 + 4x = -17.375$, and

$$gh = -\left(\tfrac{7}{2}x + \tfrac{6}{8}y + \tfrac{3}{8}z - 4x\right) \sec\theta = -17.375 \times 1.117 = -19.4.$$

For the greatest tension in *ef*, we have, in addition to the dead load of 5 tons $= x$ at every lower apex, 10 tons $= y$ at every right hand lower apex and 33 tons $= z$ at P_3. The *left* reaction is, therefore, $\frac{7}{2}x + \frac{15}{8}y + \frac{5}{8}z = 17.5 + 18.75 + 20.625 = 56.875$. The positive shear is, therefore, $56.875 - 2x = 46.875$.

For the greatest negative shear we have 10 tons $= y$ at P_1 and P_2 and 33 tons $= z$ at P_2. Hence the *right* hand reaction is $\frac{7}{2}x + \frac{3}{8}y + \frac{2}{8}z = 29.50$. The negative shear is therefore $-29.50 + 5x = -4.50$

We have, therefore,

$$ef = +46.875 \times 1.117 = +52.36 \text{ and } ef = -4.5 \times 1.117 = -5.02$$
$$de = -52.36 \qquad\qquad de = +5.02.$$

For tension in *cd* we have, in addition to the dead load, 10 tons at every right hand apex and 33 tons $= z$ at P_2 and at P_1 also. The left hand reaction is then $\frac{7}{2}x + \frac{21}{8} + \frac{7}{8}z$ $= 72.625$ tons, and the positive shear is $72.625 - x = 67.625$. Hence

$$cd = -67.625 \times 1.117 = +75.53 \text{ and } bc = -75.53.$$

For the greatest compression, if any, in *cd*, we have 10 tons at P_1 and also 33 tons at P_1. The reaction at right is then $\frac{7}{2}x + \frac{1}{8}y + \frac{1}{8}z = 22.875$. The negative shear is, therefore, $-22.875 + 6x = +7.125$. As the shear in this case comes out positive, it shows that *cd* is in tension for this loading also. In other words *cd* does not need to be counterbraced. The same holds true for all the remaining braces.

For *ab* we have finally 15 tons at every lower apex and 33 tons at P_1 and at P_6. The reaction at left end is then $52.5 + 37.125 = 89.625$. This is equal to the shear. Therefore,

$$ab = + 89.625 \times 1.117 = + 100.11 \text{ and } La = - 100.11.$$

These values agree well with those given in the Table, page 112.

We see, then, that the maximum stresses may be found directly by this method from a single equation for each member, and no Table is required. Whether a brace is to be counterbraced or not and the stress in the counter, are easily determined.

The above comprises the application of our four methods to a bridge girder sustaining a live load as well as a dead load. In the following Chapters we shall make use of one or the other of these methods, whichever may seem best adapted to the case in hand.

For the method by concentrated wheel loads see page 87.

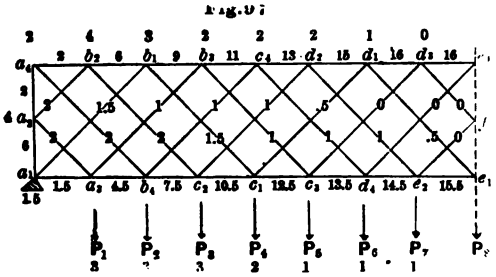
CHAPTER IV.

BRIDGE GIRDERS WITH PARALLEL CHORDS—CONTINUED.

LATTICE GIRDER—EXAMPLE FOR SOLUTION.—As the length of span becomes greater it may be advantageous to have more than one system of bracing, thus reducing the panel length. Such systems, owing to the indeterminate character of the stresses, are usually avoided in practice. Lattice girders may be regarded to-day as antiquated. No more are or will be built.

In Fig. 97 we have represented the half span of a girder with four systems of bracing, load on lower chord. We have the length $l =$ 160 feet, divided into 16 panels of 10 feet each, height of girder $=$ 20 feet, and the braces making an angle of 45° with the vertical. Therefore, $\theta =$ 45°, $\tan\theta = 1$, and $\sec\theta = 1.414$. Let the dead load $p = 0.5$ ton per foot, and the live load $m = 1$ ton per foot. Then the apex dead load is 5 tons, and the apex live load is 10 tons.*

Fig. 97.



(*a*) MAXIMUM STRESSES IN THE CHORDS.—By far the simplest method in the present case is the method by coefficients, explained in the preceding Chapter. Thus, Fig. 97, we write down the coefficients upon the diagonals, which multiplied by $P = 15$, give the shear for full load. Adding the coefficients of the two diagonals which meet at an apex, we obtain the apex coefficients as given in the Figure. Then beginning at the end and proceeding toward the centre, we find by successive addition the chord coefficients, which, multiplied by $P\tan\theta$, give the chord stresses. Since $\tan\theta = 1$ and $P = 15$, $P\tan\theta = 15$.

For the upper chords, all of which are in compression, we have, then, at once,

$$a_4 b_2 = -2 \times 15 = -30, \quad b_2 b_1 = -6 \times 15 = -90,$$

$$b_1 b_3 = -9 \times 15 = -135, \quad b_3 c_4 = -11 \times 15 = -165, \quad c_4 d_2 = -13 \times 15 = -195,$$

$$d_2 d_1 = -15 \times 15 = -225, \quad d_1 d_3 = d_3 e_4 = -16 \times 15 = -240 \text{ tons}.$$

For the lower chords, all of which are in tension, we have,

$$a_1 a_3 = 1.5 \times 15 = +22.5, \quad a_3 b_4 = 4.5 \times 15 = +67.5, \quad b_4 c_2 = 7.5 \times 15 = +112.5,$$

$$c_2 c_1 = 10.5 \times 15 = +157.5, \quad c_1 c_3 = 12.5 \times 15 = +187.5,$$

$$c_3 d_4 = 13.5 \times 15 = +202.5, \quad d_4 e_2 = 14.5 \times 15 = +217.5, \quad e_2 e_1 = 15.5 \times 15 = +232.5.$$

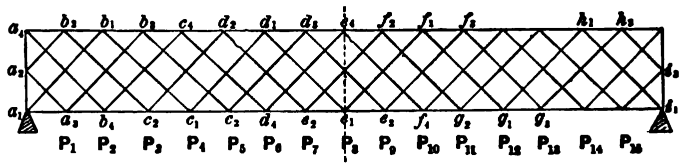
For the end post,

$$a_4 a_2 = -2 \times 15 = -30, \quad a_2 a_1 = -6 \times 15 = -90.$$

* In all our examples dead load and dimensions are assumed for convenience of illustration only, and are not to be considered as practical cases. We shall see how to estimate dead load and best dimensions hereafter. For spans less than 100 feet the method of this Chapter should not be used. For method by concentrated loads see Appendix, page 242. In all cases the method by equivalent uniform load, page 97, may be used for spans over 100 feet, instead of the method of this chapter, or the method by one locomotive excess and equivalent uniform train load, page 99.

(*b*) MAXIMUM STRESSES IN THE BRACES.—The apex live load is 10 tons.

Fig. 98



We find the maximum stresses in the braces due to it by the method of page 114.

Thus, the greatest positive shear for $d_1\ e_1$, Fig. 98, will be when P_8 and P_{12} only act, because these are the only apex weights which act on the system to which $d_1\ e_1$ belongs, on the right of $d_1\ e_1$.

This shear is $(\frac{4}{16} + \frac{8}{16})\ 10 = +7.5$. Hence

$$-d_1\ e_1 \cos\theta + 7.5 = 0, \text{ or } d_1\ e_1 = +7.5 \times 1.414 = +10.6.$$

We therefore have $d_1\ c_1 = -10.6$.

The greatest negative shear for $d_1\ e_1$ will be when P_4 only acts. This shear is $-\frac{4}{16}\ 10 = -2.5$. It causes, therefore, compression in $d_1\ e_1$ equal to $2.5 \times 1.414 = -3.53$.

In $d_1\ c_1 =$ we have then $+3.53$.

For the stress in $d_2\ e_2$, we have, from Fig. 98, the positive shear caused by P_9 and P_{13}, or equal to $(\frac{7}{16} + \frac{3}{16})\ 10 = +6.25$.

The negative shear is when P_5 and P_1 act. It is equal to $(\frac{5}{16} + \frac{1}{16})\ 10 = -3.75$. We have then $d_2\ c_2 = -6.25 \times 1.414 = -8.84$, and $d_2\ c_2 = +3.75 \times 1.414 = +5.3$, and $d_2\ e_2 = +8.84$, and -5.3.

For $e_4 f_4$, the positive shear is when P_{10} and P_{14} act, and the negative shear when P_6 and P_2 act. These shears are $(\frac{6}{16} + \frac{2}{16})\ 10 = +5$ and $(\frac{6}{16} + \frac{2}{16})\ 10 = -5$. The stresses, then, in $e_4 f_4$ are $5 \times 1.414 = -7.07$ and $+7.07$.

For $d_3\ e_3$ we have the positive shear when P_7, P_{11} and P_{15} act, and the negative shear when P_3 alone acts. These shears are $(\frac{9}{16} + \frac{5}{16} + \frac{1}{16})\ 10 = +9.375$ and $-\frac{3}{16}\ 10 = -1.875$.

We have, then,

$$d_3\ e_3 = +9.375 \times 1.414 = +13.26,$$

and

$$d_3\ e_3 = -1.875 \times 1.414 = -2.65.$$

The stresses in $d_3\ c_3$, then, are -13.26 and $+2.65$.

For $c_4\ d_4$ we have in like manner P_6, P_{10} and P_{14}, causing positive shear, and P_2 causing negative shear. The positive shear is then $(\frac{10}{16} + \frac{6}{16} + \frac{2}{16})\ 10 = +11.25$ and the negative is $-\frac{2}{16}\ 10 = -1.25$. Therefore,

$$c_4\ d_4 = +11.25 \times 1.414 = +15.91,$$
$$c_4\ d_4 = -\ \ 1.25 \times 1.414 = -\ \ 1.77.$$

The stresses in $c_4\ b_4$ are -15.91 and $+1.77$.

For $b_3\ c_3$ the positive shear is $(\frac{11}{16} + \frac{7}{16} + \frac{3}{16})\ 10 = +13.125$, and the negative shear is $-\frac{1}{16}\ 10 = -0.625$. Therefore,

$$b_3\ c_3 = +13.125 \times 1.414 = +18.56,$$

$$b_3\ c_3 = -0.625 \times 1.414 = -0.88,$$

and

$$b_3\ a_3 = -18.56, \text{ and } +0.88.$$

For $b_1\ c_1$ the positive shear is caused by P_4, P_8, P_{12}, and is $(\frac{12}{16} + \frac{8}{16} + \frac{4}{16})\ 10 = +15$. The negative shear is zero. Hence,

$$b_1\ c_1 = +15 \times 1.414 = +21.21, \text{ and } b_1\ a_1 = -21.21.$$

For $b_2\ c_2$ the positive shear is caused by P_3, P_7, P_{11}, and P_{15}, and is $(\frac{13}{16} + \frac{9}{16} + \frac{5}{16} + \frac{1}{16})$ $10 = +17.5$. The negative shear is zero. Hence,

$$b_2\ c_2 = +17.5 \times 1.414 = +24.74, \text{ and } b_2\ a_2 = -24.74.$$

For $a_4\ b_4$ the positive shear is $(\frac{4}{16} + \frac{20}{16} + \frac{6}{16} + \frac{2}{16})\ 10 = +20$. Hence,

$$a_4\ b_4 = +20 \times 1.414 = +28.28.$$

For $a_2\ a_2$ the positive shear is when the loads P_1, P_5, P_9, P_{13}, act. The shear then is $(\frac{15}{16} + \frac{11}{16} + \frac{7}{16} + \frac{3}{16})\ 10 = +22.5$. Hence,

$$a_2\ a_2 = +22.5 \times 1.414 = +31.81.$$

We can now collect these results in a Table, as follows:

TABLE OF STRESSES IN THE BRACES.*

	e_4d_4	d_3e_3	d_3c_3	d_1e_1	d_1c_1	d_2e_2	d_2c_2	c_4d_4	c_4b_4	b_3c_3	b_3a_3	b_1c_1	b_1a_1	b_2c_2	b_2a_2
Live load. Comp. −	−7.07	−5.3	−8.84	−3.53	−10.6	−2.65	−13.26	−1.77	−15.91	−0.88	−18.56	....	−21.21	...	−24.74
Live load. Tens. +	+7.07	+8.84	+5.3	+10.6	+3.53	+13.26	+2.65	+15.91	+1.77	+18.56	+0.88	+21.21	....	+24.74	....
Dead load.	0	+1.77	−1.77	+3.5	−3.5	+5.3	−5.3	+7.07	−7.07	+8.84	−8.84	+10.6	−10.6	+12.37	−12.37
Max. comp. −	−7.07	−3.53	−10.61	...	−14.1	....	−18.56	....	−22.98	....	−27.40	....	−31.81	....	−37.11
Max. tens. +	+7.07	+10.61	+3.53	+14.1	....	+18.56	....	+22.98	....	+27.40	....	+31.81	....	+37.11	....

The live load stresses just found give us the first two lines. Since the dead load is one half of the live, the algebraic sum of the first two lines divided by 2, gives the dead load stresses.

The line for dead load being thus filled out, we can find the maximum stresses. We see from the table that $f_2\ e_2$, $e_4\ d_4$, $d_3\ e_3$ and $d_3\ c_3$ are the only diagonals which require counterbracing on the left of the centre. Of course, the stresses are the same in all the corresponding members of the right half of the girder.

In a precisely similar manner we may manage any number of systems.

A double or triple system is generally used when the length of panel for a single system, owing to the increase of height due to great length, renders it advisable to support the chords at more frequent intervals.

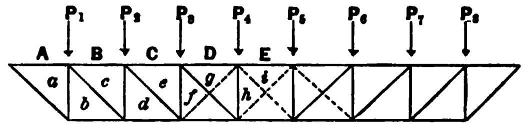
A multiple system, then, such as Fig. 98, when used for a long span, may be calculated as in the preceding pages, *disregarding locomotive excess* and considering the live load as uniformly distributed.† It is not, therefore, in general necessary to take account of locomotive excess. When, however, it is necessary so to do, the method of calculation is explained further on, when treating of the Pratt truss, double system.

* Again we call attention to the fact that a Table is unnecessary (see page 114).

† For the chords, the method of equivalent uniform load, page 97, may be used for long spans over 100 feet; for shear, the method adopted for the Pratt Truss, page 120. Or the method by one locomotive excess and equivalent uniform train load (page 99) may be used.

PRATT TRUSS.—DECK BRIDGE.—Let Fig. 99 represent a Pratt truss 90 feet long, load on the upper chord. The bridge is, therefore, a "deck" bridge. Let the depth of truss be 10 feet, and let there be 9 panels of 10 feet each in the upper chord, and 7 in the lower chord.

Fig. 99



We have then $\theta = 45°$, sec $\theta = 1.414$. Let the train load be 1 ton per foot preceded by two standard locomotives, and the dead load 0.5 ton per foot. Then we have $P = 10$ tons per live panel load, and $P = 5$ tons for uniform panel dead load. Locomotive excess 33 tons. Trains preceded by two locomotives.

In this style of truss, the verticals are to take compression only and the inclined braces tension only. Whenever the live load would tend to cause compression in any inclined brace, that piece must be counterbraced by inserting a brace uniting the other corners of the panel. Those inclined braces which are extended by the action of the dead load, or by a full load live and dead *extending over the whole truss*, are call *ties*. They are represented in Fig. 99 by full lines. The dotted lines denote *counter-ties*, which are only called into play by the live load.

When the truss is fully loaded, the centre of the girder is deflected most, and on each side of the centre the curve is the same. We can always, therefore, tell which are the ties in any case, by considering the deformed panel under full load, and remembering that the tie is the longest diagonal of the deformed panel. In Fig. 99, since we have an odd number of panels, the centre panel is not deformed, but remains a rectangle. Hence the diagonals in it are both counterbraces, and are not strained by full load, at all, but only by partial or live loads not extending over the whole truss.

(*a*) MAXIMUM STRESSES IN THE CHORDS.—Suppose at every upper apex the dead load of $x = 5$ tons, and the train load of $y = 10$ tons always acting, or 15 tons $= x + y$ at each upper apex, Fig. 99. We have, then, only to suppose, in addition to this, the locomotive excess to act at the proper apices for each chord, page 100, and we can find the maximum stresses at once.

Thus for Aa, Fig. 99, we should have the locomotive excess of $z = 33$ tons at P_1 and at P_6. The reaction at the left end due to dead and live loads is, then, $\frac{8(x+y)}{2} = 60$ tons, and due to locomotive excess $\frac{8}{9}z + \frac{3}{9}z = \frac{11}{9}z = 40.33$ tons, or altogether $\frac{8(x+y)}{2} + \frac{11}{9}z = 100.33$ tons. We have, therefore,

$$-Aa \times 10 - \left[4(x+y) + \tfrac{11}{9}z\right] \times 10 = 0 \qquad \text{or } Aa = -100.33$$

$$-Bc \times 10 - \left[4(x+y) + \tfrac{8}{9}z\right] \times 20 + (x+y) \times 10 = 0 \qquad Bc = -171$$

$$-Ce \times 10 - \left[4(x+y) + \tfrac{7}{9}z\right] \times 30 + (x+y)(20+10) = 0 \qquad Ce = -211.98$$

$$-Dg \times 10 - \left[4(x+y) + \tfrac{5}{9}z\right] \times 40 + (x+y)(30+20+10) = 0 \qquad Dg = -223.33$$

$$-Eh \times 10 - \left[4(x+y) + \tfrac{4}{9}z\right] \times 50 + (x+y)(40+30+20+10) = 0 \quad Eh = -223.33$$

It makes no difference which lower apex we take as the centre of moments for Eh the one on the right or the one on the left. Thus

$$-Eh \times 10 - \left[4(x+y) + \tfrac{5}{9}z\right] \times 40 + (x+y)(30+20+10) = 0 \qquad \text{or } Eh = -223.33$$

as before. For the lower chords we have for Lb the locomotive excess at P_1 and P_6. Therefore

$$Lb \times 10 - [4(x+y) + \tfrac{11}{9} z] \times 10 = 0 \qquad Lb = +100.33$$

$$Ld \times 10 - [4(x+y) + \tfrac{8}{9} z] \times 20 + (x+y) \times 10 = 0. \qquad Ld = +171$$

$$Lf \times 10 - [4(x+y) + \tfrac{7}{9} z] \times 30 + (x+y)(20+10) = 0 \qquad Lf = +211.98$$

$$Lh \times 10 - [4(x+y) + \tfrac{5}{9} z] \times 40 + (x+y)(30+20+10) = 0 \quad Lh = +223.33$$

These are the maximum stresses which can ever occur in the chords.

(*b*) MAXIMUM STRESSES IN THE BRACES.—We suppose each upper apex loaded with the dead load $x = 5$ tons. We take the train load $y = 10$ tons, and the locomotive excess $z = 33$ tons, at the proper apices to give the maximum stresses for each brace (page 99).

Thus for the counterbrace hi, Fig. 99, we suppose $y = 10$ tons at P_5, P_6, P_7 and P_8, and $z = 33$ tons at P_5. The left reaction is then $\dfrac{8x}{2} = 20$ tons for dead load, $(\frac{1}{9} + \frac{2}{9} + \frac{3}{9} + \frac{4}{9})$ $y = 11.11$ tons for train load, and $\frac{4}{9} z = 14.66$ tons for locomotive excess, or altogether $4x + \frac{10}{9} y + \frac{4}{9} z = 45.77$ tons. The positive shear for hi is then the left reaction minus all the weights between the left end and P_5, or $4x + \frac{10}{9} y + \frac{4}{9} z - 4x = \frac{10}{9} y + \frac{4}{9} z = 25.77$ tons. We have, therefore,

$$hi \cos \theta_{hi} + 25.77 = 0, \text{ or } hi = +25.77 \times 1.414 = +36.44 \text{ tons.}$$

If there were no other diagonal in the centre panel, the greatest compression on hi would be found by supposing $y = 10$ tons at P_4, P_3, P_2, and P_1, and $z = 33$ tons at P_4. This would cause a compression of 36.44 tons, the same as the tension in the first case. As hi cannot take compression, this stress comes as tension in the other diagonal. The two centre counterbraces are, therefore, subjected to an equal maximum stress of $+ 36.44$ tons for each; under the action of the dead load alone they are not stressed at all. This is in accordance with the principle that for uniform load over the entire span, the shear at the centre is zero (page 82).

The posts are always in compression. The greatest compression on gh will be when the train load extends over the *longer* segment, or when we have $y = 10$ tons at P_4, P_5, P_6, P_7 and P_8, and $z = 33$ tons at P_4, as well as $x = 5$ tons at every upper apex. The shear for this loading will be the greatest compression on gh, and this shear multiplied by 1.414 will be the greatest tension in fg. The left reaction is then $4x + \frac{15}{9} y + \frac{5}{9} z = 55$ tons. The shear is $4x + \frac{15}{9} y + \frac{5}{9} z - 3x = 40$ tons. Hence

$$gh = x + \tfrac{15}{9} y + \tfrac{5}{9} z = -40 \text{ tons.}$$

The same loading gives the greatest tension in fg. Hence

$$fg = +40 \times 1.414 = +56.56 \text{ tons.}$$

For the greatest compression on fg, if any, or in other words the tension in the counterbrace for fg, if any counter is needed, we must have P_1, P_2 and P_3 loaded with 10 tons, and 33 tons at P_3. The left reaction is then $20 + 23.33 + 22 = +65.33$. The shear then is $65.33 - 15 - 15 - 48 = -12.66$ tons. As the shear comes out minus it will cause compression in fg or tension in the counter. Hence

$$fg = -12.66 \times 1.414 = -17.90 \text{ tons.}$$

If the shear in the second case had also come out plus, it would have denoted that

no counter was necessary. In such case both loadings would cause tension in fg, and the greatest would be as above, $+ 56.56$ tons.

In the same way for ef, we have for left reaction $20 + 23.33 + 25.66 = + 69$. The maximum positive shear then is $+ 69 - 10 = 59$. Hence

$$ef = - 59 \text{ tons}.$$

The greatest tension in de is, therefore,

$$de = + 59 \times 1.414 = + 83.43 \text{ tons}.$$

For the load coming on from the left we have left reaction $= 20 + 16.66 + 25.66 = + 62.33$. The shear is then $62.33 - 15 - 48 = - 0.67$. As the shear thus comes out negative in this case, de requires to be counterbraced, and we have $de = - 0.67 \times 1.414 = - 0.95$ tons.

For cd we have in similar manner, left reaction $= 20 + 31.11 + 33 = + 84.11$. The greatest positive shear is, therefore, $84.11 - 5 = + 79.11$. Hence

$$cd = - 79.11.$$

The greatest tension in bc is, therefore,

$$bc = + 79.11 \times 1.414 = + 111.86 \text{ tons}.$$

For the load coming on from the left the shear is positive, and there is no counterbrace needed for bc.

For ab we have 15 tons at every upper apex and 33 tons at P_1 and P_6. Hence the reaction at left is $60 + 40.33 = + 100.33$. As there are no weights between the left end and P_1, this is also the shear. Therefore,

$$ab = - 100.33 \text{ tons}.$$

Finally, the end tie La is

$$La = + 100.33 \times 1.414 = + 141.86 \text{ tons}.$$

These are the maximum stresses in the braces.

If the girder in Fig. 99 is turned over, as shown in Fig. 100(a), the load being still on the top chord, the last vertical cd is a simple rod to support only the centre of the last end panel, which otherwise would have to be of double length. It takes no compression. The continuation of the roadway, shown by bc, is not a part of the truss, neither is the end pillar ba, which, if needed at all, takes only a compression of $\frac{15}{2} + 33 = 40.5$ tons.

Fig. 100 (a)

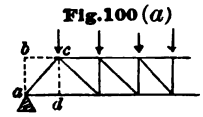
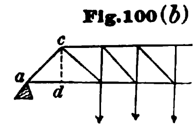
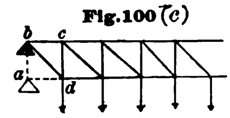


Fig. 100 (b)



If the load is on the bottom chord, Fig. 100(b), the last vertical cd takes tension only, if there is a cross girder at d, to the amount of $5 + 10 + 33 = 48$ tons. If there is no cross girder at d, it merely supports, as in the first case, the centre of the long double panel. If the girder is as in Fig. 99, but with the load on the lower chord, as shown in Fig. 100(c), the continuation of the roadway ad is not a part of the truss. The end pillar, ba, supports half the total weight of truss and train and locomotive at d. The support may be either directly under b or at a.

Fig. 100 (c)



In any of these cases there can be no difficulty experienced in calculating the stresses.

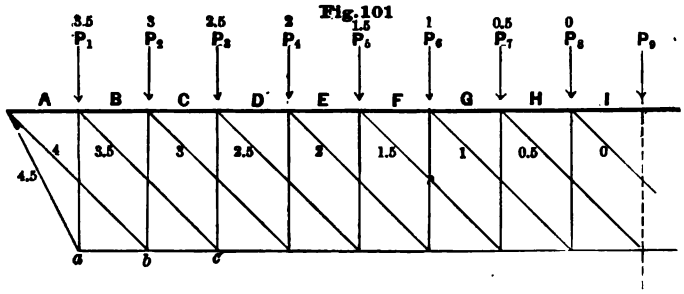
General Method for Vertical and Diagonal Bracing.—In general, then, whatever method of solution we adopt, we consider at first but one system of braces, viz.,

that strained by the dead load alone. Then if we find for any diagonal a stress of opposite kind to that which it is intended to resist, a counter must be inserted to take that stress.

When a member is thus intended to take but one kind of stress, it must be so arranged that it cannot take any other. This is easily attained in practice. Thus, if the posts merely abut on the chords and are not directly united with them, they cannot take tension under any circumstances. Or even if the posts are rigidly connected with the chords, if the ties are rods which run through the chords and are held by nuts on the outer side, they can never take compression. Or again, if posts and ties are connected with the chords and each other, still if the ties are long members of small sectional area, they will not in practice take any great amount of compression, but will bend or buckle and thus bring stress on the counters.

PRATT TRUSS—DOUBLE SYSTEM.*—Let Fig. 101 represent the half span. Let the height of truss be 20 feet, and panel length 10 feet. Length of span 180 feet, divided into 18 panels. Then $\theta = 45°$ and $\tan \theta = 1$ for all the diagonals except the ends, where $\theta = 26° \ 34'$ and $\tan \theta = 0.5$.

Fig. 101



Let the train load be 1 ton per foot, or 10 tons at each upper apex, and dead load be 0.5 ton per foot, or 5 tons at each upper apex. The locomotive excess is 33 tons (page 102). Train preceded by two locomotives.

(*a*) MAXIMUM STRESSES IN THE CHORDS.—We form a diagram of coefficients as shown in Fig. 101, precisely as directed on page 107, Fig. 93. The only difference in this case is, that as the posts are vertical, the component of their stresses in the direction of the flanges will be zero. Hence the coefficient for every post is omitted. In other respects the method is similar.

Thus the stress in A is compression and equal to

$$4.5 \ P \tan \theta + 4 \ P \tan \theta', \text{ where } P = 15 \text{ tons and } \tan \theta' = 1, \ \tan \theta = 0.5.$$

Hence

$$A = -4.5 \times 7.5 - 4 \times 15 = -93.75.$$

For B we have

$$B = -93.75 - 3.5 \times 15 = -146.25.$$

In like manner

$$C = -146.25 - 3 \times 15 = -191.25,$$

$$D = -191.25 - 2.5 \times 15 = -228.75,$$

and so on.

The stresses due to locomotive excess must now be found separately and added. In doing this we must take each system by itself. Thus, for A we have 33 tons at P_1 and at P_6. But P_1 acts on one system and P_6 on the other.

The left reaction for 33 tons at P_1 is $\frac{17}{18} \times 33 = 31.16$, and the centre of moments is at a. For P_6 the reaction is $\frac{12}{18} \times 33 = 22$, and the centre of moments is at b. If the second 33 tons were at P_7 instead of P_6 it would cause less stress at A, because the reaction would be less and its lever arm less. Hence

$$A \times 20 = -31.16 \times 10 - 22 \times 20 \text{ or } A = -37.58.$$

* All double systems, owing to indeterminate stresses, are avoided by the best practice. This system may be regarded as practically antiquated. No more will probably be built in America. When it is desirable to reduce the panel length the "sub-Pratt" is preferable. For method by concentrated loads, see page 252.

In the same way for B, we have 33 tons at P_2 and at P_7. The reaction of P_2 is $\frac{16}{18}$ 33 = 29.33, and of P_7, $\frac{11}{18}$ 33 = 20.16. The centre of moments in the first case is at b, and in the second at c. Hence

$$B \times 20 = -29.33 \times 20 - 20.16 \times 30, \text{ or } B = -59.57.$$

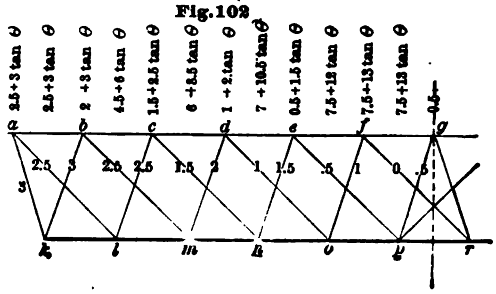
In the same way we can find the stresses in the other panels due to locomotive excess. These must be added to those already formed for dead and train loads, in order to obtain the maximum stresses.

(*b*) MAXIMUM STRESSES IN THE BRACES.—We proceed *for each system* precisely as illustrated in the preceding case, Fig. 99, and in the case of Fig. 98, page 117.

In finding the locomotive excess stresses, we must take *both the loads on the same system*. Thus for the post at P_2, Fig. 101, we have a load of 33 tons at P_2 and another at P_3, and not at P_7, because P_7 belongs to the other system. The student need find no difficulty in solving the case for himself.

POST GIRDER.—Let Fig. 102 represent a Post truss, the span being 120 feet, divided into 12 panels in the upper chord. Depth of truss, then, will be 15 feet. The angle of the ties with the vertical is 45°, and of the inclined posts 18° 26′.

Fig. 102



We have, then, tan $\theta = 1$ for the ties and tan $\theta = 0.333$ for the posts, sec $\theta = 1.414$ for the ties and sec $\theta = 1.054$ for the posts. Let the load be on the top flange and equal 1 ton per foot for live load, and 0.5 ton per foot for dead load. The apex live load is then 10 tons and the apex dead load 5 tons. Locomotive excess, as always, 33 tons (page 102). Train preceded by two locomotives.

(*a*) MAXIMUM STRESSES IN THE CHORDS.—Suppose 15 tons at each upper apex. Then write down the coefficients for each brace as always. But we cannot now add these coefficients in order to find the apex coefficients, because the post and tie do not make equal angles with the vertical.

Thus, Fig. 102, the horizontal component of ak is $3\,P \tan\theta = 3 \times 15 \times 0.333$, and that of al is $2.5\,P \tan\theta' = 2.5 \times 15 \times 1$. If, then, since tan θ for the ties is 1, we denote by θ the angle 18° 25′ of the posts, we have at the apex a the coefficient $2.5 + 3 \tan\theta$, at b, $2 + 3 \tan\theta$, etc., where each of these coefficients is to be multiplied by 15.

The stress, then, in ab is $-(2.5 + 3 \tan\theta)\,15 = -(2.5 + 3 \times 0.33)\,15 = -52.5$. In similar manner we have

$$bc = -(4.5 + 6 \times 0.33)\,15 = -97.5, \qquad cd = -(6 + 8.5 \times 0.33)\,15 = -132.5,$$

$$de = -(7 + 10.5 \times 0.33)\,15 = -157.5, \qquad ef = -(7.5 + 12 \times 0.33)\,15 = -172.5,$$

$$fg = -(7.5 + 13 \times 0.33)\,15 = -177.5.$$

In the same way we can find the stresses on the lower panels, thus:

$$kl = +(6 \times 0.33)\,15 = +30, \qquad lm = +(2.5 + 8.5 \times 0.33)\,15 = +80,$$

$$mn = +(4.5 + 10.5 \times 0.33)\,15 = +120, \qquad no = +(6 + 12 \times 0.33)\,15 = +150,$$

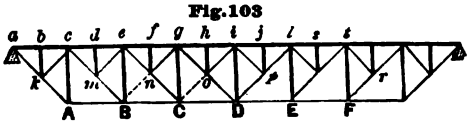
$$op = +(7 + 13 \times 0.33)\,15 = +170. \qquad pr = +(7.5 + 13.5 \times 0.33)\,15 = +180.$$

To these must be added the stresses due to locomotive excess, found precisely as in the preceding case.

(*b*) Maximum Stresses in the Braces.—In order to find the maximum stresses in the braces, we proceed precisely as in the preceding case, page **123**, only remembering to multiply the shear by 1.414 for the ties, and by 1.054 for the struts. The student can easily solve the example for himself. This type is usually used only as a "through" girder. In either case its calculation is simple.

Baltimore Bridge Company's Truss.*—Fig. 103 represents this truss. Let the load be on the upper chord. The length of each panel is 10 feet and there are 16 panels in the upper flange. The depth is 20 feet. All the verticals are posts, and all inclined members ties. The train load is 10 tons for each upper apex and dead load 5 tons. Locomotive excess 33 tons. Train preceded by two locomotives. The angle for the ties is 45°.

Fig.103



(*a*) Maximum Stresses in the Chords.—Supposing 15 tons to act at every upper apex, and taking the locomotive excess at the proper apices as required for each chord, we can easily find the maximum stresses. Thus for AB, Fig. 103, we have 33 tons at c and at h. The centre of moments is at c. Hence,

$$AB \times 20 - 159.94 \times 20 + 15 \times 10 = 0 \qquad AB = +152.44.$$

In similar manner,

$$BC \times 20 - 151.69 \times 40 + 15\,(30 + 20 + 10) = 0 \qquad BC = +258.38,$$

$$CD \times 20 - 143.44 \times 60 + 15\,(50 + 40 + 30 + 20 + 10) = 0 \qquad CD = +317.82.$$

The stresses in ab and bc, cd and de, ef and gh, etc., must always be the same, because the posts bk, dm, fn, etc., being perpendicular to the chords can cause no stress in them.

For the upper chords ab or bc, then, the centre of moments is at k. We have, therefore, 33 tons at b and g. Hence,

$$ab \times 10 = -164.06 \times 10 \qquad ab = bc = -164.06 \text{ tons.}$$

For cd and de, 33 tons at d and i will evidently give the greatest stresses. Taking, then, the centre of moments at B, the intersection of cm and AB, we have

$$cd \times 20 = -155.81 \times 40 + 15\,(30 + 20) \qquad cd = de = -274.12.$$

For ef and fg, we have the locomotive excess at f and l.
Taking the centre of moments at C, we have

$$-ef \times 20 - 147.56 \times 60 + 15\,(50 + 40 + 30 + 20) = 0 \qquad ef = fg = -337.68.$$

For gh and hi, we have 33 tons at h and at t, 50 feet to the right of h. Taking the centre of moments at D, we have

$$-gh \times 20 - 139.31 \times 80 + 15\,(70 + 60 + 50 + 40 + 30 + 20) = 0.$$

Hence, $\qquad gh = hi = -354.74$ tons.

These are the greatest stresses which can ever occur in the chords.

* This truss, or some modification of it, is now usually adopted when it is desired to reduce panel length, instead of the double system Pratt Truss, Post, or lattice. It is sometimes called the Pettit Truss, from the name of its inventor, Robert Pettit. It is more usually designated now by the name of the "sub-Pratt," or "half hitch."

(*b*) MAXIMUM STRESSES IN THE BRACES.—It is evident at once from Fig. 103, that the greatest compression in the intermediate posts, *bk, dm, fn*, etc., is equal to a full panel load, or $5 + 10 + 33 = 48$ tons.

Since *akc*, *cme*, *eng*, etc., are secondary trusses, one half the load on *bk*, *dm*, etc., is carried to *c*, *e*, *g*, etc. Thus, of any load at *d*, for instance, one half goes to the right through *me*, and one half to the left through *mc*. Of the first portion, at *e*, $\frac{12}{16}$ of $\frac{1}{2}$, or $\frac{6}{16}$ of the load at *d*, causes pressure on the left abutment. Of the second portion, at *c*, $\frac{14}{16}$ of $\frac{1}{2}$, or $\frac{7}{16}$ of the load at *d*, causes pressure at the left abutment also. The total pressure at the left abutment due to a load at *d* is, then, $\frac{6}{16} + \frac{7}{16} = \frac{13}{16}$ of that load, just as should be the case by the principle of the lever. The same holds good for any load at *b*, *f*, *h*, etc.

The maximum tension then, in all the secondary ties, *kc*, *me*, *ng*, etc., is

$$\frac{5 + 10 + 33}{2} \sec \theta = 24 \times 1.414 = + 34 \text{ tons.}$$

It remains to find the maximum stresses in the remaining web members.

For *ak* we have 48 tons at *b* and also at *g*, 50 feet back of *b*, and 15 tons at all the other upper apices.

The reaction at left end for this loading is easily found to be $+ 164.06$ tons. Hence

$$ak \cos \theta + 164.06 = 0, \text{ or } ak = + 164.06 \times 1.414 = + 232 \text{ tons.}$$

For *kA* we have 5 tons at *b*, 48 tons at *c* and *h*, and 15 tons at all the other upper apices.

The left reaction for this loading is $+ 150.56$ tons.

The *shear* just to the right of *k* is then $+ 150.56 - 5 = + 145.56$ tons. But a section to the right of *k* cuts *kc* also, as well as *kA*, and since, as we have seen, one half of any load at *b* is transmitted through *kc*, the upward shear in this case due to the stress in *kc* is 2.5 tons.

The *resultant* shear which acts in *kA*, then, is $145.56 + 2.5 = + 148.06$ tons. We have, therefore, taking a section through *bc*, *kc* and *kA*,

$$kA \cos \theta + kc \cos \theta + 145.56 = 0,$$

or, since *kc* is already known to be in tension, and therefore plus, and $kc \cos \theta = 2.5$,

$$kA \cos \theta = - 148.06, \text{ or } kA = + 148.06 \times 1.414 = + 209.36 \text{ tons.}$$

For *cA* we have the same loading as for *kA*, and the greatest compression in *cA* is equal to the shear just found for *kA*, viz.:

$$cA = - 148.06 \text{ tons.}$$

For *cm*, in like manner, we have 5 tons at *b* and *c*, 48 tons at *d* and *i*, and 15 tons at the other apices. The left reaction is $+ 137.69$ tons. The shear to the right of *c* is $137.69 - 10 = 127.69$, and the greatest tension in *cm* is

$$cm = + 127.69 \sec \theta = + 127.69 \times 1.414 = + 180.55 \text{ tons.}$$

For *mB*, we have 5 tons at *b*, *c* and *d*, 48 tons at *e* and *j*, and 15 tons at the other apices. The left reaction is then 125.44. The shear for *mB* is $125.44 - 5 - 5 - 5 + 2.5 = + 112.94$. The greatest tension in *mB* is then

$$+ 112.94 \sec \theta = + 159.7 \text{ tons.}$$

The greatest compression in eB is $-$ 112.94 tons. In the same way we can find the stresses in the other braces.

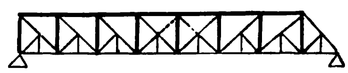
In order to find whether any diagonal, as gD, should be counterbraced, we suppose 48 tons at g and b, 15 tons at c, d, e and f, and 5 tons at the other apices. The left reaction is then $+ 136$. The shear to the right of o is $+ 136 - 48 - 15 - 15 - 15 - 15 - 48 - 5 + 2.5 = - 22.5$ tons.

Since the resultant shear for oD thus comes out negative, it shows that a counter oC is needed. The tension in this counter is $oC = + 22.5 \times 1.414 = + 31.8$ tons.

If the resultant shear had come out positive, no counter would be required. The same method is to be observed for counter Bn. Working out the numerical results, it will be found that no counter for Bn is required.

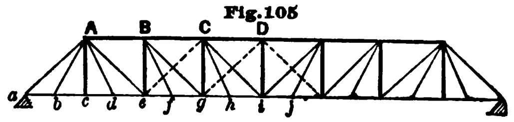
This type of truss is also usually built as a through girder, as shown in Fig. 104, and the ends may be either square or inclined. In either case the calculation offers no special difficulties in view of what has preceded.

Fig. 104



THE KELLOGG TRUSS.—We have represented this truss in Fig. 105. The verticals, except Ac, are posts, and all inclined pieces are ties, except the two ends, which are struts. The truss is 160 feet long, lower chord divided into 16 equal bays of 10 feet each. Height of truss 20 feet, and the angle for the main ties Ci, Bg and Ae is, therefore, 45°. The angle for the secondary ties Ab, Ad, Bf and Ch, is 26° 34′. Hence sec 45° = 1.414 and sec 26° 34′ = 1.118. Let the load be on the bottom chord, 1 ton per foot train load and 0.5 ton per foot dead load. Locomotive excess 33 tons. Hence the apex weights are 10 tons for train and 5 tons for dead load. The train is preceded by two locomotives.

Fig. 105



(*a*) MAXIMUM STRESSES IN THE CHORDS.—Let all the lower apices be loaded with 15 tons. Then for AB, Fig. 105, we have locomotive excess at e and at j. Hence,

$$- AB \times 20 - 151.69 \times 40 + 15\,(30 + 20 + 10) = 0 \qquad AB = - 258.38,$$

$$- BC \times 20 - 144.44 \times 60 + 15\,(50 + 40 + 30 + 20 + 10) = 0 \qquad BC = - 317.82,$$

$$- CD \times 20 - 135.18 \times 80 + 15\,(70 + 60 + 50 + 40 + 30 + 20 + 10) = 0 \qquad CD = - 330.72.$$

For the lower chord ab, bc, cd and de, the centre of moments is at A. For ef and fg at B, for gh and hi at C. For hi, then, we have a locomotive excess at g, and another 50 feet to the right of g. Hence,

$$hi \times 20 - 143.44 \times 60 + 15\,(50 + 40 + 30 + 20) = 0, \text{ or } hi = + 325.32.$$

Observe here particularly, that the moment of the weight at f balances that at h, and the moment of the weight at g is zero.

For gh we have,

$$gh \times 20 - 143.44 \times 60 + 15\,(50 + 40 + 30 + 20 + 10) = 0 \qquad gh = + 317.82$$

In similar manner,

$$fg \times 20 - 151.69 \times 40 + 15\,(30 + 20) = 0 \qquad fg = + 265.88,$$

$$ef \times 20 - 151.69 \times 40 + 15\,(30 + 20 + 10) = 0 \qquad ef = + 258.38,$$

$$de \times 20 - 159.93 \times 20 = 0 \qquad de = +159.93,$$

$$cd \times 20 - 159.93 \times 20 + 15 \times 10 = 0 \qquad bc = cd = +152.43,$$

$$bc \times 20 - 159.93 \times 20 + 15 \times 10 = 0$$

$$ab \times 20 - 159.93 \times 20 = 0 \qquad ab = de = +159.93.$$

These are the maximum stresses which can ever occur in the chords under the action of the assumed loads.

(*b*) MAXIMUM STRESSES IN THE BRACES.—The secondary ties, Ch, Bf, Ad and Ab, Fig. 105, have simply to support a full panel load, or $15 + 33 = 48$ tons. They are all in tension then, and the greatest stress which can ever occur in each of them is

$$+48 \sec \theta = +48 \times 1.118 = +53.66 \text{ tons.}$$

Ac is also a tie, and the greatest stress is a full panel load, or $+48$ tons.

For Ci we have the shears $+47.67$ and -33.56, the first when the train is on the right hand half and the locomotive excess is at i and 50 feet to the right of i, the dead load acting at every lower apex. The second when the load reaches from the left up to and including h, the locomotive excess being at h and c. Hence,

$$Ci = +47.67 \times 1.414 = +67.4, \text{ and } Ci = -33.56 \times 1.414 = -47.45.$$

Ci must, therefore, be counterbraced, and the stress in the counter gD is $+47.45$. This gives the compression in $Di = -33.56$.

For Cg the greatest compression is when the train advances to h. We have, therefore,

$$Cg = -62.43 \text{ tons.}$$

In similar manner for Bg we have the shears $+77.81$ and -7.18. Hence,

$$Bg = +77.81 \times 1.414 = +110.02 \qquad Bg = -7.18 \times 1.414 = -10.15.$$

Therefore, Bg must be counterbraced, and the stress in the counter Ce is $+10.15$. For Be, the greatest compression is when the train advances to f. Hence,

$$Be = -93.81.$$

For Ae we have the shear $+110.44$; there is no negative shear. Hence,

$$Ae = +110.44 \times 1.414 = +156.16.$$

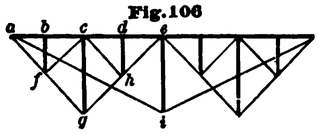
For Ac the stress as already remarked is $+48$ tons.

For Aa we have the shear $+164.06$. Hence,

$$Aa = -164.06 \times 1.414 = -232 \text{ tons.}$$

FINK TRUSS.—We have represented this truss in Fig. 106. The span is 80 feet, divided into 8 equal panels of 10 feet each. The vertical pieces, ei, dh, cg and bf, are all posts, and will take only compression. All the inclined pieces are ties. We take $ei = cg = 20$ feet. Then the angle θ which ag and eg make with the vertical is 45°, and the angle which ai makes with the vertical is 63° 26′. Hence sec 45° = 1.414, cos 45° = 0.70711, sec 63° 26′ = 2.236, cos 63° 26′ = 0.44724, sin 63° 26′ = 0.89441. The lengths of dh and bf are each 10 feet.

Fig. 106



We take 1 ton per foot train load and 0.5 ton per foot dead load, or 10 and 5 tons per apex respectively. Locomotive excess 33 tons.

We see at once from the Figure that the greatest stress which can come on the short posts, *bf* and *dh*, is a full panel load. Hence $bf = dh = 15 + 33 = -48$ tons.

We see also that every apex load causes stress in *ei* and *ai*. The greatest stresses in these pieces will then be for 15 tons at each upper apex, and 33 tons locomotive excess at *e*. We can easily find the stress in *ai* for this loading by moments. Thus, for a section cutting *de*, *he* and *ai*, the centre of moments for *ai* is at *e*. The lever arm for *ai* is

$$ei \times \sin 63^\circ\, 26' = 20 \times 0.89441 = 17.8882.$$

For train and dead load, then, since the reaction is 52.5 at the left end, we have,

$$ai \times 17.8882 - 52.5 \times 40 + 15\,(30 + 20 + 10) = 0, \text{ or}$$

$$ai = + \frac{1200}{17.8882} = + 67.07 \text{ tons.}$$

For the locomotive excess at *e*, we have

$$ai \times 17.8882 - 16.5 \times 40 = 0, \text{ or } ai = + 36.9 \text{ tons.}$$

For the stress in *ei*, we have for train and dead loads,

$$ei = -2\, ai \cos 63^\circ\, 26' = -2 \times 67.07 \times 0.44724 = -60.$$

Therefore, the load upon *ei* is equal to four apex loads. This is also evident from Fig. 106. For since the point *e* is supported by means of *ei* and *ai*, we can consider the secondary truss *age* as an independent truss supported at *a* and *e*. Therefore a load at *b* of 15 tons causes at *e* a pressure of $\frac{1}{4}$th of 15, at *c* $\frac{1}{2}$, and at *d* $\frac{3}{4}$ths of 15 tons. Hence we have at $e\,(\frac{1}{4} + \frac{1}{2} + \frac{3}{4})\,15$. The secondary truss on the right causes an equal pressure. Finally, we have 15 tons at *e*. Therefore, $2\,(\frac{1}{4} + \frac{1}{2} + \frac{3}{4})\,15 + 15 = 3 \times 15 + 15 = 4 \times 15 = 60$, is the pressure upon *ei*.

The locomotive excess at *e* causes in *ei* a compression of 33 tons.

If *ai* is known we can easily find the stress in *af*. Thus for train and dead load the reaction at left end is 52.5 tons. But of this the tie *ai* furnishes $ai \cos 63^\circ\, 26' = 30$ tons, leaving only $52.5 - 30 = 22.5$ to be supplied by *af*. We have, then,

$$af \cos 45^\circ = + 22.5, \text{ or } af = + 22.5 \times 1.414 = + 31.815 \text{ tons.}$$

For locomotive excess the stress in *af* will be greatest for 33 tons at *b*. We have, then,

$$af = + \tfrac{3}{4}\, 33 \times 1.414 = + 35 \text{ tons.}$$

The stress in *ab* is equal to the sum of the horizontal components of *ai* and *af*. Hence for train and dead loads

$$ab = -67.07 \sin 63^\circ\, 26' - 31.815 \sin 45^\circ = -82.5.$$

The stress in *bc* is evidently the same as in *ab*.

For locomotive excess, the stress in *ab* will be greatest for 33 tons at *e*. Hence,

$$bc = ab = -36.9 \sin 63^\circ\, 26' = -33 \text{ tons.}$$

We easily find *fc* by resolving b*f* into *af* and *fc*. Thus for train and dead loads,

$$fc = + 15 \cos 45^\circ = + 15 \times 0.70711 = + 10.60.$$

For locomotive excess,

$$fc = +33 \cos 45^\circ = +23.33.$$

The shear at f which causes stress in fg is the algebraic sum of the vertical components of the stresses in af, fc, and the stress in bf. We have this shear for train and dead loads equal to

$$-15 + af \cos \theta_{af} + fc \cos \theta_{cf}.$$

Substituting numerical values,

$$-15 - 31.815 \times -0.70711 - 1060 \times -0.70711 = -15 + 22.5 + 7.5 = +15.$$

The stress in fg then is,

$$fg = +15 \times 1.414 = +21.21.$$

For locomotive excess at c, we have 33 tons in cg. Hence,

$$fg = +16.5 \times 1.414 = +23.33.$$

For cg we have for train and live loads,

$$cg = -2\,fg \cos 45^\circ = -2 \times 21.21 \times 0.70711 = -30,$$

or cg sustains 2 apex weights. For locomotive excess $cg = -33$ tons. By reason of the symmetry of the Figure we have $gh = fg$, $ch = cf$, and $he = af$.

For de we can take moments about i. The stress in he is $+31.81$ for live and dead loads, and its lever arm is 14.1422. Hence,

$$-de \times 20 - 52.5 \times 40 + 15(30 + 20 + 10) - 31\ 81 \times 14.1422 = 0,$$

or

$$de = -82.5.$$

This is precisely the same as the stress already found for ab.

In this form of truss, then, *the stress in the upper chord is uniform from end to end, and the stresses in all the braces are greatest for train load over the entire span.*

To recapitulate, we have,

$bf = dh = -15 - 33 = -48$ tons,	$cg = -30 - 33 = -63$ tons,
$ei = -60 - 33 = -93$ tons,	$ai = +67.07 + 36.9 = +103.97$ tons,
$af = he = +31.815 + 35 = +66.815$ tons,	$fc = ch = +10.6 + 23.33 = +33.93$,
$fg = gh = +21.21 + 23.33 = +44.54$,	$ab = bc = cd = de = -82.5 - 33 = -115.5$ tons.

CONCLUDING REMARKS.—Our principles, if comprehended, will render easy the solution of any other form which ingenuity may suggest. The stresses found in all our examples are in excess of general practice. This is due to the assumption of an engine weighing 90,000 lbs. on a 12-foot base. The methods and principles remain the same, whatever assumption be made in this respect. The bill reported by the Joint Committee of the Ohio Legislature, appointed to investigate the Ashtabula accident, recommends the adoption of

a standard locomotive weighing 91,200 lbs. on a 12½-foot wheel base. As locomotives exceeding this in weight are in use on some roads, and the tendency is to greater loads, we do not consider our assumed load as excessive. Taking, as we do, the length of locomotive and tender at 50 feet, and 2,000 lbs. per foot over the 38 feet not covered by the drivers, we have for weight of locomotive and tender $90,000 + 38 \times 2,000 = 166,000$ lbs. This is not an excessive estimate of our large engines of to-day.

As all members expand or contract under the influence of heat and cold in direct proportion to their length, it is not customary to consider temperature as having any influence upon the stresses. It will be sufficient to rest one end of the truss upon friction rollers, so as to allow of change of length. As no deformation is caused, no stresses are caused. If it be required to find the stresses for a moving system of concentrated loads, our methods remain the same, regard being had to the principles of pages 89 and 91.

We have chosen in each case that method which seems best adapted to give the required results. But the student is by no means limited to the methods of procedure laid down. Thus it is unnecessary to form Tables as on pages 105, 109, 118, etc., giving the dead and live load stresses and locomotive excess stresses separately. The maximum stresses in any member may be found by the method of moments by a single equation for each member, the dead load, live load and locomotive excess being taken as all acting together at the proper apices to give the maximum stress. We have, as we have seen, Chapter III., page 103, four methods, either of which may be employed.

We attempt no comparison of the different types of trusses. Practical details of construction and extra material required for stiffening long struts affect the cost and quantity of materials to such an extent, that a comparison based upon stresses alone is often misleading. The bill of materials is the best means of comparison, and this the student is not yet prepared to draw up. Even then a comparison for a given length only would be imperfect, as a girder which compares unfavorably for one length may often give a better result for another. Comparison of well-executed designs and the results of practice are the only reliable tests. Estimated by this standard, the single intersection Pratt Truss is, in all respects, the best and most common. For spans up to about 65 feet the best practice gives the preference to the plate girder. This length requires two ordinary flat cars 33 feet long for transport. The span is sometimes increased to a maximum of 90, which requires three cars. Riveted Warren Girders, when used at all now, are also limited to short spans, intermediate between the longest plate girders and the shortest pin-connected spans, say between 60 and 120 feet. They possess the advantage of superior rigidity for short spans over pin-connected trusses, but less security and rigidity than plate girders, as faulty rivets make a greater reduction of strength. Plate girders are also cheaper up to 65 feet, cost less for maintenance, and possess fewer corners and recesses for the accumulation of dirt and moisture, and are therefore cleaner and less exposed to oxidation. As to pin-connected trusses, the old forms of Bollman, Fink, Kellogg, and Post have become wholly absolete. The double intersection Pratt or Whipple is disappearing also. The best practice avoids, as much as possible, all double systems of bracing, owing to the indeterminate character of the stresses. As already stated, the single intersection Pratt, or, for long spans, some modification of the Baltimore Truss or "sub-Pratt" are the standard forms.

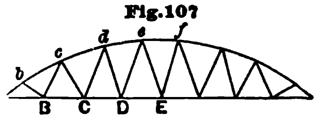
For the proper computation of the cases of this Chapter, for concentrated load system, see Appendix, page 243. The student who wishes the most recent practice ***should not proceed further*** till he has checked the results there given. The method by locomotive excess, here given, is seldom used for spans less than 100 feet.

CHAPTER V.

BRIDGE GIRDERS WITH INCLINED CHORDS.

BOWSTRING GIRDER.—In Fig. 107 we have represented a bowstring girder with isosceles bracing. The span is 120 feet, divided into 8 equal panels of 15 feet each. The bow is a polygon whose apices *Abcdef* lie upon a circle whose depth at the centre is 20 feet. As the upper panels are of course straight, the centre depth of the inscribed polygon is 19.74 feet instead of 20 feet. We take the train load at 1 ton per foot, or 15 tons per lower apex, and the dead load at 0.5 ton per foot, or 7.5 tons per lower apex. The train is preceded by two locomotives. Since the bracing is isosceles, the apices *e*, *d*, *c*, etc., are vertically over the centre of each lower panel, and the horizontal projection of each upper panel is constant and equal to 15 feet, except the two end upper panels, whose horizontal projection is 7.5 feet.*

Fig. 107



(a) The Chords.

METHOD OF CALCULATION.—The maximum stresses in the chords occur for a full load or 22.5 tons at each lower apex, together with the locomotive excess at the proper apices for each panel. Perhaps the simplest and readiest method of solution for all such cases of curved chords, is to diagram the stresses according to the method of Section I., Chapter I., page 8, as illustrated by Fig. 61, page 61. The readiest method of calculation is by moments, according to the principles of Section I., Chapter III.

(b) The Braces.

For the braces we can diagram the stresses caused by a single live load weight at *B*, and then form a Table as explained on page 105. The best method of calculation is by the principles of page 113.

We shall calculate the stresses in the members and leave the checking of them by diagram to the student.

LEVER ARMS AND ANGLES OF INCLINATION.—Before proceeding to calculate, it is necessary to know the lever arms for the panels and the angles of inclination of the various members with the vertical. This, the dimensions being given, is a simple trigonometrical operation. Much time may often be saved, however, by carefully drawing the frame in Fig. 107 to scale. The lever arms can then be measured directly from the drawing with all requisite accuracy and without the possibility of error.

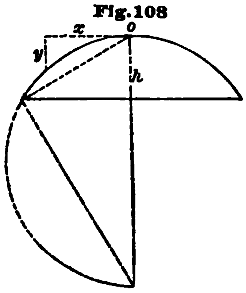
We shall, however, calculate all the necessary data in the present case, as an example for all.

---

* In all our examples dead load and dimensions are assumed for convenience of illustration only, and are not to be regarded as practical cases. We shall see how to estimate dead load and choose best dimensions hereafter. For spans less than 100 feet the method of this Chapter should not be used. For method by concentrated loads see Appendix, page 243. In all cases the method by equivalent uniform load, page 97, may be used for spans over 100 feet, instead of the method of this chapter, or the method by one locomotive excess and equivalent uniform train load (page 99).

If the curve of the bow is a circle, as shown in Fig. 108, where $S =$ the length of span and $h =$ the height of arc at centre of span, we can easily determine the radius r of the arc from the proportion

Fig. 108



$$2r - h : \frac{S}{2} :: \frac{S}{2} : h,$$

or

$$r = \frac{h}{2} + \frac{S^2}{8h}. \quad \ldots \ldots \ldots \ldots (1)$$

Taking the crown o as an origin, we have, from the well known equation of the circle, $x^2 = 2ry - y^2$,

$$y = r - \sqrt{r^2 - x^2}. \quad \ldots \ldots \ldots \ldots (2)$$

If the curve, Fig. 108, is a parabola, we have, from the well known equation of the parabola, $x^2 = 2py$, hence

$$y = \frac{x^2}{2p}. \quad \ldots \ldots \ldots \ldots (3)$$

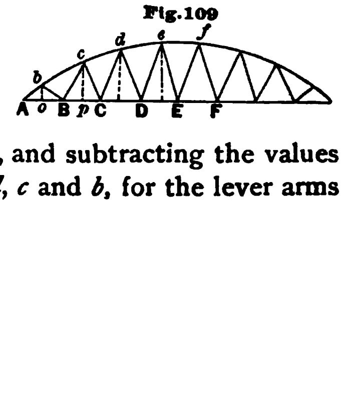
where we have

$$p = \frac{S^2}{8h}. \quad \ldots \ldots \ldots \ldots (4)$$

From these equations we can always find y for any point on the curve, that is for any apex in Fig. 107. Subtracting then y thus found from h, *we shall have the lever arms for the lower flanges.*

We find thus in the present case, Fig. 109, since $S = 120$, $h = 20$, the radius of the circle

Fig. 109



$$r = 10 + \frac{14,400}{160} = 100 \text{ feet.}$$

Making then $x = 7.5$, 22.5, 37.5, 52.5 in equation (2) above, and subtracting the values of y thus found from h, we have the verticals let fall from e, d, c and b, for the lever arms for the lower panels. Thus

lever arm for $DE = 19.74$ feet,

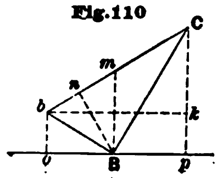
lever arm for $CD = 17.43$ feet,

lever arm for $BC = 12.7$ feet,

lever arm for $AB = 5.11$ feet.

For the upper panels, take, for instance, the panel bc, Fig. 110. We have just found $bo = 5.11$ and $cp = 12.7$. We have, then, $mB = \frac{5.11 + 12.7}{2} = 8.905$. The lever arm nB, then, is equal to $mB \cos mBn = 8.905 \cos mBn$. But the angle $bmBn$ is equal to the angle Cbk.

Fig. 110



But $\tan Cbk = \frac{Ck}{bk} = \frac{12.7 - 5.11}{15} = \frac{7.59}{15} = 0.506$. Hence the

angle $Cbk = mBn = 26° 50'$. We have, therefore, the lever arm of $bc = 8.905 \times 0.89232 = 7.95$ feet.

In similar manner we find

$$\text{lever arm of } Ab = 8.43 \text{ feet,}$$

$$\text{lever arm of } cd = 14.36 \text{ feet,}$$

$$\text{lever arm of } de = 18.37 \text{ feet,}$$

$$\text{lever arm of } ef = 19.74 \text{ feet.}$$

Denoting by θ the angle made by any member *with the vertical*, we find easily

$$\theta_{de} = 81° 15', \theta_{cd} = 72° 27', \theta_{bc} = 63° 10', \theta_{Ab} = 55° 47',$$

$$\theta_{Bb} = 55° 46', \theta_{Bc} = \theta_{Cc} = 30° 34', \theta_{Cd} = \theta_{Dd} = 23° 17, \theta_{De} = \theta_{Ee} = 20° 48'.$$

Collecting these results, we have for the data necessary for calculation, length of span = 120 feet; panel length = 15 feet; apex live load = 15 tons; apex dead load = 7.5 tons

For the lever arms of the chords we have

	DE	CD	BC	AB	Ab	bc	cd	de	ef
lever arm =	19.74 ft.	17.43	12.7	5.11	8.43	7.95	14.36	18.37	19.74

For the angle θ of pieces with the vertical

	de	cd	bc	$Ab = Bb$	$Bc = Cc$	$Cd = Dd$	$De = Ee$
$\theta =$	81° 15′	72° 27′	63° 10′	55° 47′	30° 34′	23° 17′	20° 48′
$\cos\theta =$	0.15212	0.30154	0.45140	0.56232	0.86104	0.91856	0.93483

We are now ready for the calculation.

Calculation of Stresses in the Members.

(*a*) *Maximum Stresses in the Chords.*—For the chords consider a full load of $y + x = 15 + 7.5 = 22.5$ tons at each lower apex. Then we have, since the reaction at each end is 78.75 tons, the following equations.

For the lower chord, Fig. 109, we have for the panel AB, in addition to the above load, the locomotive excess $z = 33$ tons at the apex B and 50 feet to the right of B. Fifty feet to the right of B gives a point between E and F. We take the second locomotive excess therefore at F, the first apex beyond. We have then

$$AB \times 5.11 - \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 7.5 = 0, \qquad AB = +176.12 \text{ tons,}$$

$$BC \times 12.7 - \left[\frac{7(x+y)}{2} + \frac{8}{8}z\right] \times 22.5 + (x+y) \times 7.5 = 0, \qquad BC = +184.7 \text{ tons,}$$

$$CD \times 17.43 - \left[\frac{7(x+y)}{2} + \frac{6}{8}z\right] \times 37.5 + (x+y)(22.5+7.5) = 0, \qquad CD = +183.95 \text{ tons,}$$

$$DE \times 19.74 - \left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 52.5 + (x+y)(37.5+22.5+7.5) = 0, DE = +176.38 \text{ tons.}$$

In similar manner for the upper chord we have

$$-Ab \times 8.43 - \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 15 = 0 \qquad Ab = -213.52$$

$$-bc \times 7.95 - \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 15 = 0 \qquad bc = -226.41$$

$$-cd \times 14.36 - \left[\frac{7(x+y)}{2} + \frac{8}{8}z\right] \times 30 + (x+y) \times 15 = 0 \qquad cd = -210$$

$$-de \times 18.37 - \left[\frac{7(x+y)}{2} + \frac{6}{8}z\right] \times 45 + (x+y)(30+15) = 0 \qquad de = -198.42$$

$$-ef \times 19.74 - \left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 60 + (x+y)(45+30+15) = 0 \qquad ef = -186.9$$

We see that the maximum stresses in the chords, especially in the upper chord, are very nearly uniform.

(*b*) *Maximum Stresses in the Braces.*

We find the stresses in the braces according to the method of page 16. Thus the greatest tension in Ee, Fig. 109, will be when all the lower apices on the right are loaded with $x+y = 22.5$ tons, those on the left with $x = 7.5$ tons, and the locomotive excess $z = 33$ tons is at E. When we have this loading, since ef and DE are horizontal, the vertical components of their stresses are zero, and the stress in Ee will be the shear, or the left reaction minus $3x$, multiplied by the sec θ_{Ee}. The left reaction is

$$\frac{7x}{2} + \left(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8}\right)y + \frac{4}{8}z = \frac{7}{2}x + \frac{10}{8}y + \frac{4}{8}z = 61.5 \text{ tons.}$$

The shear is, therefore, $61.5 - 3x = +39$ tons. Hence

$$Ee = +39 \sec \theta_{Ee} = +39 \times 1.0697 = +41.72 \text{ tons.}$$

The greatest compression on Ee will be when the left apices are loaded with $x+y = 22.5$ tons, the right with $x = 7.5$ and the locomotive excess $z = 33$ tons is at D. For this loading the left reaction is $\frac{7}{2}x + (\frac{7}{8} + \frac{6}{8} + \frac{5}{8})y + \frac{5}{8}z = 80.625$. The shear is $80.625 - 3(x+y) - z = -19.875$. Hence

$$Ee = -19.875 \times 1.0697 = -21.26.$$

In order to find De, consider a section cutting de, De, and DE. Then, according to the principles of page 16, the algebraic sum of the vertical components of the stresses in the cut pieces must be in equilibrium with the shear. The plus shear for De is the same as for Ee just found, viz., $+39$ tons, and the minus shear is -19.875 tons. We have, then, since DE is horizontal and the vertical component of its stress zero,

$$De \cos \theta_{De} + de \cos \theta_{de} + 39 = 0. \quad \ldots\ldots\ldots\ldots \quad (a)$$

$$De \cos \theta_{De} + de \cos \theta_{de} - 19.875 = 0 \quad \ldots\ldots\ldots\ldots \quad (b)$$

These equations give the stress in De when the train extends from the right to E, or from the left to D, provided we know the stresses in the chord de for these loadings. These we can easily find by moments. Thus in the first case,

$$de \times 18.37 = -61.5 \times 45 + 7.5(30 + 15), \qquad \text{or} \quad de = -132.28,$$

and in the second case,

$$de \times 18.37 = -80.625 \times 45 + 22.5(30 + 15), \quad \text{or} \quad de = -142.38.$$

These values inserted in equations (a) and (b) will enable us to find the stresses in De. We must remember to measure θ according to our rule, page 16. We have, then, from (a) and (b),

$$De \times +0.93483 - 132.28 \times +0.15242 + 39 = 0, \qquad \text{or} \quad De = -20.15,$$

$$De \times +0.93483 - 142.38 \times +0.15242 - 19.875 = 0, \quad \text{or} \quad De = +44.47.$$

For Dd the reactions at the left end for the two methods of loading which give maximum stresses are, when the train reaches from right end to D, $\frac{7}{2}x + \frac{15}{8}y + \frac{8}{8}z = 79.125$, and when the train reaches from left end to C, $\frac{7}{2}x + \frac{18}{8}y + \frac{8}{8}z = 75.375$ tons. The shear, then, for Dd and Cd is $79.125 - 2x = +64.125$ in the first case, and $75.375 - 2(x + y) - 33 = -2.625$.

The corresponding values of de are given by

$$de \times 18.37 = -79.125 \times 45 + 7.5(30 + 15), \qquad \text{or} \quad de = -175.15,$$

$$de \times 18.37 = -75.375 \times 45 + 22.5(30 + 15) + 33 + 15, \quad \text{or} \quad de = -102.57.$$

Observe that in the second of these equations we must introduce the moment of the locomotive excess at C, Fig. 109. Hence,

$$Dd \times -0.91856 - 175.15 \times 0.15242 + 64.125 = 0, \quad \text{or} \quad Dd = +42.9,$$

$$Dd \times -0.91856 - 102.57 \times 0.15242 - 2.625 = 0, \quad \text{or} \quad Dd = -19.8.$$

For Cd we have the same reactions and shears as for Dd. We find first cd for each case of loading. Thus,

$$cd \times 14.36 = -79.125 \times 30 + 7.5 \times 15, \quad \text{or} \quad cd = -157.46,$$

$$cd \times 14.36 = -75.375 \times 30 + 22.5 \times 15, \quad \text{or} \quad cd = -141.8.$$

Observe that since the point of moments is now at C, Fig. 109, the moment of the locomotive excess does *not* enter the second equation. Hence,

$$Cd \times +0.91856 - 157.46 \times +0.30154 + 64.125 = 0, \qquad Cd = -18.12,$$

$$Cd \times +0.91856 - 141.8 \times +0.30154 - 2.625 = 0, \qquad Cd = +49.4.$$

For Cc we have the left hand reaction $\frac{7}{2}x + \frac{21}{8}y + \frac{8}{8}z = 98.625$, and $\frac{7}{2}x + \frac{7}{8}y + \frac{7}{8}z = 68.25$, and the shears $98.625 - x = +91.125$, and $68.25 - (x + y) - z = +12.75$.

We first find *cd*. Thus,

$$cd \times 14.36 = -98.625 \times 30 + 7.5 \times 15, \qquad cd = -198.2 \text{ tons.}$$

$$cd \times 14.36 = -68.25 \times 30 + 55.5 \times 15, \qquad cd = -84.61 \text{ "}$$

Hence,

$$Cc \times -0.86104 - 198.2 \times 0.30154 + 91.125 = 0, \qquad Cc = +36.4 \text{ tons.}$$

$$Cc \times -0.86104 - 84.61 \times 0.30154 + 12.75 = 0, \qquad Cc = -14.62 \text{ "}$$

For *Bc* we have the same reactions and shears as for *Cc*. Therefore,

$$bc \times 7.95 = -98.625 \times 15, \text{ or } bc = -186.08,$$

$$bc \times 7.95 = -68.25 \times 15, \text{ or } bc = -128.77.$$

$$Bc \times +0.86104 - 186.08 \times +0.45140 + 91.125 = 0, \qquad Bc = -8.28 \text{ tons.}$$

$$Bc \times +0.86104 - 128.77 \times +0.45140 + 12.75 = 0, \qquad Bc = +52.70 \text{ "}$$

For *Bb* we have the left reaction $= 119.75$,

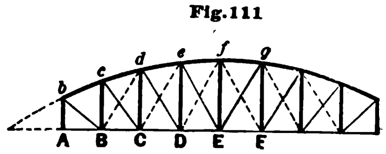
$$bc \times 7.95 = -119.75 \times 15, \text{ or } bc = -226 \text{ tons,}$$

$$Bb \times -0.56256 - 226 \times 0.45140 + 119.75 = 0, \text{ or } Bb = +32.05.$$

BOWSTRING SUITED FOR LONG SPANS.—If we were to find the stresses due to dead load alone, we should find that all the braces are in tension. As the span increases, therefore, the dead load stresses will increase while the live load remains always the same. It is evident that for a very long span the dead load tension may be greater than the compression in any brace due to live load. In such case the braces will always be in tension. Triangular bracing, such as is shown in Fig. 109, is then the best, as we thus have no long struts and can save the extra material required for stiffening. For a short span, such as the present, vertical posts and inclined ties are preferable, as then each member has to resist only one kind of stress.

TRUNCATED BOWSTRING.—In Fig. 111 we have represented a form of truss which, for lack of a better name, we shall call the "truncated bowstring," because it resembles a bowstring with the ends cut off.

Fig. 111



Let the span be 120 feet, divided into 8 panels of 15 feet each, and the bracing be vertical and diagonal, as shown in the Figure. The vertical braces take compression only, and the inclined braces tension. The load is on the lower chord, and equal to 1 ton per foot for live load and 0.5 ton per foot for dead load, or 15 tons per apex for live load, and 7.5 tons per apex for dead load. The locomotive excess is 33 tons. The train is preceded by two locomotives.

The upper chord has its apices in a parabola, the height of truss at centre being 20 feet, and at ends 10 feet. The rise of the parabola at centre, therefore, is 10 feet, and the equation of the curve, page 132, is

$$y = \frac{4hx^2}{s^2},$$

where s is the span, h the rise at centre, and x the distance of any point right or left of the highest point f. In the present case this becomes

$$y = \frac{x^2}{360}.$$

LEVER ARMS AND ANGLES OF INCLINATION.—If we make a section cutting ef, eE and ED, Fig. 111, the centre of moments for ED is at e, the intersection of the other two strained members cut. This section may really cut four pieces, viz., the counter Df, as well as the others named. We consider, however, only that system of bracing which would be called into play by the action of the dead load only, shown in the Figure by the full lines. If, then, we find any diagonal of this system in compression, the amount of compression is the tension in its counterbrace. If any post is found to be in tension, that is the compression upon it caused by the counter.

The point of moments, therefore, for CD is at d, for BC at c, etc.

We obtain, then, the lever arms for the lower panels by substituting $x = 15$, 30, 45, and 60, in the equation

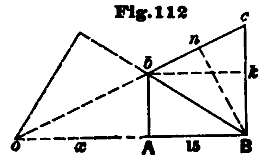
$$y = 20 - \frac{x^2}{360}.$$

We thus find for the lever arms of the lower panels,

Lower panels,	AB,	BC,	CD,	DE,
Lever arm =	10	14.375	17.5	19.375 feet.

The point of moments for the upper panel bc is at B, for cd at C, etc. The lever arm nB, then, for any upper panel, as bc, Fig. 112, is $cB \cos cBn$. But cB is already found. The angle cBn is equal to the angle cbk. The tan of this angle is $\frac{ck}{bk}$. The difference between cB and bA gives ck, and bk is known to be 15 feet. We thus find for bc, $ck = 4.375$, $cbk = cBn = 16° 14'$, hence $nB = 14.375 \times \cos 16° 14' = 14.375 \times 0.96013 = 13.8$ feet.

Fig. 112



In like manner we find

$$\text{lever arm of } cd = 17.5 \times \cos 11° 46' = 17.5 \times 0.979 = 17.13,$$

$$\text{lever arm of } de = 19.375 \cos 7° 8' = 19.375 \times 0.9923 = 19.22,$$

$$\text{lever arm of } ef = 20 \cos 2° 23' = 20 \times 0.99913 = 19.98.$$

The centre of moments for the vertical bA, Fig. 112, will be at o, where bc meets AB produced. The distance $oA = x$ may be easily found from the proportion,

$$x : Ab :: x + 15 : Bc, \quad \text{or } x : 10 :: x + 15 : 14.375.$$

Hence,

$$x = \frac{150}{4.375} = 34.285.$$

We have, then, from Fig. 111, for the lever arm for cB, $34.285 + 15 = 49.285$.

In the same way we have for the lever arm of dC,

$$x : 17.5 :: x - 15 : 14.375, \quad \text{or } x = 84 \text{ feet.}$$

Lever arm of eD,

$$x : 19.375 :: x - 15 : 17.5, \quad \text{or } x = 155 \text{ feet.}$$

Lever arm of fE,

$$x : 20 :: x - 15 : 19.375, \quad \text{or } x = 480 \text{ feet.}$$

In order to find the lever arms for the inclined braces, we see from Fig. 112 that the lever arm for $bB = (x + 15) \sin bBA$. The angle bBA is easily found to be $33° 41'$, hence for lever arm of bB,

$$(34.285 + 15) \sin 33° 41' = 49.285 \times 0.5546 = 27.33 \text{ feet.}$$

Lever arm of cC,

$$84 \sin 43° 59' = 84 \times 0.69445 = 58.33 \text{ feet.}$$

Lever arm of dD,

$$155 \sin 49° 24' = 155 \times 0.75927 = 117.69 \text{ feet.}$$

Lever arm of eE,

$$480 \sin 52° 15' = 480 \times 0.79069 = 379.53 \text{ feet.}$$

We have, then, the following lever arms, Fig. 111:

Lower chords,	AB	BC	CD	DE	
Lever arms,	10	14.375	17.5	19.375.	
Upper chords,	bc	cd	de	ef	
Lever arms,	13.8	17.13	19.22	19.98.	
Vertical braces,	bA	cB	dC	eD	fE
Lever arms,	34.285	49.285	84	155	480.
Inclined braces,	bB	cC	dD	eE	
Lever arms,	27.33	58.33	117.69	379.53.	

We are now ready for the calculation.

Calculation of Stresses.

(*a*) *Maximum Stresses in the Chords.*—Suppose at each lower apex, Fig. 111, $x + y = 22.5$ tons, and take the locomotive excess at the proper apices for each flange. Thus for the flange BC, we have $z = 33$ tons at B and at F, Fig. 111. Therefore,

$$AB \times 10 - \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 0 = 0, \qquad AB = 0.$$

$$BC \times 14.375 - \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 15 = 0, \qquad BC = +125.2.$$

$$CD \times 17.5 - \left[\frac{7(x+y)}{2} + \frac{8}{8}z\right] \times 30 + (x+y) \times 15 \times 0, \qquad CD = +172.3.$$

$$DE \times 19.375 - \left[\frac{7(x+y)}{2} + \frac{6}{8}z\right] \times 45 + (x+y)(30+15) = 0, \qquad DE = +188.1.$$

$$-bc \times 13.8 - \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 15 = 0, \qquad bc = -130.43.$$

$$-cd \times 17.13 - \left[\frac{7(x+y)}{2} + \frac{8}{8}z\right] \times 30 + (x+y) \times 15 = 0, \qquad cd = -176.$$

$$-de \times 19.2 - \left[\frac{7(x+y)}{2} + \frac{6}{8}z\right] \times 45 + (x+y)(30+15) = 0, \qquad de = -189.6.$$

$$-ef \times 19.9 - \left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 60 + (x+y)(45+30+15 = 0, \qquad ef = -185.$$

(*b*) *Maximum Stresses in the Braces.*—We shall find the stresses in the braces by moments also in this case. Thus, Fig. 111, the centre of moments for Bb is at the intersection of bc and AB, of cB at the same point, of cC at the intersection of cd and BC, and so on. Remembering, then, the rule (page 26) for positive and negative rotation, we can write down an equation of moments which shall give directly the stress in any brace for that loading which causes the maximum stress. Thus for bA we have $x + y = 22.5$ tons at every lower apex, and $z = 33$ tons at B and F. The reaction at left end is, therefore 120 tons. Hence

$$bA \times 34.285 + 120 \times 34.285 = 0, \qquad bA = -120.$$

$$-bB \times 27.33 + 120 \times 34.285 = 0, \qquad bB = +150.5.$$

For cB and cC we have the reaction at the left when the train reaches from the right end up to C, 26.25 tons for dead load, 39.375 tons for train load, and 33 tons due to locomotive excess, or 98.625 tons. Hence,

$$cB \times 49.285 + 98.625 \times 34.285 - 7.5 \times 49.285 = 0, \qquad cB = -61.13.$$

$$-cC \times 58.33 + 98.625 \times 54 - 7.5 \times 69 = 0, \qquad cC = +82.46.$$

In the same way,

$$dC \times 84 + 79.125 \times 54 - 7.5(69 + 84) = 0, \qquad dC = -37.20.$$

$$-dD \times 117.69 + 79.125 \times 110 - 7.5(125 + 140) = 0, \qquad dD = +57.06.$$

We have also for the train coming on from the other end

$$-dD \times 117.69 + 74.125 \times 110 - 22.5 \times 125 - 55.5 \times 140 = 0, \qquad dD = -19.47.$$

So also for cC we have

$$-cC \times 58.33 + 68.5 \times 54 - 55.5 \times 69 = 0, \qquad cC = -2.23.$$

Again, for eD and Ee, we have

$$eD \times 155 + 61.50 \times 110 - 7.5(125 + 140 + 155) = 0, \qquad eD = -23.32.$$

$$-eE \times 379.53 + 61.50 \times 420 - 7.5(435 + 450 + 465) = 0, \qquad eE = +41.3.$$

For train coming on from left,

$$-eE \times 379.53 + 80.625 \times 420 - 22.5(435 + 450) - 55.5 \times 465 = 0, \qquad eE = -31.2.$$

Finally, for fE the greatest compression will be when the train comes on either from right or left as far as F or D. In either case,

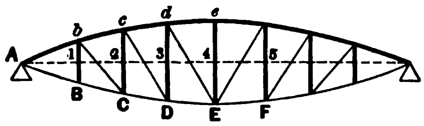
$$fE \times 480 + 49.875 \times 420 - 7.5(435 + 450 + 465 + 480) = 0, \qquad fE = -15.$$

These are the maximum stresses which can ever come upon the braces. We see that cC, dD and eE must be counterbraced, or the diagonals dB, eC and fD, Fig. 111, must be inserted, the stresses in these counters being the compression which would otherwise occur in the ties.

For any case of vertical and diagonal bracing, we simply find, as above, the stresses in the system strained by the dead load alone, and then if any tie of this system is found to have compression in it due to live load, this compression is the stress in the counter. The posts will then always be in compression, and the greatest compression will be when the train covers the longest segment of the span.

BOWSTRING SUSPENSION.—We have represented in Fig. 113 this form of truss, sometimes called the *double bow*, or *lenticular girder*.

Fig. 113



We choose the same span as in the preceding cases, viz. 120 feet, divided into 8 panels of 15 feet each. Vertical and diagonal bracing. Train load 1 ton per foot, or 15 tons per panel, dead load 0.5 ton per foot, or 7.5 tons per panel. Locomotive excess 33 tons. Train preceded by two locomotives. Each bow is a polygon inscribed in a parabola, the centre rise of which is 10 feet.

The equation (page 132)

$$h - y = 20 - \frac{x^2}{180}$$

gives, therefore, the length of the verticals eE, dD, etc., for corresponding values of x, measured horizontally from the crown e.

The load is supposed to pass along the centre line. The verticals take compression only, and the diagonals tension only.

LEVER ARMS AND ANGLES OF INCLINATION.—We find easily, by inserting $x = 0$, 15, 30, 45 in the equation

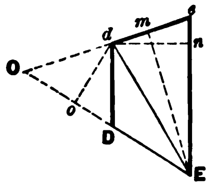
$$h - y = 20 - \frac{x^2}{180},$$

the length of the verticals, *viz.*,

eE	dD	cC	bB
20	18.75	15	8.75

The centre of moments for each panel is at the opposite upper or lower apex.

Fig. 114



For the panel de, Fig. 114, the lever arm $Em = Ee \cos eEm$. The angle $eEm = edn$, and this latter angle can be easily found, since its tangent $= \frac{en}{dn} = \frac{eE - Dd}{2 \times 15} = \frac{1.25}{30} = 0.041666$. Hence, $Em = 20 \cos 2° 23'$ $= 20 \times 0.99913 = 19.98$. We have, similarly, the lever arm of DE or do $= Dd \cos 2° 23' = 18.75 \times 0.99913 = 18.73$.

In like manner we find for

	cd	CD	bc	BC	Ab	AB
lever arm =	18.6	14.88	14.68	8.57	8.4	8.4

The distance of the point of intersection of de and DE from the vertical Ee is easily found from the proportion

$$x : 20 :: x - 15 : 18.75, \quad \text{or } x = 240 \text{ feet.}$$

In the same way we find the intersection of cd and CD distant from dD, 75 feet; of bc and BC, from cC, 36 feet; and of Ab and AB from bB, 15 feet.

The angle which dE makes with the vertical is easily found. Thus, Fig. 114, $\tan \theta_{aE} = \frac{dn}{nE} = \frac{15}{19.375} = 0.77420$, hence $\theta_{aE} = 37° 45'$. In the same way we find

$$\theta_{cD} = 41° 38', \qquad \theta_{bC} = 51° 38'.$$

We can now find the lever arms of the diagonals. Thus, for dE we have $232.258 \cos 37° 45' = 232.258 \times 0.79068 = 183.64$ feet. In the same way we find for cD, $66.66 \times \cos 41°38' = 49.83$, and for bC, $26.526 \cos 51° 38' = 16.46$.

The point of moments for the vertical bB is at the intersection of Ab and BC, because these are the two flanges cut by a section through Ab, bB and BC. The lever arm for bB, therefore, is 17.5 feet.

The point of moments for cC is at the intersection of bc and CD.* Hence the lever arm for cC is 45 feet.

The point of moments for dD is at the intersection of cd and DE. Hence the lever arm for dD is 112.5 feet.

The point of moments for eE is at the intersection of de and EF. Since these are parallel, the lever arm for eE is infinitely great.

To recapitulate, then, we have the following lever arms:

	de	DE	cd	CD	bc	BC	Ab	AB
lever arms	19.98	18.73	18.6	14.88	14.68	8.57	8.4	8.4

	bB	$bC = Bc$	cC	$cD = Cd$	dD	$dE = De$	eE
lever arms,	17.5	16.46	45	49.83	112.5	183.64	∞

Intersection of de and DE 180 feet to the left of A.
" " cd and CD 30 " " " " " "
" " bc and BC 6 " " " " " "

We are now ready for the calculation.

CALCULATION OF STRESSES IN THE MEMBERS.

(*a*) *Stresses in the Chords.*—The student should draw a Figure similar to Fig. 113 and mark upon it plainly the above lever arms and intersections. With this before him, he can check easily the following equations.

For the chords we suppose 22.5 tons at each cross-girder, 1, 2, 3, and 4, etc., and let the locomotive excess act at the proper points for each flange. Thus for de we have 33 tons at 4, Fig. 113. Hence,

$$-de \times 19.98 - 95.25 \times 60 + 22.5\,(45 + 30 + 15) = 0, \qquad de = -184.6 \text{ tons.}$$

$$-cd \times 18.6 - 103.5 \times 45 + 22.5\,(30 + 15) = 0, \qquad cd = -194.7 \;\text{"}$$

$$-bc \times 14.68 - 111.75 \times 30 + 22.5 \times 15 = 0, \qquad bc = -205.3 \;\text{"}$$

* The distance x of the point of intersection of any two panels, as bc and CD, on opposite sides of the vertical cC, is given by $x = \frac{2p\overline{c2}}{\overline{D3} - \overline{b1}}$, where p is the panel length.

$- Ab \times 8.4 - 120 \times 15 = 0,$ $\qquad Ab = - 214.3$ tons.

$DE \times 18.73 - 103.5 \times 45 + 22.5\,(30 + 15) = 0,$ $\qquad DE = + 194.6$ "

$CD \times 14.88 - 111.75 \times 30 + 22.5 \times 15 = 0,$ $\qquad CD = + 202.6$ "

$BC \times 8.57 - 120 \times 15 = 0,$ $\qquad BC = + 210$ "

$AB \times 8.4 - 120 \times 15 = 0,$ $\qquad AB = + 214.3$ "

(*b*) *Stresses in the Braces.*—The inclined braces are also easily found by moments. Thus,

$- dE \times 183.64 + 61.50 \times 180 - 7.5\,(195 + 210 + 225) = 0,$ $\qquad dE = + 34.55$ tons.

$- dE \times 183.64 + 80.625 \times 180 - 22.5\,(195 + 210) - 55.5 \times 225 = 0,$ $dE = - 39.13$ "

$- cD \times 49.83 + 79.125 \times 30 - 7.5\,(45 + 60) = 0,$ $\qquad cD = + 31.83$ "

$- cD \times 49.83 + 75.375 \times 30 - 22.5 \times 45 - 55.5 \times 60 = 0,$ $\qquad cD = - 41.78$ "

$- bC \times 16.46 + 98.625 \times 6 - 7.5 \times 21 = 0,$ $\qquad bC = + 26.38$ "

$- bC \times 16.46 + 68.5 \times 6 - 55.5 \times 21 = 0,$ $\qquad bC = - 45.84$ "

If the load occupies the axis as shown in Fig. 113, each vertical is divided into two parts, the stresses in each of which will be different.

For the maximum compression in the upper portions, we have, since the braces dE, cD, bC act for the respective loadings,

$e4 \times \infty + 49.875 \times \infty - 4 \times 7.5 \times \infty = 0,$ $\qquad e4 = - 19.875$ tons.

$d3 \times 112.5 + 61.50 \times 67.5 - 7.5\,(82.5 + 97.5 + 112.5) = 0,$ $d3 = - 17.4$ "

$c2 \times 45 + 79.125 \times 15 - 7.5\,(30 + 45) = 0,$ $\qquad c2 = - 13.87$ "

$b1 \times 17.5 + 98.625 \times 2.5 - 7.5 \times 17.5 = 0,$ $\qquad b1 = - 6.58$ "

For the lower half $E4$, we first find for the proper loading fE by moments as follows:

$+ fE \times 183.64 + 61.50 \times 300 - 7.5\,(285 + 270 + 255 + 240) - 48 \times 240 = 0,$ $fE = + 5.1404$ tons

Since then fE acts, and it is cut by a section through de, $E4$, and EF, we must take its moment into account, and thus have,

$E4 \times \infty + 61.50 \times \infty - 3 \times 7.5 \times \infty + 5.1404 \times \infty = 0,$ $\qquad E4 = - 43.64$ tons.

The tension in $e4$ for this loading is $55.5 - 43.64$, or $\qquad e4 = + 11.86$ "

For the lower half $D3$ we first find for the proper loading by moments,

$De \times 183.64 + 79.125 \times 180 - 7.5\,(195 + 210 + 225) - 48 \times 225 = 0,$ or $De = + 6.984$ tons.

Since then De acts, we must take its moment into account and have

$D3 \times 112.5 + 79.125 \times 67.5 - 7.5\,(82.5 + 97.5) + 6.984 \times 94.73 = 0,$ $D3 = - 41.35$ tons.

The tension in $d3$ for this loading is $55.5 - 41.35$, or $\qquad d3 = + 14.15$ "

In similar manner we find,

$C2 \times 45 + 98.625 \times 15 - 7.5 \times 30 + 549.287 = 0,$ $\qquad \begin{cases} C2 = - 40.08 \text{ tons.} \\ c2 = + 15.42 \text{ "} \end{cases}$

$B1 \times 17.5 + 110 \times 2.5 + 433.68 = 0,$ $\qquad \begin{cases} B1 = - 40.496 \text{ "} \\ b1 = + 15.004 \text{ "} \end{cases}$

METHOD BY DIAGRAM.—It will be seen from the preceding that the calculation of girders with curved chords, though sufficiently simple in principle, is tedious in computa-

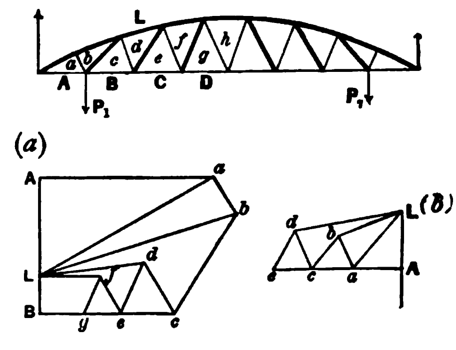
tion. There is also considerable liability to error through carelessness in writing down the equations. The student would do well to make it a rule always to check the computation by diagram.

The diagram is best applied by taking a single apex weight and finding the stresses it causes.

Thus, let Fig. 115 represent a bowstring girder; span 80 feet, divided into 8 equal panels. Bow circular, the versine being 10 feet, hence the central depth of inscribed polygon is 9.85 feet. The load is supposed to traverse the lower chord and to be equal to 1 ton per foot. Dead load 0.5 ton per foot.

First suppose only the train load P_1 of 10 tons to act, and diagram its stresses as in Fig. 115 (a).

Fig.115*



Then suppose the load P_7 = 10 tons to act, and diagram its stresses. We can now easily form a Table giving the stresses in every brace due to each separate apex live weight.

TABLE OF STRESSES IN THE BRACES.

Members.		*ab*	*bc*	*cd*	*de*	*ef*	*fg*	*gh*
P_1		+ **2.7**	+ **11.4**	– 4.8	+ 4.3	– 2.4	+ 2.3	– 1.4
P_2		+ 2.3	– 1.4	+ **8.4**	+ **8.6**	– 4.7	+ 4.6	– 2.8
P_3		+ 2.0	–1.1	+ 2.8	–2.6	+ **4.5**	+ **6.9**	–4.2
P_4		+ 1.6	– 0.9	+ 2.2	– 2.0	+ 3.6	–3.5	+ 5.6
P_5		+ 1.2	– 0.7	+ 1.7	– 1.5	+ 2.7	– 2.6	+ 4.2
P_6		+ 0.8	– 0.5	+ 1.1	– 1.0	+ 1.8	– 1.8	+ 2.8
P_7		+ 0.39	– 0.23	+ 0.56	– 0.51	+ 0.90	– 0.88	+ 1.4
Compression	Live load	....	– 4.8	– 4.8	– 7.6	– 7.1	– 8.8	– 8.4
–	Locomotive excess.	....		–15.84		– **15.51**		
Tension	Live load	+ 11.0	+ 11.4	+ 11.8	+ 12.9	+ 13.5	+ 13.8	+ 14.0
+	Locomotive excess.	+ 11.55		+ 13.07		+ **14.85**		
Dead Load		+ 5.5	+ 3.3	+ 3.5	+ 2.6	+ 3.2	+ 2.5	+ 2.8
Max. compression.		....		17.14		20		
Max. tension.		28.05		28.37		31		

Thus we set down in the Table the stresses in all the braces, caused by P_7, as found from diagram (b). Then the stresses due to P_6 will be twice those caused by P_7. Those due to P_5 and P_4, three and four times those caused by P_7, respectively. This is evident from Fig. 115, where P_6 is twice as far from the right end as P_7. Its left reaction is, there-

fore, twice as great, and causes in all the braces to the left a double stress. We can thus fill the lines for P_7, P_6, P_5 and P_4. For P_2 we see at once that *cd* and *de* will both be tension. The signs alternate both ways. The stresses in these two members for P_2 are in different type in the Table. For all braces on the right of P_2 the stresses will be twice what they were for P_1, and for all on the left six times what they were for P_7.

In the same way for P_3 the stresses on *ef* and *fg* (given in Table in black type) are both minus, and signs alternate right and left from these. For all braces on the right the stresses are three times what they were for P_1, and for all on the left five times what they were for P_7. Generally, then, the stresses are all multiples of either P_1 or P_7, and we can easily fill up the Table.

We can now fill out the lines for live load compression and tension. Then adding these algebraically and dividing by the ratio of live to dead load, we find the dead load stresses.

It remains to take account of the locomotive excess. This is easily done. Thus for *ef* the greatest tension occurs when we have 33 tons at the third apex. This weight will cause in *ef*, therefore, $\frac{33}{10} = 3.3$ times as much tension as P_3 caused, or $3.3 \times 4.5 = 14.85$. The total tension in *ef*, then, taking account of locomotive excess, is $+ 13.5 + 14.85 + 2.7 = + 31$ tons.

In the same way the compression on *ef* given by the Table, or 4.4, is to be increased by the compression in this member due to locomotive excess. This compression is 3.3 times the compression in *ef* due to P_2, or $4.7 \times 3.3 = 15.51$. Therefore, the greatest compression on *ef* is $- 7.1 - 15.51 + 2.7 = - 20$ tons.

In similar manner we can find and add the locomotive excess stresses for the other members, and thus find the maximum stresses. The student is left to fill up these lines in the Table for himself.

The chords are found by a similar Table, the locomotive excess stresses being determined in an analogous manner.

General Remarks.—The foregoing is sufficient to show the application of our principles to any bridge girder with curved or inclined chords.

In finding the lever arms the student should check the computation of each one by measuring it to scale from a properly drawn frame. In this way errors may be avoided.

Instead of finding the dead load stresses from the computed live load stresses, as is done in our Table, the dead load stresses may be easily diagrammed or computed separately, if it is thought desirable. No comparison of the girders in this Chapter has been attempted, but the double bow is easily found, so far as stresses are concerned, to be the best. This might be expected, as, both chords being curved, each acts to sustain the load, while in the bowstring, Fig. 115, the lower chord simply resists the spread of the upper chord. The bowstring ranks next, and the "truncated bowstring" last of all.

The best bracing in all cases for long span is the triangular, as in such case all the braces may always be in tension, and the material required for stiffening long struts is avoided.

The *Pauli* truss (page 58) resembles the double bow, but the chords are so curved that the stress in them is constant. Such a truss with triangular bracing is, therefore, somewhat superior to the double bow for long spans.

The student who has checked the examples in the Appendix, page 243, will have no difficulty in solving the examples of this Chapter for concentrated loads.

Advantage of Inclined Chords.—The use of inclined chords for long spans is a common practice. One advantage has already been noticed (page 136) in connection with the bowstring truss, viz., that for long spans the braces may be always in tension. Another

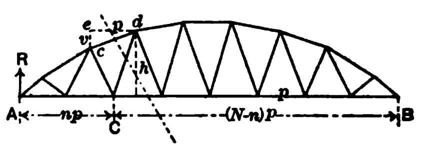
advantage has been noticed in connection with the Schwedler truss (page 56), viz., the elimination of counters.

Another advantage is that the chords may be so inclined as to take all the shear for full loading, thus reducing the bracing and avoiding reversal of stress in the braces.

FORMULA FOR INCLINATION OF CHORDS.—The following method for finding the inclination of chords is given by Prof. Benj. F. La Rue (*Engineering News*, March 19, 1896):

Let Fig. 1 represent a truss with inclined upper chord and isosceles bracing. Let W be the full panel load, dead and live, acting at the lower chord apices, let N be the number of lower chord panels and n the number of panels on the left of any apex C of the lower chord, so that the distance $AC = np$ and the length of span is Np, where p is the panel length.

Fig. 1.



Then we have for the left reaction

$$R = \frac{W(N-1)}{2}. \qquad (1)$$

The shear for the brace Cd is then

$$S = R - nW = \frac{W}{2}(N - 1 - 2n). \qquad (2)$$

The moment at the point C is

$$M = -Rnp + W(n-1)\frac{np}{2} = -\frac{Wnp}{2}(N-n). \qquad (3)$$

Let H be the horizontal component of the stress in the upper chord cd and V be its vertical component. Then taking moments about C, we have, if h is the height at d,

$$Hh - \frac{Vp}{2} = M. \qquad (4)$$

But if v is the vertical projection ec of the chord cd, we have

$$H\frac{v}{p} = V, \text{ or } H = V\frac{p}{v}.$$

Substituting this value of H in (4), and the value of M from (3), and solving for V, we have

$$V = -\frac{Wvn(N-n)}{2h-v}. \qquad (5)$$

If now the chord cd is inclined at such an angle that it takes all the shear for full loading we have

$$V + S = 0, \text{ or } \frac{Wvn(N-n)}{2h-v} = \frac{W}{2}(N - 1 - 2n.)$$

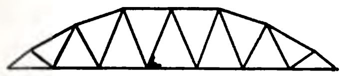
Hence we obtain for the value of v, for isosceles bracing,

$$v = \frac{h(N-1-2n)}{n(N-n) + \frac{1}{2}(N-1-2n)}. \qquad (6)$$

For an even number of panels, we have at the centre $N = 2n$ and (6) gives v negative. The formula therefore does not apply to the centre panel. For an even number of panels,

then, the top chord is horizontal at centre, as shown in Fig. 1, and (6) applies only to chords on the left.

Fig. 2.

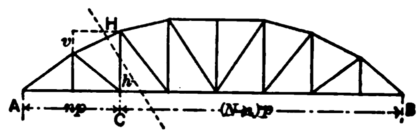


For an odd number of panels, Fig. 2, we have for the first panel point on left of centre $N = 2n + 1$, and from (6) $v = 0$. For an odd number of panels, then, the top chord is horizontal for two panels, as shown in Fig. 2, and (6) applies only to chords on the left of these.

For Pratt bracing we have, instead of equation (4), the equation

$$Hh = M$$

Fig. 3.



Hence we have, instead of (5),

$$V = -\frac{Wvn(N-n)}{2h},$$

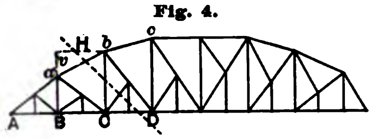
and therefore we have for the value of v for Pratt bracing

$$v = \frac{h(N - 1 - 2n)}{n(N-n)}. \quad \ldots\ldots\ldots\ldots \quad (7)$$

Here, again, for an even number of panels (7) does not apply at the centre. For an even number of panels, then, the top chord is horizontal in the *two centre panels* as shown in Fig. 3, and (7) applies only to chords on left of these.

For an odd number of panels, v in (7) is zero for the panel point on left of centre. For an odd number of panels, then, the top chord is horizontal in the *three centre panels*, and (7) applies only to chords on left of these.

Fig. 4.



For the sub-Pratt system, shown in the left half of Fig. 4, we have, instead of equation (2),

$$S = R - nW - \frac{W}{2} = \frac{W}{2}(N - 2 - 2n),$$

and therefore, in place of (7), since $H\frac{v}{2p} = V$, we have

$$v = \frac{2h(N - 2 - 2n)}{n(N-n)}, \quad \ldots\ldots\ldots\ldots \quad (8)$$

where n has the values 2, 4, 6, etc.

For even number of panels we have, as in Fig. 4, the two double centre panels at top horizontal, and for an odd number of panels we have the three double centre panels at top horizontal, and (8) applies to chords on left of these.

For the "half-hitch" system shown on the right of Fig. 4, we have, instead of equation (2),

$$S = R - nW + \frac{W}{2}\left(1 + \frac{v}{h}\right) = \frac{W}{2}\left(N - 2n + \frac{v}{h}\right).$$

We also have, in the place of equation (3),

$$M = -Rnp + W(n-1)\frac{np}{2} - Wp = -\frac{Wnp}{2}(N-n) - Wp.$$

We also have

$$Hh = M \quad \text{and} \quad H\frac{v}{2p} = V,$$

Hence we obtain

$$v = \frac{2h(N - 2n)}{n(N - n)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where n has the values 2, 4, 6, etc.

For an even number of panels we have the two double centre panels at top horizontal, and for an odd number of panels we have three double centre panels at top horizontal, and (9) applies to chords on left of these.

EXAMPLE.—Let the height of truss in Fig. 4 be 24 feet, and $N = 16$, system sub-Pratt. Then by applying (8), we have for the rise v of the chord bc, $n = 6$, and

$$v = \frac{2 \times 24(16 - 2 - 12)}{6(16 - 6)} = 1.6 \text{ ft.}$$

We have then for the height $Cb = 24 - 1.6 = 22.4$. Applying (8) again, we have for the rise v of the chord ab, $n = 4$, and

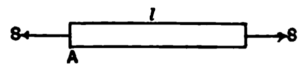
$$v = \frac{2 \times 22.4(16 - 2 - 8)}{4(16 - 4)} = 5.6 \text{ ft.}$$

The height Ba is then $Ba = 22.4 - 5.6 = 16.8$ ft.

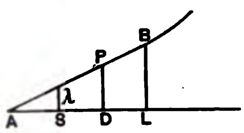
CHAPTER VI.

PRINCIPLE OF LEAST WORK—REDUNDANT MEMBERS—DEFLECTION OF A FRAMED GIRDER.

Elastic Limit.—Let a straight member of length l and constant area of cross-section A be acted upon by a stress S in its axis, and let the elongation or compression as the case may be, or in general the *strain*, be denoted by λ. We know from experiment that within a certain limit, twice, three times or four times, etc., the stress S will cause a strain of 2λ, 3λ, 4λ, etc. The limit up to which this law of proportionality of stress to strain holds true, for any material, is called the *elastic limit* for that material for the kind of stress under consideration.



Thus if we lay off the stresses to any convenient scale horizontally, and lay off the corresponding strains λ to any convenient scale vertically, we obtain within the elastic limit a straight line AB. The co-ordinates AD and DP of any point P of this line give the stress and corresponding strain for that point.



The point B at which the straight line AB begins to curve gives the stress AL, and this is the *elastic limit stress.*

Coefficient of Elasticity.—From the preceding article, we see that if S is the stress and λ the corresponding strain, we have, within the elastic limit, the ratio $\frac{S}{\lambda}$ constant. Now if A is the area of cross-section of the test piece, then $\frac{S}{A}$ is the unit stress, or stress per square inch of area. Also if l is the original length, then $\frac{\lambda}{l}$ is the unit strain, or strain per unit of length. If then the experiment were made on a test-piece of one unit area and one unit length, we should have, within the elastic limit, the ratio $\frac{S}{A} \div \frac{\lambda}{l}$, or $\frac{Sl}{A\lambda}$ constant. This constant for any material is called the *coefficient of elasticity* for that material for the kind of stress under consideration. We denote it by E. We have then, within the elastic limit,

$$E = \frac{Sl}{A\lambda}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

and can define the coefficient of elasticity in any case, as ***the unit stress divided by the unit strain.*** From (I) we can find E by experiment, S, A, and l being known, and λ measured. Values of E for different materials and different kinds of stress will be found in the Appendix to Part I, page 293.

If we assume the law of proportionality of stress to strain to hold good ***without limit,*** then we can say, that since a unit stress $\frac{S}{A}$ causes a strain λ, it will take as many times this

unit stress to cause a strain l as λ is contained in l. That is, *the coefficient of elasticity is that theoretic unit stress which would cause a strain equal to the original length, if the law of proportionality of stress to strain held good without limit.* It is therefore given in pounds per square inch.

The definition first given is, however, the best, and most easily borne in mind. That is, within the elastic limit, E is constant for the same material and same kind of stress, and is always given *by unit stress divided by unit strain.*

If then we know E, we have from (I) for the strain

$$\lambda = \frac{Sl}{AE} \quad \ldots\ldots\ldots\ldots \quad \text{(II)}$$

From (II) we can find in any case the strain λ when S, A, l, and E are known.

EXAMPLES.—(1) A wrought-iron tie-rod 30 feet long and 4 square inches in area of cross-section is subjected to a tensile stress of 40000 lbs. The elongation is found to be 0.01 ft. What is E?

Ans. The unit stress is $\frac{S}{A} = \frac{40000}{4} = 10000$ lbs. per square inch of area. The unit strain is $\frac{\lambda}{l} = \frac{1}{3000}$ ft. per foot of length. We have then from equation (I)

$$E = \frac{Sl}{A\lambda} = 10000 \times 3000 = 30{,}000{,}000 \text{ pounds per square inch.}$$

(2) A rectangular timber strut is 12 inches deep and 40 feet long. If $E = 1{,}200{,}000$ lbs. per square inch, find the width, so that its compression under a stress of 27000 lbs. may not exceed 1.2 inches, all lateral bending being prevented.

Ans. From equation (I) we have

$$A = \frac{Sl}{E\lambda} = \frac{27000 \times 40 \times 12}{1200000 \times 1.2} = 9 \text{ sq. inches.}$$

Hence the width is $\frac{A}{1.2} = 7.5$ inches.

(3) A wrought-iron bar, 2 square inches sectional area, has its ends fixed to two immovable points, when the temperature is 60° Fahr. Taking the coefficient of expansion at 0.000006944 per unit of length for one degree, and supposing all lateral bending to be prevented, what stress must be resisted by the fixed points when the temperature is raised or lowered 40 degrees?

Ans. We have $\lambda = 0.000006944l \times 40$. Therefore from equation (I)

$$S = \frac{EA\lambda}{l} = \frac{2 \times 0.000006944l \times 40}{l} = 0.00055552E.$$

If $E = 30{,}000{,}000$ pounds per square inch, $S = 16665.6$ lbs. This is compression or tension according as the temperature is raised or lowered.

WORK IN STRAINING A MEMBER.—If the stress S is gradually applied increasing from 0 to S, then $\frac{S}{2}$ is the average stress, and the work done is

$$\frac{S}{2}\lambda.$$

Substituting the value of λ from (II), we obtain

$$\text{Work} = \frac{S\lambda}{2} = \frac{S^2 l}{2AE}. \qquad \text{(III)}$$

The work then is, if the stress is gradually applied, *one half the product of stress by strain.*

PRINCIPLE OF LEAST WORK.—When a spring is compressed or extended within the elastic limit, if released it can give back the work expended in straining it. The work which a body can thus do by reason of its position or condition is called *potential energy.*

It is a principle of mechanics that if a body is in equilibrium, its potential energy is either a maximum or a minimum, and if it is in *stable* equilibrium, its *potential energy is a minimum.*

Thus let the body whose centre of mass is C be supported by the fixed point P. It is evidently in equilibrium when C is vertically above or below P. If C is vertically above P, as in Fig. 2, it is in unstable equilibrium. A slight motion to either side destroys the equilibrium. In this case, since the centre of mass has the highest possible position, the potential energy is a maximum.

Fig. 1.

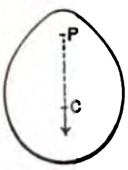
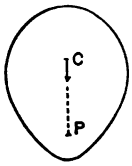


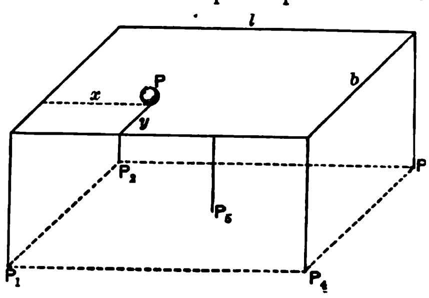
Fig. 2.



But if the centre of mass C is vertically below P, it is in stable equilibrium. If moved to either side, it returns again. In this case, since the centre of mass has the lowest possible position, the potential energy is a minimum.

We have then the general principle, that if a body is in stable equilibrium, its potential energy is a minimum. If part of that potential energy consists then of work expended in straining the body, *this work is the least possible consistent with equilibrium.*

FIVE-LEGGED TABLE.—As an illustration of the application of this principle of least work, suppose a rectangular table of length l and breadth b, to have five legs all of equal length L, and the same constant area of cross-section A, at the centre and at each corner.



Let a load P rest on the table, and let x and y be the co-ordinates of its point of application.

Let P_1, P_2, P_3, P_4, P_5 be the loads carried by each leg. We have for the conditions of equilibrium, then,

$$P_1 + P_2 + P_3 + P_4 + P_5 = P,$$

$$P_3 l + P_4 l + P_5 \frac{l}{2} = Px,$$

$$P_2 b + P_3 b + P_5 \frac{b}{2} = Py.$$

From these equations, we obtain

$$\left.\begin{aligned} P_1 &= P - P_4 - \tfrac{1}{2}P_5 - P\frac{y}{b}, \\ P_2 &= P\frac{y}{b} + P_4 - P\frac{x}{l}, \\ P_3 &= P\frac{x}{l} - P_4 - \tfrac{1}{2}P_5. \end{aligned}\right\} \qquad (1)$$

From equations (1) we see that P_1, P_2, and P_3 are given in terms of P_4 and P_5. If then P_4 and P_5 are known we can find P_1, P_2, and P_3. But P_4 and P_5 are not known. We have five

unknown quantities, and the conditions of equilibrium give us only three equations of condition between them. We need then two more equations. These two equations are furnished by the principle of least work.

Thus from equation (III) we have for the work done in compressing the legs, assuming the floor and table-top to be rigid,

$$\text{work} = \frac{L}{2AE}[P_1^2 + P_2^2 + P_3^2 + P_4^2 + P_5^2].$$

If in this we substitute the values of P_1, P_2, and P_3 as given by (1), we have

$$\text{work} = \frac{L}{2AE}\Big[P^2 + 4P_4^2 + \frac{3}{2}P_5^2 + \frac{2y^2}{b^2}P^2 + \frac{2x^2}{l^2}P^2 - 2PP_4 - PP_5 - \frac{2y}{b}P^2$$
$$+ 2P_4P_5 + \frac{4y}{b}PP_4 + \frac{y}{b}PP_5 - \frac{2xyP^2}{bl} - \frac{4x}{l}PP_4 - \frac{x}{l}PP_5\Big].$$

We thus have the work given in terms of P_4 and P_5 and known quantities. Now P_4 and P_5 must have such values that the work shall be a minimum. We therefore put the differential of the work with reference to P_4 and P_5 equal to zero. We thus obtain

$$\frac{d(\text{work})}{dP_4} = 0 = 8P_4 - 2P + 2P_5 + \frac{4y}{b}P - \frac{4x}{l}P, \quad \text{or} \quad 8P_4 + 2P_5 = 2P - \frac{4y}{b}P + \frac{4x}{l}P. \quad (2)$$

Also

$$\frac{d(\text{work})}{dP_5} = 0 = 3P_5 - P + 2P_4 + \frac{y}{b}P - \frac{x}{l}P, \quad \text{or} \quad 2P_4 + 3P_5 = P - \frac{y}{b}P + \frac{x}{l}P. \quad (3)$$

We thus have two more equations of condition. From (2) and (3) we obtain

$$P_5 = \tfrac{1}{5}P,$$
$$P_4 = \tfrac{1}{5}P - \frac{y}{2b}P + \frac{x}{2l}P = P\Big[\frac{1}{5} - \frac{y}{2b} + \frac{x}{2l}\Big].$$

That is, *the load carried by the centre leg is always $\frac{1}{5}P$ no matter where the load P is placed.*

Now substituting these values of P_4 and P_5 in (1), we have

$$P_1 = P\Big[\frac{7}{10} - \frac{y}{2b} - \frac{x}{2l}\Big],$$
$$P_2 = P\Big[\frac{1}{5} + \frac{y}{2b} - \frac{x}{2l}\Big],$$
$$P_3 = P\Big[\frac{y}{2b} + \frac{x}{2l} - \frac{3}{10}\Big].$$

If P is at the centre, we have $x = \frac{l}{2}$, $y = \frac{b}{2}$, and

$$P_1 = P_2 = P_3 = P_4 = P_5 = \tfrac{1}{5}P.$$

If P is at the middle of the side l, $x = \frac{l}{2}$, $y = 0$, and

$$P_1 = \tfrac{9}{20}P, \quad P_2 = -\tfrac{1}{20}P, \quad P_3 = -\tfrac{1}{20}P, \quad P_4 = \tfrac{9}{20}P, \quad P_5 = \tfrac{1}{5}P.$$

In this case, legs 2 and 3 must be fastened to the floor, and are then in tension. If not fastened they are lifted and the table is supported on three legs only.

REMARKS ON THE PRECEDING PROBLEM.—The preceding problem of the five-legged table illustrates the principle of least work. It also illustrates much more. It furnishes a good example of the *misapplication of theory*. The theory is sound and the results are

therefore correct, provided the assumptions are realized. But these assumptions cannot be realized by any actual table. For instance, it is assumed that the floor is absolutely rigid and level, and the table-top the same; also, every leg is of exactly the same length and the same constant area of cross-section. Such a table is an ideal and cannot really exist. The theory is then misapplied, since its assumptions are not in accord with fact. The strain of a leg is very small, and a small discrepancy in length of legs, such as must be expected in practice, would entirely change the results.

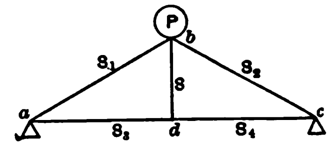
Evidently, then, the results are practically worthless. The legs should be designed for three only. Then if any others are desired, they can be added of the same size. This practical solution is not only simpler, but it is actually more accurate and more scientific.

The results of the first case are the results of sound principles logically applied to a fiction. The results of the second case are the results of sound principles logically applied to an actual table.

The student then must regard the example as an illustration simply of the principle of least work. He should also note that sound principles need care in their application. In order to apply them to a practical case certain assumptions must be made, and these assumptions should accord with the facts of that case. Otherwise the results are worthless, even though the principles be sound.

REDUNDANT MEMBERS.—The same principle of least work is illustrated by the calculation of redundant members in a truss. The method of procedure is the same as for the case of the five-legged table.

Thus, take the simple truss shown in the accompanying figure, consisting of two rafters *ab*, *bc*, a tie-rod *ac*, and a strut *bd*.



If a load P is supported at b, the strut bd is superfluous.

Let the angle of the rafters with the vertical be θ, and the length, stress, and area of cross-section of ab be l_1, S_1, a_1; of bc be l_2, S_2, a_2; of ad be l_3, S_3, a_3; of dc be l_4, S_4, a_4; of bd be l, S, a.

We first find S_1, S_2, S_3, S_4 in terms of S. Thus, for equal length of rafters we have at apex b

$$-S_1 \cos\theta - S_2 \cos\theta - P + S = 0,$$

or, since $S_1 = S_2$,

$$S_1 = S_2 = \tfrac{1}{2}(S - P)\sec\theta. \quad . \quad . \quad . \quad (1)$$

At apex a we have

$$S_3 = S_4 = S_1 \sin\theta = \tfrac{1}{2}(S - P)\tan\theta. \quad . \quad . \quad . \quad (2)$$

From equation (III) we have

$$\text{Work} = \frac{1}{2E}\left[\frac{S_1^2 l_1}{a_1} + \frac{S_2^2 l_2}{a_2} + \frac{S_3^2 l_3}{a_3} + \frac{S_4^2 l_4}{a_4} + \frac{S^2 l}{a}\right].$$

Since we have

$$l_1 = l_2 = l\sec\theta, \qquad l_3 = l_4 = l\tan\theta,$$
$$a_2 = a_1, \qquad a_3 = a_4,$$

the expression for the work becomes, when we substitute these values and the values of S_1, S_2, S_3, S_4 in terms of S,

$$\text{Work} = \frac{l}{2E}\left[\frac{S^2 - 2SP + P^2}{2a_1}\sec^2\theta + \frac{S^2 - 2SP + P^2}{2a_3}\tan^2\theta + \frac{S^2}{a}\right].$$

Now the stress in S must make the work a minimum. Hence

$$\frac{d(\text{work})}{dS} = 0 = \frac{S}{a_1}\sec^2\theta - \frac{P}{a_1}\sec^2\theta + \frac{S}{a_3}\tan^2\theta - \frac{P}{a_3}\tan^2\theta + \frac{2S}{a}$$

or

$$S = \frac{Paa_2 (\sec^2\theta + \tan^2\theta)}{2a_1a_2 + aa_2 \sec^2\theta + aa_1 \tan^2\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We thus know S and can find from (1) and (2) the values of S_1, S_2, S_3, S_4.

It will be noted that the cross-sections must be known for each member in advance.

If the cross-sections are all equal we have

$$S = \frac{P(\sec^2\theta + \tan^2\theta)}{2 + \sec^2\theta + \tan^2\theta}.$$

REMARKS ON THE PRECEDING PROBLEM.—The principle of least work has been extensively employed for the calculation of redundant members. It will be seen that though the method is simple, its application to a truss with many members becomes tedious.

It will also be seen that the same remarks apply here as in the case of the five-legged table. Every member must be of absolutely true length and of the exact cross-section assigned. Any variation from these ideal conditions invalidates the result.

Such applications of the principle of least work we regard therefore as of no practical value, and they will not be elaborated here. Applications of real value will be made in subsequent Chapters.

DEFLECTION OF A FRAMED GIRDER.—By the application of equations (II) and (III) the deflection at any point of a framed girder may be calculated.

Thus let S be the stress in any member due to the *actual loading* of the truss, and l and a the length and area of cross-section. Then from (II) the strain of the member due to the actual loading is

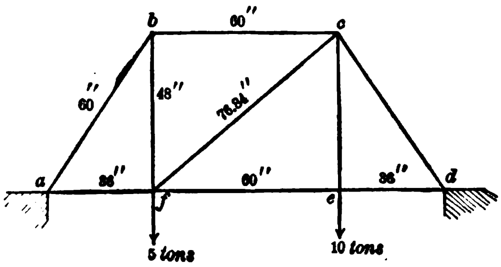
$$\lambda = \frac{Sl}{aE}.$$

Now let s be the stress in the same member due to any arbitrary assumed load p supposed to rest at the panel point for which the deflection is desired. This load we may assume of any convenient amount. Then the work due to this load in the member is

$$\frac{s\lambda}{2} = \frac{Ssl}{2aE}.$$

The total work in all the members due to this load is then

$$\sum \frac{Ssl}{2aE}.$$



But if Δ is the deflection at the panel point where p is supposed to act, then the work done by p is $\frac{p\Delta}{2}$, and hence we have

$$\frac{p\Delta}{2} = \sum \frac{Ssl}{2aE},$$

or

$$\Delta = \frac{1}{pE} \sum \frac{Ssl}{a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV)$$

EXAMPLE.—Suppose a girder (see Figure) consisting of two inclined rafters of length 60 inches, two vertical ties of length 48 inches, an upper chord of length 60 inches, and a lower tie of length 132 inches, the two end panels 36 inches and the centre 60 inches. Let there be a diagonal strut cf, whose length is 76.84 inches. Suppose a load of 5 tons at f and 10 tons at e.

Required the deflection at e, the areas of cross-section being supposed to be known.

Let E be 12500 tons per square inch, and the areas of cross-section as given in the following table:

Member	Length l in inches.	E in tons per sq. in.	S in tons.	s in tons.	Cross-section a in sq. in.	$\frac{l}{pE}$	$\frac{Ss}{a}$	Δ in inches.
ab	60	12500	− 7.9545	− 3.4091	1.85	$\frac{3}{6250}$	+ 14.6582	
bc	60	12500	− 4.7727	− 2.0454	1.00	$\frac{3}{6250}$	+ 9.7621	0.0372
cd	60	12500	− 10.7954	− 9.0909	1.85	$\frac{3}{6250}$	+ 53.0486	
de	36	12500	+ 6 4772	+ 5.4545	1.5	$\frac{9}{31250}$	+ 23.5532	
ef	60	12500	+ 6.4772	+ 5.4545	1.5	$\frac{3}{6250}$	+ 23.5532	0.0199
af	36	12500	+ 4.7727	+ 2.0454	1.5	$\frac{9}{31250}$	+ 6.5080	
bf	48	12500	+ 6.3636	+ 2.7272	2.0	$\frac{6}{15265}$	+ 8.6777	
ce	48	12500	+ 10.0000	+ 10.0000	2.0	$\frac{6}{15265}$	+ 50.0000	0.0308
fc	........76.84	12500	− 2.1829	− 4.36579	0.75	$\frac{76.84}{125000}$	+ 12.7067	

Deflection at right hand weight = 0.0879

We take for the value of p the load of 10 tons at e, and find the stresses s in every member due to this single load. We also find the stresses S in every member due to the actual loading. In the product Ss these stresses must be taken with their proper signs. Thus if s is compression or minus and S is also compression or minus, the product Ss is positive. If one is tension and the other compression, the product Ss is negative. If the signs of S and s are carefully observed, the sign of the products Ss will thus take care of itself.

If we take E in pounds or tons per square inch, S, s and p must be taken in pounds or tons and l in inches, and a in square inches.

Note that we have taken p at e equal to 10 tons, the load actually acting there. If, however, there were no load acting there, we could assume a load of $p = 1$ ton, or any convenient amount, and proceed as before.

The stresses S due to actual loading are, strictly speaking, affected by the change of shape. This can, however, be disregarded without perceptible error, as the deflection in all practical cases is very small.

Remarks on the Preceding Example.—In our example we assume E as constant for all members. Every member has its accurate length and area of section. All pins at the apices are presumed to fit tight, and all adjustable members, if any, to be accurately adjusted.

A girder after erection may then be tested by calculating the deflection at the centre for a given load and comparing with the actual observed deflection for this load.

A good agreement is then a test of the close fit of all pins, of the proper adjustment of all adjustable members, of the agreement of the lengths and sections of the members with those called for by the design, of the constant value of E and its proper assumption as to magnitude, and finally of the fact that *the elastic limit is not exceeded.*

It is evident that when so many conditions must concur, a discrepancy between the observed and calculated deflection has little practical significance. The last-mentioned fact, that the elastic limit is not exceeded, is the most important, and this is proved, not by any close agreement between actual and calculated deflections, but by observing whether the deflection is constant under repeated applications of the same loading, after the structure has attained its permanent set from the first application.

Calculations of deflection are then of little value as a means of testing framed structures, and the calculated result cannot be expected to agree very closely with the actual deflection.

CHAPTER VII.

SWING BRIDGES.

PIVOT OR SWING SPANS.—The pivot or swing span is a girder continuous or partially continuous over three or four supports.

If over three supports, it is a "pivot span." If over four supports, the length of the small intermediate span is the width of the turn-table. Loads in the centre span act directly upon the turn-table, and hence cause no stresses in the members.

The reaction at any support is the sum of the shears on each side of that support. For an end support, the reaction and shear are the same.

Our formulas give the shears at a support, and these must not be confounded with the reactions.

In the case of the pivot span, it is evident that if the end shear, due to a load placed anywhere, is known, then, since at the end there is no moment, we have all we need in order to find the stresses. The centre span, if there is a turn-table, is not affected by loads placed in it, since these loads act directly on the turn-table.

RAISING OF CENTRE SUPPORT.—The centre supports should be above the level of the ends by the amount of deflection of the open span, or else when the span is open it would deflect and it would be difficult to shut it again.

If the end supports are not raised after the draw is shut, then the dead load stresses for the draw open *exist just the same when the draw is shut.* The apex live loads can then be considered, each by itself, for draw shut, and the fact that the supports are out of level *does not affect our formulas.* They hold good just as if the supports were on level. Moreover, it is not necessary to enter into elaborate computations as to the precise amount by which the centre supports must be raised. It is only necessary in practice to raise the ends till they just bear when the bridge is empty. Thus even when shut there is no pressure on the end supports except when the live load comes on.

It may seem strange at first sight that under these circumstances the live load pressures are just what they would be for level supports. If the girder, originally straight, were *held* down at the ends, then the end reactions would have to be computed for supports out of level. These reactions would be negative (downward), and a live load coming on would diminish them, or, if great enough, reverse them. But such is not the state of things. The end reactions are zero in the beginning, and any live load gives therefore at the end the same reactions as for level supports.

An analytical discussion would be out of place here, but assuming the expression to which such a discussion would lead us, we may show that such is the case.

Thus for a beam over three supports, A, B, and C, *not* on a level, h_1 being the distance of A below B, and h_2 the distance of C below B, the coefficient of elasticity being E and the moment of inertia I, we have for the moment M_2 at the centre support due to any number of loads in both spans *

$$4\,M_2 l = \left[\frac{h_1 + h_2}{l}\right] 6\,EI + \frac{1}{l}\,\Sigma Pz\,(l-z)\,(l+z) + \frac{1}{l}\,\Sigma Pz\,(l-z)\,(2l-z)$$

z being the load distance from the left end of the loaded span.

* See Appendix to Part I, page 340.

Now in this expression the last two terms are precisely the same as for supports on level. The influence of the different levels is contained in the first term on the right only. But in our case the differences of level h_1 and h_2 are due entirely to the dead load, and the value of this term is then independent of the live load.

RAISING OF ENDS.—In the case of the pivot span, if the ends just bear when draw is closed, any live load in one span lifts the other end. The ends must then be latched down, and there is considerable vibration. To avoid this, it is the practice to *raise the ends* after the draw is closed, so that the supports are all on level. The dead load thus causes positive end reactions, and unless the live load over one span preponderates, the ends need not be latched down. Our formulas are not affected in their application by this practice.

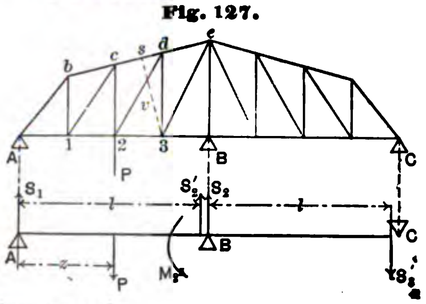
METHOD OF CALCULATION.—We have then to make two calculations, one for draw open and stresses due to dead load, the other for draw shut and stresses due to live load, and, for raised ends, dead load also. The union of the two will give the maximum stresses. For the draw open, the stresses are very easily found. For the draw shut, we have simply to find the shears at the supports for each apex load. It therefore only remains to give the formulas which give these shears and an example illustrative of their use.

These formulas are new and we believe an advance upon those heretofore in use, as will be shown later by comparison. The deduction of the formulas is given at the end of this Chapter.

THE CENTRE-BEARING PIVOT SPAN—THREE SUPPORTS.

FORMULAS.—Let the length of span $AB = BC = l$, Fig. 127. Let the apex load be P and its distance from the left end A be z. Let M_2 be the moment at centre support. When this is positive it indicates counter clock-wise rotation as in the figure, and causes tension in upper and compression in lower chord. The pressure at A is denoted by S_1. At B we have S_2' on left and S_2 on right. At C we have S_3'. The reactions are then S_1 and S_3' at the ends and $R_2 = S_2' + S_2$ at the centre support. A positive value for any S indicates that it acts upwards, a negative value downwards.

Fig. 127.



We have then

$$\left.\begin{aligned} S_1 &= -\frac{M_2}{l} + \frac{P}{l}(l-z), \\ S_2' &= P - S_1 = \frac{M_2}{l} + \frac{Pz}{l}, \\ S_2 &= \frac{M_2}{l} = -S_3'. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

From equations (1) we see that we can find S_1 and S_2' for any position of P, just as soon as we know M_2. It is also evident that if we know S_1 and S_2' we can compute the stresses for any position of P.

It therefore remains to give the value of M_2.

Let s be the length of any chord member, upper or lower. Thus in the figure the length of cd or 2–3 is s. Draw the system of braces for dead load, draw open, as shown in the figure, and let x be the distance from left end A or B to the centre of moments for any chord member, upper or lower. Thus for cd centre of momenst is at 2 and x is the

distance $A2$, for 2–3 centre of moments is at d and x is the distance $A3$. Let v be the lever arm for any chord member upper or lower. Thus for de the lever arm v is $s3$, and for 2–3 the lever arm v is $d3$. Let a be the area of cross-section of any chord member, upper or lower. Then we have for M_2

$$M_2 = \frac{P\left[\sum_0^z \frac{sx^2}{av^2} - \frac{z}{l}\sum_0^l \frac{sx^2}{av^2} + z\sum_z^l \frac{sx}{av^2}\right]}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This equation is general, provided that the two spans are equal and symmetrical on each side of the centre. A positive value for M_2 indicates counter clock-wise rotation, or tension in upper chord.

If a is constant it cancels out. So also if v is constant it cancels out.

It will be noted that equation (2) requires that the area of cross-section a for each chord member shall be known, while it is the object of our investigation to determine these areas by first finding the stress and then dividing this stress by the allowable unit stress. If then we first assume a as constant it will cancel out, and we can use (2) to determine the provisional stresses and areas. We can then use these areas in (2). It will, however, be found unnecessary to make a second computation.

EXAMPLE.—A short example will thoroughly illustrate our method and the use of the formulas.

In the preceding Fig. 127 let the length of span be $l = 80$ ft., divided into four panels of 20 ft. each. Let the centre height be $Be = 10$ ft. and the height at end be $b1 = 7$ ft. For these dimensions the lever arm v for any upper chord member as de will not differ appreciably from the height $d3$, and the length s for any chord member as cd will not differ appreciably from the panel lengths. We can therefore readily form the following table:

(1)	(2) s	(3) v	(4) v^2	(5) $\frac{s}{v^2}$	(6) x	(7) $\frac{sx}{v^2}$	(8) $\frac{sx^2}{v^2}$	(9) $\sum\frac{sx}{v^2}$	(10) $\sum\frac{sx^2}{v^2}$
$A1$	20	7	49	0.40815	20	8.163	163.26	8.163	163.26
bc	20	7	49	0.40815	20	8.163	163.26	16.326	326.52
1–2	20	8	64	0.31250	40	12.500	500.00	28.826	826.52
cd	20	8	64	0.31250	40	12.500	500.00	41.326	1326.52
2–3	20	9	81	0.24690	60	14.815	888.90	56.141	2215.42
de	20	9	81	0.24690	60	14.815	888.90	70.956	3104.32
$3B$	20	10	100	0.20000	80	16.000	1280.00	86.956	4384.32
				2.1351		86.956	4384.32		

In the first column we place the designation of each chord member. In column (2) the value of s, in column (3) the value of v, and in column (4) the value of v^2 for each chord member. Then we compute $\frac{s}{v^2}$ for each chord member and put the results in column (5). In column (6) we place the value of x for each chord member. We can now compute the values of $\frac{sx}{v^2}$ and $\frac{sx^2}{v^2}$ and put the results in columns (7) and (8). In (9) and (10) we place the summation of the values in (7) and (8). The last values in (9) and (10) then should check with the sum of the values in (7) and (8).

We can now easily apply equation (2) if we take the area of cross-section a of the chord members constant, so that a cancels out.

Thus for the first apex load P at 1 (see preceding figure) we have $z = 20$ and

$$M_2 = \frac{P(326.52 - 1096.08 + 1412.60)}{109.608 + 170.808 - 173.912} = \frac{643.04}{106.504}P = +6.0377P.$$

For the second apex load P at 2 we have $z = 40$, and

$$M_2 = \frac{P(1326.52 - 2192.16 + 1825.20)}{106.504} = +9.0096P.$$

For the third apex load P at 3 we have $z = 60$, and

$$M_2 = \frac{P(3104.32 - 3288.24 + 960.00)}{106.504} = +7.287P.$$

For the next apex load P in the next span the reaction S_1 at A is of course the same as the reaction S_2' at C for P_2, and so on.

We have then, from equations (1),

For P at $z = 20$	$S_1 = +0.67453P,$
" P " $z = 40$	$S_1 = +0.38738P,$
" P " $z = 60$	$S_1 = +0.1589P,$
" P " $z = 100$	$S_1 = -0.0911P,$
" P " $z = 120$	$S_1 = -0.11262P,$
" P " $z = 140$	$S_1 = -0.07547P.$

Negative values denote that S_1 acts downwards.

We can thus in any case find the reaction S_1 at the end A for every apex load.

Comparison with Formulas Heretofore in Use.—If in equation (2) we assume the area a of cross-section of chord members as constant, *and also assume the height as constant*, then av^2 will cancel out. If, further, *we assume the panel length small in comparison to length of span*, we can put dx in place of s. In accordance with these assumptions we can then write

$$M_2 = \frac{\int_0^z x^2dx - \frac{z}{l}\int_0^l x^2dx + z\int_z^l x\,dx}{\frac{2}{l}\int_0^l x^2dx + l\int_0^l dx - 2\int_0^l x\,dx} \,.\, P.$$

If we perform these integrations, and put k for the ratio $\frac{z}{l}$, we obtain

$$M_2 = \frac{Pl}{4}(k - k^3).$$

This is the formula heretofore in use for the Pivot span.

It is a special case of our equation (2) under the assumption not only of constant chord section a, but also of constant height v, that is, parallel chords, and also of small panel length compared to span. *It is therefore not applicable to most practical cases.*

In the present case we give a comparison of the results of this formula with those of equation (2).

	M_2 Equation (2).	M_2 Ordinary Formula.	Error of Ordinary Formula, per cent.
$z = 20$	$6.0377P$	$4.688P$	22.4
$z = 40$	$9.0096P$	$7.5P$	16.8
$z = 60$	$7.287P$	$6.562P$	10.0

We see then that, even on the assumption of constant chord section, the error due to disregarding change of depth is very considerable.

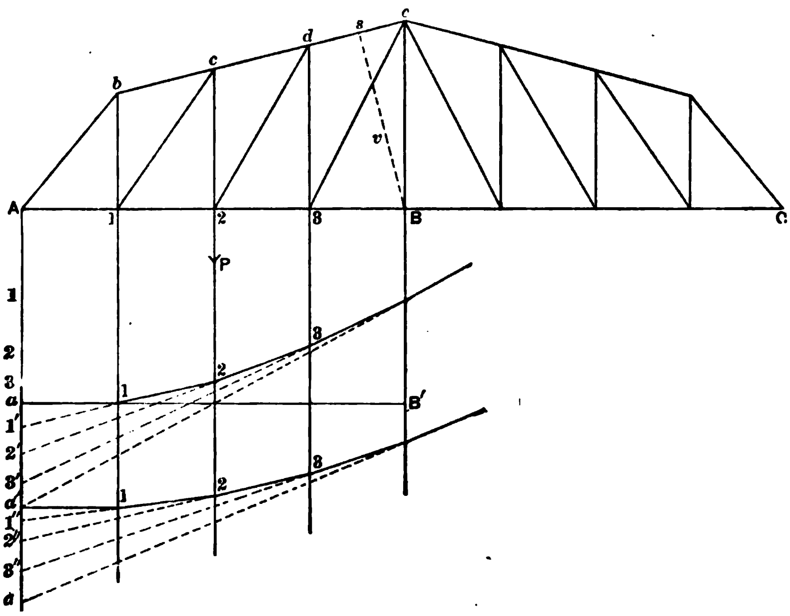
Method by Diagram.—The solution of equation (2) by means of a table as illustrated is sufficiently simple.

If, however, it is considered tedious, the following method by diagram may be employed for making the different summations:

Let the quantity $\frac{s}{av^3}$ for each chord member be considered as a fictitious load acting at the centre of moments for that member. If a is constant it is, of course, disregarded,—and it must be disregarded in determining the provisional areas, as pointed out on page 157.

Lay off (Fig. 128) these fictitious loads to any convenient scale along the line Aa. (See next Figure.) These loads are $A1$, 1–2, 2–3, $3a$. Take a pole at B so that the pole distance $AB = l$, draw the rays BA, $B1$, $B2$, etc., and construct the equilibrium polygon a–1–2–3. Produce the sides of this polygon to their intersections $1'$, $2'$, $3'$, a'.

Fig. 128.



Then in the horizontal through a take the pole distance $aB' = l$, draw the rays $B'a$, $B'1'$, $B'2'$, etc., and construct the second equilibrium polygon a'–1–2–3, etc. Produce the sides of this polygon to their intersections, $1''$, $2''$, $3''$, a''.

Then we have

$$Aa = \sum_0^l \frac{s}{av^3}, \qquad l \times aa' = \sum_0^l \frac{sx}{av^3}, \qquad l^2 \times a'a'' = \sum_0^l \frac{sx^2}{av^3}.$$

Also, for an apex load P at any apex, as, for instance, at 2, we have

$$l \times 2'a' = \sum_z^l \frac{sx}{av^3} \quad \text{and} \quad l^2 \times 2''a' = \sum_0^z \frac{sx^2}{av^3}.$$

We can then write in the place of equation (2), for P at 2

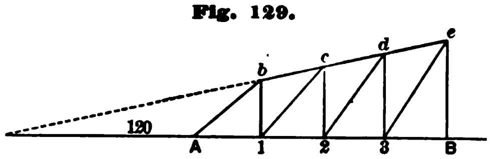
$$M_z = \frac{P[l \times 2''a' - z \times a'a'' + z \times 2'a']}{2 \times a'a'' + Aa - 2 \times aa'}.$$

For P at any other apex, as 3, we put $3'$ and $3''$ in place of $2'$ and $2''$.

There is very little, if any, advantage in this method by diagram over the tabulation of page 157, and the latter is more accurate.

CALCULATION OF STRESSES FOR PIVOT SPAN.—Let us now calculate the stresses in the example of page 157. Length of span $l = 80$ ft., divided into four panels of 20 ft. each; centre height $Be = 10$ ft., and height at end $b1 = 7$ ft. Let the train load be, say, 1 ton per foot; dead load, $\frac{1}{2}$ ton per foot. Locomotive excess, as on page 102, say 30 tons. Then $P = 20$ tons.

Fig. 129.



The upper chord, if prolonged, intersects the lower at a point 120 ft. from A, Fig. 129.

We have then the following lever arms:

	$A1$	1–2	2–3	$3B$	bc	cd	de	Ab	$b1$	$c1$	$c2$	$d2$	$d3$	$e3$
lever-arm =	7	8	9	10	7	8	9	39.64	140	52	160	65.66	180	80.5

1st. Draw Open.—For draw open, we have 5 tons at A and 10 tons at 1, 2, and 3. The stresses can be easily diagrammed by the method of Chapter I, page 8, or calculated as follows:

$A1 \times 7 + 5 \times 20 = 0,$	$A1 = -\ 14.286$ tons.
$1\text{–}2 \times 8 + 5 \times 40 + 10 \times 20 = 0,$	$1\text{–}2 = -\ 50$ "
$2\text{–}3 \times 9 + 5 \times 60 + 10(40 + 20) = 0,$	$2\text{–}3 = -\ 100$ "
$3B \times 10 + 5 \times 80 + 10(60 + 40 + 20) = 0,$	$3B = -\ 160$ "
$-\ bc \times 7 + 5 \times 20 = 0,$	$bc = +\ 14.286$ "
$-\ cd \times 8 + 5 \times 40 + 10 \times 20 = 0,$	$cd = +\ 50$ "
$-\ de \times 9 + 5 \times 60 + 10(40 + 20) = 0,$	$de = +\ 100$ "
$Ab \times 39.64 - 5 \times 120 = 0,$	$Ab = +\ 15.13$ "
$c1 \times 52 - 5 \times 120 - 10 \times 140 = 0,$	$c1 = +\ 38.46$ "
$d2 \times 65.66 - 5 \times 120 - 10(140 + 160) = 0,$	$d2 = +\ 54.82$ "
$e3 \times 80.5 - 5 \times 120 - 10(140 + 160 + 180) = 0,$	$e3 = +\ 67.08$ "
$-\ b1 \times 140 - 5 \times 120 = 0,$	$b1 = -\ 4.286$ "
$-\ c2 \times 160 - 5 \times 120 - 10 \times 140 = 0,$	$c2 = -\ 12.5$ "
$-\ d3 \times 180 - 5 \times 120 - 10(140 + 160) = 0,$	$d3 = -\ 20.0$ "

$$eB = -\ 70 \text{ tons.}$$

A negative sign denotes compression, a positive sign tension.

2d. Draw Shut.—We have found by the application of equation (2) (page 157),

For $P = 20$ tons at 1,	$S_1 = +\ 13.49$ tons.
" $P = 20$ " " 2,	$S_1 = +\ 7.75$ "
" $P = 20$ " " 3,	$S_1 = +\ 3.18$ "

And on the other span,

For $P = 20$ tons at 4,	$S_1 = -\ 1.822$ tons.
" $P = 20$ " " 5,	$S_1 = -\ 2.25$ "
" $P = 20$ " " 6,	$S_1 = -\ 1.51$ "

We have then, for P at 1,

$A1 \times 7 - 13.49 \times 20 = 0,$ $A1 = +38.54$ tons, $bc = -38.54$ tons.

$1\text{-}2 \times 8 - 13.49 \times 40 + 20 \times 20 = 0,$ $1\text{-}2 = +17.45$ " $cd = -17.45$ "

$2\text{-}3 \times 9 - 13.49 \times 60 + 20 \times 40 = 0,$ $2\text{-}3 = +1.04$ " $de = -1.04$ "

$3B \times 10 - 13.49 \times 80 + 20 \times 60 = 0,$ $3B = -12.08$ "

$Ab \times 39.64 + 13.49 \times 120 = 0,$ $Ab = -40.83$ tons.

$c1 \times 52 + 13.49 \times 120 - 20 \times 140 = 0,$ $c1 = +22.71$ "

$d2 \times 65.66 + 13.49 \times 120 - 20 \times 140 = 0,$ $d2 = +17.98$ "

$e3 \times 80.5 + 13.49 \times 120 - 20 \times 140 = 0,$ $e3 = +14.67$ "

$b1 \times 140 + 13.49 \times 120 = 0,$ $b1 = +11.56$ "

$-c2 \times 160 + 13.49 \times 120 - 20 \times 140 = 0,$ $c2 = -7.38$ "

$-d3 \times 180 + 13.49 \times 120 - 20 \times 140 = 0,$ $d3 = -6.56$ "

$$eB = -(S_1' + S_1) = -\left(\frac{2M_1}{l} + \frac{Pz}{l}\right) = -8.02 \text{ tons.}$$

A negative sign denotes compression, a positive sign tension.

In similar manner we find the stresses for P at 2, 3, and on the other span at 4, 5, 6. We can then draw up the following table. This table gives the stress in each member for each apex live load. The locomotive excess stresses can then be entered for each member. Thus for 1–2 we see that a load at 2 gives the greatest tension, and a load at 5 gives the greatest compression. If the locomotive excess is taken at 30 tons, then, since $P = 20$ tons, we have for the locomotive excess at 2 the stress in 1–2 equal to $+38.75 \times \frac{3}{2}$ $= \times 58.12$ tons, and for locomotive excess at 5 the stress in 1–2 is equal to $-11.25 \times \frac{3}{2}$ $= -16.87$ tons.

Since the dead load is a certain proportion of the live, in this case $\frac{1}{2}$, we can find the dead load stresses by taking one half the algebraic sum of the live load stresses. Thus for 1–2 we have dead load stress equal to $\dfrac{+72.10 - 27.91}{2} = +22.09$ tons.

Taking the dead load stresses in combination with the live load stresses, we find the maximum stresses for draw shut. Thus for 1–2 we have for maximum tension $+72.10$ for train, $+58.12$ for locomotive excess, and $+22.09$ for dead load. Total $+152.31$ tons. For maximum compression we have -27.91 for train and -16.87 for locomotive excess. But since the dead load acts also, the resultant is $-27.91 - 16.87 + 22.09 = -22.69$ tons.

If now we insert the stresses for draw open, we can pick out for any member the maximum stresses that can ever occur. Thus for 1–2 we have the greatest tension $+152.31$ tons, and the greatest compression is for draw open, viz., -50 tons.

If the ends are not raised to level with centre when the draw is shut, we have only to omit from the table the line for dead load stresses.

We see from the table just what loads and where placed give the greatest stress in any member.

We see also from the table that the stress in Ab is compression only. If, however, the dead load had been smaller we might have had tension in Ab also, for P_4, P_5, and P_6, and locomotive excess at P_4 acting. In such case the ends would have to be latched down.

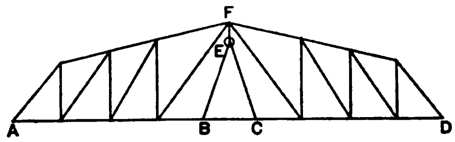
GENERAL METHOD FOR ANY SWING SPAN.—In the preceding we have given in detail the method of calculation for the centre-bearing pivot span. The same method is to be used for any case of swing span. Only the formulas will vary. It will therefore be sufficient if we give for other cases simply the formulas which apply to the case.

STRESSES IN PIVOT SPAN.

		$A1$	1–2	2–3	$3B$	bc	cd	de	Ab	$c1$	de		$b1$	$c2$	$d3$	eB
	P_1	+ 38.54	+ 17.45	+ 1.04	− 12.08	− 38.54	− 17.45	− 1.04	− 40 83	+ 22.71	+ 17.98	+ 14.67	+ 11.56	− 7.38	− 6.56	− 8 02
	P_2	+ 22.14	+ 38.75	+ 7.22	− 58.00	− 22.14	− 38.75	− 7.22	− 23.46	− 17.88	+ 34.57	+ 28.20	+ 6.64	+ 5 81	− 12.61	− 14.5
	P_3	+ 9.08	+ 15 90	+ 21.20	− 14.56	− 9.08	− 15.90	− 21.20		− 7.34	− 5.81	+ 39.98	+ 2.72	+ 2.38	+ 2.12	− 18.64
	P_4	− 5.20	− 9.11	− 12.15	− 14.58	+ 5.20	+ 9.11	+ 12.15	− 9.63 + 5.51	+ 4.20	+ 3.33	+ 2.71	− 1.56	− 1.36	− 1.21	− 18.69
	P_5	− 6.43	− 11.25	− 15.00	− 18.00	+ 6.43	+ 11.25	+ 15.00	+ 6.81	+ 5.19	+ 4.11	+ 3.35	− 1.92	− 1.68	− 1.50	− 14.5
	P_6	− 4.31	− 7.55	− 10.06	− 12.08	+ 4.31	+ 7.55	+ 10.06	+ 4.57	+ 3.48	+ 2.76	+ 2.25	− 1.29	− 1.13	− 1.00	− 8.02
Live Load Stresses.	+	+ 69.76	+ 72.10	+ 29.46	....	+ 15.94	+ 27.91	+ 37.21	+ 16.89	+ 35.58	+ 62.75	+ 91.16	+ 20.92	+ 8.19	+ 2.12	....
	−	− 15.94	− 27.91	− 37.21	− 129.30	− 69.76	− 72.10	− 29.46	− 73.92	− 25.22	− 5.81	....	− 4.77	− 11.55	− 22 88	− 82.37
Locomotive Excess Stresses	+	+ 57.81	+ 58.12	+ 31.80	....	+ 9.64	+ 16.87	+ 22.5	+ 10.21	+ 34.06	+ 51 85	+ 59.97	+ 17.34	+ 8.71	+ 3.18	....
	−	− 9.64	− 16.87	− 22.5	− 87.0	− 57.81	− 58.12	− 31.80	− 61.24	− 26.82	− 8.71	....	− 2.88	− 2.52	− 18.91	− 28.03
Dead load stresses...		+ 26.91	+ 22.09	− 3.87	− 64.65	− 26.91	− 22.09	+ 3.87	− 28.51	+ 5.18	+ 28.47	+ 45.58	+ 8.07	− 1.68	− 10.38	− 41.18
Maximum Stresses...		+ 154.48	+ 152.31	+ 57.39	....	....	+ 22.69	+ 63.58	....	+ 74.82	+ 143.07	+ 196.71	+ 46.33	+ 15.22	....	....
Draw Shut..........		....	− 22.69	− 63.58	− 280.95	− 154.48	− 152.31	− 57.39	− 163 67	− 46.86	....	....	....	− 15.75	− 52.17	− 69.21
Draw Open..........		− 14.29	− 50.00	− 100.00	− 160.00	+ 14.29	+ 50.0	+ 100.00	+ 15.13	+ 38.46	+ 54.82	+ 67.08	− 4.29	− 12.5	− 20.0	− 70.0
Total Maximum Stresses		+ 154 48	+ 152.31	+ 57.39	....	+ 14.29	+ 50.0	+ 100.0	+ 15.13	+ 74.82	+ 143.07	+ 196.71	+ 46.33	+ 15.22	....	....
		− 14.29	− 50.00	− 100.0	− 280.95	− 154.48	− 152.31	− 57.39	− 163.67	− 46.86	....	....	− 4.29	− 15 75	− 52.17	− 70.0

RIM-BEARING TURN-TABLE—THREE SUPPORTS.—Instead of turning on a pivot, a turn-table is often used, Fig. 130. Thus in the Figure the frame BEC rests on the turn-table. The short link FE carries the load to the frame at E. The calculation is then precisely the same as for pivot span, the length of each span being the horizontal distance from A to E or D to E.

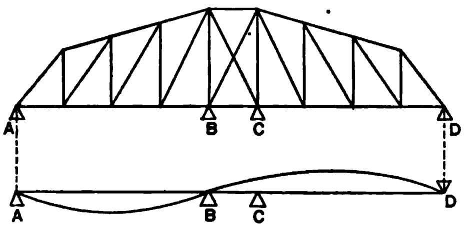
Fig. 130.



RIM-BEARING TURN-TABLE—FOUR SUPPORTS.—When a turn-table is used instead of a pivot, we have three spans continuous or partially continuous over four supports. The two long end spans are of equal length, the small centre span is the width of the turn-table.

If the bracing is carried through the centre span, as shown in the accompanying Fig. 131, it is evident that a load over one end span as AB tends to lift the span from the support C. It will be found in general to be impracticable to hold the span down at C.

Fig. 131.

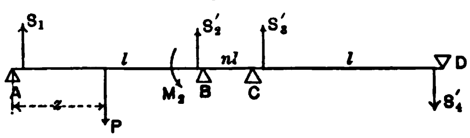


For this reason *the bracing in the centre span is omitted.* The continuity in such case is only partial, but the span can then be held down at C, and the calculation of the stresses is then readily made.

For this case with the centre span without bracing we have the following formulas:

Let the length of span AB, Fig. 132, or CD be l, the centre span BC be nl, the apex load P, and its distance from the end A be z. Let M_2 be the moment at B (counter clockwise rotation, positive). The moment at C is the same as at B. We have then

Fig. 132.



$$S_1 = -\frac{M_2}{l} + \frac{P}{l}(l - z), \quad S_2' = \frac{M_2}{l} + \frac{Pz}{l}, \quad S_3 = +\frac{M_2}{l} = -S_4'. \quad . \quad . \quad . \quad . \quad (1)$$

From equations (1) we can find the pressures at the supports for any position of P if M_2 is known

Let s, a, v, and x, have the same signification as before (page 157). Then we have

$$M_2 = \frac{P\left[\sum_0^z \frac{sx^2}{av^2} - \frac{z}{l}\sum_0^l \frac{sx^2}{av^2} + z\sum_z^l \frac{sx}{av^2}\right]}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^{nl} \frac{s}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It will be seen that equation (2) is the same as for the pivot span, except for the second term in the denominator. That is, for $nl = 0$ we have the pivot span.

COMPARISON WITH FORMULAS HERETOFORE IN USE.—If in equation (2) we assume the area a of cross-section of chord members as constant, also assume parallel chords and panel length small in comparison to length of span l, then av^2 cancels out, we can put dx for s, and obtain

$$M_2 = \frac{\int_0^z x^2 dx - \frac{z}{l}\int_0^l x^2 dx + z\int_z^l x\,dx}{\frac{2}{l}\int_0^l x^2 dx + l\int_0^{nl} dx + l\int_0^l dx - 2\int_0^l x\,dx} \cdot P$$

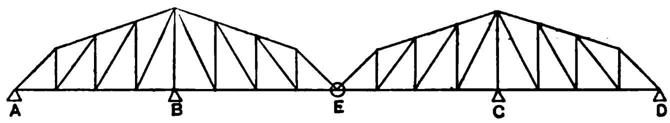
If we perform the integrations and put k for the ratio $\frac{z}{l}$, we obtain

$$M_2 = \frac{Pl(k - k^3)}{2(2 + 3n)}.$$

This is the ordinary formula. It is based, as we see, upon assumptions not in accord with fact.

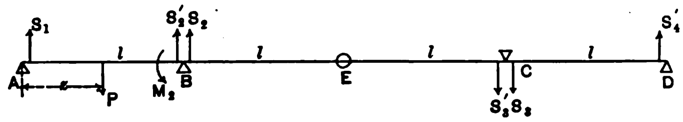
DOUBLE PIVOT SPAN.—The Fig. 133 represents a double pivot span, or it may be a rolling span. The span is opened either by swinging or rolling back the trusses. When closed it is rendered partially continuous by a pin at E.

Fig. 133.



We have evidently two cases, load P in span AB and in span BE. Let the spans AB, BE, EC, CB be equal and denoted by l.

Case I.—Load P in span AB. We have for this case for the end shears



$$\left.\begin{aligned} S_1 &= -\frac{M_2}{l} + \frac{P}{l}(l - z), \\ S_2' &= \frac{M_2}{l} + \frac{Pz}{l}, \\ S_2 &= \frac{M_2}{l} = -S'_3 = -S_3 = S'_4, \end{aligned}\right\} \quad (1)$$

and for M_2

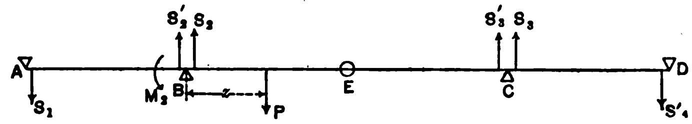
$$M_2 = \frac{P}{2} \cdot \frac{\left[\sum_0^z \frac{sx^2}{av^2} - \frac{z}{l}\sum_0^l \frac{sx^2}{av^2} + z\sum_z^l \frac{sx}{av^2}\right]}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

It will be seen by comparing with the single pivot span, page 157, that this value of M_2 is one half of the value of M_2 there given.

If then we assume a as constant, parallel chords, and panel length small compared to length of span l, we obtain as before if we put $\kappa = \frac{z}{l}$,

$$M_2 = \frac{Pl}{8}(k - k^3).$$

Case II.—Load P in span BC. We have for this case



$$\left.\begin{aligned} S_1 &= -\frac{M_2}{l} = -S'_2, \\ S_2 &= \frac{M_2}{l} + \frac{P(l - z)}{l}, \\ S'_3 &= -\frac{M_2}{l} + \frac{Pz}{l} = S_3 = -S'_4, \end{aligned}\right\} \quad (1)$$

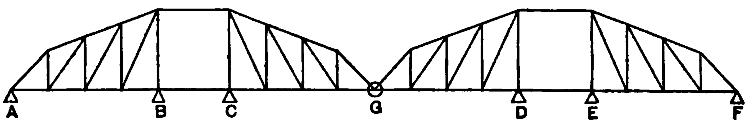
and for M_2

$$M_2 = \frac{P}{2} \cdot \frac{\left[l\sum_0^z \frac{sx}{av^2} - \sum_0^z \frac{sx^2}{av^2} + \frac{3z}{l}\sum_0^l \frac{sx^2}{av^2} - 3z\sum_0^l \frac{sx}{av^2} - z\sum_z^l \frac{sx}{av^2} + lz\sum_z^l \frac{s}{av^2} + lz\sum_0^l \frac{s}{av^2}\right.}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}}. \quad (2)$$

If we take a as constant, assume parallel chords and panel lengths small compared to l, we obtain, if we put $k = \frac{z}{l}$,

$$M_2 = \frac{Pl(6k - 3k^2 + k^3)}{8}.$$

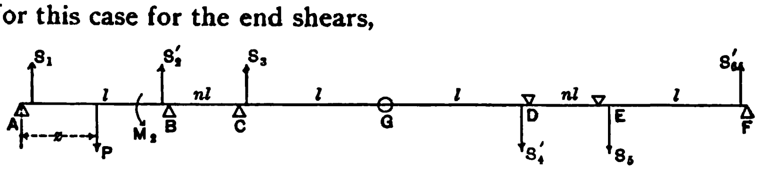
DOUBLE RIM-BEARING TURNTABLE.—The following Figure represents a double swing span with rim-bearing turn-tables. When shut it is rendered partially continuous by a pin at G. There is no bracing in the turn-table spans.



We have evidently two cases—load P in span AB and in span CG. Let the spans AB, CG, GB, EF be equal and denoted by l; the turn table spans BC and DE be nl.

CASE I.—*Load P in span AB.*

We have for this case for the end shears,



$$\left.\begin{aligned} S_1 &= -\frac{M_2}{l} + \frac{P}{l}(l - z), \qquad S_2' = \frac{M_2}{l} + \frac{Pz}{l}, \\ S_3 &= \frac{M_2}{l} = -S_4' = -S_5 = S_6'; \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

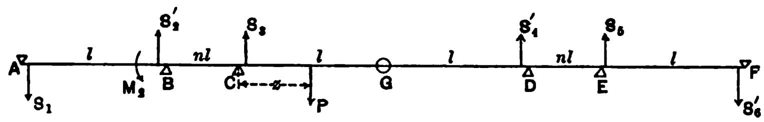
and for M_2,

$$M_2 = \frac{P}{2} \cdot \frac{\left[\sum_0^z \frac{sx^2}{av^2} - \frac{z}{l}\sum_0^l \frac{sx^2}{av^2} + z\sum_z^l \frac{sx}{av^2}\right]}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^{nl} \frac{s}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}}.$$

If we take a as constant, assume parallel chords and panel length small compared to l, and put $k = \frac{z}{l}$, we obtain

$$M_2 = \frac{Pl(k - k^3)}{4(2 + 3n)}.$$

CASE II.—*Load P in span CG.*



We have in this case for the end shears

$$S_1 = -\frac{M_2}{l} = -S_2', \quad S_3 = \frac{M_2}{l} + \frac{P(l - z)}{l}, \quad S_4' = -\frac{M_2}{l} + \frac{Pz}{l} = S_5 = -S_6'; \quad . \, (1)$$

and for M_2,

$$M_2 = \frac{P}{2} \cdot \frac{\begin{aligned}\Big[l\sum_0^z \frac{sx}{av^2} - \sum_0^z \frac{sx^2}{av^2} + \frac{3z}{l}\sum_0^l \frac{sx^2}{av^2} - 3z\sum_0^l \frac{sx}{av^2} - z\sum_z^l \frac{sx}{av^2} \\ + lz\sum_z^l \frac{s}{av^2} + lz\sum_0^{nl} \frac{s}{av^2} + lz\sum_0^l \frac{s}{av^2}\Big]\end{aligned}}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^{nl} \frac{s}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}}$$

If we take a as constant, assume parallel chords and panel lengths small compared to l we obtain, if we put $k = \frac{z}{l}$,

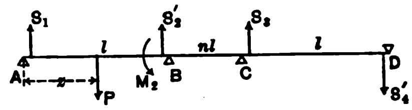
$$M_2 = \frac{Pl[k(6 + 6n) - 3k^2 + k^3]}{4(2 + 3n)}$$

Deduction of the Formulas.

We shall now give the demonstration of the formulas used in this Chapter.

Rim-bearing Turntable—Four Supports—No Shear in Centre Span.

Let the length of span AB or CD be l, of the centre span BC be nl, the apex load P and its distance from the end A be z.



Let M_2 be the moment at B (counter clockwise rotation positive). Since there is no shear in the centre span, the moment at C is also M_2, the same as at B.

Let the pressure or shear at support A due to P be S_1 and at B on left S_2', at C on right S_3, at D on left S_4'; positive values denote upward direction and negative downward.

Taking moments about B we have

$$- S_1 l + P(l - z) = M_2.$$

Taking moments about D we have

$$- S_3 l + M_2 = 0.$$

From the first of these equations we obtain

$$\left.\begin{aligned} S_1 &= - \frac{M_2}{l} + \frac{P}{l}(l - z), \\ S_2' &= P - S_1 = \frac{M_2}{l} + \frac{Pz}{l}, \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

and from the second,

$$S_3 = \frac{M_2}{l} = -S_4',$$

These are equations (1), page 163.

Let v be the lever arm for any chord member and M the moment at the centre of moments for that member. Then the stress in that member is $\frac{M}{v}$. Let a be the area of cross-section of the member, and s its length.

Then from (III.) of the preceding Chapter the work done in straining the member is

$$\text{work} = \frac{M^2 s}{2Eav^2}.$$

The total work of straining all the chord members is then

$$\text{work} = \sum \frac{M^2 s}{2Eav^2}, \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{I})$$

and this work, as we have seen from the preceding Chapter, according to the principle of least work, *must be a minimum.*

Now for any point distant x from A, between A and P, we have

$$M = -S_1 x = \frac{M_2 x}{l} - \frac{P(l - z)x}{l}.$$

For any point between P and B we have

$$M = - S_1 x + P(x - z) = \frac{M_2 x}{l} - \frac{Pz}{l}(l - x).$$

For any point between B and C we have

$$M = M_2.$$

For any point distant x from C, between C and D, we have

$$M = M_2 - S_2 x = \frac{M_2(l-x)}{l}.$$

We have then, for the work in straining all the chord members from (I.),

$$\text{Work} = \sum_0^z \left[\frac{M_2 x - P(l-z)x}{l}\right]^2 \frac{s}{2Eav^2} + \sum_0^l \left[\frac{M_2 x - Pz(l-x)}{l}\right]^2 \frac{s}{2Eav^2} + \sum_0^{nl} \frac{M_2^2 s}{2Eav^2}$$
$$+ \sum_0^l \left[\frac{M_2(l-x)}{l}\right]^2 \frac{s}{2Eav^2}.$$

Since the work is here given in terms of M_2 and known quantities, and the work must be a minimum, we differentiate with reference to M_2 and put the differential coefficient equal to zero.

We thus obtain

$$\frac{d(\text{work})}{dM_2} = 0 = \sum_0^z \left[\frac{M_2 x^2 - P(l-z)x^2}{l^2}\right] \frac{s}{Eav^2} + \sum_z^l \left[\frac{M_2 x^2 - Pz(l-x)x}{l^2}\right] \frac{s}{Eav^2} + \sum_0^{nl} \frac{M_2 s}{Eav^2} + \sum_0^l \frac{M_2(l-x)^2 s}{l^2 Eav^2}$$

Hence

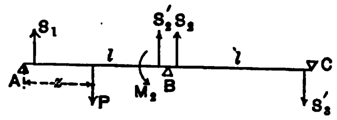
$$M_2 = \frac{P\left[\sum_0^l \frac{z(l-x)xs}{av^2} + \sum_0^z \frac{(l-z)x^2 s}{av^2}\right]}{\sum_0^l \frac{sx^2}{av^2} + \sum_0^{nl} \frac{l^2 s}{av^2} + \sum_0^l \frac{(l-x)^2 s}{av^2}};$$

or, by reduction,

$$M_2 = \frac{P\left[\sum_0^z \frac{sx^2}{av^2} - \frac{z}{l}\sum_0^l \frac{sx^2}{av^2} + z\sum_z^l \frac{sx}{av^2}\right]}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^{nl} \frac{s}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}}. \quad . \quad . \quad . \quad . \quad (2)$$

This is equation (2), page 163.

Centre-bearing Pivot Span—Three Supports.—This is only a special case of the preceding, when the centre span is zero, or $n = 0$.



We have then from equations (1) already deduced

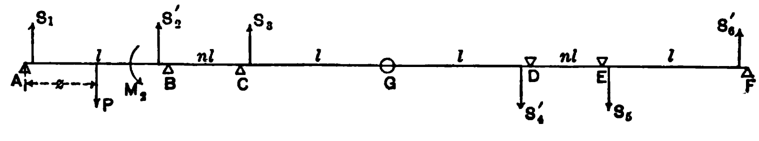
$$\left.\begin{aligned} S_1 &= -\frac{M_2}{l} + \frac{P}{l}(l-z), \\ S_2' &= \frac{M_2}{l} + \frac{Pz}{l}, \\ S_2 &= \frac{M_2}{l} = -S_2'; \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and making $n = 0$ in (2) preceding, we obtain

$$M_2 = \frac{P\left[\sum_0^z \frac{sx^2}{av^2} - \frac{z}{l}\sum_0^l \frac{sx^2}{av^2} + z\sum_z^l \frac{sx}{av^2}\right]}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is equation (2), page 157.

Double Rim-bearing Turntable.—The method is the same for this case also. We have evidently two cases, load P in span AB and in span CG.



CASE I.—*Load P in Span AB.*—Since there is no shear in the span BC or DE, the moment M_2 at B and C is the same. We have also S_4' and S_6' equal and opposite to S_2'.

Taking moments then about B and about G, we have

$$-S_1 l + P(l-z) = M_2. \qquad M_2 - S_2 l = 0.$$

Hence

$$\left.\begin{aligned} S_1 &= -\frac{M_2}{l} + \frac{P}{l}(l-z), \quad S_2 = \frac{M_2}{l} = -S_4' = -S_6 = S_6', \\ S_2' &= P - S_1 = \frac{M_2}{l} + \frac{Pz}{l}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

These are equations (1), page 165.

For any point distant x from A, between A and P, we have

$$M = -S_1 x = \frac{M_2 x}{l} - \frac{P(l-z)x}{l}.$$

For any point between P and B we have

$$M = -S_1 x + P(x-z) = \frac{M_2 x}{l} - \frac{Pz(l-x)}{l}.$$

For any point between B and C we have

$$M = M_2.$$

For any point between C and G we have

$$M = M_2 - S_2 x = \frac{M_2(l-x)}{l}.$$

For any point between G and D we have

$$M = -\frac{M_2 x}{l}.$$

For any point between D and E we have

$$M = -M_2.$$

For any point between E and F we have

$$M = -\frac{M_2(l-x)}{l}.$$

Hence we obtain

$$\text{Work} = \sum_0^z \left[\frac{M_2 x - P(l-z)x}{l}\right]^2 \frac{s}{2Eav^2} + \sum_z^l \left[\frac{M_2 x - Pz(l-x)}{l}\right]^2 \frac{s}{2Eav^2}$$
$$+ 2\sum_0^{nl} \frac{M_2^2 s}{2Eav^2} + 2\sum_0^l \frac{M_2^2(l-x)^2 s}{2l^2 Eav^2} + \sum_0^l \frac{M_2^2 x^2 s}{2l^2 Eav^2}.$$

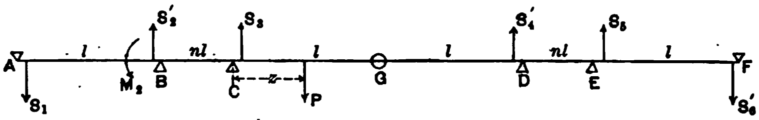
Differentiating with reference to M_2 and placing the differential coefficient equal to zero we obtain

$$M_2 = \frac{P\left[\sum_0^z \frac{(l-z)x^2 s}{av^2} + \sum_z^l \frac{z(l-x)xs}{av^2}\right]}{2\sum_0^l \frac{x^2 s}{av^2} + 2l^2 \sum_0^{nl} \frac{s}{av^2} + 2\sum_0^l \frac{(l-x)^2 s}{av^2}}$$

$$= \frac{P}{2} \cdot \frac{\sum_0^z \frac{sx^2}{av^2} - \frac{z}{l}\sum_0^l \frac{sx^2}{av^2} + z\sum_z^l \frac{sx}{av^2}}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^{nl} \frac{s}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}}. \quad . \quad . \quad (2)$$

This is the same as equation (2), page 165.

CASE II.—*Load P in Span CG.*—In this case we have, taking moments about B and G,



$$-S_1 l = M_2, \qquad M_2 - S_2 l + P(l - z) = 0.$$

Hence

$$\left.\begin{array}{l} S_1 = -\dfrac{M_2}{l} = -S_2', \qquad S_2 = \dfrac{M_2}{l} + \dfrac{P(l-z)}{l}, \\ S_4' = P - S_2 = -\dfrac{M_2}{l} + \dfrac{Pz}{l} = S_5 = -S_6'. \end{array}\right\} \quad (1)$$

These are equations (1), page 165.

For any point between A and B we have

$$M = -S_1 x = \frac{M_2 x}{l}.$$

For any point between B and C we have

$$M = M_2.$$

For any point between C and P we have

$$M = M_2 - S_2 x = \frac{M_2(l-x)}{l} - \frac{P(l-z)x}{l}.$$

For any point between P and G we have

$$M = M_2 - S_2 x + P(x - z) = \frac{M_2(l-x)}{l} - \frac{Pz(l-x)}{l}.$$

For any point between G and D we have

$$M = S_4' x = -\frac{M_2 x}{l} + \frac{Pzx}{l}.$$

For any point between D and E we have

$$M = -M_2 + Pz.$$

For any point between E and F we have

$$M = -M_2 + Pz - S_5 x = -\frac{M_2(l-x)}{l} + \frac{Pz(l-x)}{l}.$$

Hence we obtain

$$\begin{aligned} \text{Work} = & \sum_0^l \frac{M_2^2 x^2 s}{2l^2 Eav^2} + \sum_0^{nl} \frac{M_2^2 s}{2Eav^2} + \sum_0^z \left[\frac{M_2(l-x) - P(l-z)x}{l}\right]^2 \frac{s}{2Eav^2} \\ & + \sum_z^l \left[\frac{M_2(l-x) - Pz(l-x)}{l}\right]^2 \frac{s}{2Eav^2} + \sum_0^l \left[\frac{M_2 x - Pzx}{l}\right]^2 \frac{s}{2Eav^2} \\ & + \sum_0^{nl} \frac{(M_2 - Pz)^2 s}{2Eav^2} + \sum_0^l \left[\frac{M_2(l-x) - Pz(l-x)}{l}\right]^2 \frac{s}{2Eav^2}. \end{aligned}$$

Differentiating with reference to M_2 and placing the first differential coefficient equal to zero, we obtain, after reduction,

$$M_2 = \frac{P}{2} \cdot \frac{l\sum_0^z \frac{sx}{av^2} - \sum_0^z \frac{sx^2}{av^2} + \frac{3z}{l}\sum_0^l \frac{sx^2}{av^2} - 3z\sum_0^l \frac{sx}{av^2} - z\sum_z^l \frac{sx}{av^2} + lz\sum_z^l \frac{s}{av^2} + lz\sum_0^{nl} \frac{s}{av^2} + lz\sum_0^l \frac{s}{av^2}}{\frac{2}{l}\sum_0^l \frac{sx^2}{av^2} + l\sum_0^{nl} \frac{s}{av^2} + l\sum_0^l \frac{s}{av^2} - 2\sum_0^l \frac{sx}{av^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is the same as equation (2), page 165.

Double Pivot Span.—This is only a special case of the preceding when the turn-table spans are zero, or $n = 0$. Making this change, we obtain at once equations (1) and (2), page 164.

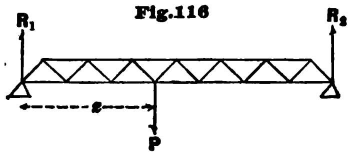
Work in the Braces.—It will be noted that the work of straining the braces is neglected. This could also be inserted. But the resulting formulas, while much longer, would give practically the same result.

CHAPTER VIII.

THE CONTINUOUS GIRDER.

Definition of Shear—Reaction.—A continuous girder is one which rests upon more than two supports. When a girder rests upon two supports only, a weight placed anywhere upon it causes pressures or reactions at the two supports, which may be at once determined from the law of the lever. Thus in Fig. 116, a weight P, placed at a distance, z, from the left end, causes the reactions $R_1 = \frac{P(l-z)}{l}$ and $R_2 = \frac{Pz}{l}$. These reactions being thus known, the stresses in every member can be readily calculated by moments, or otherwise.

Fig. 116

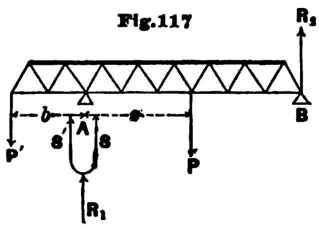


But suppose one end of this girder to overhang the support, as in Fig. 117, and to have a weight P' at the end, as well as the weight P, as before. The reaction R_2 at the right end will then be found by moments, as follows:

$$+R_2 \times l - Pz + P'b = 0, \quad \text{or} \quad R_2 = \frac{Pz}{l} - \frac{P'b}{l}.$$

The reaction R_2 is, therefore, no longer the same as before, but is diminished by $\frac{P'b}{l}$.

Fig. 117



The reaction at A is also no longer the same as before, but is composed of two parts, viz.: the shear S at A due to P, and the shear S' at A due to P'. The shear due to P, or the portion of P which goes toward the left, is equal to $S = P - R_2$, or $S = \frac{P(l-z)}{l} + \frac{P'b}{l}$. The shear at A due to P' is $S' = P'$. Hence the entire reaction at A is $R_1 = S' + S = P' + \frac{P(l-z)}{l} + \frac{P'b}{l}$. The same result can also be found by moments. Thus,

$$-R_1 l + P(l-z) + P'(b+l) = 0, \quad \text{or} \quad R_1 = P' + \frac{P(l-z)}{l} + \frac{P'b}{l}.$$

We see, then, and the above is simply intended to illustrate this point, that the reaction at a support, when the girder extends past this support, is composed of two parts, viz., the shear due to loads on the right, and the shear due to loads on the left. *Shear* and *reaction*, then, must now be distinguished from each other and never be confounded. In the case of the simple girder upon two supports only, the shears and reactions at the supports are the same, but in a continuous girder they are not.

Now in Fig. 117, the weight P' and the shear $S' = P'$, form a couple, the moment of which is,

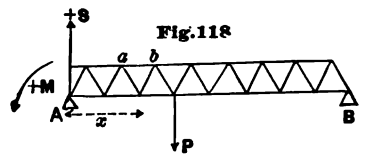


Fig. 118

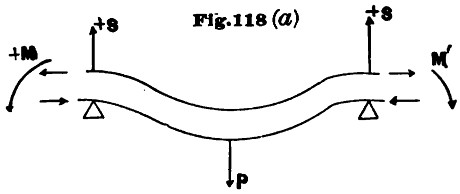


Fig. 118 (a)

therefore, constant and equal to $P'b$ for all points of the truss to the right of A (see page 25). If, then, Fig. 118, we suppose acting at A the shear S due to P, viz., $\frac{P(l-z)}{l} + \frac{P'b}{l}$, and *in addition* the moment $+M = +P'b$, we can find the stresses in every member just as for the simple girder, the only difference being that we have the moment $+M$ at the support, whereas in the simple girder we have the shear or reaction at the support only.

Thus let ab, Fig. 118, be any panel, the point of moments for which is distant x from A, and let d be the depth of girder.

Then for the simple girder the stress in ab would be $ab \times d = -Sx$, or $ab = -\frac{Sx}{d}$, where S would be, as in Fig. 116, equal to $R_1 = \frac{P(l-z)}{l}$.

But for the overhanging girder, we should have $ab \times d = -Sx + M$, or $ab = -\frac{Sx}{d} + \frac{M}{d}$, where now $S = \frac{P(l-z)}{l} + \frac{P'b}{l}$ and $M = P'b$. If, therefore, S and M can be found for any loading, the calculation of the stresses offers no difficulty.

Continuous Girder—Exterior and Interior Loading.—Now Fig. 118 (a) represents precisely the state of a span of a continuous girder. A load placed anywhere upon the span causes at each end *positive* shears and *positive* moments. One portion of the problem, therefore, which we must solve, is to find for any position of the load what these shears and moments are. Any system of loading in the span itself we call *interior loading*.

But in the case of the continuous girder, not only do loads in the span itself cause stresses in all the members of that span, but also loads in other spans. We have, therefore, to find the moment and shear at the ends of any span caused by loads in any of the others.

In Fig. 119 let there be a weight in the span AB. As we have seen, this causes positive shears at A and B. But as the other spans are unloaded the curve of the girder must be as shown in the figure. That is, *the shears at both supports of any loaded span are positive, and are alternately minus and plus either way from that span.*

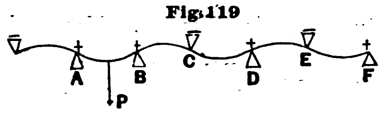


Fig. 119

In the same way we see that the *moments at the ends of a loaded span are both positive, that is, cause tension in the upper chord, and are alternately minus and plus either way from that span.*

For any span, then, as DE, Fig. 120, the greatest positive shear and moment at the end D, due to exterior loading, will be caused when the spans AB, CD, FG, etc., are fully loaded and the others are empty. The greatest negative shear and moment at D will be when BC, EF, GH, etc., are loaded.

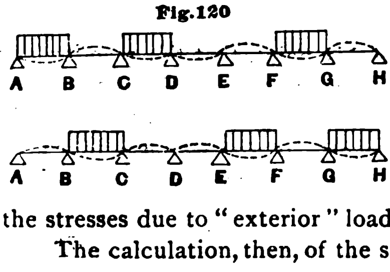


Fig. 120

The second part of our problem is, then, to determine for any span the shear and moment at the end of that span caused by a full load over any other span. We can thus find the stresses due to "exterior" loading.

The calculation, then, of the stresses in any span of a continuous girder offers no especial diffi-

culty, provided we can find, 1st, the shear and moment at the end of that span due to a concentrated load placed anywhere within it, and 2d, the shear and moment at the same end for a full load over any other span.

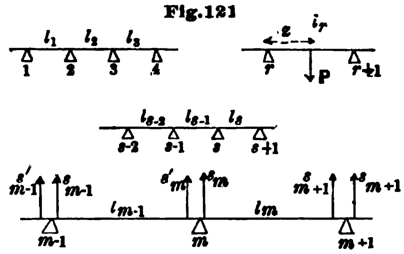


Fig. 121

GENERAL FORMULAS.*—We give here, therefore, the formulas which will enable us to determine the shears and moments. The development of these formulas is given in the Appendix to Part I. page 340.

NOTATION.—The notation we adopt is as follows, Fig. 121:

Whole number of spans is indicated by s.

Hence, whole number of supports is $s + 1$, numbered from left to right. Number of any support in general, always from left, is m.

The supports *adjacent to the loaded span*, left and right, are indicated by r and $r + 1$.

The length of span is denoted by l. The subscript denotes which span is referred to. Thus l_2 is the second span, l_3 the third from left, and so on. l_r is the length of the loaded span, l_r any span in general. The subscript is thus always the number of the left hand support.

A concentrated load is denoted by P.

Its distance from the left hand support is z.

The ratio of z to length of loaded span l_r is $k = \frac{z}{l_r}$.

The moment at any support in general is M_m, where m may be 1, 2, 3, r, $r + 1$, s, etc., indicating in every case the moment at corresponding support from left.

In same way the shear just to the right of any support is denoted by S_m. Thus S_r is the shear just to the right of the left end of the loaded span. The shear just to the left of any support is denoted by S'_m.

The uniform live load is w per unit of length.

These comprise all the symbols we shall have occasion to use. By reference to Fig. 121, the reader can familiarize himself with their signification, and will then find no difficulty in understanding and using the following formulas.

FORMULAS FOR MOMENTS AND SHEARS.—ALL SUPPORTS ON LEVEL.—For the moment at any support to the left of the loaded span, or 1st when $m < r + 1$,

$$M_m = c_m \frac{A_r d_{s-r+2} + B_r d_{s-r+1}}{l_2 d_{s-1} + 2(l_1 + l_2) d_s} \quad \text{. . . . .} \quad \text{(I.)}$$

For the moments on left at any support on the right of the loaded span, or 2d when $m > r$,

$$M_m = d_{s-m+2} \frac{A_r c_r + B_r c_{r+1}}{l_{s-1} c_{s-1} + 2(l_s + l_{s-1}) c_s} \quad \text{. . . . .} \quad \text{(II.)}$$

For the shear at the left support of loaded span,

$$S_r = \frac{M_r - M_{r+1}}{l_r} + q \quad \text{. . . . .} \quad \text{(III.}a\text{)}$$

at the right support of loaded span,

$$S'_{r+1} = \frac{M_{r+1} - M_r}{l_r} + q' \quad \text{. . . . .} \quad \text{(III.}b\text{)}$$

For *un*loaded spans,

$$S_m = \frac{M_m - M_{m+1}}{l_m}, \quad S'_m = \frac{M_m - M_{m-1}}{l_{m-1}} \quad \text{. . . . .} \quad \text{(IV.)}$$

* These formulas were first given by Prof. Merriman, *London Phil. Magazine*, Sept 1875.

In these formulas we have for concentrated loading,

$$q = P\,(1 - k), \qquad q' = Pk, \qquad k = \frac{z}{l_r},$$

$$A_r = P_r l_r^2\,(2k - 3k^2 + k^3), \qquad B_r = P_r l_r^2\,(k - k^3).$$

For uniform load entirely covering one span, we have $q = q' = \frac{1}{2}\,w_r l_r$ and $A = B = \frac{1}{4}\,w_r l_r^3$.
The numbers c and d have the following values:

$$c_1 = 0, \qquad c_2 = 1, \qquad c_3 = -\frac{2\,(l_1 + l_2)}{l_2}, \qquad c_4 = -2\,c_3\,\frac{l_2 + l_3}{l_3} - c_2\,\frac{l_2}{l_3},$$

$$c_5 = -2c_4\,\frac{l_3 + l_4}{l_4} - c_3\,\frac{l_3}{l_4}, \qquad c_6 = -2c_5\,\frac{l_4 + l_5}{l_5} - c_4\,\frac{l_4}{l_5},$$

and so on.

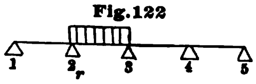
$$d_1 = 0, \qquad d_2 = 1, \qquad d_3 = -2\,\frac{l_s + l_{s-1}}{l_{s-1}}, \qquad d_4 = -2d_3\,\frac{l_{s-1} + l_{s-2}}{l_{s-2}} - d_2\,\frac{l_{s-1}}{l_{s-2}},$$

$$d_5 = -2d_4\,\frac{l_{s-2} + l_{s-3}}{l_{s-3}} - d_3\,\frac{l_{s-2}}{l_{s-3}}, \qquad d_6 = -2d_5\,\frac{l_{s-3} + l_{s-4}}{l_{s-4}} - d_4\,\frac{l_{s-3}}{l_{s-4}}.$$

and so on. The numbers can be written out by simple inspection to any extent desired.

These formulas hold good for any number of spans of different lengths, *provided all the supports are on the same level, or at a constant elevation.* They are also based upon the supposition of a constant coefficient of elasticity and constant moment of inertia of cross-section.

Fig. 122



A few examples will make the use and application of the preceding formulas clear.

EXAMPLE 1. *A continuous girder of four equal spans has the second span from the left covered with the live load. What are the moments and shears at the supports?*

We have in this case, Fig. 122,

$$s = 4, \qquad r = 2, \qquad c_1 = 0, \qquad c_2 = 1, \qquad c_3 = -4, \qquad c_4 = +15, \text{ and } d_1 = 0,$$
$$d_2 = 1, \qquad d_3 = -4, \qquad d_4 = +15.$$

For supports 1 and 2, we have $m < r + 1$, hence from equation (I.), p. 173,

$$M_1 = 0, \quad M_2 = c_2\,\frac{Ad_4 + Bd_3}{ld_3 + 4ld_4} = \frac{15A - 4B}{56l} = +\frac{11}{224}\,wl^2.$$

For supports 3, 4, and 5, we have $m > r$, hence from equation (II.), page 173,

$$M_3 = d_3\,\frac{Ac_3 + Bc_3}{lc_3 + 4lc_4} = -4\,\frac{A - 4B}{-4l + 60l} = +\frac{12}{224}\,wl^2,$$

$$M_4 = d_2\,\frac{Ac_2 + Bc_3}{lc_3 + 4lc_4} = \frac{A - 4B}{56l} = -\frac{3}{224}\,wl^2, \quad M_5 = 0.$$

For the shear (or reaction) at the first support, we have from equation (IV.),

$$S_1 = \frac{M_1 - M_2}{l} = -\frac{11}{224}\,wl.$$

For the second support, the shear on the *left* is,

$$S'_2 = \frac{M_2 - M_1}{l} = +\frac{11}{224}\,wl.$$

The shear on the *right* of the second support is from equation (III.*a*),

$$S_2 = \frac{M_3 - M_2}{l} + \tfrac{1}{2}wl = -\frac{1}{224}wl + \tfrac{1}{2}wl = +\frac{111}{224}wl.$$

The shear on the *left* of the third support is from equation (III.*b*),

$$S_3' = \frac{M_2 - M_3}{l} + \tfrac{1}{2}wl = +\frac{1}{224}wl + \tfrac{1}{2}wl = +\frac{113}{224}wl.$$

The shear on the right of the third support is from equation (IV.),

$$S_3 = \frac{M_3 - M_4}{l} = +\frac{15}{224}wl.$$

In the same way,

$$S_4' = \frac{M_4 - M_3}{l} = -\frac{15}{224}wl,$$

$$S_4 = \frac{M_4 - M_5}{l} = -\frac{3}{224}wl,$$

$$S_5' = \frac{M_5 - M_4}{l} = +\frac{3}{224}wl.$$

A positive shear acts upward, a negative shear downward.

A positive moment causes tension in the upper chord, a negative moment compression in the upper chord above the support.

EXAMPLE 2. *In the preceding case, what is the moment and shear at the second su^port for a concentrated load P, placed anywhere on the span?*

Answer

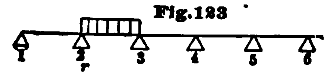
$$M_2 = \frac{1}{56}(26k - 45k^2 + 19k^3)Pl,$$

$$S_2 = \frac{P}{56}(56 - 38k - 57k^2 + 39k^3).$$

EXAMPLE 3. *A continuous girder of five spans, the centre and adjacent spans being* 100 *feet and the end spans each* 75 *feet long, has a uniform load extending over the second span. What are the moments at the supports? What is the shear on the right of the* 4*th support?*

In this case, Fig. 123, we have

Fig.123



$s = 5, \quad l_1 = l_5 = 75 = \tfrac{3}{4}l_2, \quad l_2 = l_3 = l_4 = 100, \quad r = 2,$

also

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -\tfrac{7}{2}, \quad c_4 = +13, \quad c_5 = -48.5, \quad \text{and } d_1 = 0,$$
$$d_2 = 1, \quad d_3 = -\tfrac{7}{2}, \quad d_4 = +13, \quad d_5 = -48.5.$$

Since then $A = B = \frac{1}{4}wl_2^3$ for uniform load, we have from equation (I.),

$$M_1 = 0, \qquad M_2 = +\frac{35.5}{627}wl_2^2,$$

and from equation (II.),

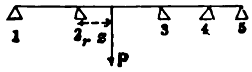
$$M_3 = +\frac{65}{1254}wl_2^2, \qquad M_4 = -\frac{35}{2508}wl_2^2, \qquad M_5 = +\frac{5}{1254}wl_2^2, \qquad M_6 = 0.$$

For the shear on the right of the 4th support, we have from equation (IV.),

$$S_4 = \frac{M_4 - M_5}{l_4} = -\frac{45}{2508} wl_2.$$

EXAMPLE 4. *A continuous girder of four spans, $l_1 = 80$, $l_2 = 100$, $l_3 = 50$, and $l_4 = 40$ feet, has a load of* 10 *tons in the second span, at a distance of* 40 *feet from the second support. What are the moments at the supports? What is the shear just on the right of the second support?*

Fig. 124



In this case, Fig. 124, we have

$$s = 4, \quad r = 2, \quad z = 40, \quad k = \frac{z}{l_2} = \frac{40}{100} = 0.4, \quad c_1 = 0,$$

$$c_2 = 1, \quad c_3 = -3.6, \quad c_4 = +19.6, \quad c_5 = -83.7, \quad d_1 = 0, \quad d_2 = 1,$$
$$d_3 = -3.6, \quad d_4 = +10.3, \quad d_5 = -41.85.$$

We have then from equations (I.) and (II.),

for $m < 3$

$$M_m = \frac{c_m}{3348}(Ad_4 + Bd_3),$$

for $m > 2$

$$M_m = \frac{d_{6-m}}{3348}(Ac_2 + Bc_3).$$

Hence,

$$M_1 = 0, \quad M_2 = \frac{10.3\,A - 3.6\,B}{3348} = \frac{Pl_2^2}{3348}(17\,k - 30.9\,k^2 + 13.9\,k^3) = +82.01,$$

$$M_3 = -3.6\frac{A - 3.6\,B}{3348} = +\frac{3.6\,Pl_2^2}{3348}(1.6\,k + 3\,k^2 - 4.6\,k^3) = +88.77,$$

$$M_4 = \frac{A - 3.6\,B}{3348} = -\frac{Pl_2^2}{3348}(1.6\,k + 3\,k^2 - 4.6\,k^3) = -24.65, \qquad M_5 = 0.$$

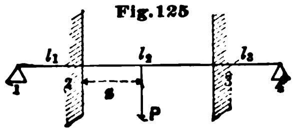
For the shear S_2, we have from equation (III.*a*)

$$S_2 = \frac{M_3 - M_2}{l_2} + P(1 - k) = +5.9324 \text{ tons}.$$

CONTINUOUS GIRDER WITH FIXED ENDS.—It is worthy of remark that if we make l_1 or $l_s = 0$ our formulas still hold good for a girder with either or both ends fastened or walled in horizontally. We must, however, remember that when we thus make l_1 or l_s or both equal to zero, the value of s must still remain unchanged, *and the supports must be numbered as they were before the end spans were taken away.*

EXAMPLE 1. *A beam of one span is fixed horizontally at the ends. What are the end moments and shears for a concentrated load distant $z = kl$ from left end?*

Fig. 125



In this case, Fig. 125, the two outer spans l_1 and l_3 are zero. But we have still $s = 3$ and $r = 2$, just the same as if the outer spans still existed.

We have then,

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -2, \quad d_1 = 0, \quad d_2 = 1, \quad d_3 = -2.$$

Hence for $m = 2$ we have

$$M_2 = c_2\frac{Ad_2 + Bd_2}{ld_2 + 2ld_2},$$

and for $m = 3$ we have

$$M_3 = d_3 \frac{Ac_2 + Bc_3}{lc_2 + 2lc_3}.$$

Inserting the values of c and d, we have

$$M_2 = + Pl\,(k - 2\,k^2 + k^3) \quad \text{and} \quad M_3 = + Pl\,(k^2 - k^3).$$

For the shear at the left end, we have

$$S_2 = \frac{M_2 - M_3}{l} + P\,(1 - k), \quad \text{or} \quad S_2 = P(1 - 3\,k^2 + 2\,k^3).$$

For a load anywhere, we have simply to give proper values to k, and we have at once the moment and shear (which in this case is the same as the reaction) at the end.

Thus for a load in the centre, $k = \frac{1}{2}$ and

$$M_2 = M_3 = + \tfrac{1}{8}\,Pl, \qquad S_2 = S_3 = \tfrac{1}{2}\,P, \quad \text{as should be.}$$

EXAMPLE 2. *For a uniform load over the same beam, what are the end moments and shears?*

We have simply to introduce the proper values of A and B for this case, and we have at once

$$M_2 = + \tfrac{1}{12}\,wl^2 = M_3 \quad \text{and} \quad S_2 = S_3 = \tfrac{1}{2}\,wl.$$

EXAMPLE 3. *A girder of three equal spans is "walled in" at the ends, and has a concentrated load in the first span. What are the moments and shears at the ends and intermediate supports?*

In this case we have $s = 5$, $r = 2$, and hence

$$M_2 = + \frac{Pl}{45}(45\,k - 78\,k^2 + 33\,k^3), \qquad M_3 = + \frac{21}{45}\,Pl\,(k^2 - k^3),$$

$$M_4 = - \frac{2Pl}{45}(3\,k^2 - 3\,k^3), \qquad M_5 = + \frac{Pl}{45}(3\,k^2 - 3\,k^3).$$

For the shears we have,

$$S_2 = \frac{P}{45}(45\,k - 99\,k^2 + 54\,k^3),$$

$$S_3' = \frac{P}{45}(99\,k^2 - 54\,k^3), \qquad S_4' = - \frac{P}{45}(27\,k^2 - 27\,k^3),$$

$$S_3 = \frac{P}{45}(27\,k^2 - 27\,k^3), \qquad S_4 = - \frac{P}{45}(9\,k^2 - 9\,k^3),$$

$$S_5' = \frac{P}{45}(9\,k^2 - 9\,k^3).$$

The *reactions* are equal to the sum of the shears at each support. Thus the reaction at the second support is

$$R_3 = S_3' + S_3 = \frac{P}{45}(126\,k^2 - 81\,k^3).$$

Observe that the moments are positive at each end of the loaded span, and alternate in sign each way. A positive moment always denotes tension in the upper chord.

The shears are positive at the ends of the loaded span, and alternate in sign each way. A

positive shear acts upward, and requires the support to be *below* the girder. Disregarding, then, the weight of the girder itself, it would have to be *held down* at the first pier from the right end.

Since the sum of all the reactions should equal the weight, this fact affords a ready check upon the accuracy of our results.

EXAMPLE 4. *A beam of one span is fixed horizontally at the right end, what are the shears and moments for concentrated load?*

We have here,

$$s = 2, \quad r = 1, \quad l_2 = 0, \quad c_1 = 0, \quad c_2 = 1, \quad d_1 = 0, \quad d_2 = 1, \quad d_3 = -2.$$

Hence,

$$M_1 = 0, \quad M_2 = d_2 \frac{Ac_1 + Bc_2}{lc_1 + 2lc_2} = + \frac{Bc_2}{2lc_2} = + \frac{B}{2l} = + \frac{Pl}{2}(k - k^3),$$

$$S_1 = - \frac{M_2}{l} + P(1 - k) = \frac{P}{2}(2 - 3k + k^3),$$

$$S_2' = \frac{P}{2}(3k - k^3).$$

If the beam is uniformly loaded, we have

$$M_1 = 0, \qquad M_2 = + \tfrac{1}{8} wl_2, \qquad S_1 = \tfrac{3}{8} wl, \qquad S_2' = \tfrac{5}{8} wl.$$

EXAMPLE 5. *A beam of three spans of* 25, 50, *and* 40 *feet respectively, is fixed horizontally at the right end and has a concentrated load of* 10 *tons at* 12 *feet from the third support from the left. What are the moments at the supports?*

Here,

$$l_1 = 25, \quad l_2 = 50, \quad l_3 = 40, \quad l_4 = 0, \quad P = 10, \quad kl_3 = 12, \quad k = 0.3, \quad s = 4, \quad \text{and} \quad r = 3.$$

Also,

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -3, \quad c_4 = 12.25, \quad d_1 = 0, \quad d_2 = 1, \quad d_3 = -2, \quad d_4 = 6.4, \quad d_5 = -34.4.$$

When, then, $m < 4$,

$$M_m = \frac{c_m}{860}(-2A + B) = - \frac{c_m Pl_3^2}{860}(3k - 6k^2 + 3k^3).$$

Inserting $k = 0.3$, and the values of c, we have,

$$\text{for } m = 1, \quad M_1 = 0; \quad m = 2, \quad M_2 = -8.20; \quad m = 3, \quad M_3 = +24.62.$$

When $m = 4$,

$$M_4 = \frac{d_3}{860}(-3A + 12.25B) = + \frac{Pl_3^2}{860}(6.25k + 9k^2 - 15.25k^3).$$

Or

$$M_4 = + 42.29 \text{ foot-tons}.$$

Find the shears. Also moments and shears for uniform load over the third span.

UNIFORM LOAD OVER ENTIRE LENGTH OF GIRDER.—Our formulas, page 173, enable us to find the moment and shear at any support, for a uniform load over any single span. If, then, we suppose each span in turn uniformly loaded, the algebraic sum of the moments and shears thus obtained at each support will give the moments and shears for any uniform load over the entire length of girder.

This is, however, unnecessarily tedious. We can find the moment at any support in this case directly by the following formulas:*

$$M_m = \frac{u}{4}\left[b_m - \frac{c_m[(l^3_{s-1} + l^3_s)d_2 + (l^3_{s-2} + l^3_{s-1})d_3 + \ldots (l_1^3 + l_2^3)d_s]}{d_{s-1}l_2 + 2d_s(l_2 + l_1)}\right],$$

where u is the uniform load per unit of length, and the numbers c and d are the same as in the general formulas, page 174. The numbers b are as follows:

$$b_1 = 0, \quad b_2 = 0, \quad b_3 = \frac{l_1^3 + l_2^3}{l_2}, \quad b_4 = \frac{l_2^3 + l_3^3}{l_3} + 2b_3\frac{l_2 + l_3}{l_3},$$

$$b_5 = \frac{l_3^3 + l_4^3}{l_4} + 2b_4\frac{l_3 + l_4}{l_4} + b_3\frac{l_3}{l_4},$$

$$b_6 = \frac{l_4^3 + l_5^3}{l_5} + 2b_5\frac{l_4 + l_5}{l_5} + b_4\frac{l_4}{l_5}, \text{ etc.}$$

Here again we may fix one or both ends horizontally by making l_1 or l_s or both equal to zero, and proceeding as directed on page 176.

If the spans *are all equal*, the preceding formula becomes much simpler. Thus *for equal spans:*

$$M_m = \frac{A}{3c_{s+1}}[c_m(1 - c_{s+2}) - c_{s+1}(1 - c_{m+1})],$$

where $A = \frac{ul^2}{4}$, and the numbers indicated by c are

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -4, \quad c_4 = +15, \quad c_5 = -56, \quad c_6 = +209, \text{ etc.},$$

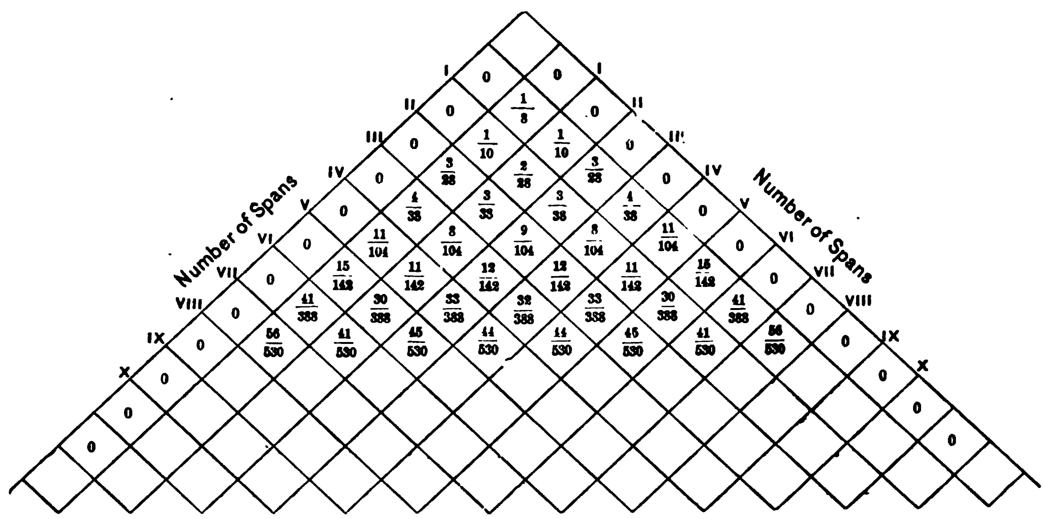
alternating in sign, and each one being numerically equal to four times the preceding, *minus* the next preceding.

The shears at any support are given in any case, whether the spans are equal or not, by the general formulas (III.a) and (III.b) of page 173.

In the case of equal spans, the moment at any support can be easily found without formulas or calculation. Thus, the following Table gives the coefficients of $+ ul^2$ for any number of spans.

The Roman numerals at the sides indicate the number of spans, and the horizontal line to which they belong gives the moments.

MOMENTS AT SUPPORTS—TOTAL UNIFORM LOAD—ALL SPANS EQUAL—COEFFICIENTS OF $+ ul^2$ GIVEN IN TABLE.



* These formulas and the following for equal spans are here given for the first time.

The Table may be easily continued to any number of spans desired. Thus for any *even* number of spans, as VIII. for example, the coefficients are obtained by multiplying the fraction preceding in the same diagonal row, both numerator and denominator, by 2, and adding the numerator and denominator of the fraction preceding that. Thus

$$\frac{15 \times 2 + 11}{142 \times 2 + 104} = \frac{41}{388}, \qquad \frac{11 \times 2 + 8}{142 \times 2 + 104} = \frac{30}{388}.$$

In like manner,

$$\frac{12 \times 2 + 9}{142 \times 2 + 104} = \frac{33}{388} \quad \text{or} \quad \frac{11 \times 2 + 11}{142 \times 2 + 104} = \frac{33}{388}$$

in the other diagonal row.

For any *odd* number of spans, as IX. for instance, we have simply to add, numerator to numerator and denominator to denominator, the two preceding fractions in the same diagonal row.

Thus,
$$\frac{41 + 15}{388 + 142} = \frac{56}{530}, \qquad \frac{33 + 12}{388 + 142} \text{ or } \frac{30 + 15}{388 + 142} = \frac{45}{530}$$

and so on. We can thus, independently of the formula, produce the table to any required number of spans.

For seven equal spans, then, we have at once from the table,

$$M_1 = M_8 = 0, \qquad M_2 = M_7 = \frac{+15}{142} ul^2, \qquad M_3 = M_6 = \frac{+11}{142} ul^2, \qquad M_4 = M_5 = \frac{+12}{142} ul^2.$$

The moments are all positive, showing that the upper chord is in tension over every support.

Similar tables* may easily be drawn up for shears and reactions. It is unnecessary to give them here.

The moments being known, the shears can easily be found by the formulas of page 173.

General Method of Calculation Indicated.—Thus we see that the simple formulas of page 173 are all that we need for the complete solution of any case of level supports—whether the spans be all equal, or the end ones only different, or all different; whether the girder merely rests on the end supports or is fastened horizontally at one or both ends. We have only to remember that a positive moment causes tension, and a negative moment compression, in the *upper* chord. Also, that a positive shear acts upward and a negative shear downward. Also, that the moment and shear are positive at the supports of the loaded span, and alternate in sign both ways. This is all we need in order to form properly the equation of moments for any apex, and determine the character of the stresses in chords and diagonals. We can thus solve any practical case of framed continuous girder with the same ease as the simple girder.

Thus, for any span, as DE, Fig. 120, we have only to find by the formulas of page 173 the moments at D and E due to every position of the apex load P in the span DE, and the corresponding shears at D. These once known, we can find and tabulate the stresses in every member due to each apex weight. An addition of these stresses gives then the maxima of each kind due to interior loading.

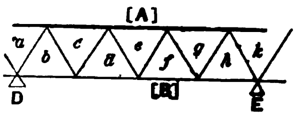
We have then to find, in like manner, the stresses due to the two cases of *exterior loading*, as

* Many such tables will be found in "*Elements of Graphical Statics*," Du Bois: Wiley & Sons, 1879.

represented in Fig. 120. An addition of these stresses gives the maxima of each kind due to exterior loading. We can then deduce the dead load stresses, and finally the total maximum stresses of each kind for every member.

EXAMPLE.—Let us take as an illustration of the preceding, a continuous girder of seven equal spans, and seek the maximum stresses which can ever occur in the middle span. Let Fig. 126 represent the centre span DE. Length 80 feet, divided into 4 panels, and let the live load per panel be, for instance, 40 tons or $w = 2$ tons, the uniform or dead load being *half* as much, or 20 tons per panel. Height of truss 10 feet. Load on top chord.

Fig. 126



We have then, by the application of our formulas, page 173.

For 1st span loaded (Fig. 120) . . . . . . moment at $D = + 61.56$ ft. tons
shear at $D = + 0.97$ tons.

For 2d span loaded . . . . moment at $D = - 184.68$ ft. tons
shear at $D = - 2.93$ tons

For 3d span loaded . . . . moment at $D = + 677.15$ ft. tons
shear at $D = + 10.73$ tons.

For 5th span loaded . . . . moment at $D = - 181.38$ ft. tons
shear at $D = - 10.73$ tons.

For 6th span loaded . . . . moment at $D = + 49.47$ ft. tons
shear at $D = + 2.93$ tons.

For 7th span loaded . . . . moment at $D = - 16.49$ ft. tons
shear at $D = - 0.97$ tons.

Also for the loads in the span DE itself,

For the 1st load P_1 . . . . moment at $D = + 158.97$ ft. tons
shear at $D = + 36.2$ tons.

For the 2d load P_2 . . . . moment at $D = + 271.97$ ft. tons
shear at $D = + 25.86$ tons.

For the 3d load P_3 . . . . moment at $D = + 203.2$ ft. tons
shear at $D = + 14.14$ tons.

For the 4th load P_4 . . . . moment at $D = + 62.89$ ft. tons
shear at $D = + 3.8$ tons.

These quantities are easily found from the formulas of page 173. The student, if he has followed our explanation of the use of the formulas, will have no difficulty in checking the above results. Once known, the complete calculation of the continuous girder offers no special difficulty.

Thus for live load over the 1st, 3d, and 6th spans, the others being unloaded, we have the moment at $D = + 788.18$ and the shear $+ 14.63$. We have, therefore, for the stresses in the upper chord of the 4th span, due to this loading,

$Aa \times 10 = +\ 788.18$ or, $Aa = +\ 78.82$ tons,

$Ac \times 10 = +\ 788.18 - 14.63 \times 20$ or, $Ac = +\ 49.56$ tons,

$Ae \times 10 = +\ 788.18 - 14.63 \times 40$ or, $Ae = +\ 20.30$ tons,

$Ag \times 10 = +\ 788.18 - 14.63 \times 60$ or, $Ag = -\ 8.96$ tons,

$Ak \times 10 = +\ 788.18 - 14.63 \times 80$ or, $Ak = -\ 38.22$ tons.

In similar manner, for the lower chord,

$Bb \times 10 = -\ 788.18 + 14.63 \times 10$ or, $Bb = -\ 64.19$ tons,

$Bd \times 10 = -\ 788.18 + 14.63 \times 30$ or, $Bd = -\ 34.93$ tons,

$Bf \times 10 = -\ 788.18 + 14.63 \times 50$ or, $Bf = -\ 5.67$ tons,

$Bh \times 10 = -\ 788.18 + 14.63 \times 70$ or, $Bh = +\ 23.59$ tons.

For the diagonals, since the angle made by these with the vertical is $\theta = 45°$, we have sec $\theta = 1.414$, and hence, $ab = 14.63 \times 1.414 = +\ 20.68$, $bc = -\ 20.68$, etc.

In this way we can fill up the column for L_1, in the table which follows. This column gives the stresses in every member in the span, due to the first, third, and sixth spans loaded. In the same way we can easily calculate the stresses in every member of the span DE, due to the live load extending over the second, fifth, and seventh spans. We thus find the columns L_1 and L_2 of the table.

We can now find the stresses in every member due to each apex load in the span DE. Thus, for P_1 we have moment at $D = +\ 158.92$, and shear at $D = +\ 36.17$. We have, then,

$Aa \times 10 = +\ 158.97$ or, $Aa = +\ 15.9$ tons,

$Ac \times 10 = +\ 158.97 - 36.2 \times 20 + 40 \times 10$ or, $Ac = -\ 16.5$ tons,

$Ae \times 10 = +\ 158.97 - 36.2 \times 40 + 40 \times 30$ or, $Ae = -\ 8.9$ tons,

and so on.

So also for the lower chord,

$Bb \times 10 = -\ 158.97 + 36.2 \times 10$ $Bb = +\ 20.3$,

$Bd \times 10 = -\ 158.97 + 36.2 \times 30 + 40 \times 20$ $Bd = +\ 12.7$,

and so on.

Also, for the diagonals, we have

$$ab = -\ 36.2 \times 1.414 = -\ 51.19, \quad bc = -\ (40 - 36.2)\ 1.414 = -\ 5.37, \quad cd = +\ 5.37, \text{ etc.}$$

We can thus fill out the column for P_1, and in similar manner the columns for P_2, P_3, etc.

The shear at any point is equal to the shear just to the right of the left support, minus all the weights between the support and the point. Thus, for P_1 we have for diagonal bc, the shear $36.2 - 40$, or a downward force of 3.8, since the weight 40 tons acts down. This downward shear causes compression in bc, since it acts at the upper end. For ab we have 36.2 acting up at the foot of ab, and, therefore, also causing compression. The diagonals which meet at the weight are always either both tension or both compression, according as the weight acts at the bottom or top. Right and left from the weight the diagonals alternate in sign.

We have thus the following Table for the stresses in the various members of the span DE. Fig. 126. The stresses due to a locomotive excess at any panel points can be easily inserted. Thus, for an excess of 33 tons at P_1, all the stresses in the column for P_1 will be increased by $\frac{33}{40}$ths

TABLE OF STRESSES IN THE MEMBERS.

MEMBERS.	LIVE LOADS IN FOURTH SPAN.				EXTERIOR LOADING.		LIVE LOAD STRESSES.		DEAD LOAD = ½ LIVE.	TOTAL MAXIMUM STRESSES.	
	P_1	P_2	P_3	P_4	L_1	L_2	COMP. −	TENS. +		COMP. −	TENS. +
Aa	+ 15.90	+ 27.20	+ 20.32	+ 6.30	+ 78.82	− 38.22	38.22	148 54	+ 55.16	......	203.70
Ac	− 16.50	− 24.52	− 7.96	− 1.30	+ 49.56	− 8.96	59.24	49.56	− 4.84	64.08	44.72
Ae	− 8.90	− 36.24	− 36.24	− 8.90	+ 20 30	+ 20.30	90.28	40.60	− 24.84	115.12	15.76
Ag	− 1.30	− 7.96	− 24.52	− 16.50	− 8.96	+ 49.56	59.24	49.56	− 4.84	64.08	44.72
Ak	+ 6.30	+ 20.32	+ 27.20	+ 15.90	− 38.22	+ 78.82	38.22	148.54	+ 55.16	.... .	203.70
Bb	+ 20.30	− 1.34	− 6.18	− 2.50	− 64.19	+ 23.59	74.21	43.89	− 15.16	89.37	28.73
Bd	+ 12.70	+ 50.38	+ 22.10	+ 5.10	− 34.93	− 5.67	40.60	90.28	+ 24.84	15.76	115.12
Bf	+ 5.10	+ 22.10	+ 50 38	+ 12.70	− 5.67	− 34.93	40.60	90.28	+ 24.84	15.76	115.12
Bh	− 2.50	− 6.18	− 1.34	+ 20.30	+ 23.59	− 64.19	74.21	43.89	− 15.16	89.37	28.73
ab	− 51.19	− 36.57	− 20.00	− 5.37	− 20.68	+ 20 68	133.81	20.68	− 56.56	190.37	......
bc	− 5.37	+ 36.57	+ 20.00	+ 5.37	+ 20.68	− 20.68	26.05	82.62	+ 28.28	......	110.90
cd	+ 5.37	− 36.57	− 20.00	− 5.37	− 20.68	+ 20.68	82.62	26.05	− 28.28	110.90	......
de	− 5.37	− 20.00	+ 20.00	+ 5.37	+ 20.68	− 20.68	46.05	46.05	0	46.05	46.05
ef	+ 5.37	+ 20.00	− 20 00	− 5.37	− 20.68	+ 20.68	46.05	46.05	0	46.05	46.05
fg	− 5.37	− 20.00	− 36.57	+ 5.37	+ 20.68	− 20.68	82.62	26.05	− 28.28	110.90	......
gh	+ 5.37	+ 20.00	+ 36.57	− 5.37	− 20.68	+ 20.68	26.05	82.62	+ 28.28	......	110.90
hk	− 5.37	− 20.00	− 36 57	− 51.19	+ 20.68	− 20 68	133.81	20.68	− 56.56	190.37	......

For the dead load stresses we simply have to add algebraically all the other columns horizontally, and divide by 2 in this case, or by the proper number in any case, whatever that is. The Table then gives at once the maximum stresses in every member, as well as the position of the loads which cause these maximum stresses. We can also tell at once whether any member needs to be counterbraced, or is subject to stresses of two kinds. Thus the dead load acts always and causes in *Aa*, for instance, a tension of 55.16 tons. All the interior loads, P_1, P_2, P_3, etc., also cause tension in *Aa*, as do also the live loads of the 1st, 3d, and 6th spans. The maximum tension, since all these loads *may* act together, is, therefore, the sum, or 203.70 tons tension. On the other hand, the only loads which can cause compression in *Aa* are those in the 2d, 5th, and 7th spans. If these three spans are all loaded simultaneously, the united compression in *Aa* is less than the tension due to the dead load. This member, then, does not need to be counterbraced. It is always in tension, and the greatest tension upon it is 203.79 tons.

Again, for *Ac* we have a dead load compression of 4.84 tons, which may be increased by all the interior loads, and by the live load in the 2d, 3d, and 7th spans to 64.08 tons. The live load in 1st, 3d, and 6th spans causes tension in *Ac* of 49.56 tons. The sum of all these tensions is given in L_1, and subtracting the dead load compression, we have 44.72 tons tension remaining. The member *Ac*, then, is subjected to 44.72 tons tension and 64.08 tons compression, and must be made to resist both. So for each and every member, the two columns for total maximum stresses are easily made out. We also see at once from the Table what weights, and where placed, give these two stresses.

Continuous Girder—Supports not on a Level.—We have, then, on page 173, all the formulas required for the solution of the continuous girder for supports on a level, or all on line, when the deviation from level is small, whatever may be the number or relative length of the spans. If for a continuous girder of *constant cross-section* the supports are properly lowered, a considerable saving, of 20 per cent. or more over the same girder with supports on a level, may be obtained. If, however, the cross-section varies according to the stress—in other words, if the girder is of constant strength—no advantage is thus gained from lowering intermediate supports. Such disposition of the supports may even act injuriously.

The formulas for shear and moments, which we have given, are, indeed, based upon the hypoth-

esis of constant cross-section; but if the stresses in every member are found for the shears and moments thus obtained, and each member is proportioned to its stress, the actual girder erected is not of constant cross-section, but more nearly one of uniform strength. *Formulas for the case of supports out of level, as well as determinations of the best differences of level, are, hence, of little practical importance.* If, however, it is desired to find the effect due to a change of level of any one pier, we may make use of the following formulas :

Let the nth support be out of level by the distance h_n. Then the moments at all the supports are changed. The moments at n, and at each alternate support from n, are diminished, and at the others increased.

For the sake of convenience, let

$$H = -\frac{36\, h_n\, EI}{l^2},$$

where E is the coefficient of elasticity, and I the moment of inertia of the cross-secsion. When the support is lowered h_n is minus ; when raised, h_n is plus. For the moment due to the lowering of the supports alone, we have, then, *when all the spans are equal,*

for $m < n$

$$M_m = \frac{c_m c_{s-n+2}}{c_{s+1}} H,$$

for $m = n$

$$M_n = \frac{H}{6} + \frac{c_n c_{s-n+2}}{c_{s+1}} H$$

for $m > n$

$$M_m = \frac{c_{s-m+2} c_n}{c_{s+1}} H,$$

where n is the number of the lowered support from the left, and

$$c_1 = 0, \quad c_2 = +1, \quad c_3 = -4, \quad c_4 = +15, \quad c_5 = -56, \quad c_6 = +209, \text{ etc.},$$

the numbers alternating in sign, and each one being equal to four times the preceding, minus the one preceding that.

From the moments at the supports the shears can be determined by the formulas III. and IV. of page 173.

2d. *When the spans are unequal*

for $m < n$,

$$M_m = \frac{c_m\left(\frac{d_{s-n+1} - d_{s-n+2}}{l_n} + \frac{d_{s-n+3} - d_{s-n+2}}{l_{n-1}}\right)6\, h_n\, EI}{d_{s+1}\, l_1};$$

for $m = n$,

$$M_n = -\frac{6\, EI\, h_n}{l^2_{n-1}} + \frac{6\, EI\, c_n\, h_n}{d_{s+1}\, l_1}\left[\frac{d_{s-n+1} - d_{s-n+2}}{l_n} + \frac{d_{s-n+3} - d_{s-n+2}}{l_{n-1}}\right];$$

for $m > n$,

$$M_m = \frac{d_{s-m+2}\left(\frac{c_{n-1} - c_n}{l_{n-1}} + \frac{c_{n+1} - c_n}{l_n}\right)6\, h_n\, EI}{c_{s+1}\, l_s};$$

where

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -2\frac{l_1 + l_2}{l_2}, \quad c_4 = -2\, c_3 \frac{l_2 + l_3}{l_3} - c_2 \frac{l_2}{l_3}, \quad c_5 = -2c_4 \frac{l_3 + l_4}{l_4} - c_3 \frac{l_3}{l_4}, \text{ etc.},$$

and

$$d_1 = 0, \quad d_2 = 1, \quad d_3 = -2\frac{l_s + l_{s-1}}{l_{s-1}}, \quad d_4 = -2\,d_3\frac{l_{s-1} + l_{s-2}}{l_{s-2}} - d_2\frac{l_{s-1}}{l_{s-2}}$$

$$d_5 = -2d_4\frac{l_{s-2} + l_{s-3}}{l_{s-3}} - d_3\frac{l_{s-2}}{l_{s-3}}, \text{ etc.}$$

The reader who has learned the use of the formulas of page 173 will have no difficulty in applying the above to any particular case. In the same way as explained on page 176, by making l_1 or l_n or both zero, we may fix the girder horizontally at one or both ends. The formulas for shear at any support are the same as on page 173.

EXAMPLE 1.—*Let a beam of two equal spans be uniformly loaded throughout its whole length, and let the centre support be lowered by an amount* $h_2 = -\dfrac{wl^4}{48\,EI}$. *What are the moments, shears and reactions?*

The moments due to the full load alone, before the support is lowered, are $M_1 = 0$, $M_2 = +\dfrac{wl^2}{8}$, $M_3 = 0$. For the moment due to the lowering of the support alone, we have from the formulas just given, since $H = \dfrac{3\,wl^2}{4}$, $s = 2$, $m = n = 2$,

$$M_1 = 0, \quad M_2 = -\frac{wl^2}{16}, \quad M_3 = 0$$

Hence the total moment is

$$M_2 = +\frac{wl^2}{8} - \frac{wl^2}{16} = +\frac{wl^2}{16},$$

or only one half as much as before the support was lowered. For the shears we have,

$$S_1 = \frac{M_2}{l} + \frac{wl}{2} = \frac{7}{16}\,wl, \qquad S_2' = \frac{9}{16}\,wl.$$

$$S_2 = \frac{9}{16}\,wl\,, \qquad S_3' = \frac{7}{16}\,wl.$$

Hence,

$$R_1 = \frac{7}{16}\,wl, \qquad R_2 = \frac{18}{16}\,wl, \qquad R_3 = \frac{7}{16}\,wl.$$

EXAMPLE 2.—*How much must we lower the second support in the preceding example, in order that the reaction at the centre support may be just zero?*

In this case we have,

$$M_2 = -\frac{H}{12} + \frac{wl^2}{8} = \frac{3h_2EI}{l^2} + \frac{wl^2}{8}$$

$$R_2 = \frac{2M_2}{l} + wl = +\frac{wl}{8} + \frac{3h_2EI}{l^3} + \frac{wl}{2} = \frac{5}{8}\,wl + \frac{3h_2EI}{l^3}.$$

If this is to be zero, we have,

$$h_2 = -\frac{5\,wl^4}{24\,EI},$$

and hence

$$M_2 = -\tfrac{1}{2}\,wl^2, \quad \text{and} \quad R_1 = wl, \quad R_2 = 0, \quad R_3 = wl,$$

or precisely as for a beam of single span and length $2l$.

EXAMPLE 3.—*A beam of four equal spans is unloaded, and the third support is lowered by an amount* $h_3 = -\frac{wl^4}{24\,EI}$. *What are the reactions?*

Answer:

$$R_1 = -\frac{12}{112}\,wl, \quad R_2 = +\frac{44}{112}\,wl, \quad R_3 = -\frac{64}{112}\,wl, \quad R_4 = R_2, \quad R_5 = R_1.$$

EXAMPLE 4.—*A beam of five equal spans rests as a continuous girder over six supports. Having given the dimensions of the beam, length of span, and coefficient of elasticity, to determine the reactions due to a sinking of the third support one eighth of an inch.*

Let the beam be of wood, 1 foot wide and 1.5 deep.

$$l = 20 \text{ feet}, \quad s = 5, \quad n = 3, \quad E = 288{,}000{,}000 \text{ lbs. per sq. foot}, \quad h_3 = \tfrac{1}{8} \text{ in.} = 0.010417 \text{ feet},$$

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -4, \quad c_4 = 15, \quad c_5 = -56, \quad c_6 = 209.$$

$$M_2 = +\frac{540\,EIh_3}{209\,l^2}, \quad M_3 = -\frac{906\,EIh_3}{209\,l^2}, \quad M_4 = +\frac{576\,EIh_3}{209\,l^2}, \quad M_5 = -\frac{144\,EIh_3}{209\,l^2},$$

$$M_1 = M_6 = 0.$$

Or inserting the constants above, and $I = \frac{1}{12}\,bd^3 = \frac{3.375}{12}$,

$$M_1 = M_6 = 0, \quad M_2 = +5{,}448, \quad M_3 = -9{,}142, \quad M_4 = +5{,}812, \quad M_5 = -1{,}453 \text{ ft. lbs.}$$

when the spans are unloaded. For the reactions necessary to bend the beam down and keep it to its supports,

$$R_1 = -272 \text{ lbs}, \quad R_2 = +1{,}002, \quad R_3 = -1{,}477, \quad R_4 = +1{,}111, \quad R_5 = -436, \quad R_6 = +73.$$

If unloaded, then, the beam must be fastened down at the first, third, and fifth supports.

If the beam weighs 75 *pounds per foot, what deflection of the third support will raise the left end off the abutment?*

Ans.

$$R_1 = \frac{15}{30}\,wl = \frac{540\,EIh_3}{209\,l^3} \quad \text{or } h_3 = 0.0287 \text{ feet} = 0.3712 \text{ inches}.$$

It will be observed that a small difference of level in the supports has a very considerable effect.

EXAMPLE 5.—*Two equal spans are uniformly loaded. How high must the centre be raised in order that the ends may just touch the supports?*

This is the case of the pivot span when the centre support is properly raised.

The reactions at the end are zero. At the centre $R_2 = 2\,wl$, hence $M_2 = +\frac{1}{2}\,wl^2$. But the moment when the supports are on level is $M_2 = +\frac{1}{8}\,wl^2$, hence $+\frac{3}{8}\,wl^2$ must be due to the elevation of the support. From our formulas

$$+\tfrac{3}{8}\,wl^2 = \frac{3\,EIh_2}{l^2} \quad \text{or } h_2 = +\frac{wl^4}{8\,EI}.$$

This is precisely the same as the deflection of a horizontal beam, fastened at one end and free at the other. (See page 298.)

ECONOMY OF THE CONTINUOUS GIRDER.—Although a comparison of stresses alone is not sufficient to demonstrate economy in all cases, owing to increased cost of construction, etc., yet when the stress sheet shows a great saving, it may point the way to improvement.

Upon page 183 we have given the stresses for a continuous girder, the centre span of seven. For the girder discontinuous, we find for the same load,

	Aa	*Ac*	*Ae*	*Ag*	*Ak*	*Bb*	*Bd*	*Bf*	*Bh*
Continuous		− 64.08	− 115.12	− 64.08		− 89.37	− 15.76	− 15.76	− 89.37
	+ 203.7	+ 44.72	+ 15.76	+ 44.72	+ 203.7	− 28.73	+ 115.12	+ 115.12	+ 28.73
Simple	0	− 180	− 180	0	+ 90	+ 210	+ 210	+ 210	+ 90

	ab	*bc*	*cd*	*de*	*ef*	*fg*	*gh*	*hk*
Continuous	− 190.37	+ 110.9	− 110.9	± 46.05	± 46.05	− 110.9	+ 110.9	− 190.37
Simple	− 127.3	+ 127.3	− 56.5	− 56.5	+ 56.5	− 56.5	+ 127.3	− 127.3

It will be seen at once that there is a saving in the flanges—about 11 per cent. in all—but the bracing is heavier, giving little or no saving. The span is too short to bring out the relative economy of the continuous girder.

For a girder of 200 feet, height 20 feet, 10 panels, double system of triangulation, live load 20 tons per panel, dead load 10 tons, we have the following results :

	ONE SPAN.	TWO SPANS.	FIVE SPANS CENTRE.
Bracing,	1,398.6	1,428.2	1,596.2
Lower Chord,	2,400.	1,793.2	1,395.7
Upper Chord,	2,550.	1,981.6	1,622.6
Total	6,348.6	5,203.	4,614.5
Per cent. saving		18 per cent.	27 per cent.

We see, then, that the saving increases rapidly with length of span. Although undoubtedly the full theoretical saving cannot be obtained in practice, the above is sufficient to point out that for very long girders the system may have its legitimate place amongst bridge constructions.

Disadvantages of the Continuous Girder.—In order, then, that we may properly estimate this system and be able to make use of it in such circumstances as render its use desirable, it will be well to consider the objections to it as contrasted with the simple girder.

1st. The chords at certain points undergo stresses of opposite character. Stresses of alternating kind have a more injurious effect than stresses of one kind only, and require a greater area of cross section for the same safety. This tends to reduce our theoretical saving. It must, however, be borne in mind, that these alternating stresses in the chords occur at those points where the stress is least, and where the cross section in the simple girder is generally considerably larger than the stress sheet demands. This tends to balance the above objection.

2d. Extra work and cost of chords and chord connections necessary to secure chords against both compressive and tensile stress. For long spans this objection decreases in force.

3d. The changes of stresses, unforeseen and often considerable, which a small settling of the piers or change of level of supports may occasion.

This is a strong objection. As we have seen a very slight change of level causes great changes in the stresses. It is, therefore, indispensable that the supports of the continuous girder should be invariable. All cases where the piers are iron columns are, then, unsuitable for the employment of this system, as a slight change of temperature would affect the system. Even for masonry supports the girder cannot be erected until after the first season, when the piers have settled. Any subsequent change due to insecure foundations would be disastrous. We recognize, therefore, another of the necessary conditions which must be complied with in all cases where the continuous girder is used. The span must not only be long, *but the supports must be practically immovable.*

4th. Changes of temperature. The greater the number and length of spans the greater is the elongation due to rise of temperature. It would seem advisable, therefore, to limit the use of the continuous girder to three or four long spans.

5th. Difficulty of calculation. The method given in the preceding pages involves no special difficulty. Its application to any special case involves considerable labor, but in a construction costing many thousands of dollars, the labor involved in calculation is not a legitimate question. The

only points of interest are whether the method of calculation is clear, simple, easily systematized and reliable in its results and not liable to errors of computation. The method here given is believed to answer satisfactorily all these requirements.

6th. Accuracy of formulas. Our formulas are based upon the assumption of constant moment of inertia of cross-section and constant coefficient of elasticity of the material. Neither of these assumptions are strictly correct. The theory of flexure as applied to girders of variable cross-section shows that the results obtained by assuming a constant cross-section err upon the safe side, in giving somewhat greater stresses than the actual ones. The slight gain in accuracy is more than counterbalanced by the increased complexity of the resulting formulas. As to the second objection, while it is true that the coefficient of elasticity varies in the case of iron between tolerably wide limits, these limits become consideraby less in iron manufactured for a special purpose, as nearly as possible of uniform quality, by the same establishment, from the same ore. An assumption of a constant mean value is allowable in such case, and leads to no errors of practical importance.

ADVANTAGES OF THE CONTINUOUS GIRDER.—The principal advantages of the continuous girder are:

1st. Saving in width of piers as compared with width required for separate successive spans. The girder may, indeed, theoretically be set upon knife edges at the piers. In fact such a construction would be preferable, as better insuring the calculated stresses. Width of piers is undesirable.

2d. Ease of erection, where false works are difficult or expensive. The girder may be put together on shore and pushed out over the piers.

3d. Saving of material, which for long spans would appear to be considerable.

SUMMARY.—It will be seen, then, that of the objections or disadvantages enumerated, 3 and 4 have considerable weight. The use of the continuous girder must, therefore, be confined to the comparatively rare cases of a number of successive very long spans. Even in such cases the question of economy is of less importance than that of ease of erection. The remaining objections have less weight. The proper employment of the continuous girder may then be stated as confined to a few occasional situations. When the situation justifies its use, it offers special advantages well worth consideration.

BEST RATIO OF SPANS.—The length of the various spans has some influence upon the economy of construction. The best ratio of spans for minimum material is given by Winkler* in the following Table:

LENGTH OF SPAN.	FOR 3 SPANS.	FOR 4 SPANS.
150 feet.	1 : 1.111 : 1	1 : 1.129 : 1.129 : 1
300 feet.	1 : 1.125 : 1	1 : 1.136 : 1.136 : 1
450 feet.	1 : 1.148 : 1	1 : 1.168 : 1.168 : 1
Or about	7 : 8 : 7	7 : 8 : 8 : 7

Deviations from the above ratios have, however, but slight effect. Thus, if for 3 spans we choose the ratios 1 : 1 : 1, 1 : 1.1 : 1, 1 : 1.2 : 1, 1 : 1.3 : 1, we have for 150 feet span, respectively 1.6, 0.1, 0.9, 1.6 per cent., for 300 feet span, 1.6, 0.1, 0.5, 1.1 per cent., and for 450 feet span, 1.4, 0.1, 0.2, 0.7 per cent. more material than for the ratios given above. These ratios, can, therefore, be deviated from, when circumstances render it advisable, without much loss.

HINGED CONTINUOUS GIRDERS.—The hinged continuous girder of Gerber, page 57, is free from all the objections which apply to the continuous girder proper, and has its principal advantage. That is, it may be built on shore and pushed out over the piers, and the chords afterward cut or

* Vorträge über Brückenbau.—Wien, 1875.

hinged. A calculation of the stresses in such a girder shows considerable saving when the spans are long, over the simple girder, and the system is worth more attention than it has yet received. The most remarkable girder of this kind in this country is the Kentucky River Bridge, designed by C. Shaler Smith, consisting of three spans of 375 feet each. It was erected without scaffolding, the girder being pushed out from each end and united at the centre. The chords were afterwards cut at a distance of 75 feet on the land side of each of the central piers, thus making the middle portion a continuous girder 525 feet long, with two discontinuous spans, each 300 feet in length, at the ends of the projecting cantilevers, extending 75 feet from each pier.

Literature upon the Continuous Girder.

We give for the benefit of students and those interested a short list of the more important works which treat the continuous girder. The literature is very extended, and no attempt is made at completeness; only a few of the more important works are cited. For a much fuller list we refer to the author's "Elements of Graphical Statics."

CLAPEYRON.—"*Calcul d'une poutre élastique reposant librement sur des appuis inégalement espacés.*" Comptes Rendus, 1857. [Giving the well-known Clapeyronian method and "Theorem of three moments."]

MOHR.—"*Beitrag zur Theorie der Holz- und Eisenconstructionen.*" Zeitschr. des Hannöv. Arch u. Eng. Ver., 1860. [Theory of continuous girder with reference to relative height of supports. Application to girders of two or three spans. Best sinking of supports for constant cross-section. Disadvantage of accidental changes of height of supports. Influence of breadth of piers.]

WINKLER.—"*Beiträge zur Theorie der continuirlichen Brückenträger.*" Civil Ingénieur, 1862. [General Theory. Determination of methods of loading, causing maximum stresses.]

WINKLER.—"*Die Lehre von der Elasticitaet und Festigkeit,*" 1867. [Complete treatment of continuous girder for all spans equal and unequal, uniform and concentrated loading.]

WINKLER.—"*Vorträge über Brückenbau,*" 1875. [Complete graphic and analytic treatment. Also discussions of girder of varying cross-section.]

CULMANN.—"*Die graphische Statik,*" 1866. [Graphical treatment of simple and continuous girder of constant and variable cross-section.]

WEYRAUCH.—"*Allgemeine Theorie und Berechnung der continuirlichen und einfachen Träger,*" 1873. [Gives the general theory for constant and variable cross-section for any number of spans and all kinds of loading. Difference of level of supports; most unfavorable position of load; examples illustrating use of formulas.]

GREENE, CHAS. E.—"*Graphical method for the analysis of Bridge Trusses.*" Van Nostrand, 1875. [Application of equilibrium polygon by balancing of moment areas.]

DU BOIS.—"*Elements of Graphical Statics.*" Wiley, 1877. [Graphic and analytic treatment.]

MERRIMAN.—"*On the Theory and Calculation of Continuous Bridges.*" Van Nostrand, 1876. [Analytic treatment, with illustrations of method of using formulas.]

CHAPTER IX.

THE BRACED ARCH.

THREE KINDS OF BRACED ARCH.—We may distinguish three kinds of braced arch, viz.: 1st, hinged at crown and at ends; 2d, hinged at ends only; 3d, without hinges.

The stresses in the various members can be found, either by diagram or calculation, for any loading, if all the external forces acting upon the arch for that loading are known; that is, so soon as in addition to the load we know the horizontal thrust and vertical reactions at the ends, and the moments, if any, which exist at the ends.

ARCH HINGED AT CROWN AND ENDS.

This form of construction, Fig. 134, owing to the hinges at crown and ends, is an arch only in form, but in principle is simply two braced rafters, the thrust of which is taken by the abutments instead of by a tie rod.

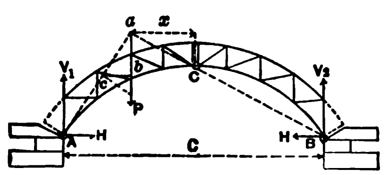


Fig. 134

The determination of the stresses presents, then, no special difficulty. They may be found by diagram or calculation, provided not more than two members, the stresses in which are unknown, meet at any apex.

HORIZONTAL THRUST AND REACTIONS.—The resultant at the crown for the unloaded half, Fig. 134, must pass through the hinges at C and B. Its direction is, therefore, constant for every weight P upon the other half. The resultant for the loaded half must, then, pass through the hinge at A and the point of intersection a of the load P with the line through C and B.

We have, then, simply to draw the line CB and prolong it to intersection a with P, and then draw aA. Aa and Ba are the directions of the resultant pressures at A and B. By resolving P along these lines we can find the magnitude of these pressures, viz., ac and Pc. Resolving these pressures vertically and horizontally, we obtain the vertical reactions $V_1 = ab$, $V_2 = Pb$, and the horizontal thrust $H = cb$, for a load P at any apex. We can thus find H and V_1 by graphic construction, without calculation.

We can also readily compute V_1 and H. Thus let the span AB, or chord of the arc, be denoted by c, and let the rise, measured always from the chord *AB to the hinge C* at the crown, be denoted by r, and the distance of any weight P from the left end be denoted by x_0. Then, taking moments about the right-end hinge B, we have for the reaction V_1 at the left end (or, in general, the *end nearest the load P*),

$$-V_1c + P(c - x_0) = 0, \quad \text{or} \quad V_1 = P - \frac{x_0}{c}P; \quad \therefore \; V_2 = \frac{x_0}{c}P. \quad . \quad . \quad . \quad . \quad (1)$$

Also, taking moments about the hinge C at the crown,

$$-V_1\frac{c}{2} + Hr + P\left(\frac{c}{2} - x_0\right) = 0, \quad \text{or} \quad H = \frac{V_1 c}{2r} - P\frac{\left(\frac{c}{2} - x_0\right)}{r} = \frac{Px_0}{2r}. \quad . \quad . \quad (2)$$

These values of V_1, V_2, and H are independent of the shape of the arch.

DETERMINATION OF STRESSES.—BY DIAGRAM.—Having thus found V_1, V_2, and H, either by graphic construction or by computation, we can find the stresses in the members.

By far the easiest method is to diagram the stresses in the members for *each apex train load separately*, by the method of diagram of Chapter I, Section I, page 8. A tabulation of the stresses for each load will then give the stresses for dead load and locomotive excess as well as for train load, as illustrated by the table on page 105.

In making the tabulation labor can often be saved by noting that if the stresses in the left-hand half are found for the first apex train load from the right on the right-hand half, the stresses in this left-hand half for the next equal train load from right will be twice as much, *if the horizontal apex distances are equal*; for the next, three times, etc. Because for each of these loads the reaction at A has always the same direction AC, and hence for all loads on the right-hand half, the reaction R_1 for the left-hand half follows the law of the lever, and hence for equal loads is twice as great for a load twice as far from B, etc. For the stresses in the members of the left-hand half for apex loads on the left half, we must, however, make a separate diagram for each load, as the reactions Aa for these loads vary in direction, and therefore do not follow the law of the lever.

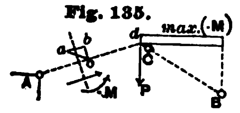
We can then fill out our Table as illustrated fully on page 105, find the stresses for dead load, put in the locomotive excess stresses, and thus determine the maximum stress in every member. This method is simple and free from tedious computations. It is recommended in the case of all arches, whether hinged or not.

LOADING GIVING MAXIMUM STRESSES.—We might in the same way compute the stresses for each apex train load separately and then tabulate as before. The labor of computation may, however, be lessened by first determining for each member that loading which causes the greatest stress, and thus find the maximum stress for each member by one computation for that member. The method by diagram and tabulation just described will, however, be found preferable.

CHORDS.—For any chord panel, as ab, Fig. 135, we find the loading which gives the maximum stress as follows:

The centre of moments is at o, the intersection of the other members cut by a section through the arch at ab.

Fig. 135.



Now it is evident that if we draw a line through A and o, and produce it to intersection d with BC, that a weight at d causes no stress in ab, because the resultant for that weight passes through the point of moments o for ab. Any weight to the right of P, in Fig. 135, has a resultant which lies to the right of Ad and causes then a *positive* moment (counter clockwise rotation), that is, tension in an upper or compression in a lower chord. The maximum positive moment for ab will occur, then, when all the loads from the right end up to d act at once. When all the loads from the left end up to d act, we have the greatest compression in ab.

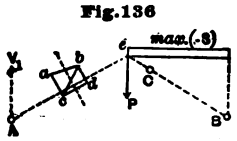
So for any chord panel, whether upper or lower, we have simply to draw a line through A and the point of moments for that panel and produce to intersection with BC. When the load reaches from the right up to this point we have the greatest positive moment (counter clockwise rotation), and when it reaches from the left up to this point we have

the greatest negative moment (clockwise rotation). If d falls on the right of the hinge C at the crown, then, since the resultant through A for every load will lie above Ad, every load causes a negative moment. A negative moment (clockwise rotation) causes compression, a positive moment (counter clockwise rotation) tension, in the *upper chord*. The moment divided by the lever arm gives then at once the maximum stress.

BRACES.—We find the position of the loading which gives the maximum shear for any brace as follows:

Let bc, Fig. 136, be any brace, and ab and cd the two chords cut by a transverse section through the arch and brace.

Fig.136



If ab and cd are parallel their intersection is at an infinite distance. If then we draw through the hinge A the line Ae parallel to ab and cd, this line will pass through the point of moments or point of intersection for ab and cd. If ab and cd are not parallel, produce them to their point of intersection, and through this point draw the line Ae. In either case let e be the intersection of Ae with BC, the line through the other two hinges.

Then a load P at e has a resultant Ae which, as it passes through the point of moments for ab and cd, is wholly sustained by ab and cd, and therefore causes no shear at c.

For all loads right of e the shear at c is downward or negative. For all loads left of c, since the reaction V_1 must be less than the loads which cause it, the shear at c is also evidently downward or negative. For all loads between e and d the shear is upward or positive.

When the load extends then from the right end to e, and also from A to c, we have he greatest negative or downward shear. If e falls *on the right* of the hinge at crown, then, since the resultant through A for every weight will lie above Ae, every weight will cause a positive or upward shear.

DETERMINATION OF STRESSES—BY COMPUTATION.—We can now find the maximum stress in any member by computation as follows:

1*st*. *Chords*.—For any chord panel, as ab, Fig. 135, find the loading which gives the greatest positive and negative moment. If ab is an upper chord panel, a positive moment (counter clockwise rotation) causes tension and a negative moment (clockwise rotation) compression. If ab is a lower chord panel, a positive moment causes compression and a negative moment tension. Then find, by the successive application of the formulas of page 191, V_1 and H for the entire loading giving the maximum positive and negative moment for ab. Let x, y be the co-ordinates from the left end A of the point of moments o for ab, Fig. 135. Then for the maximum positive moment at o we have

$$\text{max.}\ (+M) = +Hy - V_1x,$$

where V_1 and H are the values for the loading giving maximum positive moment at o. We have also, for the maximum negative moment at o,

$$\text{max.}\ (-M) = +Hy - V_1x + \sum_A^o P(x - x_0),$$

where V_1 and H are the values for the loading giving maximum negative moment at o, x_0 is the distance of any load P from the left end A, and $\sum_A^o P(x - x_0)$ is the sum of the moments about o of all loads from o to A.

Divide the maximum moment by the lever arm for ab, and we obtain the maximum stress, compression or tension, according to the rule just stated.

We may avoid computation by making a diagram for each loading for each member, according to the method of Chapter I, section 1, page 8.

2d. Braces.—For any brace, as bc, Fig. 136, find the loading which gives the greatest positive and negative shear. Then find, by successive application of the formulas of page 191, V_1 and H for these loadings. Let i be the angle of inclination which cd, the chord through *the left end* c of the brace bc in question, makes with the horizontal. Then we have for the maximum positive shear at c

$$\text{max.}\ (+S) = V_1 - H \tan i,$$

where V_1 and H are the values for the loading giving maximum positive shear. Also for the maximum negative shear at c

$$\text{max.}\ (-S) = V_1 - H \tan i - \sum_A^c P,$$

where V_1 and H are the values for the loading giving maximum negative shear, and $\sum_A^c P$ denotes the sum of all loads from the point c to A.

If now we find by moments the stresses in ab and cd for the two loadings, and if θ_{ab}, θ_{bc}, θ_{cd} are the angles with the vertical through c of the respective members, we have the stress in bc given by

$$ab \cos \theta_{ab} + bc \cos \theta_{bc} + cd \cos \theta_{cd} + S = 0.$$

In writing this algebraic sum of the vertical components of the stresses in the members, we must measure the angles from the vertical through the *left* ends and follow the rule for signs of page 16. That is, the maximum shear S must have its proper sign as determined, the cosines have their proper signs according to the quadrant in which the members lie, compression in ab or cd is negative, tension positive.

We may also determine the stress in the brace by the method of moments by taking moments about the point of intersection of ab and cd. Or we may avoid computation by making a diagram for each loading for each member, according to the method of Chapter I, Section 1, page 8. It is unnecessary to give a numerical example. The student familiar with our methods should find no difficulty in applying them in the present case.

As already remarked, the method by diagram and tabulation described on page 105 will be found much the easier of application, involving no tedious calculations.

UNNECESSARY MEMBERS.—There should be only two members meeting at the abutments. Thus the members in Fig. 134, represented by broken lines, belong to the superstructure, and serve only to transmit load to the arch. They should not be members of the arch proper.

In the same way at the crown the upper chords are not connected.

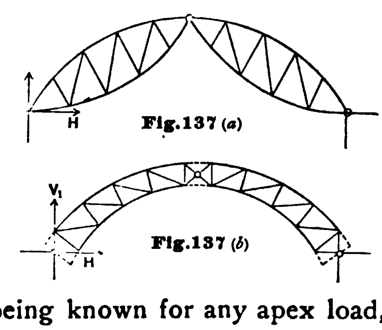


Fig. 137 (a)
Fig. 137 (b)

The hinges may be in either chord at the crown and at the abutments, provided we have regard to the preceding. The shape and depth of the arch may vary at will. This will affect the lever arms of the members, but V_1 and H remain unchanged. Thus in Figs. 137, whatever the shape of arch and character of the bracing, V_1 and H can always be found from our formulas page 191, and these two forces being known for any apex load, the resulting stresses can be found in any case.

BEST FORM AND BEST DEPTH OF ARCH.—If the arch is hinged in the lower chord, as shown in Fig. 134, the resultant for loads on the right half passes through A and C. Loads on the right half will then cause tension in the upper chord. It is therefore well to

hinge the arch at the upper chord, or at the centre line, as shown in Fig. 137 (*a*). In such case, if the depth is made large enough at the quarters, we may have both chords always in compression.

As to this proper depth for this, let ACB, Fig. 138, be the centre line. If we draw the line AC, and find the greatest vertical ordinate mn, the depth at this point should be greater than mn. For a circular arch this ordinate mn is

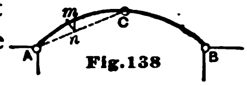


Fig. 138

$$mn = \frac{2R}{c}\sqrt{\frac{c^2}{4} + r^2} - R.$$

For a parabola we have

$$mn = \frac{1}{4} r.$$

The vertical depth of the arch at the quarters should not be less than this if hinged in the upper chord, nor less than twice this if hinged at the centre, for constant depth. If the depth is variable, it should at any point be greater than the ordinate to AC at that point. In other words, the resultant AC should always lie *inside of the chords.*

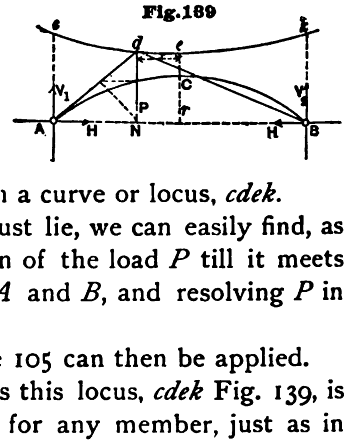
Such a construction as Fig. 137 (*b*) would therefore seem well adapted to long spans.

TEMPERATURE STRESSES.—Changes of temperature occasion no stresses in the arch hinged at crown and ends. Each half is free to turn about the hinges and accommodate itself to any change of shape due to temperature.

ARCH HINGED AT ENDS ONLY.

If we suppose the hinge at the crown removed, those at the ends being retained, then for any position of a weight P, Fig. 139, the resultant pressures must, for equilibrium, pass, as in the preceding case, through the end hinges. This case differs, then, from the preceding case of three hinges only in that now the intersection d of the load P and resultant reactions has a different locus. Thus, in the case of three hinges, the intersection d, Fig. 139, of Ad and P was always in the line BC for P on the left half and in the line AC for P on the right half. Now it is no longer in these lines, but is situated in a curve or locus, *cdek.*

Fig. 139

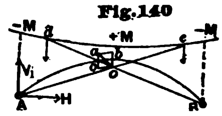


If we can find this locus or curve in which the point d must lie, we can easily find, as before, the reactions by simply prolonging the line of direction of the load P till it meets this locus, drawing from the point of intersection d, lines to A and B, and resolving P in these directions. We thus find V_1, V_2, and H as before.

The method by diagram and tabulation described on page 105 can then be applied.

LOADING GIVING MAXIMUM STRESSES.—Also, as soon as this locus, *cdek* Fig. 139, is known, we can find the loading giving the maximum stress for any member, just as in Figs. 135 and 136, and can then apply the method of calculation described on page 192.

Fig. 140



Chords.—Thus for any chord panel as *ab*, Fig. 140, we find the loading which gives the maximum stress as follows:

The centre of moments is at *o*, the intersection of the other members *ao* and *co*, cut by a section through the end at *ab*. Through *o* draw *Ao* and *Bo* and produce to intersections *e* and *d* with the locus. We see at once that all loads between *d* and *e* cause a negative moment (clock-wise rotation) at *o*, or compression in an upper chord and tension in a lower.

All loads right of e and left of d cause a positive moment (counter clockwise rotation) at o, or tension in an upper chord and compression in a lower. We can find V_1 and H for these loadings, by the construction of Fig. 139, for each apex load, and can then find by computation or diagram, as directed on page 105, the maximum stress for any chord panel.

Thus, if x and y are the co-ordinates from the left end A of the point of moments o, we have for the maximum positive moment at o

$$\max.\,(+M) = +Hy - V_1 x + \sum_d^o P(x - x_0),$$

where V_1 and H are the values for the loading giving maximum positive moment at o, x_0 is the distance of any load P from the left end A, and $\sum_d^o P(x - x_0)$ is the sum of the moments about o of all loads from o to d.

We have also

$$\max.\,(-M) = +Hy - V_1 x + \sum_A^d P(x - x_0),$$

where V_1 and H are the values for the loading giving maximum negative moment at o, and $\sum_A^d P(x - x_0)$ is the sum of the moments about o of all loads from d to A.

Divide the maximum moment by the lever arm for ab and we obtain the maximum stress, compression or tension according to the rule just stated.

We may avoid computation by making a diagram for each loading for each member according to the method of Chapter I, Section I, page 8.

BRACES.—Let ac be any brace, Fig. 141, and ab, cd the two chords cut by a transverse section through the arch and brace. If ab and cd are parallel, their intersection is at an infinite distance. If then we draw through the hinge A the line Ae parallel to ab and cd, this line will pass through the point of moments for ab and cd. If ab and cd are not parallel, produce them to their point of intersection, and through this point draw the line Ae. In either case, let e be the intersection of Ae with the locus.

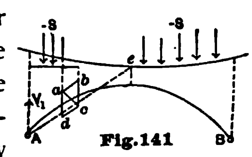


Fig. 141

Then a load P at e has a resultant Ae, which, as it passes through the point of moments for ab and cd, causes no shear at a. For all loads right of e the shear at a is downward or negative. For all loads left of a, since the reaction V_1 must be less than the loads which cause it, the shear at a is also downward or negative. For all loads between e and a the shear is upward or positive.

As before, then (page 193), we have for the maximum positive shear at a

$$\max.\,(+S) = V_1 - H \tan i,$$

where i is the angle of inclination made by ab with the horizontal, and V_1 and H are the values for the loading, giving maximum positive shear.

For the maximum negative shear at a we have

$$\max.\,(-S) = V_1 - H \tan i - \sum_A^a P,$$

where V_1 and H are the values for the loading giving maximum negative shear, and $\sum_A^a P$ the sum of all loads between a and A. We now proceed precisely as on page 193. Or we

may avoid computation by making a diagram for each loading for each member, according to the method of Chapter I, page 8.

The method by diagram and tabulation described on page 105 will be found much the easier of application, involving no tedious calculation.

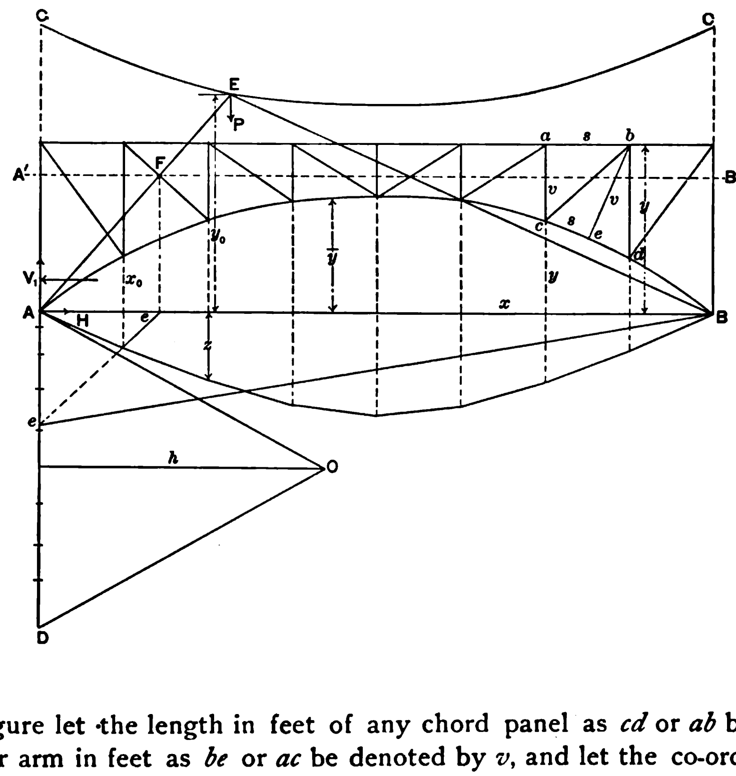
DETERMINATION OF V_1 AND H.—We see then that we can readily find the stresses, provided we can find the values of V_1 and H, or can determine the locus *cdek*, Fig. 139, page 194.

The vertical reaction V_1 can be easily found, just as for the case of three hinges, by taking moments about the right end B. Thus, if c is the span or chord of the arc, and x_0 is the distance of any load P from the left end A, we have for the vertical reaction V_1 at the left end

$$-V_1c + P(c - x_0) = 0 \quad \text{or} \quad V_1 = P - \frac{x_0}{c}P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The value of V_1 then is easily found, and is independent of the shape of the arch.

General Formulas for H and the Locus.—The value of H depends on the shape of the arch, and its determination requires the application of the principle of least work, already explained in Chapter VI. This is given at the end of this chapter. We give here the result of the discussion in the form of a general method, which holds good for *any form of arch.*



In the Figure let the length in feet of any chord panel as cd or ab be denoted by s, and let its lever arm in feet as be or ac be denoted by v, and let the co-ordinates from the left end A of the point of moments b or c for cd or ab be x and y in feet, and the area of cross-section *in square inches* of any chord panel be a. Let the co-ordinates of the point E in the load P from the left end A be x_0 and y_0 in feet. Let the length of span AB, or chord of the arc, be c in feet.

Now find for ***every chord panel ab, cd,*** etc., both upper and lower, the quantities

$$\frac{ys}{av^2},\quad \frac{y^2s}{av^2},\quad \frac{xys}{av^2},\ \ldots\ldots\ldots\ldots \quad (a)$$

where a is in *square inches* and y, s, and v *in feet.*

Then the horizontal thrust H at A, due to a load P at the distance x_0 from the left end A, is given by the equation

$$H\sum_0^c \frac{y^2s}{av^2} = Px_0\sum_{x_0}^c \frac{ys}{av^2} - P\frac{x_0}{c}\left[\sum_0^c \frac{xys}{av^2} - \frac{c}{x_0}\sum_0^{x_0}\frac{xys}{av^2}\right], \quad \ldots \quad (2)$$

where $\sum_0^c \frac{y^2s}{av^2}$ is the sum of all the quantities $\frac{y^2s}{av^2}$ for every chord panel, upper and lower; $\sum_0^c \frac{xys}{av^2}$ is the sum of all the quantities $\frac{xys}{av^2}$ for every chord panel, upper and lower; $\sum_0^{x_0} \frac{xys}{av^2}$ is the sum of all the quantities $\frac{xys}{av^2}$ for every chord panel, upper and lower, between the load P and the left end A; $\sum_{x_0}^c \frac{ys}{av^2}$ is the sum of all the quantities $\frac{ys}{av^2}$ for every chord panel, upper and lower, between the right end B and the load P.

Equation (2) is general. If the arch has a constant depth, v is constant and cancels out. It need not then be considered in finding the quantities (a) and their summations. If the chords have a constant area of cross-section, a in like manner cancels out.

If the arch is carefully drawn to scale, we can measure off directly s, y, and v, for each chord panel, and thus find the quantities (a) for each chord panel. The summations indicated can then be easily made.

For the distance y_0 of the point E of the locus CEC (Figure, page 196) above the horizontal line AB through the hinges we have then

$$y_0 = x_0\frac{V_1}{H},\ \ldots\ldots\ldots\ldots\ldots \quad (3)$$

where V_1 is given by (1) and H by (2). We can thus find as many points E in the locus CEC as may be desired, and can then draw it. The value of H for any position of P can then be at once found by construction, as in Fig. 139, page 194.

It will be noted that equation (2) requires that the area of cross-section a for each chord panel shall be known in advance, while it is the object of our investigation to determine these areas by first finding the stress and then dividing this stress by the allowable unit stress. It will in general then be necessary to first find provisional values for H and y_0 by assuming a to be constant in (2). It then cancels out. We thus determine the locus and the provisional stresses for *constant area of chord cross-section.* From these stresses we determine provisional areas a. These areas we can then use in (2), and thus find the actual stresses and areas.

GENERAL CONSTRUCTION FOR H AND THE LOCUS.*—If the summations indicated by (2) are considered tedious, much of the summation may be performed graphically, as follows:

*** This construction is given by Prof. W. Ritter in *Luegers Lexikon der gesamten Technik,* Stuttgart.**

Perform the summations denoted by $\sum_0^c \frac{ys}{av^2}$ and $\sum_0^c \frac{y^2 s}{av^2}$ and determine the distance $\bar{y}$ in feet given by

$$\bar{y} = \frac{\sum_0^c \frac{y^2 s}{av^2}}{\sum_0^c \frac{ys}{av^2}}.$$

At this distance $\bar{y}$ above AB draw a line $A'B'$ parallel to AB. (Figure on page 196.)

Consider the quantity $\frac{ys}{av^2}$ for each chord panel, as a weight acting at the centre of moments for that chord. Lay off these fictitious weights on a vertical AD through A to any convenient scale. Take the pole distance h through the centre of the load line and equal to AD or $\sum_0^c \frac{ys}{av^2}$, and construct the equilibrium polygon having AB for its closing line.

For a load P placed at any apex let z be the corresponding ordinate of this equilibrium polygon. Then we have (proof to follow)

$$H\bar{y} = Pz.$$

We have also from (1)

$$V_1 = P\frac{(c - x_0)}{c}.$$

Hence we have the proportion

$$H : V_1 = \frac{cz}{c - x_0} : \bar{y}.$$

If now we draw the line Be through the foot of the ordinate z, we have the distance $Ae = \frac{cz}{c - x_0}$. Hence

$$\frac{H}{V_1} = \frac{Ae}{\bar{y}}$$

If then we lay off the distance Ae horizontally from A, and draw the vertical eF, the intersection F with $A'B'$ is a point on the line AE, and AF prolonged to intersection with P gives the point E.

As soon, therefore, as we have found $\bar{y}$ and thus located $A'B'$, and constructed the equilibrium polygon, we can easily find the point E for any position of P. Any number of points on the locus CEC can thus be found and the locus drawn.

[*Proof of* $H\bar{y} = Pz$.—Equation (2) can be written in the form

$$H\sum_0^c \frac{y^2 s}{av^2} = Px_0\sum_0^c \frac{ys}{av^2} - P\frac{x_0}{c}\sum_0^c \frac{xys}{av^2} - P\sum_0^{x_0} \frac{(x_0 - x)ys}{av^2}.$$

Let $\sum_0^c \frac{ys}{av^2}$ be denoted by w. Then since $w\bar{y} = \sum_0^c \frac{y^2 s}{av^2}$ and $\sum_0^c \frac{xys}{av^2} = w\frac{c}{2}$, we have

$$Hw\bar{y} = P\left[\frac{wx_0}{2} - \sum_0^{x_0} \frac{(x_0 - x)ys}{av^2}\right].$$

Now in the equilibrium polygon hz is equal to the quantity in the parenthesis, for $\frac{w}{2}$ is the reaction of all the fictitious weights $\frac{ys}{av^2}$. But we have taken $h = w$. Hence

$$H h\bar{y} = Phz \quad \text{or} \quad H\bar{y} = Pz.$$

ARCH—CONSTANT DEPTH—CONSTANT CHORD-SECTION.—There are certain cases in which for constant depth simple formnlas can be derived and the preceding general methods thus greatly abridged.

From equation (2) we have for constant depth *and constant chord-section* the general expression

$$H\sum_0^c y^2 s = Px_0 \sum_{x_0}^c ys - P\frac{x_0}{c}\left[\sum_0^c xys - \frac{c}{x_0}\sum_0^{x_0} xys\right].$$

This general expression we must solve by the general method already given. But if the number of chord panels is very large we can put practically ds for s. Also, if the depth of arch is small compared to the rise, we can put for y and x the *co-ordinates of any point of the centre line or axis.*

In such case, then, we have the approximate expression

$$H\int_0^c y^2 ds = Px_0\int_{x_0}^c yds - P\frac{x_0}{c}\left[\int_0^c xyds - \frac{c}{x_0}\int_0^{x_0} xyds\right]. \quad . \quad . \quad . \quad . \quad (4)$$

This expression holds good, then, practically for framed arches of small constant depth compared to rise, where the length of each chord does not differ practically from the length of the circumscribed curve. These conditions are generally complied with in most practical cases, since the number of chord panels is great for long span.

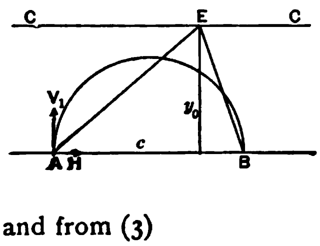
Semicircle—Constant Depth and Chord Cross-section.—Let R be the radius of the axis or centre line, and x and y the co-ordinates of any point of the axis. Then we have by similar triangles

$$\frac{ds}{dx} = \frac{R}{y} \quad \text{or} \quad ds = \frac{Rdx}{y}$$

Inserting this value of ds in (4), we obtain

$$H\int_0^c ydx = Px_0\int_{x_0}^c dx - P\frac{x_0}{c}\left[\int_0^c xdx - \frac{c}{x_0}\int_0^{x_0} xdx\right].$$

But $\int_0^c ydx$ is the area $A = \frac{\pi R^2}{2}$ of the semicircle. Performing the other integrations we obtain, since $c = 2R$,



$$HA = \frac{Px_0(2R - x_0)}{2}, \quad \text{or} \quad H = \frac{Px_0(2R - x_0)}{2A}. \quad . \quad (5)$$

From (1) we have

$$V_1 = \frac{P(2R - x_0)}{2R},$$

and from (3)

$$y_0 = x_0\frac{V_1}{H} = \frac{A}{R} = \frac{\pi R}{2} = 1.5708R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We also have $y_0 R = A$. But $y_0 R$ is the area of the triangle AEB. Hence *the area AEB is equal to the area of the semicircle.*

We see also that the locus CEC is a straight line parallel to AB at a distance $1.5708R$ above AB, or at a distance $0.5708R$ above the crown.

FLAT ARCH—CONSTANT DEPTH.—If the arch is flat, we can assume the cross-section to increase from the crown to the ends in the same ratio as sec i, where i is the inclination of any chord panel to the horizontal. If then a_0 is the chord-section at the crown, we have

$$a = a_0 \sec i.$$

Since in practice the ratio $\frac{r}{c}$ of the rise to span is rarely $\frac{1}{10}$, this supposition is in such cases admissible. Also, if the number of chord panels is large, we can put ds for s, and if the depth of arch is small compared to rise, we can put for y and x the *co-ordinates of any point of the centre line or axis.*

We have, then, from equation (2) for constant depth v, putting ds for s, and a_0 sec i for a, since v and a_0 cancel out and $\frac{ds}{\sec i} = dx$,

$$H\int_0^c y^2 dx = Px_0\int_{x_0}^c ydx - P\frac{x_0}{c}\left[\int_0^c xydx - \frac{c}{x_0}\int_0^{x_0} xydx\right].$$

Now if A is the area between the chord AB and the centre line, we have

$$\int_0^c ydx = A, \qquad \int_0^c xydx = A\frac{c}{2}.$$

Also, if $\bar{y}$ is the distance of the centre of mass of this area above AB, we have

$$\int_0^c y^2 dx = 2A\bar{y}.$$

Hence in general, for flat arches of constant depth, small as compared to rise, and numerous panels,

$$2HA\bar{y} = \frac{PAx_0}{2} - P\int_0^{x_0}(x_0 - x)ydx. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Flat Parabola—Constant Depth.—For a flat parabola, if r is the rise and c the span of the centre line, $A = \frac{2}{3}cr$, $\bar{y} = \frac{2}{5}r$, and the equation of the parabola referred to the origin A is

$$y = \frac{4rx}{c^2}(c - x).$$

We have then

$$\int_0^x (x_0 - x)ydx = \frac{rx_0^3}{3c^2}(2c - x_0).$$

Inserting these values in (7), we have

$$H = \frac{5P}{8rc^3}x_0(c - x_0)\left[c(c + x_0) - x_0^2\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

We have then from (1) and (3)

$$y_0 = x_0\frac{V_1}{H} = \frac{8rc^2}{5[c(c + x_0) - x_0^2]}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In the following table we give the values of $\frac{y_0}{r}$ for various values of $\frac{x_0}{c}$.

TABLE OF VALUES $\frac{y_0}{r}$ FOR VARIOUS VALUES OF $\frac{x_0}{c}$.

$\frac{x_0}{c}$	0.0	0.1	0.2	0.3	0.4	0.5
$\frac{y_0}{r}$	1.600	1.468	1.379	1.322	1.290	1.280

From this table we can determine the centre point of the locus *CEC* in the figure, page 196, and five points on the left. Since the locus is symmetrical with repect to the centre, the other half is similar.

Flat Circle—Constant Depth.—We see from (7) that if the area A is unchanged, only the last term varies with the form of the arch. For a flat circular arch, then, we may without practical error use the same formulas (8) and (9) as for a parabola *of same span and same area* between the curve and chord AB.

SOLID ARCH.—If the arch is solid instead of framed, we have in (2) ds in place of s, and in place of av^2 the moment of inertia I of the cross-section (Appendix, page 270). Equation (2) then becomes

$$H\int_0^c \frac{y^2ds}{I} = Px_0\int_{x_0}^c \frac{yds}{I} - P\frac{x_0}{c}\left(\int_0^c \frac{xyds}{I} - \frac{c}{x_0}\int_0^x \frac{xyds}{I}\right), \quad . \quad . \quad . \quad . \quad (10)$$

where x and y are the co-ordinates of any point of the neutral axis.

Here I is supposed to be known, whereas it is our object to determine I at every point. We therefore first suppose I or the cross-section constant. For this case we have

$$H\int_0^c y^2ds = Px_0\int_{x_0}^c yds - P\frac{x_0}{c}\left[\int_0^c xyds - \frac{c}{x_0}\int_0^{x_0} xyds\right]. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

We determine H from (11) and thus find provisional values for I. We can then use (10) by dividing the arch into a sufficient number of segments, letting the length of each segment be ds and the moment of inertia of its centre section I, and then proceed as for equation (2), page 197, or the construction page 198.

We see that (11) is precisely the same as (4). Hence for solid arch *the same formulas* (5), (6), (7), (8), *and* (9) *hold as for framed arch under the same conditions.*

TEMPERATURE STRESSES.—While in the arch with three hinges there are no temperature stresses, in the arch with only two hinges at the ends the stresses due to temperature may be considerable. The effect of a change of temperature is to cause a horizontal force at the ends. For a rise of temperature we have a positive thrust H_t as in Fig. 142, causing a positive moment at any point or tension in the upper and compression in the lower chords. If there is a fall of temperature, the arch contracts and H_t acts in the opposite direction, causing a negative moment at every point, or compression in the upper and tension in the lower chords. If H_t is known, the resulting stresses are easily found for every member directly, by a single diagram, according to the method of Chapter I, page 8.

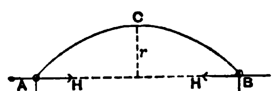


Fig. 142

The temperature stresses thus found for greatest rise and fall of temperature, above and below the mean, must be taken in connection with the maximum stresses already found for the live and dead loads in order to find the total maximum stresses.

The thrust H_t is easily shown (page 215) to be given by

$$\frac{H_t}{E}\sum_0^c \frac{y^2s}{av^2} = c\epsilon t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where the summation $\sum_0^c \frac{y^2s}{av^2}$ is made as directed, page 197; t is the rise or fall of temperature above or below the mean temperature of erection, for which there is no stress; ϵ is the coefficient of expansion, or the change of length per unit of length for one degree; and E is the coefficient of elasticity (page 148).

We have for one degree Fahrenheit,

For cast iron	$\epsilon = 0.00000617$;
wrought iron	$\epsilon = 0.00000686$;
steel (untempered)	$\epsilon = 0.00000599$.

Values for E are given in the Appendix, page 270.

Equation (12) requires that the areas of chord sections should be known in advance, whereas these are what we desire to find. We must in general then find provisional values of a by first assuming a constant value for a in (12) equal to the chord section at the crown, and thus determining provisional areas. These areas we then use in (12), and thus find the actual areas.

Denote this assumed constant area of chord section at the crown by a_0. We then have for our first calculation, for constant chord section,

$$\frac{H_t}{Ea_0}\sum_0^c \frac{y^2s}{v^2} = c\epsilon t \ , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

or for constant depth v and constant chord section,

$$\frac{H_t}{Ea_0v^2}\sum_0^c y^2s = c\epsilon t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

TEMPERATURE THRUST—SOLID ARCH.—For a solid arch we have instead of (12) the general formula

$$\frac{H_t}{E}\int_0^c \frac{y^2ds}{I} = c\epsilon t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where I is the moment of inertia at any point, and x and y are co-ordinates of any point of the neutral axis.

For our first approximation this becomes, for constant cross-section,

$$\frac{H_t}{EI_0}\int_0^c y^2ds = c\epsilon t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where I_0 is the moment of inertia at the crown, and x, y co-ordinates for any point of the neutral axis.

TEMPERATURE THRUST—CONSTANT DEPTH—CONSTANT CROSS-SECTION.—There are certain cases in which for constant depth we can perform the integrations denoted in (14) and (16) and get simple expressions for H_t. If the number of chord panels is large, we can put ds for s in (14), and if the depth of arch is small compared to the rise, we can put for y and x, in (14), the co-ordinates of any point of the centre line.

We have then for framed arches of constant cross-section and constant depth under these conditions which are usually complied with in practice

$$\frac{H_t}{Ea_0v^2}\int_0^c y^2 ds = c\epsilon t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

For solid arches, as we see from (16), we simply have to replace a_0v^2 by I_0.

Semicircle—Constant Depth—Constant Cross-section—Temperature Thrust.—Let R be the radius of the centre line, or neutral axis. Then we have

$$\frac{ds}{dx} = \frac{R}{y}, \quad \text{or} \quad ds = \frac{Rdx}{y}.$$

Substituting in (17), we have

$$\frac{H_tR}{Ea_0v^2}\int_0^c ydx = c\epsilon t.$$

But $\int_0^c ydx$ is the area $A = \frac{\pi R^2}{2}$ of the semicircle. Hence

$$H_t = \frac{4 \cdot Ea_0l^2\epsilon t}{\pi R^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

For solid arch replace a_0v^2 by I_0.

FLAT ARCH—CONSTANT DEPTH—TEMPERATURE THRUST.—If the arch is flat we can (as on page 200) put dx in place of ds in (17). We have then

$$\frac{H_t}{Ea_0v^2}\int_0^c y^2dx = c\epsilon t.$$

If A is the area between the chord AB and the centre line or neutral axis, and $\bar{y}$ is the distance of the centre of mass of this area above AB, we have

$$\int_0^c y^2dx = 2A\bar{y}.$$

Hence, in general for flat arches of constant depth under the conditions specified, of small depth compared to rise, and numerous panels,

$$H_t = \frac{Ea_0v^2c\epsilon t}{2A\bar{y}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

For a solid arch replace a_0v^2 by I_0.

Flat parabola.—For a flat parabola, if r is the rise and c the span of centre line,

$$A = \frac{2}{3}cr, \quad \bar{y} = \frac{2}{5}r, \qquad \text{and} \qquad H_t = \frac{15Ea_0v^2\epsilon t}{8r^2}.$$

For a solid arch replace a_0v^2 by I_0.

Flat Circle.—For a flat circle of radius R, rise r, and span c, if the half angle subtended at the centre by the span is α°, we have

$$A = \frac{\alpha}{360}\pi R^2, \quad \text{and} \quad \bar{y} = \frac{c^2}{12A} - (R - r).$$

Hence we have

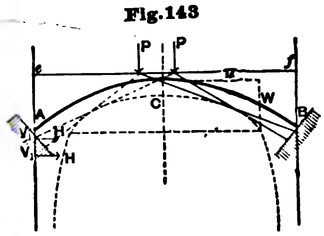
$$H_t = \frac{6Ea_0v^2cet}{c^2 - \frac{\alpha^0}{30}\pi R^2(R - r)}.$$

For a solid arch replace a_0v^2 by I_0.

Arch with Fixed Ends.

In this case, as in the preceding, the intersection of the load P with the reactions lies in a curve or locus. But the present case differs from the preceding in that the reactions *no longer pass through the ends* of the arch A and B, Fig. 143, but pass through points above or below the ends. This is the same thing as saying that we have at each end not only a horizontal thrust and vertical reaction, but also a moment which varies with the position of the load, and is always of such magnitude as to keep the tangents at the ends of the arch constant in direction.

Fig. 143



If then we can find the locus and the directions of the reactions, we can resolve P in these directions and find V_1 and H. We also have the point above or below the end through which the reactions pass. The end moment at the left end M_1 is equal to Hy_1 where y_1 is the vertical distance of this point from the end. As we shall see hereafter, this locus, for instance, *for constant depth and constant chord section and flat parabolic arch*, is a straight line ef, at $\frac{1}{5}$ the rise of the arch above the crown of the centre line, as shown in Fig. 143.

If we draw the reactions for various positions of P, we shall find that they envelop or are tangent to a curve. In the case, for instance, of a *flat parabolic arch, of constant depth and constant cross-section*, this curve, shown dotted in Fig. 143, is a hyperbola given by the equation

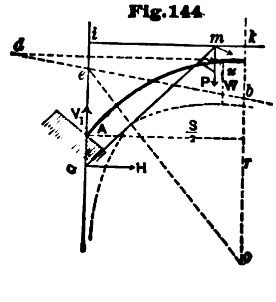
$$w = \frac{5c^2 - 10cu + 8u^2}{15c(c - 2u)}r,$$

where u is the abscissa on either side of the vertical through the crown of the centre line, and w is the ordinate measured from the horizontal through the crown of the centre line.

This curve we call the *guide curve*. We can always find it in any case, as soon as we know the locus ef and the directions of the reactions, by drawing the reactions for a number of positions of P.

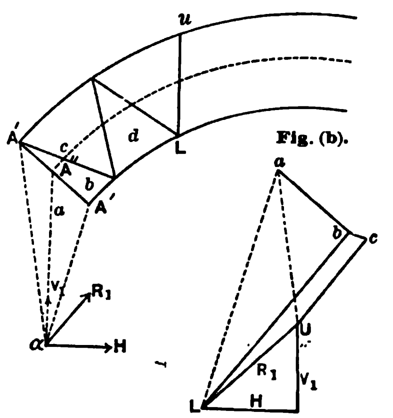
Thus in Fig. 144, the locus ik and guide curve being known, for any position of P intersecting the locus at m we can draw the reaction $m\alpha$ tangent to the guide curve. We thus have at once its direction and the point α at which it intersects the vertical through A. We can also draw the other reaction through m, tangent to the guide curve on the other side. Then resolving P in these two directions, we obtain at once V_1 and H at A. The moment M_1 at A is equal to H multiplied by the distance $A\alpha$. It is positive or causes counter clockwise rotation when the point α, as shown in Fig. 144, falls below A; negative, or causes clockwise rotation, when α falls above A.

Fig. 144



In other words, the horizontal and vertical reactions H and V_1 must be considered as acting not at the point A of the centre line of the arch, *but at the point of intersection α of the reactions with the vertical through the end A.*

Fig. (a). Fig. (b).



The reactions V_1 and H being thus known for any position of P and *taken as acting at α*, we can find the stresses by the method of diagram and tabulation already described (page 105).

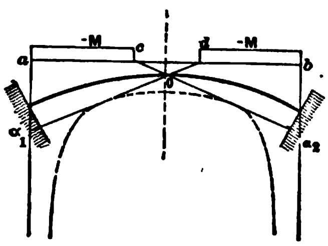
The reader may be in some doubt as to how to start the diagram. In any case proceed as follows:

Join the point α, Fig. (*a*), where H and V_1 act by fictitious members to A' and A'' the ends of the upper and lower chords. Then starting at α, diagram the stresses as for any frame.

Thus in Fig. (*b*) we lay off H and V_1 and obtain R_1. Then R_1 is in equilibrium with La and Ua, the stresses in the fictitious members. We can now go to apex A'' and find ab and Lb; then to apex A' and find bc and Uc, and so on.

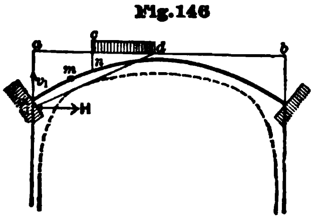
LOADING GIVING MAXIMUM STRESSES.—As soon as the locus for P and the guide curve are known, we can, as before, find the loading which gives the maximum stress for any member, and can then apply the method of calculation described on page 195, only remembering that H and V_1 are applied at α and not at the end A.

Fig. 145



Chords.—Thus for any chord panel, if o, Fig. 145, is the point of moments for that panel, we proceed just as in Fig. 140, page 194, for two hinges. Thus we draw through o the resultants oα_1 and oα_2, which meet the locus ab at the points c and d. Then every load to the right of d or left of c causes a positive moment about o (counter clockwise rotation). Loads at d and c cause no moment about o. Loads between c and d cause a negative moment about o (clockwise rotation).

Fig. 146

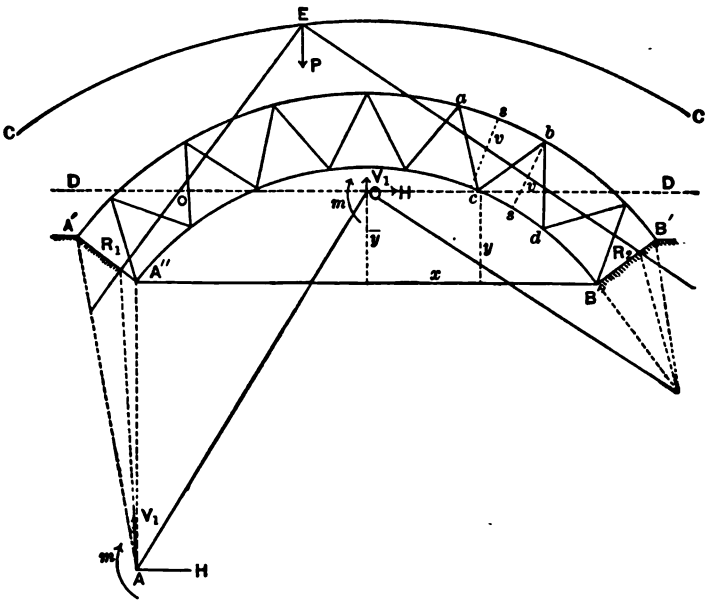


Braces.—For the greatest shear at any point n, draw $\alpha_1 d$, Fig. 146, through the point of intersection of the two chords cut by a section through n, or if these chords are parallel, then $\alpha_1 d$ is parallel to them, just as in Fig. 141, page 195. Then a load at d causes no shear at n. Any load on the right of d or left of c causes negative (downward) shear at n, and any load between c and d causes positive (upward) shear at n.

GENERAL FORMULAS.—The developmentof these formulas is given at the close of this chapter. We give here the result in the form of a general method, which holds good for *any form of arch.* For any load P, let CEC be the locus and ER_1 the direction of the left reaction.

Let the span $A''B''$ as always be denoted by c in feet. As before, page 196, let the length in feet of any chord panel be s, its lever arm be v in feet, the co-ordinates from A'' of its point of moments be x and y in feet, and its area of cross-section be a in *square inches.* Thus for any chord panel ab the length is s feet, the lever arm $cs = v$ feet, the point of moments is c, and the co-ordinates of c from A'' are x and y in feet. The area of cross-section is a square inches. So also for the chord panel cd, the length is s feet, the lever arm $bs = v$ feet, the point of moments is b, and x, y are the co-ordinates from A'' of this point.

Let the co-ordinates of the point E of the locus in the load P, from A'', be x_0 and y_0 in feet.



Now find for every chord panel the quantities

$$\frac{s}{av^2}, \quad \frac{ys}{av^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

where a is in square inches and s, y, v in feet (page 196).

Then find the distance

$$\bar{y} = \frac{\sum_0^c \frac{ys}{av^2}}{\sum_0^c \frac{s}{av^2}},$$

where $\sum_0^c \frac{ys}{av^2}$ is the sum of all the quantities $\frac{ys}{av^2}$ for every chord panel, upper and lower, and $\sum_0^c \frac{s}{av^2}$ is the sum of all the quantities $\frac{s}{av^2}$ for every chord panel, upper and lower.

At this distance $\bar{y}$ above $A''B''$, draw a parallel DD to $A''B''$, and let o be the intersection of this line with ER_1.

Let O be the point whose co-ordinates are $\bar{x} = \frac{c}{2}$ and $\bar{y}$. Through this point O draw a parallel OA to ER_1, and let the point A be vertically below R_1. Join A', A'' to A by the fictitious members $A'A$ and $A''A$.

If now we consider O as a fixed point, and introduce the members OA and $A'A$ and $A''A$, and suppose a certain moment m to act at O, we can remove the abutment $A'A''$ and equilibrium will still exist.

Then, as is proved by the principle of least work (page 215), we shall have

$$m\sum_0^c \frac{s}{av^2} = -P\sum_{x_0}^c \frac{s(x - x_0)}{av^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$V_1 \sum_0^c \frac{s\left(x - \frac{c}{2}\right)}{av^2} = P \sum_{x_0}^c \frac{s(x - x_0)\left(x - \frac{c}{2}\right)}{av^2}, \quad \ldots \ldots \quad (2)$$

$$H \sum_0^c \frac{s(y - \bar{y})^2}{av^2} = - P \sum_{x_0}^c \frac{s(x - x_0)(y - \bar{y})}{av^2}. \quad \ldots \ldots \quad (3)$$

These summations can be made for any position of P. In making them, we must remember that when x is less than $\frac{c}{2}$, or y less than $\bar{y}$, $\left(x - \frac{c}{2}\right)$ and $\left(y - \bar{y}\right)$ will be negative.

If the arch is carefully drawn to scale, we can measure off directly s, y and v, for each chord panel, and thus find the quantities (a) for each chord panel. The summations for $\bar{y}$ are then readily made and $\bar{y}$ determined.

Then drawing the line DD, we can measure off directly the quantities $(y - \bar{y})$ and $\left(x - \frac{c}{2}\right)$ for each point of moments for each chord panel. The summations indicated are thus easily made and m, V_1 and H_1 thus determined for any position of P. Then, $V_2 = P - V_1$.

Let the distance Oo be denoted by u. Then we have

$$u = \frac{m}{V_1}. \quad \ldots \ldots \ldots \ldots \quad (4)$$

If then through o, thus located, we draw a line ER_1, making with DD an angle *whose tangent is* $\frac{V_1}{H}$, and produce it to intersection E and P, we obtain the point E of the locus CEC, and we also have ER_1 in true position.

Then through E we draw ER_2, making an angle with DD whose tangent is $\frac{V_2}{H}$, and we have ER_2 in true position.

Thus a number of points E in the locus CEC can be found, and the guide curve (page 204) can also be drawn. The value of H and V_1, and the directions of the reactions can then be found for any position of P as in Fig. 144, page 204.

It will be noted that equations (1), (2), and (3) require that the area of cross-section a for each chord panel shall be known in advance, while it is the object of our investigation to determine these areas. It will in general, then, be necessary to first find provisional values by assuming a to be constant. It then cancels out. We thus determine provisional stresses and the corresponding provisional areas. These areas we can then use in (1), (2), and (3).

GENERAL CONSTRUCTION.*—Much labor may be saved by the following construction —notation as in the preceding article:

Draw the arch carefully to scale, and find as before for every chord panel the quantities

$$\frac{s}{av^2}, \quad \frac{ys}{av^2}.$$

Then determine the distance

$$\bar{y} = \frac{\sum_0^c \frac{ys}{av^2}}{\sum_0^c \frac{s}{av^2}},$$

and draw as before the line DD parallel to $A''\ B''$ at the distance $\bar{y}$.

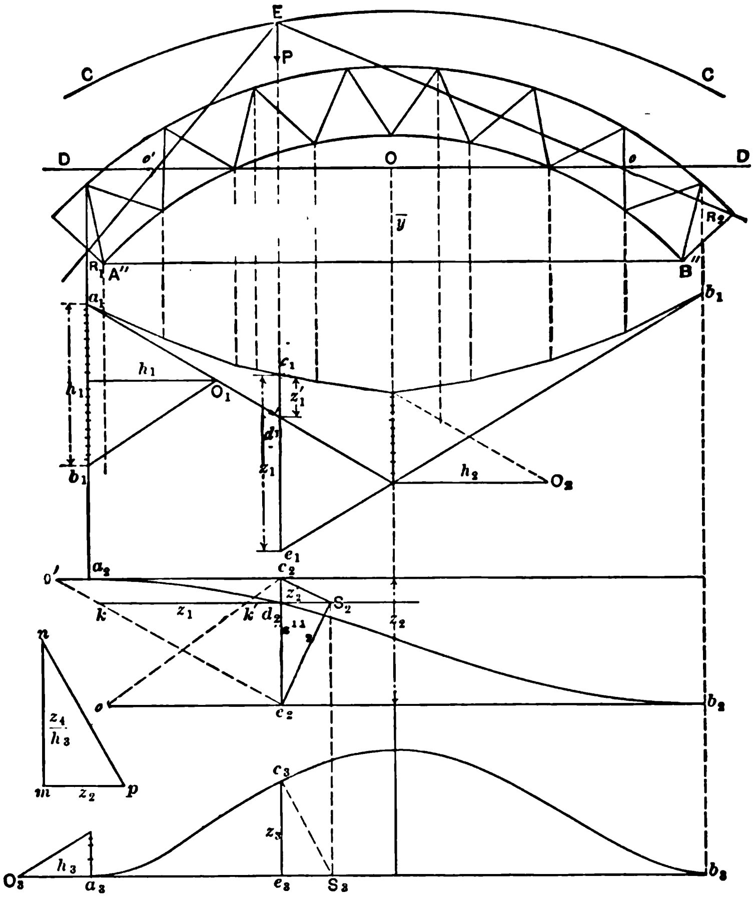
* This construction is given by Prof. W. Ritter in *Luegers Lexikon der gesamten Technik*, Stuttgart.

Find now the quantity

$$\frac{s(y-\bar{y})^2}{av^2}$$

for each chord panel, and determine the quantity

$$z_4 = \sum_0^c \frac{s(y-\bar{y})^2}{av^2}.$$



Consider the quantity $\frac{s}{av^2}$ for each chord panel, as a weight acting at the centre of moments for that chord panel. Lay off these fictitious weights on a vertical line a_1b_1 to any convenient scale. Take the pole O_1 in the horizontal through the middle point of a_1b_1 and take the pole distance h_1 equal to a_1b_1 or $\sum_0^c \frac{s}{av_2}$, and construct the first equilibrium polygon $a_1c_1b_1$. Produce its end lines, and let z_1 be the ordinate c_1e_1 and z_1' the ordinate c_1d_1 corresponding to any position of P. Then by the property of the equilibrium polygon

$$h_1z_1 = \sum_{x_0}^c \frac{s(x-x_0)}{av^2},$$

and from equation (1), page 206, since $h_1 = \sum_0^c \frac{s}{av^2}$,

$$mh_1 = Ph_1z_1 \quad \text{or} \quad m = Pz_1.$$

Again, produce each side of the polygon $a_1c_1b_1$ to intersection with the centre ordinate. Then $h_1 \times$ each intercept gives $\frac{s}{av^2}\left(x - \frac{c}{2}\right)$ for each chord panel.

Choose then a new pole O_2 with any convenient pole distance h_2, and considering $\frac{s}{av^2}\left(x - \frac{c}{2}\right)$ for each chord panel, as a force acting at the centre of moments for that panel, construct the second equilibrium polygon $a_2d_2b_2$. Let z_2 be the centre ordinate between its two horizontal end lines, and z_2'' the ordinate d_2e_2, and z'_2 the ordinate c_2d_2 corresponding to any position of P.

Then, by the property of the equilibrium polygon,

$$h_1h_2z_2'' = \sum_{x_0}^{c} \frac{s(x - x_0)\left(x - \frac{c}{2}\right)}{av^2}, \quad h_1h_2z_2 = \sum_{0}^{c} \frac{s\left(x - \frac{c}{2}\right)^2}{av^2}.$$

and from equation (2), page 207,

$$V_1h_1h_2z_2 = Ph_1h_2z_2'', \text{ or } V_1 = \frac{z_2''}{z_2}P.$$

Find now the quantity

$$\frac{s(\bar{y} - y)}{av^2}$$

for each chord panel. Consider these quantities as forces acting at the centre of moments for each chord panel, lay them off to any convenient scale, positive up and negative down, choose a pole O_3, take any convenient pole distance h_3, and construct the third equilibrium polygon $a_3c_3b_3$. Let z_3 be the ordinate c_3e_3 corresponding to any position of P. Then, by the property of the equilibrium polygon,

$$h_3z_3 = \sum_{x_0}^{c} \frac{s(\bar{y} - y)(x - x_0)}{av^2},$$

and from equation (3), page 207,

$$Hz_4 = Ph_3z_3, \text{ or } H = Ph_3\frac{z_3}{z_4}.$$

We have then

$$\frac{H}{V_1} = \frac{z_3z_2h_3}{z_4z_2''} \text{ and } \frac{m}{V_1} = z_2\frac{z_1}{z_2''}.$$

If we draw a right-angled triangle mnp, having z_2 for base and $\frac{z_4}{h_3}$ for altitude, and draw c_2S_1 parallel to its hypothenuse np, then the distance $e_2S_1 = \frac{z_3z_2h_3}{z_4}$. If then we project S_1 to S_2 upon the horizontal d_2S_2, we have $\frac{H}{V} = \frac{d_2S_2}{d_2e_2}$. The line S_2e_2 is then parallel to ER_1 and the line c_2S_2 is parallel to ER_2. Also, since the distance $Oo' = \frac{m}{V_1} = z_2\frac{z_1}{z_2''}$, if we lay off d_2k equal to z_1 and d_2k' equal to z_1' and draw e_2k and c_2k' and produce to intersections o' and o, the distance c_2o' is equal to Oo' and the distance e_2o to Oo.

We thus find by construction for any position of P the points o' and o and the directions of ER_1 and ER_2. As many points E as desired can thus be found in the locus CEC,

and the reactions ER_1, ER_2 for each point can be drawn, thus determining the guide curve (page 204).

FLAT ARCH—CONSTANT DEPTH.—If the arch is flat we can assume the cross-section to increase from the crown to the ends in the same ratio as sec i, where i is the inclination of any chord panel to the horizontal. If then a_0 is the chord section at the crown, we have

$$a = a_0 \sec i.$$

Since in practice the ratio $\frac{r}{c}$ of the rise to the span is rarely over $\frac{1}{10}$, this supposition is in such cases admissible. Also if the number of chord panels is large we can put ds for s, and if the depth of arch is small compared to rise, we can put for y and x *the co-ordinates of any point of the centre line or axis.*

We have then from equations (1), (2), (3), page 207, for constant depth v, putting ds for s and a_0 sec. i for a since v and a_0 cancel out and $\frac{ds}{\sec i} = dx$,

$$m\int_0^c dx = P\int_{x_0}^c dx\,(x - x_0),$$

$$V_1\int_0^c dx\left(x - \frac{c}{2}\right)^2 = P\int_{x_0}^c dx\,(x - x_0)\left(x - \frac{c}{2}\right),$$

$$H\int_0^c dx(y - \bar{y})^2 = P\int_{x_0}^c dx\,(x - x_0)\,(\bar{y} - y),$$

In the last equation

$$\bar{y} = \frac{\int_0^c ydx}{\int_0^c dx}$$

is the distance of the centre of mass of the centre line above the chord of the centre line. Performing the integrations, we have from the first two equations

$$m = -\frac{P}{2c}(c - x_0)^2, \quad \ldots \quad (5)$$

$$V_1 = \frac{(c + 2x_0)\,(c - x_0)^2}{c^3}P, \quad V_2 = P - V_1 = \frac{x_0^2\,(3c - 2x_0)\,P}{c^3}. \quad \ldots \quad (6)$$

Equations (5) and (6) hold good *whatever the form of arch.* If A is the area between the centre line and its chord, we have

$$\int_0^c ydx = A, \quad \int_0^c xy\,dx = A\frac{c}{2}.$$

Also, if $\bar{\bar{y}}$ is the distance of the centre of mass of this area above the chord of the centre line

$$\int_0^c y^2dx = 2\,A\bar{\bar{y}}.$$

Hence, performing the integrations of the third equation,

$$H[2A\bar{\bar{y}} - 2\,A\bar{y} + c\bar{y}^2] = \left[\frac{\bar{y}}{2}(c - x_0)^2 - \int_{x_0}^c (x - x_0)\,ydx\right]P. \quad . \quad . \quad (7)$$

FLAT PARABOLA—CONSTANT DEPTH.—For a flat parabola, if r is the rise and c the span of the centre line, we have $A = \frac{2}{3}cr$, $\bar{y} = \frac{2}{3}r$, $\bar{\bar{y}} = \frac{4}{5}r$, and the equation of the parabola referred to the origin at left end of span is

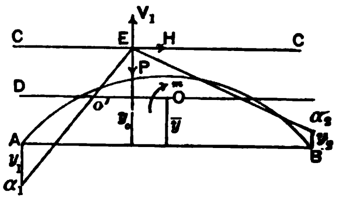
$$y = \frac{4rx}{c^2}(c - x).$$

We have then

$$\int_{x_0}^{c}(x - x_0)\, y dx = \frac{r}{3c^2}(c - x_0)^3 (c^2 - x_0^2).$$

Substituting in (6), we obtain

$$H = \frac{15 x_0^2 (c - x_0)^2 P}{4rc^3}, \quad \ldots\ldots\ldots\ldots \quad (8)$$



If E is a point of the locus CEC and y_0 is the ordinate of E, we have, taking moments about O,

$$- H(y_0 - \bar{y}) - V_1\left(\frac{c}{2} - x_0\right) = m,$$

or

$$y_0 = \bar{y} - \frac{m}{H} - \frac{V_1}{H}\left(\frac{c}{2} - x_0\right).$$

Inserting the values of $\bar{y}$ and $\frac{m}{H}$, $\frac{V_1}{H}$, from (5), (6), and (8), we obtain

$$y_0 = \frac{2r}{3} + \frac{2rc^2}{15x_0^2} - \frac{2r(c^2 - 4x_0^2)}{15x_0^2}, \quad \text{or} \quad y_0 = \frac{6}{5}r.\ \ldots\ldots\ldots \quad (9)$$

That is, *the locus CEC is a straight line at a distance of $\frac{1}{5}r$ above the crown of the centre line.*

Since the reactions $E\alpha_1$ and $E\alpha_2$ make the angles with the horizontal whose tangents are $\frac{V_1}{H}$ and $\frac{V_2}{H}$, we have for the distances $A\alpha_1 = y_1$ and $B\alpha_2 = y_2$,

$$(y_0 - y_1)\frac{H}{V_1} = x_0, \quad (y_0 - y_2)\frac{H}{V_2} = c - x_0.$$

Inserting the values of H_1, V_1, V_2, and y_0 from (6), (8), and (9), we have

$$\left.\begin{aligned} y_1 &= \frac{2r(5x_0 - 2c)}{15x_0}, \\ y_2 &= \frac{2r(3c - 5x_0)}{15(c - x_0)}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (10)$$

Positive values are laid off above A and B, negative below. In the Figure, y_1 is negative and y_2 positive.

The tangent of the angle which $E\alpha_1$ makes with the horizontal is

$$\frac{V_1}{H} = \frac{4r(c + 2x_0)}{15x_0^2}, \quad \ldots\ldots\ldots\ldots \quad (11a)$$

and the tangent of the angle which $E\alpha_2$ makes with the horizontal is

$$\frac{V_2}{H} = \frac{4r(3c - 2x_0)}{15(c - x_0)^2}. \quad \ldots \ldots \ldots \ldots \quad (11b)$$

We can thus draw the reactions for any position of P and determine the guide curve (page 204, Fig. 143).

Flat Circle—Constant Depth.—We see from (7) that if the area A is unchanged, only the last term varies with the form of arch. For a flat circular arch, then, we may without practical error use the same formulas *as for a parabola of same span and area* between the centre line and chord AB.

Solid Arch.—If the arch is solid instead of framed, we have in (1), (2), and (3), page 207, ds in place of s, and in place of av^2 the moment of inertia I of the cross-section (page 270). Since I is thus supposed to be known, whereas it is our object to determine it at every point, we first suppose I or the cross-section constant. Having thus determined provisional values of I, we can use these values.

For flat solid arch the same results hold as for flat framed arch under the same conditions.

TEMPERATURE STRESSES.—The temperature thrust H_t is shown by the principle of least work (page 216) to be given by

$$\frac{H_t}{E}\sum_0^c \frac{s(y - \bar{y})^2}{av^2} = c\epsilon t, \quad \ldots \ldots \ldots \ldots \quad (12)$$

where E is the coefficient of elasticity and ϵ the coefficient of expansion and t the rise or fall of temperature above or below the mean temperature of erection. Values of ϵ are given page 202; values of E on page 270. For rise of temperature, as explained, page 201, H_t is positive, or is a thrust; for a fall it is negative, or is a pull.

The force H_t does not act at the ends A and B, however, as in Fig. 142, but at the point O in the preceding Figure given by

$$\bar{x} = \frac{c}{2}, \quad \bar{y} = \frac{\sum_0^c \frac{ys}{av^2}}{\sum_0^c \frac{s}{av^2}}.$$

Equation (12) requires that the areas of chord sections should be known, whereas these are what we desire to find. We must, in general, then, find provisional values of a by first assuming a constant value for a in (12) equal to the chord section of the crown, and thus determining provisional areas. These areas we then use in (12), and thus find the actual areas.

Denote this assumed constant area of chord section at the crown by a_0. We then have for our first calculation, for constant chord section,

$$\frac{H_t}{Ea_0}\sum_0^c \frac{s(y - \bar{y})^2}{v^2} = c\epsilon t, \quad \ldots \ldots \ldots \ldots \quad (13)$$

or for constant depth v and constant chord section

$$\frac{H_t}{Ea_0 v^2}\sum_0^c s(y - \bar{y})^2 = c\epsilon t. \quad \ldots \ldots \ldots \ldots \quad (14)$$

TEMPERATURE THRUST—SOLID ARCH.—For a solid arch we have instead of (12) the general formula

$$\frac{H_t}{E}\int_0^c \frac{(y-\bar{y})^2 ds}{I} = c\epsilon t, \quad \ldots \ldots \ldots \ldots \quad (15)$$

where I is the moment of inertia at any point and x and y are co-ordinates of any point of the neutral axis.

For our first approximation this becomes, for constant cross-section,

$$\frac{H_t}{EI_0}\int_0^c (y-\bar{y})^2 ds = c\epsilon t \quad \ldots \ldots \ldots \ldots \quad (16)$$

where I_0 is the moment of inertia at the crown.

FLAT ARCH—CONSTANT DEPTH—TEMPERATURE THRUST.—If the number of chord panels is large, we can put ds for s in (14); and if the depth of arch is small compared to the rise, we can put y and x in (14), the co-ordinates of any point of the centre line.

We have then for framed arches of constant cross-section and constant depth, under these conditions, which are usually complied with in practice,

$$\frac{H_t}{Ea_0v^2}\int_0^c (y-\bar{y})^2 ds = c\epsilon t. \quad \ldots \ldots \ldots \ldots \quad (17)$$

For solid arches we simply have to replace a_0v^2 by I_0.

If now the arch is flat, we can put dx in place of ds. We have then

$$\frac{H_t}{Ea_0v^2}\int_0^c (y-\bar{y})^2 dx = c\epsilon t. \quad \ldots \ldots \ldots \ldots \quad (18)$$

If A is the area between the chord AB and the centre line or neutral axis, and $\bar{y}$ is the distance of the centre of mass of this area above AB, we have

$$\int_0^c y^2 dx = 2A\bar{y}, \qquad \int_0^c y\,dx = A.$$

Hence in general, for flat arches of constant depth, small as compared to rise, and numerous panels,

$$H_t = \frac{Ea_0v^2c\epsilon t}{2A\bar{y} - 2A\bar{y} + c\bar{y}^2} \quad \ldots \ldots \ldots \ldots \quad (19)$$

For a solid arch replace a_0v^2 by I_0.

Flat parabola.—For a flat parabola, if r is the rise and c the span of centre line, $A = \frac{2}{3}cr$, $\bar{y} = \frac{2}{5}r$, $\bar{y} = \frac{2}{3}r$, and

$$H_t = \frac{45Ea_0v^2\epsilon t}{4r^2}.$$

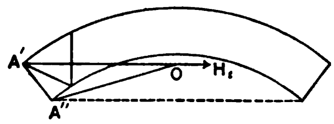
For a solid arch replace a_0v^2 by I_0.

Flat Circle.—For a flat circle, we see from (19) that if the area A is unchanged we may use the same formula as for a flat parabola of same area and span.

Diagram for H_t.—If H_t is known in any case, the resulting stresses are easily found for every member directly, by a single diagram, according to the method of Chap. I, page 12.

The reader may be in some doubt as to how to start the diagram. In any case. proceed as follows:

Join the point O where H_t acts by fictitious members $A'O$ and $A''O$ to the ends of the upper and lower chords.

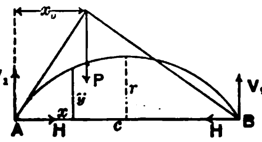


Then starting at O diagram the stresses as for any frame.

DEMONSTRATION OF THE PRECEDING FORMULAS.

We shall now give the derivation of the formulas given in this chapter.

ARCH HINGED AT ENDS ONLY.—Let the load P be at a distance x_0 from the left end A; V_1 and V_2 be the vertical reactions at A and B, and H the horizontal thrust Let the span $AB = c$. Then, taking moments about B,



$$\left.\begin{aligned} -V_1c + P(c - x_0) = 0. \therefore\ V_1 &= P - \frac{x_0}{c}P; \\ V_2 = P - V_1 &= \frac{x_0}{c}P. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The value of V_1 is then independent of the shape of the arch.

Let v be the lever arm for any chord member and M the moment at the centre of moments for that member. Then the stress in that member is $\frac{M}{v}$ Let a be the area of cross-section of the member, and s its length. Then from Chapter VI the work of straining the member is,

$$\text{Work} = \frac{M^2s}{2Eav^2}.$$

The total work of straining all the chord members is then

$$\text{Work} \doteq \sum \frac{M^2s}{2Eav^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{I})$$

and this work, as we have seen from Chapter VI, according to the principle of least work, must be a minimum.

Now for any point distant x from A, between A and P, we have

$$M = Hy - V_1x = Hy - Px + \frac{Px_0x}{c}.$$

For any point between P and B,

$$M = Hy - V_1x + P(x - x_0) = Hy - Px_0 + \frac{Px_0x}{c}.$$

We have then for the work of straining all the chord members, from (I),

$$\text{Work} = \sum_0^{x_0}\left[\frac{Hyc - P(c - x_0)x}{c}\right]^2\frac{s}{2Eav^2} + \sum_{x_0}^{c}\left[\frac{Hyc - Px_0(c - x)}{c}\right]^2\frac{s}{2Eav^2}.$$

Since the work is thus given in terms of H and known quantities, and the work must be a minimum, we differentiate with reference to H, and put the differential coefficient equal to zero.

We thus obtain

$$\frac{d(\text{work})}{dH} = 0 = \sum_0^{x_0}\left[\frac{Hy^2c^2 - Pcy(c - x_0)x}{c^2}\right]\frac{s}{Eav^2} + \sum_{x_0}^{c}\left[\frac{Hy^2c^2 - Pcx_0y(c - x)}{c^2}\right]\frac{s}{Eav^2}.$$

Hence, by reduction,

$$H \sum_0^c \frac{y^2 s}{av^2} = Px_0 \sum_{x_0}^c \frac{ys}{av^2} - \frac{Px_0}{c}\left[\sum_0^c \frac{xys}{av^2} - \frac{c}{x_0}\sum_0^{x_0} \frac{xys}{av^2}\right]. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is equation (2), on page 197.

Temperature Thrust.—If ϵ is the coefficient of expansion and t the rise or fall of temperature, the change of span is $c\epsilon t$. The work is then

$$\frac{H_t c\epsilon t}{2}.$$

The moment at any point is $H_t y$, and from (I) we have the work

$$\sum_0^c \frac{H_t^2 y^2 s}{2Eav^2}.$$

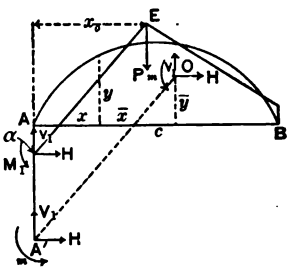
Hence,

$$H_t \frac{c\epsilon t}{2} = \sum_0^c \frac{H_t^2 y^2 s}{2Eav^2} \qquad \text{or} \qquad \frac{H_t}{E}\sum_0^c \frac{y^2 s}{av^2} = c\epsilon t.$$

This is equation (12), page 202.

The other equations for this case are deduced in the text.

ARCH WITH FIXED ENDS.—Let O be the centre of mass of the arch, and its co-ordinates be $\bar{x}, \bar{y}$. Then we must have



$$\Sigma_0^c s(y - \bar{y}) = 0, \quad \Sigma_0^c s(x - \bar{x}) = 0, \quad \bar{x} = \frac{c}{2}.$$

We have at the left end the reactions V_1, H, and the moment M_1. The resultant pressure is in the direction $E\alpha$. Draw through O a parallel OA' to $E\alpha$. If now we consider O as a fixed point, rigidly connected to the arch at A' by members OA' and OA, we can remove the abutment at A and equilibrium will still exist. We shall then have at O the reactions V_1 and H, and the moment m.

For any point distant x from A, between A and P, we have

$$M = H(y - \bar{y}) - V_1(x - \bar{x}) + m.$$

For any point between P and B we have

$$M = H(y - \bar{y}) - V_1(x - \bar{x}) + P(x - x_0) + m.$$

We have then as in the preceding case, for the work of straining all the chord members,

$$\text{Work} = \sum_0^{x_0} \frac{[H(y - \bar{y}) - V_1(x - \bar{x}) + m]^2 s}{2Eav^2} + \sum_{x_0}^c \frac{[H(y - \bar{y}) - V_1(x - \bar{x}) + P(x - x_0) + m]^2 s}{2Eav^2}$$

Since H, V_1, and m must have such values as to make the work a minimum, we place the first differential coefficients of the work with reference to H, V_1, and m equal to zero.

Hence,

$$\frac{d(\text{work})}{dm} = 0 = \sum_0^c \frac{[m + H(y - \bar{y}) - V_1(x - \bar{x})]s}{Eav^2} + \sum_{x_0}^c \frac{P(x - x_0)s}{Eav^2},$$

$$\frac{d(\text{work})}{dV_1} = 0 = \sum_0^c \frac{[V_1(x - \bar{x}) - H(y - \bar{y}) - m](x - \bar{x})s}{Eav^2} - \sum_{x_0}^c \frac{P(x - \bar{x})(x - x_0)s}{Eav^2},$$

$$\frac{d(\text{work})}{dH} = 0 = \sum_0^c \frac{[H(y - \bar{y}) - V_1(x - \bar{x}) + m](y - \bar{y})s}{Eav^2} + \sum_{x_0}^c \frac{P(y - \bar{y})(x - x_0)s}{Eav^2}.$$

But since $\sum_0^c (y - \bar{y})s = 0$ and $\sum_0^c (x - \bar{x})s = 0$ and $\bar{x} = \frac{c}{2}$, these equations reduce to

$$m\sum_0^c \frac{s}{av^2} = -P\sum_{x_0}^c \frac{s(x - x_0)}{av^2}, \quad \ldots \ldots \ldots \ldots \quad (1)$$

$$V_1\sum_0^c \frac{s\left(x - \frac{c}{2}\right)}{av^2} = P\sum_{x_0}^c \frac{s(x - x_0)\left(x - \frac{c}{2}\right)}{av^2}, \quad \ldots \ldots \ldots \quad (2)$$

$$H\sum_0^c \frac{s(y - \bar{y})^2}{av^2} = -P\sum_{x_0}^c \frac{s(x - x_0)(y - \bar{y})}{av^2}. \quad \ldots \ldots \ldots \quad (3)$$

These are equations (1), (2), and (3), page 207.

Temperature Thrust.—For the temperature thrust H_t acting at O, we have the moment at any point,

$$M = H_t\,(y - \bar{y}).$$

Hence from (I) the work is

$$\sum_0^c \frac{H_t^2(y - \bar{y})^2 s}{2Eav^2}.$$

We have also the work given by

$$H_t \frac{c\epsilon t}{2}.$$

Hence,

$$H_t \frac{c\epsilon t}{2} = \sum_0^c \frac{H_t^2(y - \bar{y})^2 s}{2Eav^2}, \quad \text{or} \quad \frac{H_t}{E}\sum_0^c \frac{s(y - \bar{y})^2}{av^2} = c\epsilon t.$$

This is equation (12), page 212.

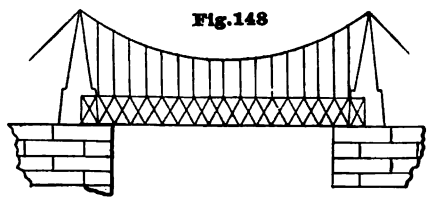
The other equations for this case are deduced in the text.

CHAPTER X.

COMPOSITE STRUCTURES—SUSPENSION SYSTEM WITH STIFFENING TRUSS.

EACH of the structures of the preceding chapter may be inverted, and constitutes in such case an inverted arch or rigid suspension system. The method of calculation is then precisely the same, the only difference being that the horizontal thrust at the end of the arch becomes a horizontal pull at the ends of the cable, and therefore members which were in compression are now in tension, and *vice versa.*

SUSPENSION SYSTEM.—A common construction for long spans, however, is that shown in Fig. 148. Such a structure we may call a "composite" system, that is, it consists of two different systems which act together. Fig. 148 represents the most important of these, known as the "suspension system." It consists of a flexible chain or cable which is stiffened under the action of partial loads by a truss. The truss is slung from the cable by suspenders, and may be of any design, either double or single intersection, Pratt, etc. The cable carries the entire dead weight, that is, the suspenders are screwed up until the ends of the truss just bear on the abutments. The office of the truss is thus to stiffen the cable and prevent change of shape and oscillation due to partial and moving loads. It also acts to support its share of the moving load. There are usually side spans at each end. In any case the cable passes over rollers on top of the towers, and is carried on beyond and firmly fastened to large anchorages of masonry.

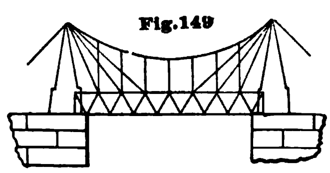


DEFECTS OF THE SYSTEM.—The principal defect of this system is its lack of rigidity. The cable possesses little inherent rigidity, and the stiffness is due almost entirely therefore to the truss.

A second disadvantage is that a rise of temperature, by increasing the deflection, throws considerable load on the truss. To obviate this objection, the truss may be hinged at the centre and placed on rollers at the ends.

ADVANTAGES OF THE SYSTEM.—It is evident from the preceding that the system is best applied to long spans. The cable, then, carries the dead weight, and by reason of its own very considerable weight in such case resists in some degree the deforming action of partial loads. The truss can thus be very light compared to what it would have to be if there were no cable.

STAYS UNNECESSARY.—The system is accordingly in practice applied only to very long spans. In such case, with cables made of steel wire it admits of great economy. But, owing to lack of rigidity, additional stiffness is sought to be obtained by the introduction of *stays* reaching from the top of the tower to various points of the truss, as shown in Fig. 149. The use of these is not to be recommended. They render the correct determination of the stresses indeterminate. A load at any point may be carried entirely by the suspender and stay at that point, or by the suspender and truss, or by the stay and truss. It is impossible to tell exactly the duty performed by each; and even if it were not, it would be impossible to so adjust the several systems that each shall take its proper share. If such adjustment

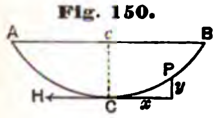


could be made, it would not last. Variations of stress, set, and elongation of members, shocks and vibrations, rise and fall of temperature, would constantly disturb such adjustment.

The stays are also superfluous. The truss is a rigid construction. It ought to render rigid the system of which it forms a part, and should be so designed as to perform its duty without help. If such superfluous members are introduced, they can then be considered as an extra addition, contributing to strength and stiffness. But the truss should be designed without reference to their action.

SHAPE OF CABLE.—The curve of the cable for a uniformly distributed load per unit of horizontal length is a parabola. This is easily proved.

Fig. 150.



Let the span or chord of the cable AB be c, Fig. 150, and the rise be r. Then if w be the load per unit of horizontal length, we have, taking moments about B, if H is the horizontal pull at the bottom C,

$$-Hr + \frac{wc^2}{8} = 0 \quad \text{or} \quad H = \frac{wc^2}{8r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Equation (1) gives the horizontal pull H of the cable.* This is evidently the same at every point.

If now we take moments about any point P distant x from C, we have, if y is the ordinate of this point for origin at C,

$$-Hy + \frac{wx^2}{2} = 0 \quad \text{or} \quad y = \frac{wx^2}{2H}.$$

Inserting the value of H from (1) we have

$$y = \frac{4rx^2}{c^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which is the equation of a parabola.

Hence the curve of a flexible string uniformly loaded along the horizontal is a parabola.

LENGTH OF CABLE.—In the system shown in Fig. 148, the cable carries the entire dead load of the truss, as the suspenders are supposed to be so adjusted during erection that the truss just bears. The weight of truss, flooring, suspenders, etc., may be taken as very nearly uniform, as the variation of weight due to variation of cross-section of chords and braces, and difference in length of suspenders, can be neglected in comparison with the uniform dead weight of flooring, wind bracing, etc. Moreover, this uniform load is very great compared to weight of cables. The actual curve of the cable may then be considered as very closely a parabola, as given by equation (2).

Differentiating (2) we obtain

$$\frac{dy}{dx} = \frac{8rx}{c^2}.$$

If the length of the cable is s, we have

$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

Inserting the value for $\frac{dy}{dx}$, we obtain

$$ds = dx\sqrt{1 + \frac{64r^2x^2}{c^4}} = dx\left[1 + \frac{32r^2x^2}{c^4} - \frac{512r^4x^4}{c^8} + \ldots\right].$$

* If w is taken for the whole bridge, the value of H must be divided among the number of cables.

For long spans the ratio $\frac{r}{c}$ of versed sine to span is small, and $\frac{r^4}{c^4}$ can be neglected. We have then approximately

$$ds = dx\left(1 + \frac{32r^2x^2}{c^4}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Integrating between the limits $x = +\frac{c}{2}$ and $x = -\frac{c}{2}$, we have for the length of cable

$$s = c\left(1 + \frac{8r^2}{3c^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equation (4) gives the length of cable when the span c and versine r are given, and the ratio $\frac{r}{c}$ is small; that is, for long spans.

DEFLECTION OF CABLE.—Let the cable carry a uniformly distributed load. The new curve will be a parabola of same span whose new length s_1 will be from (4)

$$s_1 = c\left(1 + \frac{8r_1^2}{3c^2}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The distance of any point P of the cable (Fig. 150) below the horizontal AB before deflection is, from (2),

$$r - y = r - \frac{4rx^2}{c^2},$$

where x is the distance of the point P from the centre C.

The distance of the same point of the deflected cable below AB is then

$$r_1 - \frac{4r_1x^2}{c^2},$$

where r_1 is the new versine.

The difference of these two distances is the deflection δ_1 at any point. Hence

$$\delta_1 = r - r_1 - \frac{4x^2}{c^2}(r - r_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

It remains to find the value of $(r - r_1)$ and substitute in (6).

Let e be the strain per unit of length at any point where the area of cross-section is A. The stress at any point is $H\frac{ds}{dx}$. If E_1 is the coefficient of elasticity for the cable, we have (Chapter VI, page 149)

$$e = \frac{Hds}{E_1Adx}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The cable may be composed of links and pins, or of wires. The last is more common. In the first case the cross-section should vary so that the unit stress at every point shall be constant. In the second case the cross-section at every point must be constant and equal to the cross-section required by the greatest stress, that is, to the cross-section at the ends.

Let us take these two cases separately.

1st. Cross-section Constant.—For this case we have from (3), by making $x = \frac{c}{2}$ for the secant of the angle of inclination at the ends,

$$\sec i = 1 + \frac{8r^2}{c^2}.$$

The stress at the end is then

$$H \sec i = H\left(1 + \frac{8r^2}{c^2}\right).$$

If σ_1 is the allowable unit stress for which the cable is designed, then the constant area of cross-section must be

$$A = \frac{H\left(1 + \frac{8r^2}{c^2}\right)}{\sigma_1}.$$

Substituting this value of A in (7), we have the strain per unit of length

$$e = \frac{\sigma_1}{E_1\left(1 + \frac{8r^2}{c^2}\right)} \cdot \frac{ds}{dx} = e_0 \frac{dx}{ds},$$

where e_0 denotes the strain per unit of length at the centre C, so that

$$e_0 = \frac{\sigma_1}{\left(1 + \frac{8r^2}{c^2}\right)E_1}. \quad \ldots \ldots \ldots \ldots \quad (8)$$

We have then from (3)

$$e = e_0\left(1 + \frac{32r^2x^2}{c^4}\right).$$

The strain of any element ds of the cable is then eds, and the new length of this element is $ds + eds$. We have then from (3)

$$ds_1 = ds(1 + e) = dx\left[1 + e_0 + \frac{32r^2x^2}{c^4} + \frac{64e_0r^2x^2}{c^4} + \frac{1024e_0r^4x^4}{c^8}\right].$$

Integrating between $x = +\frac{c}{2}$ and $x = -\frac{c}{2}$, we have

$$s_1 = c(1 + e_0)\left[1 + \frac{8r^2}{3c^2} + \frac{8r^2e_0}{3c^2(1 + e_0)} + \frac{64e_0r^4}{5c^4(1 + e_0)}\right].$$

Since $\frac{r}{c}$ is a small fraction for long spans, and $\frac{e_0}{1 + e_0} = \frac{1}{1 + \frac{1}{e_0}}$ is also a small fraction, we can neglect the last two terms, and have approximately for long spans

$$s_1 = c(1 + e_0)\left(1 + \frac{8r^2}{3c^2}\right).$$

Equating this with (5), we have

$$c(1 + e_0)\left(1 + \frac{8r^2}{3c^2}\right) = c\left(1 + \frac{8r_1^2}{3c^2}\right). \quad \ldots \ldots \ldots \quad (9)$$

From this equation we find the new versine v_1. Thus

$$r_1 = r\sqrt{1 + e_0\left(1 + \frac{3c^2}{8r^2}\right)} = r\left[1 + \frac{e_0}{2}\left(1 + \frac{3c^2}{8r^2}\right) - \frac{e_0^2}{8}\left(1 + \frac{3c^2}{8r^2}\right)^2 + \ldots\right].$$

Since e_0 is a small fraction, this becomes approximately

$$r_1 = r\left[1 + \frac{e_0}{2}\left(1 + \frac{3c^2}{8r^2}\right)\right],$$

or

$$r_1 - r = \frac{e_0 r}{2}\left(1 + \frac{3c^2}{8r^2}\right) = \frac{3e_0 c^2}{16r}\left(1 + \frac{8r^2}{3c^2}\right);$$

or, since $\frac{r}{c}$ is a small fraction, approximately

$$r_1 - r = \frac{3e_0 c^2}{16r}.$$

Substituting this in (6), we obtain for the deflection of the cable at any point distant x from the centre, for *constant cross-section*,

$$\delta_1 = -\frac{3e_0}{16r}(c^2 - 4x^2); \qquad (10)$$

or, putting for e_0 its value from (8),

$$\delta_1 = -\frac{3\sigma_1}{16r\left(1 + \frac{8r^2}{c^2}\right)E_1}(c^2 - 4x^2), \qquad (11)$$

where σ_1 is the unit stress at the end for which the cable is designed. The minus sign denotes that the deflection is below the original unloaded curve.

If we put for σ_1 its value

$$\sigma_1 = \frac{H\left(1 + \frac{8r^2}{c^2}\right)}{A},$$

and put for H its value from (1),

$$H = \frac{wc^2}{8r},$$

we have

$$\delta_1 = -\frac{3wc^3}{128r^2AE_1}(c^2 - 4x^2). \qquad (12)$$

Equation (12) gives the deflection when the load w per unit of length and the constant area of cross-section A are known; equation (11) the deflection when the cable is loaded up to its allowable unit stress σ_1 at the ends; equation (10) when the strain per unit of length at the centre is known.

2d. *When the unit stress is constant.*—In this case if the unit stress for which the cable is designed is as before σ_1, and A_0 is the area of cross-section at the centre, we have

$$\frac{H}{A_0} = \sigma_1 \quad \text{and} \quad e = \frac{\sigma_1}{E_1} = \frac{H}{E_1 A_0}. \qquad (13)$$

Since then e is constant, the new length of the cable will be $e + es$, and equating with (5),

$$s + es = c\left(1 + \frac{8r_1^2}{3c^2}\right),$$

or from (4),

$$c\left(1 + \frac{8r^2}{3c^2}\right)(1 + e) = c\left(1 + \frac{8r_1^2}{3c^2}\right).$$

This is the same as equation (9), except that we have e in place of e_0. We proceed then just as before, and obtain for *constant unit stress*,

$$\delta_1 = -\frac{3e}{16r}(c^2 - 4x^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

or, putting for e its value from (13),

$$\delta_1 = -\frac{3\sigma_1}{16rE_1}(c^2 - 4x^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\delta_1 = -\frac{3wc^2}{128r^2E_1A_0}(c^2 - 4x^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

DEFLECTION OF CABLE DUE TO TEMPERATURE.—Let ϵ be the coefficient of expansion for the cable for one degree and t the number of degrees above or below the temperature of erection. Then the elongation or contraction per unit of length is ϵt. If we substitute this in (10) or (14) in place of e_0 or e, we have for the deflection at any point due to a rise or fall of temperature,

$$\delta_1 = -\frac{3\epsilon t}{16r}(c^2 - 4x^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

For a rise of temperature t is to be taken positive; for a fall of temperature t is to be taken negative. If δ_1 comes out negative, it indicates that the deflection is below the mean position of cable. If positive, above.

WORK OF LIVE LOAD IN STRAINING CABLE.—The stress at any point of the cable is

$$S = H\frac{ds}{dx},$$

where H is the horizontal pull.

Let w be the uniform load per unit of horizontal length when the live load covers the whole span. The cable carries a portion of this load, which we shall denote by ϕw. Therefore ϕ is a fraction which we shall determine later by the principle of least work.

We have from (1), for live load,

$$H = \frac{\phi wc^2}{8r},$$

and hence

$$S = \frac{\phi wc^2 ds}{8rdx}.$$

From Chapter VI, page 150, the work of the live load in straining the cable is then, in general,

$$\text{work} = \int_{-\frac{c}{2}}^{+\frac{c}{2}} \frac{\phi^2 w^2 c^4 ds^3}{128r^2 A_1 E_1 dx^2},$$

where A_1 is the area of cross-section at any point of the cable and E_1 is the coefficient of elasticity for the cable.

If the cross-section is constant A_1 is constant. We have then for this case

$$\text{work} = \frac{\phi^2 w^2 c^4}{128 r^2 A_1 E_1} \int_{-\frac{c}{2}}^{+\frac{c}{2}} \frac{ds^3}{dx^2}.$$

If the cross-section varies so that the unit stress is constant, let A_1 be the cross-section at bottom. Then the cross-section at any other point is $A_1 \frac{ds}{dx}$, and we have for this case

$$\text{work} = \frac{\phi^2 w^2 c^4}{128 r^2 A_1 E_1} \int_{-\frac{c}{2}}^{+\frac{c}{2}} \frac{ds^2}{dx}.$$

Now we have from (3)

$$ds = \left(1 + \frac{32 r^2 x^2}{c^4}\right) dx.$$

Hence, neglecting higher powers of $\frac{r^2}{c^4}$,

$$\frac{ds^2}{dx} = \left(1 + \frac{64 r^2 x^2}{c^4}\right) dx, \qquad \frac{ds^3}{dx^2} = \left(1 + \frac{96 r^2 x^2}{c^4}\right) dx,$$

$$\int_{-\frac{c}{2}}^{+\frac{c}{2}} \frac{ds^2}{dx} = \frac{3c^2 + 16 r^2}{3c}, \qquad \int_{-\frac{c}{2}}^{+\frac{c}{2}} \frac{ds^3}{dx^2} = \frac{c^2 + 8 r^2}{c}.$$

Substituting, we have

for constant cross-section,

$$\text{work} = \frac{\phi^2 w^2 c^3 (c^2 + 8 r^2)}{128 r^2 A_1 E_1}; \quad \ldots \ldots \quad (18)$$

for constant unit stress,

$$\text{work} = \frac{\phi^2 w^2 c^3}{384 r^2 A_1 E_1}(3c^2 + 16 r^2). \quad \ldots \ldots \quad (19)$$

If we have a concentrated load P at any point on the truss, let the portion of this load carried by the cable be $\phi_1 P$. Then, since this load is uniformly distributed over the cable by the truss, the load per unit of horizontal length is $\frac{\phi_1 P}{c}$. Equations (18) and (19) become for this case:

for constant cross-section,

$$\text{work} = \frac{\phi_1^2 P^2 c}{128 r^2 A_1 E_1}(c^2 + 8 r^2); \quad \ldots \ldots \quad (20)$$

for constant unit stress,

$$\text{work} = \frac{\phi_1^2 P^2 c}{384 r^2 A_1 E_1}(3c^2 + 16 r^2). \quad \ldots \ldots \quad (21)$$

WORK OF LIVE LOAD IN STRAINING SUSPENDERS.—Let the panel length be p and the length of a suspender be l. Then we have for any suspender the stress

$$S = \phi wp$$

and for the work of all the suspenders,

$$\text{work} = \frac{\phi^2 w^2 p^2 \Sigma l}{2E_2A_2}, \quad \ldots \ldots \ldots \ldots \quad (22)$$

where A_2 is the area of cross-section of a suspender, E_2 the coefficient of elasticity for the suspenders, and Σl the sum of the lengths of all the suspenders.

If we have a concentrated load P at any point on the truss, let the portion of this load carried by the cable be $\phi_1 P$ as before. Then the load of the cable per unit of horizontal length is $\frac{\phi_1 P}{c}$, and the stress of a suspender is

$$S = \frac{\phi_1 Pp}{c}.$$

The work, then, in this case is

$$\text{work} = \frac{\phi_1^2 P^2 p^2 \Sigma l}{2c^2 E_2A_2}. \quad \ldots \ldots \ldots \ldots \quad (23)$$

WORK OF LIVE LOAD IN STRAINING TRUSS.—Let the live load w per unit of length cover the whole span, and the portion carried by the truss be $(1 - \phi)w$. If h is the height of truss and M the moment at any point, then the stress in the chord at that point is

$$S = \frac{M}{h}.$$

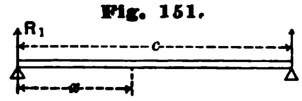
We have, then, considering the panel length as small compared to the whole span c (Chapter VI, page 150),

$$\text{work} = \int_0^c \frac{M^2 dx}{2E_3A_3h^2},$$

where E_3 is the coefficient of elasticity for the truss, and A_3 the chord section, supposed constant.

Now the reaction at the left end is, Fig. 151,

Fig. 151.



$$R_1 = \frac{(1 - \phi)wc}{2},$$

and the moment at any point distant x from the left is

$$M = -R_1x + \frac{(1 - \phi)wx^2}{2} = \frac{(1 - \phi)wx}{2}(x - c).$$

Substituting this value of M_1, we have

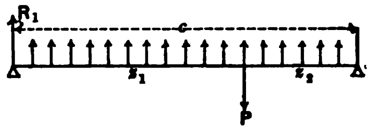
$$\text{work} = \int_0^c \frac{(1 - \phi)^2 w^2}{8E_3A_3h^2}(x - c)^2 x^2 dx.$$

Performing the integration, we obtain

$$\text{work} = \frac{(1 - \phi)^2 w^2 c^5}{240E_3A_3h^2}. \quad \ldots \ldots \ldots \ldots \quad (24)$$

Let a concentrated load P be placed on the truss at a distance z_1 from the left end and z_2 from the right end, Fig. 152. Let the cable carry the portion $\phi_1 P$ of this load. The load of the cable per unit of horizontal length is then $\frac{\phi_1 P}{c}$ uniformly distributed by means of the truss.

Fig. 152.



This, therefore, acts as a uniform *upward* load on the truss.

The reaction at the left end is, then,

$$R_1 = \frac{Pz_2}{c} - \frac{\phi_1 P}{2},$$

and the reaction R_2 at the right end is

$$R_2 = \frac{Pz_1}{c} - \frac{\phi_1 P}{2}.$$

The moment for any point between the left end and P is then

$$x < z_1, \quad M = -R_1 x - \frac{\phi_1 P x^2}{2c} = \frac{\phi_1 P x}{2c}(c - x) - \frac{Pz_2 x}{c}.$$

For any point between P and the right end we have, taking x from the right end,

$$x < z_2, \quad M = -R_2 x - \frac{\phi_1 P x^2}{2c} = \frac{\phi_1 P x}{2c}(c - x) - \frac{Pz_1 x}{c}.$$

We have, then, for the work,

$$\text{work} = \int_0^{z_1} \left[\frac{\phi_1 P x}{2c}(c - x) - \frac{Pz_2 x}{c}\right]^2 \frac{dx}{2A_2E_2h^2} + \int_0^{z_2} \left[\frac{\phi_1 P x}{2c}(c - x) - \frac{Pz_1 x}{c}\right]^2 \frac{dx}{2A_2E_2h^2}.$$

Performing the integrations, we obtain

$$\text{work} = \frac{P^2}{240A_2E_2h^2c^2}\Big[[10c^2(z_1^3 + z_2^3) - 15c(z_1^4 + z_2^4) + 6(z_1^5 + z_2^5)]\phi_1^2 + 40z_1^2z_2^2(z_1 + z_2)$$
$$+ [3(z_1^3 + z_2^3) - 4c(z_1^2 + z_2^2)](10\phi_1 z_1 z_2)\Big]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

DETERMINATION OF ϕ AND ϕ_1 BY THE PRINCIPLE OF LEAST WORK.—In the preceding articles we have found the work of the live load in straining the cable, suspenders, and truss, both for a uniform live load w per unit of horizontal length and for a concentrated live load P. In each case the work is given in terms of known quantities and the fractions ϕ and ϕ_1, or the portions of w and P carried by the cable. The portions carried by the truss are, then, $(1 - \phi)$ and $(1 - \phi_1)$.

According to the principle of least work (Chapter VI) the values of ϕ and ϕ_1 must make the work on the system a minimum, in each case.

1st. Uniform Live Load—Constant Cable Section.—For live load over the whole span and constant cable cross-section, the work on the system is the sum of (18), (22), and (24). Hence we have

$$\text{work} = \frac{\phi^2 w^2 c^3 (c^2 + 8r^2)}{128r^2 A_1 E_1} + \frac{\phi^2 w^2 p^2 \Sigma l}{2E_2 A_2} + \frac{(1 - \phi)^2 w^2 c^5}{240E_2A_2h^2}. \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The stress at the centre of the cable is, from (1),

$$\frac{(u+\phi w)c^2}{8r}.$$

At the end, we have by making $x=\frac{c}{2}$ in (3), the secant of the angle of inclination i,

$$\sec i=\frac{ds}{dx}=1+\frac{8r^2}{c^2}.$$

Hence the stress at the end is

$$\frac{(u+\phi w)c^2}{8r}\sec i=\frac{(u+\phi w)(c^2+8r^2)}{8r}.$$

If σ_1 is the unit stress for which the cable is designed, we have

$$A_1=\frac{(u+\phi w)(c^2+8r^2)}{8r\sigma_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

If σ_2 is the unit stress for the suspenders, we have

$$A_2=\frac{(u+\phi w)p}{\sigma_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The stress at the centre of the truss is

$$\frac{(1-\phi)wc^2}{8h}.$$

We may take, then, for the average value of A_3,

$$A_3=\frac{(1-\phi)wc^2}{16h\sigma_3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

where σ_3 is the unit stress for the truss.

Inserting the values of A_1, A_2, A_3 in (26), differentiating with reference to ϕ and putting the first differential coefficient equal to zero, we obtain for the value of ϕ which makes the work on the system a minimum,

$$\phi=-\frac{u}{w}+\frac{u}{w}\sqrt{\frac{1}{1-\dfrac{16c^3}{15c^3\dfrac{h}{r}\dfrac{\sigma_1E_3}{\sigma_3E_1}+120ph\Sigma l\dfrac{\sigma_2E_3}{\sigma_3E_2}}}}. \quad . \quad . \quad . \quad (30)$$

2d. Uniform Live Load—Constant Unit Stress in the Cable.—For live load over the whole span and cross-section of the cable varying so that the unit stress is constant, the work on the system is the sum of (19), (22), and (24). But in this case if σ_1 is the unit stress for the cable

$$A_1=\frac{(u+\phi w)c^2}{8r\sigma_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

and A_2, A_3 are the same as given by (28) and (29). We have, then,

$$\text{work}=\frac{\phi^2w^2c\sigma_1(3c^2+16r^2)}{48E_1r(u+\phi w)}+\frac{\phi^2\sigma_2w^2\Sigma pl}{2(u+\phi w)E_2}+\frac{(1-\phi)\sigma_3wc^3}{15E_3h}.$$

Differentiating with reference to ϕ and putting the first differential coefficient equal to zero, we obtain for the value of ϕ, which makes the work on the system a minimum,

$$\phi = -\frac{u}{w} + \frac{u}{w}\sqrt{\frac{1}{1 - \dfrac{48c^3}{15c(3c^2 + 16r^2)\dfrac{h}{r}\dfrac{\sigma_1 E_2}{\sigma_2 E_1} + 360ph\Sigma l\dfrac{\sigma_3 E_2}{\sigma_2 E_3}}}. \quad . \quad . \quad . \quad (32)$$

3*d*. *Concentrated Live Load P—Constant Cable Cross-section.*—For a concentrated load P at any point of the truss distant z_1 from the left end and z_2 from the right end, and constant cable cross-section, the work on the system is the sum of (20), (23), and (25).

Differentiating with respect to ϕ_1, and putting the first differential coefficient equal to zero, we obtain for the value of ϕ_1

$$\phi_1 = \frac{5z_1z_2[4c(z_1^2 + z_2^2) - 3(z_1^3 + z_2^3)]}{10c^3(z_1^3 + z_2^3) - 15c(z_1^4 + z_2^4) + 6(z_1^5 + z_2^5) + 120h^2p^2\Sigma l\dfrac{E_3A_3}{E_2A_2} + \dfrac{15}{8}c^2(c^2 + 8r^2)\dfrac{h^2 E_2A_2}{r^2 E_1A_1}},$$

where the values of A_1, A_2, A_3 are given by (27), (28), and (29).

We can put this equation in a more convenient shape for computation as follows : Let N be the number of panels, and

$$\frac{z_1}{c} = \frac{n_1}{N} \quad \text{or} \quad z_1 = \frac{n_1}{N}c,$$

$$\frac{z_2}{c} = \frac{n_2}{N} \quad \text{or} \quad z_2 = \frac{n_2}{N}c,$$

so that for any panel load P, n_1 is simply the number of panels from the left end and n_2 the number of panels from the right end. Inserting these values of z_1 and z_2, we obtain

$$\phi_1 = \frac{5n_1n_2[4N(n_1^2 + n_2^2) - 3(n_1^3 + n_2^3)]}{10N^2(n_1^3+n_2^3) - 15N(n_1^4+n_2^4) + 6(n_1^5+n_2^5) + N^5\left[\dfrac{120h^2p^2\Sigma lE_3A_3}{c^6E_2A_2} + \dfrac{15(c^2 + 8r^2)h^2E_2A_2}{8c^2r^2E_1A_1}\right]}.$$

But we have

$$N = n_1 + n_2,$$

and hence

$$n_1^2 + n_2^2 = N^2 - 2n_1n_2,$$

$$n_1^3 + n_2^3 = N[N^2 - 3n_1n_2],$$

$$n_1^4 + n_2^4 = N^2[N^2 - 3n_1n_2] - n_1n_2[N^2 - 2n_1n_2],$$

$$n_1^5 + n_2^5 = N^3[N^2 - 3n_1n_2] - n_1n_2N[N^2 - 3n_1n_2] - n_1n_2N[N^2 - 2n_1n_2].$$

Substituting these values and reducing, we have

$$\phi_1 = \frac{5n_1n_2(N^2 + n_1n_2)}{N^4\left[1 + \dfrac{120h^2p^2\Sigma lE_3A_3}{c^6E_2A_2} + \dfrac{15(c^2 + 8r^2)h^2E_2A_2}{8c^2r^2\,E_1A_1}\right]} \quad . \quad . \quad . \quad . \quad (33$$

where N is the number of panels, n_1 the number of panels from the left end, and n_2 from the right end, to any load P, and A_1, A_2, A_3 are given by (27), (28), (29).

Equation (33) is easy and rapid of application in any special case. (See example, page 234.)

4th. Concentrated Live Load P—Constant Unit Stress in Cable.—For this case the work on the system is the sum of (21), (23), and (25). Hence, proceeding just as before, we obtain

$$\phi_1 = \frac{5n_1n_2(N^2 + n_1n_2)}{N^4\left[1 + \frac{120h^2p^2\Sigma lE_2A_2}{c^6\,E_2A_2} + \frac{5(3c^2 + 16r^2)h^2E_2A_2}{8c^2r^2\,E_1A_1}\right]}, \quad \ldots \quad (34)$$

where A_1 is given by (31), and A_2, A_2 by (28) and (29).

LOAD OF TRUSS DUE TO CHANGE OF TEMPERATURE.—From (17) we have for the deflection Δ_1 of cable at centre, due to change of temperature,

$$\Delta_1 = -\frac{3\epsilon tc^2}{16r},$$

where ϵ is the coefficient of expansion and t the number of degrees above or below the mean temperature of erection.

The deflection Δ_2 of the truss at centre for a uniform load u per unit of length, is given by the Theory of Flexure (Appendix, page 303).

$$\Delta_2 = \frac{5uc^4}{24E_2A_2h^2};$$

or, inserting the value of A_2 from (29),

$$\Delta_2 = \frac{10uc^2\sigma_2}{3h(1-k)wE_2}.$$

If we equate Δ_1 and Δ_2, we have, for the uniform load u_t of the truss due to change of temperature,

$$u_t = -\frac{9\epsilon th(1-\phi)wE_2}{160r\sigma_2}. \quad \ldots \quad (35)$$

This load is upwards or positive for a fall of temperature, or t minus, and downwards for a rise of temperature, or t plus.

RECAPITULATION.—We group together here for convenience of use and reference the formulas necessary for the calculation of the stresses in a suspension system in proper order.

From (2) we have, for the equation of the cable,

$$y = \frac{4rx^2}{c^2}, \quad \ldots \quad (I)$$

where r is the versine of the cable, c the chord or span, x and y the co-ordinates for any point, for origin at centre of cable.

By the application of (I) we can find the length of each suspender, and thus find Σl, the sum of the length of all the suspenders.

We can now find ϕ, the fraction of the live load carried by the cable. Thus, from (30), we have, for cable of constant cross-section,

$$\phi = -\frac{u}{w} + \frac{u}{w}\sqrt{\frac{1}{1 - \frac{16c^2}{15c^2\frac{h}{r}\frac{\sigma_1E_2}{\sigma_2E_1} + 120ph\Sigma l\frac{\sigma_2E_2}{\sigma_2E_2}}}}. \quad \ldots \quad (II)$$

For cable of constant unit stress we have, from (32),

$$\phi = -\frac{u}{w} + \frac{u}{w}\sqrt{\frac{1}{1 - \dfrac{16c^3}{15c(3c^2 + 16r^2)\dfrac{h}{r}\dfrac{\sigma_1 E_3}{\sigma_3 E_1} + 360ph\Sigma l\dfrac{\sigma_2 E_3}{\sigma_3 E_2}}}}, \quad . \quad . \quad (\text{II}')$$

where u is the dead load and w the live load per unit of horizontal length; c the chord or span; r the versine of cable; h the depth of truss; p the panel length; Σl the sum of the lengths of the suspenders; E_1, E_2, E_3 the coefficients of elasticity for cable, suspenders, and truss respectively; and σ_1, σ_2, σ_3 the unit stress for cable, suspenders, and truss respectively.

From (35) we find the equivalent load per unit of horizontal length u_t due to temperature,

$$u_t = \frac{9\epsilon th(1 - \phi)wE_3}{160r\sigma_3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{III})$$

where ϵ is the coefficient of expansion; t the number of degrees above or below the mean temperature of erection; r the versine of cable; h the depth of truss; w the live load per unit of horizontal length; E_3 and σ_3 the coefficient of elasticity and the unit stress for the truss, and ϕ is given by (II) or (II′).

From (27) we have now for the area of the cable for constant cross-section *

$$A_1 = \frac{(u_t + u + \phi w)(c^2 + 8r^2)}{8r\sigma_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{IV})$$

or for cable of constant unit stress from (31) the area at the centre is *

$$A_1 = \frac{(u_t + u + \phi w)c^2}{8r\sigma_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{IV}')$$

and at any other point from (3),

$$A = A_1\frac{ds}{dx} = A_1\left(1 + \frac{32r^2x^2}{c^4}\right),$$

where x is the distance from centre.

For the area of a suspender,* we have from (28)

$$A_2 = \frac{(u_t + u + \phi w)p}{\sigma_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{V})$$

From (29) we can now find the average cross-section for the truss†

$$A_3 = \frac{(1 - \phi)wc^2}{16h\sigma_3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{VI})$$

and therefore we know the values of

$$\frac{A_2}{A_3} \text{ and } \frac{A_3}{A_1}.$$

---

* If u and w are taken for the whole bridge, the area must be divided among the number of cables and their suspenders.

† If w is taken for the whole bridge, the truss area must be divided among the number of trusses.

We can now find ϕ_1, the fraction of any panel live load P carried by the cable, for cable of constant cross-section from (33)

$$\phi_1 = \frac{5n_1n_2(N^2 + n_1n_2)}{N^4\left[1 + \frac{120h^2p^2\Sigma lE_2A_2}{c^4E_2A_2} + \frac{15(c^2 + 8r^2)h^2E_2A_2}{8c^2r^2E_1A_1}\right]}, \quad \ldots \quad \text{(VII)}$$

where N is the number of panels in the truss; n_1 and n_2 the number of panels left and rignt of the panel live load.

For cable of constant unit stress we have from (34)

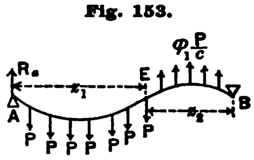
$$\phi_1 = \frac{5n_1n_2(N^2 + n_1n_2)}{N^4\left[1 + \frac{120h^2p^2\Sigma lE_2A_2}{c^4E_2A_2} + \frac{5(3c^2 + 16r^2)h^2E_2A_2}{8c^2r^2E_1A_1}\right]}. \quad \ldots \quad \text{(VII')}$$

We now have for the left reaction of the truss for any panel live load P,

$$R_1 = \frac{Pn_2}{N} - \frac{\phi_1 P}{2}. \quad \ldots \ldots \quad \text{(VIII)}$$

Maximum Moment and Shear in Truss.—By the application of (VII) or (VII') we can find ϕ_1 for each panel live load P, and then by the application of (VIII) we can find the left reaction R_1 for each panel live load. In any special case we should find that for a certain distance $z_2 = BE$, Fig. 153, from the right end, every panel live load causes a negative reaction, $-R_1$, at the left end, while for the remaining distance, $z_1 = AE$, every panel live load causes a positive reaction, $+R_1$, at the left end. The truss must then be held down at the ends.

Fig. 153.



Maximum Negative Moment.—The maximum negative moment, then, for any point of the left half span will be as shown in Fig. 153, when the live load covers the distance AE only. The reaction for this loading will then be

$$\sum_A^E R_1,$$

or the sum of the reactions for all the panel live loads P between E and A. This reaction will be increased by the uniform temperature load u_t due to rise of temperature, so that the total reaction at the left end for the loading of Fig. 153 is

$$R_A = \sum_A^E R_1 + \frac{(N-1)u_tp}{2},$$

where N is the total number of panels in the truss and p is the panel length.

The upward uniform load due to the cable is

$$\frac{P}{c}\sum_A^E \phi_1,$$

or $\frac{P}{c}$ multiplied by the sum of the values of ϕ_1 for all the panel live loads between E and

A. This is diminished by the temperature load u_t due to rise of temperature, so that the resultant uniform upward load w is

$$w_1 = \frac{P}{c}\sum_A^B \phi_1 - u_t.$$

If p is the panel length, this gives an upward panel load at every panel, since $Np = c$, of

$$w_1 p = \frac{P}{N}\sum_A^B \phi_1 - u_t p.$$

We have then for the maximum negative moment at any panel point of the left half span, if n is the number of panels from the left end to the point,

$$M = -R_A np + \frac{n(n-1)p}{2}P - \frac{n(n-1)}{2}w_1 p^2,$$

or

$$M = -np\left[\sum_A^B R_1 + \frac{(N-1)u_t p}{2}\right] + \frac{n(n-1)p}{2}\left[P - \frac{P}{N}\sum_A^B \phi_1 + u_t p\right]. \quad . \quad . \quad (\mathrm{IX})$$

Maximum Positive Moment.—Since all loads from B to E cause a negative reaction at the left end, the maximum positive moment for any point of the left half span will be as shown in Fig. 154, when the live load covers the distance BE only.

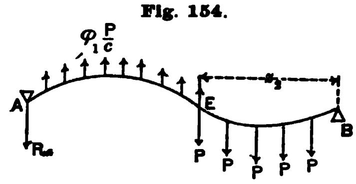
The negative reaction for this loading will then be

$$\sum_E^B R_1.$$

This negative reaction is increased by the temperature load u_t due to fall of temperature, so that the total negative reaction for the loading of Fig. 154 is

$$R_A = \sum_E^B R_1 - \frac{(N-1)u_t p}{2}.$$

Fig. 154.



The upward uniform load due to the cable is

$$\frac{P}{c}\sum_E^B \phi_1.$$

This is increased by the temperature load u due to fall of temperature, so that the resultant uniform upward load is

$$w_1 = \frac{P}{c}\sum_E^B \phi_1 + u_t.$$

We have then for the maximum positive moment at any panel point of the left half span, if n is the number of panels from the left end to the point,

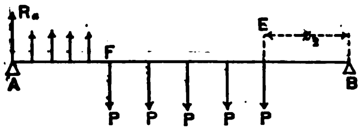
$$M = -R_A np - \frac{n(n-1)}{2}w_1 p^2;$$

or, since $c = Np$,

$$M = -np\left[\sum_E^B R_1 - \frac{(N-1)u_t p}{2}\right] - \frac{n(n-1)p}{2}\left[\frac{P}{N}\sum_E^B \phi_1 + u_t p\right]. \quad . \quad . \quad . \quad (\mathrm{X})$$

Maximum Positive Shear.—The maximum positive shear at any point of the left half span is when we have the greatest positive reaction and have no loads on the left of the point. This loading for any point F is shown by Fig. 155.

Fig. 155.



The reaction for this loading is

$$\Sigma_F^B R_1,$$

and this reaction is increased by the temperature load u_t due to rise of temperature, so that the total positive reaction is

$$R_A = \Sigma_F^B R_1 + \frac{(N-1)u_t p}{2}$$

The upward uniform load due to the cable is

$$\frac{P}{c}\Sigma_F^B \phi_1,$$

and this is diminished by the temperature load, so that the resultant uniform upward load is

$$w_1 = \frac{P}{c}\Sigma_F^B \phi_1 - u_t.$$

The maximum positive shear at any panel point of the left half span, if n is the number of panels from the left end to the point, is

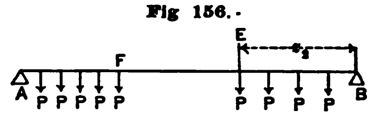
$$S = R_A - (n-1)w_1 p,$$

or

$$S = \Sigma_F^B R_1 + \frac{(N-1)u_t p}{2} - (n-1)\left[\frac{P}{N}\Sigma_F^B \phi_1 - u_t p\right]. \quad . \quad . \quad . \quad . \quad \text{(XI)}$$

Maximum Negative Shear.—The maximum negative shear at any point of the left half span is when we have the greatest negative reaction and the truss is loaded from the left end up to the point. This loading is shown by Fig. 156. The reaction for this loading is

Fig. 156.



$$\Sigma_E^B R_1 + \Sigma_A^1 R_1.$$

This reaction is increased by the temperature load due to fall of temperature, so that the total reaction is

$$R_A = \Sigma_E^B R_1 + \Sigma_A^F R_1 - \frac{(N-1)u_t p}{2}.$$

The upward uniform load due to the cable is

$$\frac{P}{c}\Sigma_E^B \phi_1 + \frac{P}{c}\Sigma_A^F \phi_1,$$

and for fall of temperature an upward load u_t, so that the resultant upward load is

$$w_1 = \frac{P}{c}\Sigma_E \phi_1 + \frac{P}{c}\Sigma_A^F \phi_1 + u_t.$$

Hence the maximum negative shear at any panel point of the left half of span, if n is the number of panels from the left end to the point, is

$$S = R_A + (n - 1)w_1 p - \Sigma_A^F P,$$

or

$$S = -\Sigma_A^F P + \Sigma_E^B R_1 + \Sigma_A^F R_1 - \frac{(N-1)u_t p}{2} + \frac{(n-1)P}{N}\left[\Sigma_E^B \phi_1 + \Sigma_A^F \phi_1\right] + (n-1)u_t p. \quad \text{(XII)}$$

From (IX) and (X) we can find the maximum chord stresses, and form (XI) and (XII) the maximum stresses in the bracing. If u and w are taken for the whole bridge, the stresses must be divided among the number of trusses.

EXAMPLE.—We are now able to find the stresses in a suspension system. In order to abridge the work of computation we take a short span. Our formulas do not hold strictly for short spans, but the method of procedure is the same as for a long span.

Data.—Let the span be $c = 120$ ft.; the versine of cable, $r = 15$ ft.; depth of truss, $h = 12$ ft.; panel length, $p = 12$ ft.; $N = 10$; cable of wire, coefficient of elasticity, $E_1 = 30{,}000{,}000$ lbs. per sq. in.; working unit stress, $\sigma_1 = 30{,}000$ lbs. per sq. in.

Suspenders and truss, $E_2 = E_3 = 30{,}000{,}000$ lbs. per sq. in.; working stress, $\sigma_2 = \sigma_3 = 10{,}000$ lbs. per sq. in.

Dead load of structure, $u = 2000$ lbs. per ft. of length; live load on structure, $w = 2000$ lbs. per ft. of length. Hence $P = 24{,}000$ lbs.

These data are taken for the sake of illustration and mainly to facilitate computation. References to formulas are by the numbers of the two preceding articles.

Calculation.—We first find Σl, or the sum of the length of the suspenders. From (I) we have, for the equation of the cable,

$$y = \frac{x^2}{240}.$$

Let the length of the suspender at the centre be 6 ft.; then the length of any other is

$$y = 6 + \frac{x^2}{240},$$

where x has the values 12, 24, 36, 48, both plus and minus. We have then

$$\Sigma l = 6 + 2(6.6 + 8.4 + 11.4 + 15.6) = 90 \text{ ft.}$$

We can now find ϕ or the fraction of the live load carried by the cable from (II), which becomes for this case

$$\phi = -1 + \sqrt{\frac{1}{1 - \frac{160}{369}}} = 0.33.$$

We see then that the cable carries the dead load and one third of the live load.

Taking $\epsilon = 0.00000686$ and $t = 80°$, we have from (III) for the temperature load

$$u_t = 97.79 \text{ lbs. per ft.}$$

From (IV) we have then for the area of cross-section of the cable

$$A_1 = 12.43 \text{ sq. inches.}$$

This area is to be divided among the number of cables.

For the area of the suspenders, we have from (V)

$$A_2 = 3.30 \text{ sq. inches.}$$

This area is to be divided among the sets of suspenders.

For the average cross-section of trusses we have, from (VI),

$$A_3 = 10 \text{ sq. inches.}$$

We have now

$$\frac{A_3}{A_2} = \frac{100}{33}, \quad \frac{A_3}{A_1} = \frac{1000}{1243}.$$

Inserting these values in (VII), we have

$$\phi_1 = \frac{n_1 n_2 (100 + n_1 n_2)}{4239}.$$

Let the panel live loads be P_1, P_2, P_3, etc., counting from the left end. Taking, then, $n_1 = 1, 2, 3$, etc., and $n_2 = 9, 8, 7$, etc., we obtain the following values for ϕ_1, the fraction of each panel load, carried by the cable:

TABLE I.—VALUES OF ϕ_1.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
ϕ_1	0.231	0.438	0.599	0.702	0.737	0.702	0.599	0.438	0.231

From (VIII) we have for the reaction R_1 at left end of truss for any panel live load

$$R_1 = \frac{n_2}{N}P - \phi_1 \frac{P}{2}.$$

The panel live load is $P = 2000 \times 12 = 24000$ lbs. We have then the following values of R_1 for each panel live load, taking ϕ_1 for that load from Table I.

TABLE II.—VALUES OF R_1.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
R_1	+ 18840	+ 13944	+ 9624	+ 5976	+ 3168	+ 1176	0	− 456	− 360

Positive values act up, negative values act down.

We see from Table II that the maximum positive reaction at the left end is for loads P_1 to P_8 inclusive, all acting at once—that is, *for live load covering two thirds of the span from the left end.* The maximum negative reaction is for loads P_9, P_{10}, P_{11} acting, or *for live load covering one third of the span from the right end.* Hence $z_1 = \frac{2}{3}c$, $z_2 = \frac{1}{3}c$. We have then from (IX) and Tables I and II, for the maximum negative moment at any point of the left half span distant n panels from the left end, taking u_t at 100 lbs. per ft.,

$$M = -685536n + 102110n(n-1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We have from (X), for the maximum positive moment at any point of the left half span distant n panels from the left end,

$$M = +74592n - 25459n(n-1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We have from (XI), for the maximum positive shear at any point of the left half span distant n panels from the left end,

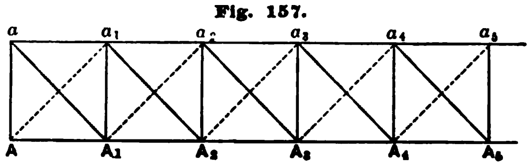
$$S = \sum_{n}^{n=6} R_1 + 5400 - 1200(n-1)\left(2\sum_{n}^{n=6}\phi_1 - 1\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We have from (XII), for the maximum negative shear at any point of the left half span distant n panels from the left end,

$$S = -nP - 6216 + \Sigma_0^n R_1 + 4243(n-1) + 2400(n-1)\Sigma_0^n \phi_1. \quad . \quad . \quad . \quad (4)$$

Stresses in Truss.—The maximum stress in a chord panel is equal to the maximum moment at its point of moments, divided by the depth, or equal to $\frac{M}{h}$. A negative moment (clockwise) causes compression in top chord and tension in bottom. A positive moment (counter clockwise) causes tension in top chord and compression in bottom.

Fig. 157.



We have then from (1), for the stresses in the chords as denoted by the notation of Fig. 157, by making $n = 1, 2, 3, 4, 5$

$$aa_1 = -\frac{685536}{12} = -57128 \text{ lbs.} \qquad A_1A_2 = +57128 \text{ lbs.}$$

$$a_1a_2 = \frac{-1371072 + 204220}{12} = -97238 \text{ lbs.} \qquad A_2A_3 = +97238 \text{ lbs.}$$

$$a_2a_3 = \frac{-2056608 + 612660}{12} = -120329 \text{ lbs.} \qquad A_3A_4 = +120329 \text{ lbs.}$$

$$a_3a_4 = \frac{-2742144 + 1225320}{12} = -126402 \text{ lbs.} \qquad A_4A_5 = +126402 \text{ lbs.}$$

$$a_4a_5 = \frac{-3427680 + 2042200}{12} = -115457 \text{ lbs.}$$

From (2) we have

$$a_1a_2 = \frac{+74592}{12} = +6216 \text{ lbs.} \qquad AA_1 = -6216 \text{ lbs.}$$

$$a_2a_3 = \frac{+149184 - 50918}{12} = +8188 \text{ lbs.} \qquad A_1A_2 = -8188 \text{ lbs.}$$

$$a_3a_4 = \frac{+223776 - 152754}{12} = +5918 \text{ lbs.} \qquad A_2A_3 = -5918 \text{ lbs.}$$

The angle of the braces with the vertical is $\theta = 45°$, and hence $\sec\theta = 1.414$. The stress in a brace is the shear $\times \sec\theta$. Hence from (3), making $n = 5, 4, 3, 2, 1$, we have

$n = 5$, $S = +4344 + 5400 - 4800 \times 1.878 = +730$ lbs.;
$A_4a_4 = -730$ lbs.; $A_3a_4 = +730 \times 1.414 = +1032$ lbs.

$n = 4$, $S = +10320 + 5400 - 3600 \times 3.282 = +3905$ lbs.;
$A_3a_3 = -3905$ lbs.; $A_4a_3 = +3905 \times 1.414 = +5522$ lbs.

$n = 3$, $S = +19944 + 5400 - 2400 \times 4.480 = +14592$ lbs.;
$A_2a_2 = -14592$ lbs.; $A_3a_2 = +14592 \times 1.414 = +20633$ lbs.

$n = 2$, $S = +33888 + 5400 - 1200 \times 5.356 = +32861$ lbs.;
$A_1a_1 = -32861$ lbs.; $A_2a_1 = +32861 \times 1.414 = +46465$ lbs.

$n = 1$, $S = +52728 + 5400 = +58128$ lbs.;
$Aa = -58128$ lbs.; $A_1a = +58128 \times 1.414 = +82193$ lbs.

From 4) we have, making $n = 4, 3, 2, 1, 0$,

$n = 4$, $S = -96000 - 6216 + 48384 + 12729 + 7200 \times 1.970 = -26919$ lbs.;
$A_4a_3 = -26919$ lbs.; $A_4a_3 = +26919 \times 1.414 = +38063$ lbs.

$n = 3$, $S = -72000 - 6216 + 42408 + 8486 + 4800 \times 1.268 = -21236$ lbs.;
$A_4a_4 = -21236$ lbs.; $A_3a_4 = +21236 \times 1.414 = +30028$ lbs.

$n = 2$, $S = -48000 - 6216 + 32784 + 4243 + 2400 \times 0.669 = -15584$ lbs.;
$A_3a_3 = -15584$ lbs.; $A_2a_3 = +15584 \times 1.414 = +22036$ lbs.

$n = 1$, $S = -24000 - 6216 + 18840 = -11376$ lbs.;
$A_2a_2 = -11376$ lbs.; $A_1a_2 = +11376 \times 1.414 = +16086$ lbs.

$n = 0$, $S = -6216 - 4243 = -10459$ lbs.;
$A_1a_1 = -10459$ lbs.; $Aa_1 = +10459 \times 1.414 = +14789$ lbs.

Taking the maximum values for the posts, we have the following maximum stresses, which are to be divided among the number of trusses:

Upper chord panels,

$$aa_1 = -57128 \text{ lbs.} \qquad a_1a_2 = \begin{cases} -97238 \text{ lbs.} \\ +6216 \text{ ''} \end{cases} \qquad a_2a_3 = \begin{cases} -120329 \text{ lbs.} \\ +8188 \text{ ''} \end{cases}$$

$$a_3a_4 = \begin{cases} -126402 \text{ lbs.} \\ +5918 \text{ ''} \end{cases} \qquad a_4a_5 = -115457 \text{ lbs.}$$

Lower chord panels,

$AA_1 = -6216$ lbs. $\quad A_1A_2 = \begin{cases} -8188 \text{ lbs.} \\ +57128 \text{ "} \end{cases} \quad A_2A_3 = \begin{cases} -5918 \text{ lbs.} \\ +97238 \text{ "} \end{cases}$

$A_3A_4 = +120329$ lbs. $\quad A_4A_5 = +126402.$

Braces:

$A_1a = +82{,}193$ lbs., $\quad A_2a_1 = +46{,}465$ lbs., $\quad A_3a_2 = +20{,}633$ lbs.,
$A_4a_3 = +\ 5{,}522$ lbs., $\quad A_5a_4 = +\ 1{,}032$ lbs.

Counters:

$Aa_1 = +14{,}789$ lbs., $\quad A_1a_2 = +16{,}086$ lbs., $\quad A_2a_3 = +22{,}036$ lbs.,
$A_3a_4 = +30{,}028$ lbs., $\quad A_4a_5 = +38{,}063$ lbs.

Posts:

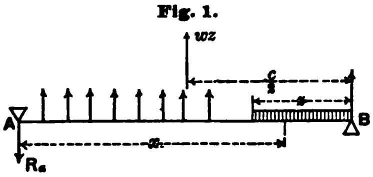
$Aa = -58{,}128$ lbs., $\quad A_1a_1 = -32{,}861$ lbs., $\quad A_2a_2 = -14{,}592$ lbs.,
$A_3a_3 = -15{,}584$ lbs., $\quad A_4a_4 = -21{,}236$ lbs., $\quad A_5a_5 = -26{,}919$ lbs.

Old Theory of Suspension System.—The theory of the suspension system given in the preceding is new, and is believed to be based upon correct principles and assumptions.

The theory of the suspension system heretofore in use is due to Rankine, and is based upon the assumption *that the cable carries the entire load, dead and live*, the office of the truss being simply to distribute a partial loading over the cable, and thus prevent change of shape.

Maximum Shear in Truss—Old Method.—Let the uniform live load w per unit of length extend over the distance z from the right end (see Fig. 1).

Fig. 1.



Then the load is wz, and since by the assumption of the old theory the cable carries all this load, the upward load of the cable is wz, or $\frac{wz}{c}$ per unit of length.

Let R_A be the reaction at the left end A.

Taking moments about the right end, we have

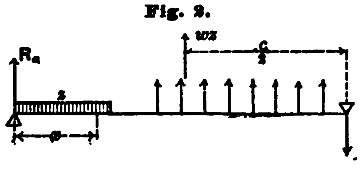
$$R_Ac = -wz \times \frac{c}{2} + wz \times \frac{z}{2},$$

or

$$R_A = -\frac{wz(c-z)}{2c}. \qquad (1)$$

Since this is negative, the truss should be tied down at the ends.

Fig. 2.



If the load w extends over the distance z from the left end (Fig. 2), we have

$$R_Ac = wz\left(c - \frac{z}{2}\right) - wz \times \frac{c}{2},$$

or

$$R_A = +\frac{wz(c-z)}{2c}. \qquad (2)$$

In the first case the shear at any point distant x from the left end (Fig. 1) is

$$S = R_A + \frac{wz}{c}x - w[x - (c - z)],$$

or, inserting the value of R_A from (1),

$$S = \frac{wz}{2c}[2x - (c - z)] - w[x - (c - z)].$$

This is a positive maximum when $z = c - x$, that is, the shear for the unloaded portion is a positive maximum at the head of the load.

It is a negative maximum when $z = \frac{c}{2} - x$. In both cases, then, the shear in general is

$$S = \frac{wz}{2c}[2x - (c - z)].$$

Inserting the values of $z = c - x$ and $z = \frac{c}{2} - x$, we have for any point of the unloaded portion distant x from the left end

$$\text{unloaded portion}\left\{\begin{aligned} \max(+S) &= +\frac{w(c - x)x}{2c}, \\ \max(-S) &= -\frac{w(c - 2x)^2}{8c}. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (3)$$

In the second case (Fig. 2) the shear at any point distant x from the left end is

$$S = R_A + \frac{wz}{c}x - wx;$$

or. inserting the value of R_A from (2),

$$S = + \frac{wz(c - z)}{2z} + \frac{wz}{c}x - wx.$$

This is a negative maximum for $z = x$; that is, the shear for the loaded portion is a negative maximum at the head of the load.

It is a positive maximum when $z = \frac{c}{2} + x$. In both cases, then, the value of S is as given. Inserting the values of $z = \frac{c}{2} + x$ and $z = x$, we have, for any point of the loaded portion distant x from the left end,

$$\text{loaded portion}\left\{\begin{aligned} \max(+S) &= +\frac{w(c - 2x)^2}{8c}, \\ \max(-S) &= -\frac{w(c - x)x}{2c}. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (4)$$

We see from equations (3) and (4), that we have for

$$x = 0, \qquad \frac{1}{4}c, \qquad \frac{1}{2}c,$$

$$S = \pm\frac{wc}{8}, \quad \pm\frac{3wc}{32}, \qquad \pm\frac{wc}{8}.$$

It is therefore customary by the old method *to design every brace for the maximum shears*

$$S = \pm\frac{wc}{8}, \quad \ldots \ldots \ldots \quad \text{(I)}$$

thus giving uniform sizes throughout the entire truss.

Maximum Moment in Truss—Old Method.—For the moment at any point of the unloaded portion (Fig. 1) distant x from the left end, we have

$$M = -R_A x - \frac{wzx^2}{2c};$$

or, substituting the value of R_A from (1),

$$M = -\frac{wzx}{2c}[x - (c - z)]. \quad \ldots \ldots \ldots \quad (5)$$

For any point of the loaded portion (Fig. 2) we have

$$M = -R_A x - \frac{wzx^2}{2c} + \frac{wx^2}{2},$$

or, substituting the value of R_A from (2),

$$M = -\frac{wzx}{2c}[x + (c - z)] + \frac{wx^2}{2}. \quad \ldots \ldots \ldots \quad (6)$$

In (5), $M = 0$ for $x = c - z$, and in (6), $M = 0$ for $x = z$. That is, the moment at the head of the load is zero. Also, if x is less than $c - z$ in (5) the moment is positive, and if greater than $c - z$ the moment is negative. *The head of the load is then a point of inflection, and* both loaded and unloaded portions may be considered as simple trusses uniformly loaded. The greatest moment for each portion will then always be at the centre of that portion. Making, then, $x = \frac{c - z}{2}$ in (5) and $x = \frac{z}{2}$ in (6), we have for the moment at the centre of each portion

$$+\frac{wz(c - z)^2}{8c}, \qquad -\frac{wz^2}{8c}(c - z).$$

These are a maximum, respectively, for $z = -\frac{1}{3}c$ and $z = \frac{2}{3}c$.

Hence, *the maximum positive moment is at the middle of the unloaded portion when the load extends over one third the span, and the maximum negative moment is at the middle of the load when it covers two thirds the span.*

We have, then, for the maximum positive moment at any point of the unloaded left half span, distant x from the left end,

$$M = \frac{w}{18}(2c - 3x)x. \quad \ldots \ldots \ldots \quad (7)$$

For the maximum negative moment at any point of the loaded left half span, distant x from the left end,

$$M = -\frac{w}{18}(2c - 3x)x. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

From equations (7) and (8) we have for

$$x = 0, \qquad \tfrac{1}{6}c, \qquad \tfrac{1}{3}c, \qquad \tfrac{1}{2}c,$$

$$M = 0, \qquad \pm\frac{wc^2}{72}, \qquad \pm\frac{wc^2}{54}, \qquad \pm\frac{wc^2}{72}.$$

It is therefore customary by the old method to *design every chord panel for the maximum moments*

$$M = \pm\frac{wc^2}{54}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

thus giving uniform sizes throughout the entire truss.

Cable.—If σ_2 is the unit stress for the truss, the chord area is then

$$A_2 = \frac{wc^2}{54h\sigma_2},$$

From (35), then, we have for the temperature load u_t, if E_2 is the coefficient of elasticity for the truss, h its depth, r the versine of cable, ϵ the coefficient of expansion, and t the number of degrees above or below the mean,

$$u_t = \frac{\epsilon t h w E_2}{60 r \sigma_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (III)$$

The total load of the cable is then

$$u + u_t + w;$$

and if σ_1 is the unit stress for the cable and A_1 its area of cross-section, we have for constant cross-section

$$A_1 = \frac{(u + u_t + w)(c^2 + 8r^2)}{8r\sigma_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV)$$

and for constant unit stress the area at centre

$$A_1 = \frac{(u + u_t + w)c^2}{8r\sigma_1},$$

and at any other point

$$A = A_1\left(1 + \frac{32r^2x^2}{c^4}\right) = \frac{(u + u_t + w)c^2}{8r\sigma_1}\left(1 + \frac{32r^2x^2}{c^4}\right). \quad . \quad . \quad . \quad . \quad (IV')$$

SUSPENDERS.—For the area of a suspender, if σ_2 is the unit stress,

$$A_2 = \frac{(u_t + u + w)p}{\sigma_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

where σ_2 is the unit stress for the suspenders.

Equations (I) to (V) in order are then all we need for the calculation of the suspension system by the old method. If u and w are taken for the whole bridge, the truss stresses must be divided among the number of trusses and the cable stresses among the number of cables.

COMPARISON OF OLD AND NEW METHODS.—The old theory of the suspension system just given is based upon the assumption *that the cable carries the entire load*, dead and live,

the office of the truss being simply to distribute a partial live load over the cable, and thus prevent change of shape.

This assumption cannot be practically realized. If the suspenders are properly adjusted during erection the cable can be made to take the entire dead load. But if a live load now comes on the cable cannot carry it all—the truss must carry its share. By the new method we determine the portion carried by the cable by the principle of least work. That is, the portions carried by cable and truss are such that the work upon cable, truss, and suspenders is a minimum. Thus in our example, page 233, we have seen that the cable carries only one third of the live load, whereas by the old method we should assume it as carrying the whole.

The old method evidently gives a heavier cable and much lighter truss than the new method.

A comparison of the example, page 233, will show how much in this case the difference amounts to. By the old method we have

$$u_t = 44 \text{ lbs. per ft.}$$

Hence $u + u_t + w = 4044$ lbs. per ft., and the area of the cable is

$$A_1 = 18.2 \text{ sq. inches,}$$

as against 12.4 sq. inches by the new method, or the cable is about 47 per cent heavier.

We have for the suspenders

$$A_2 = 4.85 \text{ sq. inches,}$$

as against 3.30 sq. inches by the new method, or the suspenders are about 47 per cent heavier.

Tne stress in every chord panel by the old method is

$$\pm \frac{wc^2}{54h} = \pm 44444 \text{ lbs.}$$

The stress in every post is

$$-\frac{wc}{8} = -30000 \text{ lbs.}$$

The stress in every brace and counter is

$$\pm \frac{wc}{8} \times 1.414 = \pm 42420 \text{ lbs.}$$

Comparing with the results of page 236, as given by the following Table, we see that

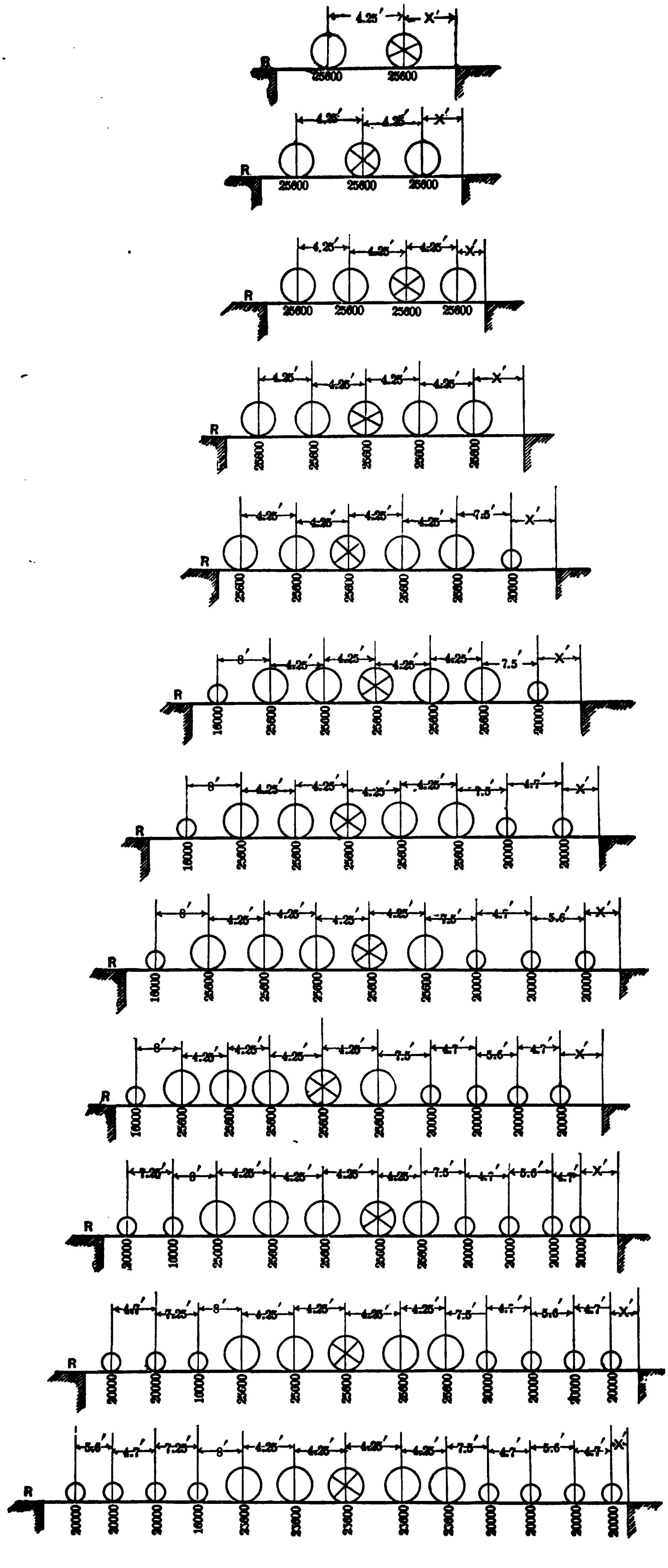
Member Top Chord.	New Method.	Old Method.	Member Brace.	New Method.	Old Method.
aa_1	-57128	∓ 44444	A_1a	$+82193$ -14789	± 42420
a_1a_2	-97238 $+6216$	∓ 44444	A_2a_1	$+46465$ -16086	± 42420
a_2a_3	-120329 $+8188$	∓ 44444	A_3a_2	$+20633$ -22036	± 42420
a_3a_4	-126402 $+5918$	∓ 44444	A_4a_3	$+5522$ -30028	± 42420
a_4a_5	-115457	∓ 44444	A_5a_4	$+1032$ -38063	± 42420

the chord stress by the old method is less than the least chord stress in one direction by over 22 per cent. and less than the greatest chord stress in this direction by over 61 per cent. In the other direction the chord stress by the old method is over five times as great as the greatest stress by the new method.

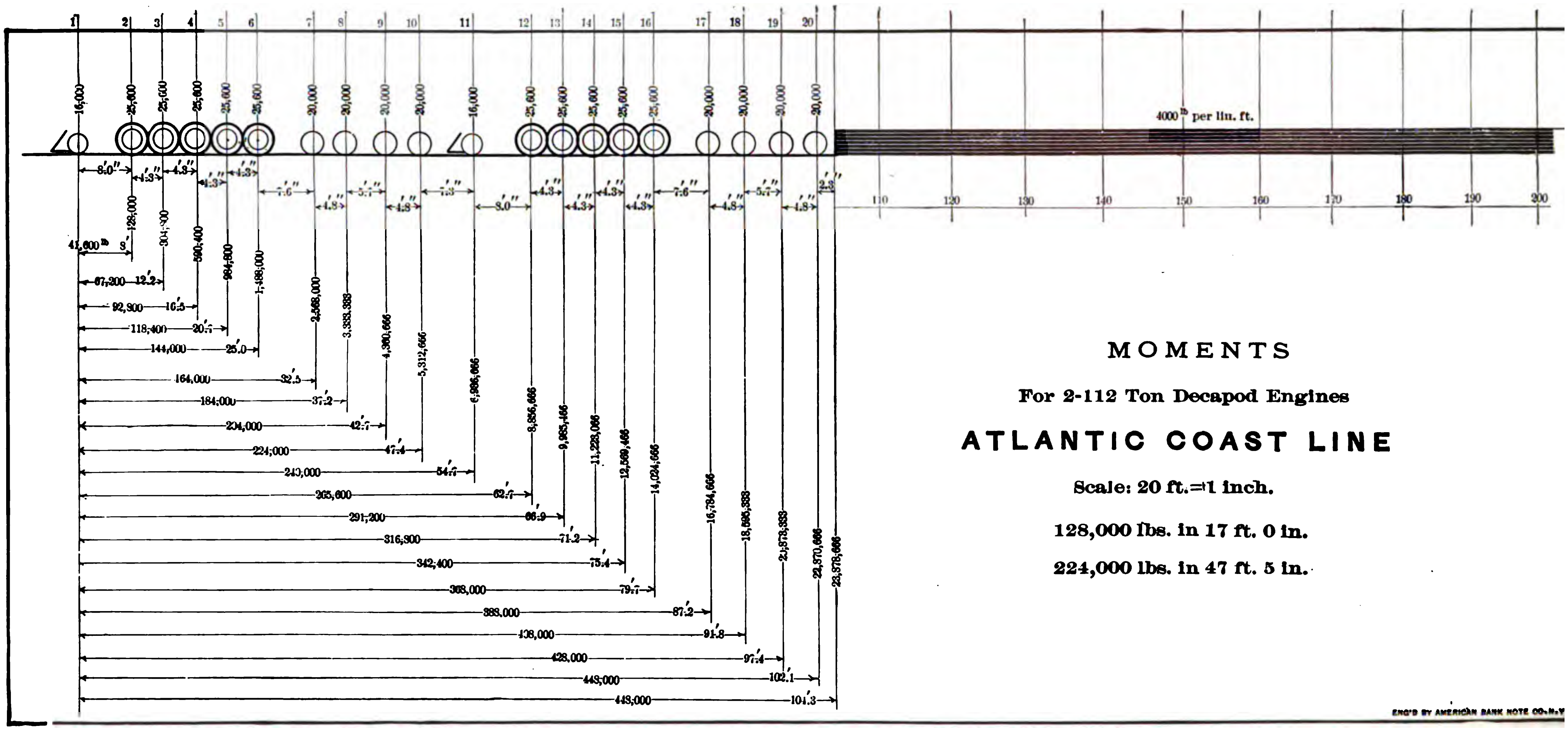
The stresses in end and centre chord panels are not reversed by the new method.

The chords are heavier by the new method, and the distribution of stress entirely different. The chords by the old method are all too light.

The braces are too light by the old method for the two end panels and too heavy for the three centre panels. The counters are all too heavy.



Span in feet.	Maximum Bending Moment per track.	Limits.	
8	55200	7.2′	$x = \frac{l}{2} - 1.0625$
9	67200	to	$R = 25600 + \frac{54400}{l}$
10	83200	9.4′	$M = 12800l - 54400 + \frac{57800}{l}$
11	102400		
12	121600		
13	140800	9.4′	$x = \frac{l}{2} - 4.25$
14	160000	to	$R = 38400$
15	179200	15.8′	$M = 19200l - 108800$
16	199200		
17	224400		
18	249600	15.8′	$x = \frac{l}{2} - 5.3125$
19	281600	to	$R = 51200 + \frac{108800}{l}$
20	313600	18.0′	$M = 25600l - 217600 + \frac{115600}{l}$
21	345600		
22	377600		
23	409600		
24	441600	18.0′	$x = \frac{l}{2} - 8.5'$
25	473600	to	$R = 64000$
26	505600		
27	537600	30.9′	$M = 32000l - 326400$
28	569600		
29	601600		
30	633600	30.9′	$x = \frac{l}{2} - 14.93$
31	666200	to	$R = 74000 - \frac{160900}{l}$
32	703000		
33	739800	34.2′	$M = 37000l - 486400 + \frac{172960}{l}$
34	776700		
35	816700		
36	857700	34.2′	$x = \frac{l}{2} - 15.83$
37	898700	to	$R = 82000 - \frac{28120}{l}$
38	939700		
39	980700	39.9′	$M = 41000l - 618400 + \frac{4780}{l}$
40	1022400		
41	1068200		
42	1114000	39.9′	$x = \frac{l}{2} - 19.4$
43	1159900	to	$R = 92000 - \frac{230800}{l}$
44	1205700		
45	1251600	45.9′	$M = 46000l - 825100 + \frac{299360}{l}$
46	1298000		
47	1349000	45.9′	$x = \frac{l}{2} - 21.688$
48	1399900	to	$R = 102000 - \frac{63686}{l}$
49	1450900		
50	1501900	51.5′	$M = 51000l - 1048470 + \frac{19870}{l}$
51	1552900		
52	1606200		
53	1662000	51.5	$x = \frac{l}{2} - 25.192$
54	1717900		
55	1773700	to	$R = 112000 - \frac{330343}{l}$
56	1829500		
57	1885400	57.6	$M = 56000l - 1315200 + \frac{487256}{l}$
58	1943000		
59	2004000	57.6	$x = \frac{l}{2} - 26.46$
60	2065000		
61	2126000	to	$R = 122000 - \frac{50240}{l}$
62	2187000		
63	2248000	63.7	$M = 61000l - 1595174 + \frac{10400}{l}$
64	2310400		
65	2376300		
66	2442200	63.7	$x = \frac{l}{2} - 29.838$
67	2508200	to	$R = 132000 - \frac{284567}{l}$
68	2574100	68.9	$M = 66000l - 1918445 + \frac{307050}{l}$
69	2640700		
70	2711700		
71	2782700	68.9	$x = \frac{l}{2} - 31.124$
72	2853700		
73	2924700	to	$R = 142000 + \frac{51800}{l}$
74	2995700		$M = 71000l - 2258447 + \frac{10719}{l}$
75	3066700		



MOMENTS

For 2-112 Ton Decapod Engines

ATLANTIC COAST LINE

Scale: 20 ft.=1 inch.

128,000 lbs. in 17 ft. 0 in.

224,000 lbs. in 47 ft. 5 in.

APPENDIX.

CHAPTER I.

CONCENTRATED LOAD SYSTEM.

We have already given (page 87) the general method of dealing with a system of concentrated loads, and explained the formation and method of use of our diagram.

The criterions given (pages 90 and 93) are for the special cases of horizontal chords and vertical and diagonal bracing. We can now deduce the general criterions.

General Criterion for Maximum Moment.—Let the maximum moment be required at the point O, Fig. (a), of the panel HK, whose length is p. Let b be the distance of any wheel from the right end of span, B, and k the distance of any wheel from the point K, these distances always to be taken without reference to sign or direction. Let l be the length of span AB, and e be the distance of the point O from H. The sum of all the wheels between A and H we denote by $\sum_H^A P = P_2$, and their moment with reference to H is M_2. The sum of all the wheels from H to K, in the panel HK, is $\sum_K^H P = P_1$, and their moment with reference to K is M_1. The sum of all the wheels on the span is $\sum_B^A P = P_n$. Let M_r be the moment at the right end of the span of all the loading on the span, including the uniform train load, if any, so that $M_r = \sum_B^A Pb + \frac{wy_n^2}{2}$.

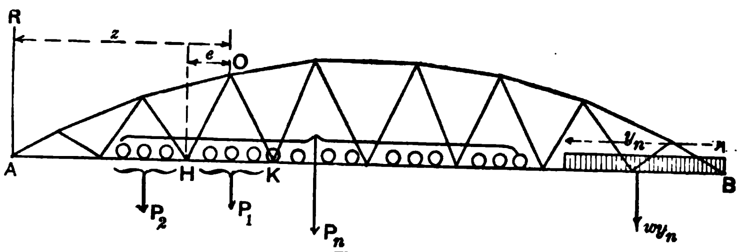


Fig. (a.)

Then the reaction R at the left end of the span is $\frac{1}{l}\sum_B^A Pb + \frac{wy_n^2}{2l}$, and the portion of the load in the panel HK, which takes effect at H, is $\frac{1}{p}\sum_K^H Pk$. Let the distance of the point O from the left end be z.

Then we have for the moment at O

$$M = -\frac{z}{l}M_r + M_2 + P_2 e + \frac{e}{p}M_1 = -\frac{z}{l}\sum_B^A Pb - \frac{z}{2l}wy_n^2 + \sum_H^A Po + \frac{e}{p}\sum_K^H Pk, \quad . \quad . \quad (1)$$

where o is the distance of any wheel from O.

If now the system is moved a very small distance, δx, to the left, we have

$$M + \delta M = -\frac{z}{l}\sum_B^A P(b+\delta x) - \frac{zw}{2l}(y_n + \delta x^2) + \sum_H^A P(o+\delta x) + \frac{e}{p}\sum_K^H P(k+\delta x).$$

Expanding $(y_n + \delta x)^2$ and neglecting higher powers of δx, subtracting (1) and dividing by δx we have

$$\frac{\delta M}{\delta x} = -\frac{z}{l}\sum_B^A P - \frac{z}{l}wy_n + \sum_H^A P + \frac{e}{p}\sum_K^H P = -\frac{z}{l}P_n - \frac{z}{l}wy_n + P_2 + \frac{e}{p}P_1.$$

The general criterion for maximum moment is then

$$\frac{P_n + wy_n}{l} \genfrac{}{}{0pt}{}{=}{>} \frac{1}{z}\left(P_2 + \frac{eP_1}{p}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This criterion is general whatever the bracing may be, and we see it is independent of the inclination of the chords. If the braces are equally inclined, $\frac{e}{p} = \frac{1}{2}$. If the braces are vertical and diagonal, $e = o$, and $P_2 = P_0$, or the load from A to H is the same as from A to O, and we have at once the special case of page 92. If we wish the moment at K we have $e = p$, and the criterion reduces to that of page 92.

Now $\frac{P_1}{p}$ is the average load in the panel, and $\frac{eP_1}{p}$ is therefore the load on the distance e.

We have, then, the moment in general a maximum, *when a wheel is at the point and when the average load upon the span is equal to or just greater than the average load beyond the point O.*

We can easily shift our diagram on the span, just as explained in the case of vertical and inclined braces (page 93) until this condition is satisfied, and find the maximum moment for this position from (1). We should try, as on page 93, for all maximums, and take the largest. In using (1) remember that M_r is the moment at the right end of the span of all the loads on the span, including the uniform train load, if any. M_2 is the moment at H_1 of all wheels on left of H; P_2 is the sum of these wheels, M_1 is the moment at K of all wheels in the panel, $HK = p$, Fig. (a).

An example in illustration will be given, page 249.

General Criterion for Maximum Shear.—When the chords are inclined they will take a portion of the shear, and only the residual shear (page 79) takes effect in the web.

On this account the position of the load system for maximum stress in any web member is not the same as for horizontal chords.

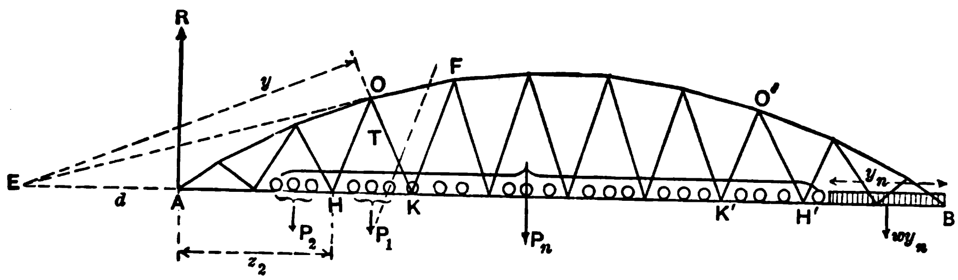


Fig. (b)

Let T, Fig. (b), be the stress in any web member OK, and y be its lever arm with reference to the intersection E of the chords OF and HK, and d the distance EA of this intersection from the end of the span A.

The rest of our notation is the same as before, except that z_2 is the distance AH from the left end to the left end of the panel $HK = p$, and a is the distance of any wheel from the left end A of the span.

Taking moments about E, we have

$$Ty = +\frac{d}{l}\sum_B^A Pb - \frac{d+z_2}{p}\sum_K^H Pk - \sum_H^A P(d+a). \quad \ldots \ldots \quad (1)$$

If the system is moved a very small distance, δx, to the left, b will be $b + \delta x$, k will be $k + \delta x$, and $d + a$ will be $d + a - \delta x$. Subtracting the value of Ty from its new value, we have

$$\frac{y\delta T}{\delta x} = +\frac{d}{l}\sum_B^A P - \frac{d+z_2}{p}\sum_K^H P + \sum_H^A P = +\frac{d}{l}P_n - \frac{d+z_2}{p}P_1 + P_2.$$

The criterion, then, for maximum stress in OK, or for maximum residual shear at O, if wy_n is the train load, is given by

$$\frac{wy_n + P_n}{l} \overset{=}{>} \frac{d+z_2}{d}\cdot\frac{P_1}{p} - \frac{P_2}{d}. \quad \ldots \ldots \ldots \quad (2)$$

This criterion enables us to find by trial the position of the load system for maximum resultant shear at O, just as on page 92. It is independent of the web system.

If the chords are horizontal, $d = d + z_2 = \infty$, and the criterion (2) becomes the same as already found, page 91.

Since $\sum_H^A P(d + a) = P_2(d + z_2) - M_2$, where M_2 is the moment of all the wheels between A and H, with reference to H, we have from (1)

$$T = \text{brace stress} = \mp \frac{1}{y}\left[\frac{M_r d}{l} - \frac{M_1(d + z_2)}{p} - P_2(d + z_2) + M_2\right], \quad . \quad . \quad . \quad (3)$$

where the minus sign denotes compression in OK, and the plus sign tension in OH, remembering that we must take y and d *for the brace required.* M_r is the moment at the right end, B, of the span of all loads on the span, including uniform train load if any; M_1 is the moment at K of all the wheels in the panel $HK = p$; and P_2 is the sum of all wheels between A and H, and M_2 their moment with reference to H.

In general, in all practical cases there will be no wheels between A and H, and P_2, M_2 will be zero.

For the shear at any point O, Fig. (b), we have

$$\text{Shear} = T \cos \theta = \frac{1}{d + p + z_2}\left[\frac{M_r d}{l} - \frac{M_1(d + z_2)}{p} - P_2(d + z_2) + M_2\right],$$

where θ is the angle of OK with the vertical, and d is taken for OK. A positive (upward) shear gives tension in OK. For horizontal chords, $d = d + z_2 = d + p + z_2 = \infty$, and the shear becomes the same as on page 91.

For the counter stress in OK, we can take the train coming on from the left, or, what is the same thing, we can find the stress in the corresponding brace $O'K'$ on the right of the centre for a train coming on from the right.

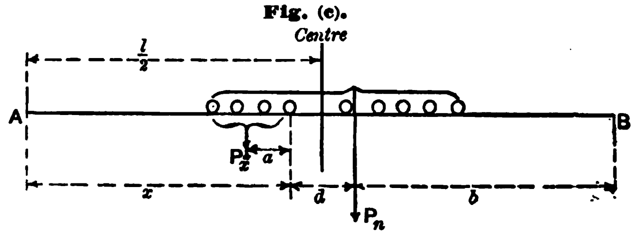
In the latter case, we should put $d + l$ in place of d, and remember that z_2 is now the distance of H' from the right end.

MAXIMUM MOMENT IN A PLATE GIRDER.—In designing plate girders and stringers, we need to know the position of the train which causes the greatest maximum moment at or near the centre of the span.

The maximum moment at any point always occurs when some wheel is at that point.

Let x, Fig. (c), be the distance of the point from the left end, let the sum of the wheel loads on left of this point be P_x, let a wheel be at the point, and let the distance of the centre of gravity of the wheel loads P_x from the wheel at the point be a. Let P_n be the sum of all the loads on the girder, and the distance of the resultant P_n from the right end be b.

Fig. (c).



Then the moment at the point is

$$M = -\frac{P_n b}{l}x + P_x a = -M_r\frac{x}{l} + M_x.$$

If the system moves a very little to the left, the distance a is unchanged, but x becomes $x - \delta x$, and b becomes $b + \delta x$.

We have by subtraction, neglecting δx^2,

$$\frac{\delta M}{\delta x} = \frac{P_n}{l}(b - x).$$

Hence for the maximum, $b = x$. That is, the moment is a maximum, *when the system is so placed that the wheel at the point, causing the maximum, is as far on one side of the centre of the span as the centre of gravity of the total load is on the other.*

If any uniform train load comes on the span, it must be included in the value of the total load P_n.

The distance of the centre of gravity of the total load from the centre of the span is then

$$\frac{d}{2} = \frac{l}{2} - x = \frac{l}{2} - \frac{M_r}{P_n},$$

where M_r is the moment of all the loads on the span with reference to the right end, and is easily found from our diagram.

We can, in general, find a maximum for each one of a number of wheels on the left of the centre. Judgment must be exercised, therefore, in selecting the wheels to test for, in order to determine the greatest maximum moment. In general, we should select that position which *brings the greatest load on the girder*, and at the same time brings the resultant of the total load *nearest the centre of the span.*

Guided by this, we can usually select not more than three wheels, one of which will give the greatest maximum moment, and can be found by trial.

EXAMPLE.—A PLATE GIRDER IS 60 FEET LONG. REQUIRED THE MAXIMUM MOMENT FOR THE SYSTEM OF LOADS OF OUR DIAGRAM.*

Mark the points 30 and 60 feet on a strip of paper to the scale of the diagram, and apply to the diagram as follows:

We see, by shifting the strip so that each weight is successively at the centre of the span, that we bring the greatest load on the span, and at the same time the resultant of the total load is nearest the centre of the span, either for p_{14}, p_{16}, or p_{18} at the centre. We have, therefore, only to test for these loads.

For p_{14} at the centre we have $y_n = 30 + 71.2 - 97.4 = 3.8$ feet; loads $p_1 - p_8$ are off the span, p_8 is distant $30 + 71.2 - 37.2 = 64$ feet from the right end. There is no uniform train load on the span; the total load is 428000 − 184000 = 244000 lbs., and $M_r = 20873333 + 428000 \times 3.8 - 3333333 - 184000 \times 64 = 7390400$ ft. lbs. Hence

$$\frac{d}{2} = \frac{l}{2} - \frac{M_r}{244000} = -0.3 \text{ ft.}$$

As this shows that the resultant of the total load is already left of the centre, we must, for a maximum, have p_{14} on right of centre. If we shift p_{14} a distance 0.15 ft. on right of centre, then, since during the shifting no wheels come on or go off, the resultant will be 0.15 ft. on left. This position then gives a maximum moment.

For this position, we have, $y_n = 3.65$, p_8 is 63.85 feet from right end, $M_r = 20873333 + 428000 \times 3.65 - 3333333 - 184000 \times 63.85 = 7353800$ ft. lbs. Hence the moment at p_{14} is

$$-\frac{M_r \times 30.15}{60} + (11223066 - 3333333 - 184000 \times 34) = -2061557 \text{ lbs.}$$

Let us try p_{16} at the centre. For this position $y_n = 30 + 75.4 - 104.3 = 1.1$ ft.; loads $p_1 - p_9$ are off the span; p_9 is distant $30 + 75.4 - 42.7 = 62.7$ feet from right end. The total load is $448000 - 204000 + 4000 \times 1.1 = 248400$ lbs.

$$M_r = 23878666 + 448000 \times 1.1 + \frac{4000(1.1)^2}{2} - 4360666 - 204000 \times 62.7 = 7222420 \text{ ft. lbs.,}$$

and

$$\frac{d}{2} = \frac{l}{2} - \frac{M_r}{248400} = 0.93 \text{ ft.}$$

If, now, we should shift p_{16} to the left of the centre, a distance of 0.465 ft., if no load passed off or came on during the shifting, we would have $\frac{l}{2} - x = \frac{d}{2}$. But as we shift the train load comes on, and this moves the resultant a little to the right. Let us therefore shift p_{16} a distance of, say, 0.6 ft. $= \frac{l}{2} - x$ to left of centre.

For this position $y_n = 30.6 + 75.4 - 104.3 = 1.7$ ft.; loads $p_1 - p_9$ are off; p_9 is distant $30.6 + 75.4 - 42.7 = 63.3$ from right end; total load = 250800 lbs.

$$M_r = 7372180, \quad \frac{d}{2} = \frac{l}{2} - \frac{M_r}{250800} = 0.61 = \frac{l}{2} - x.$$

This position, therefore, gives a maximum.

For this position, we have $M_x = 12569466 - 4360666 - 204000 \times 32.7 = 1538000$, and the moment at p_{16} is

$$M = -\frac{M_r \times 29.4}{60} + 1538000 = -2074368 \text{ ft. lbs.}$$

Let us try p_{18} at the centre. For this position $y_n = 30 + 79.7 - 104.3 = 5.4$ feet; wheels p_1 to p_{10} are off; p_{10} is distant $30 + 79.7 - 47.4 = 62.3$ feet from right end; the total load is $448000 - 224000 + 4000 \times 5.4 = 245600$;

$$M_r = 23878666 + 448000 \times 5.4 + \frac{4000 \times (5.4)^2}{2} - 5312666 - 224000 \times 62.3 = 7088320,$$

and

$$\frac{d}{2} = \frac{l}{2} - \frac{M_r}{245600} = 1.14 \text{ ft.}$$

* The table given at page 243 gives at once the position and maximum moment for the assumed load system for any length of span. This table is due to P. Q. Szlapka, C.E., and, as is evident, saves much time in computation. Similar tables for other load systems can easily be made out.

If we shift p_{16} a distance $\frac{l}{2} - x = 0.75$ on left of centre, we have $y_n = 6.15$, p_{16} distant 63.05 feet from right end; total load = 248600 lbs.

$M_r = 23878666 + 448000 \times 6.15 + \frac{4000\,(6.15)^2}{2} - 5312666 - 224000 \times 63.05 = 7273645$, $\frac{d}{2} = \frac{l}{2} - \frac{M_r}{248600} = 0.74$, or very nearly $= \frac{l}{2} - x$.

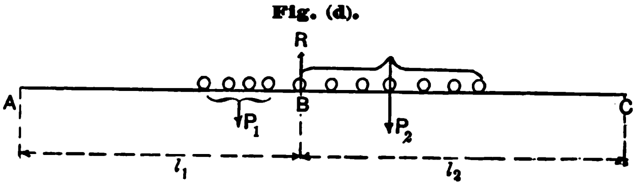
For the moment at this point, we have $M_x = 14024666 - 5312666 - 224000 \times 32.3 = 1476800$, and

$$M = -\frac{M_r \times 29.25}{60} + M_x = -2069102.$$

We see, therefore, that the greatest maximum is for p_{15}, a distance 0.6 on left of the centre, and is 2074368 ft. lbs. at this point.

Maximum Load on a Cross-girder.—Let AB, BC, Fig. (d), be two consecutive panels of length l_1 and l_2.

Fig. (d).



The greatest reaction R at B will occur when a wheel is at B. Let a be the distance of any wheel from A, and c the distance of any wheel from C.

Then the reaction at B is

$$R = \frac{1}{l_1}\sum_B^A Pa + \frac{1}{l_2}\sum_C^B Pc.$$

If the system is moved a very small distance δx to the left, we shall have $a - \delta x$ instead of a, and $c + \delta x$ instead of c, and hence by subtraction

$$\frac{\delta R}{\delta x} = \frac{1}{l_2}\sum_C^B P - \frac{1}{l_1}\sum_B^A P = \frac{P_2}{l_2} - \frac{P_1}{l_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Any wheel, therefore, at B, which, when moved just to the left of B, makes the value of (1) pass from positive to negative, gives a maximum R at B, and the criterion is,

$$\frac{P_2}{l_2} \overset{=}{>} \frac{P_1}{l_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

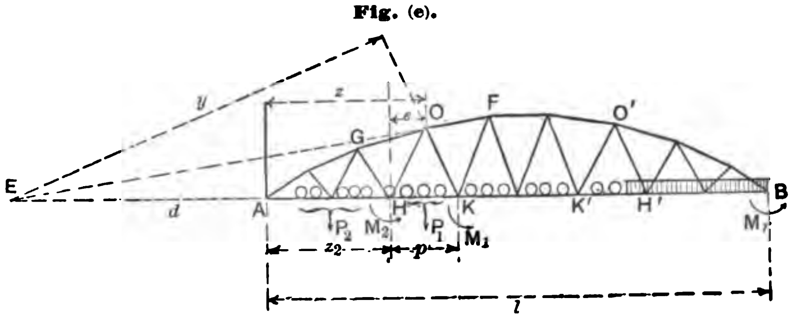
That is, R is a maximum *when a wheel is at B and when the average load in the right panel, including the wheel at B, is equal to or just greater than the average load in the left panel.*

When the panels are of equal length we have simply

$$P_2 \overset{=}{>} P_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Recapitulation.—In applying our diagram for concentrated load system, we have then two general criterions, one for shear, and one for moments.

Fig. (e).



Shear. — The general criterion for maximum shear is, Fig. (e),

$$\frac{P_n + wy_n}{l} \overset{=}{>} \frac{d + z_2}{d}\frac{P_1}{p} - \frac{P_2}{d} \quad . \quad . \quad . \quad . \quad (1)$$

and the maximum stress in any brace is

$$\text{Brace stress} = \mp \frac{1}{y}\left[M_r\frac{d}{l} - \frac{M_1(d + z_2)}{p} - P_2(d + z_2) + M_2\right] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where M_r is the moment at the right end of all loads in the span, including the uniform train load, if any. The minus sign indicates compression in OK, the plus sign tension in OH. We must take d and *y for the brace desired.* Since y for OK is $(d + z_2 + p) \cos \theta$ where θ is the angle of OK with the vertical, the shear at O, which causes the stress in OK, is,

$$\text{Shear} = \frac{1}{d + z_2 + p}\left[M_r\frac{d}{l} - \frac{M_1(d + z_2)}{p} - F_2(d + z_2) + M_2\right] \quad \ldots \ldots \quad (3)$$

For the shear at O which causes the stress in OH, we put $d + z_2$ in place of $d + z_2 + p$, and remember that d must be taken for the intersection of OG and HK.

In most cases, P_2 and M_2 will be zero. These equations are independent of the character of the bracing, and depend only upon the inclination of the chords.

For the counter $O'K'$, we put $d + l$ in place of d, and z_2 is the distance from the right end to H'.

When the chords are horizontal, $d = d + z_2 = d + z_2 + p = \infty$, and we have at once from (1), as on page 91.

$$\frac{P_n + wy_n}{l} \overset{=}{>} \frac{P_1}{p} \quad \ldots \ldots \ldots \ldots \quad (4)$$

and from (3),

$$S = \frac{M_r}{l} - \frac{M_1}{p} - P_2 \quad \ldots \ldots \ldots \ldots \quad (5)$$

In all practical cases, where the panel length is not very short, P_2 is zero.

Moments.—The general criterion for moments is, Fig. (e),

$$\frac{P_n + wy_n}{l} \overset{=}{>} \frac{1}{z}\left(P_2 + \frac{eP_1}{p}\right) \quad \ldots \ldots \ldots \quad (6)$$

and for the moment itself,

$$M = \frac{z}{l} M_r - (M_2 + P_2 e) - \frac{e}{p} M_1 \quad \ldots \ldots \ldots \quad (7)$$

These equations (5) and (6) are independent of the inclination of the chords, and depend upon the character of the bracing.

When the bracing is vertical and inclined, $e = o$, and we have, as found on page 92,

$$\frac{P_n + wy_n}{l} \overset{=}{>} \frac{P_2}{z} \quad \ldots \ldots \ldots \ldots \quad (8)$$

This holds in all cases, for a point in the *loaded* chord. We only need (5) and (6), therefore, for a point in the *unloaded* chord, *when the bracing is triangular.*

Also, when $e = o$, we have for the moment for vertical and inclined bracing, from (6), as on page 92,

$$M = \frac{z}{l} M_r - M_2 \quad \ldots \ldots \ldots \ldots \quad (9)$$

Plate girder.—For a plate girder, in order to find the maximum moment, the system must be so placed *that the wheel which causes the maximum is as far on one side of the centre of the span as the centre of gravity of the total load is on the other.*

Cross-girder.—Finally, for the maximum load on a cross-girder, we have, Fig. (d), page 219,

$$\frac{P_2}{l_2} \overset{=}{>} \frac{P_1}{l_1} \quad \ldots \ldots \ldots \ldots \quad (10)$$

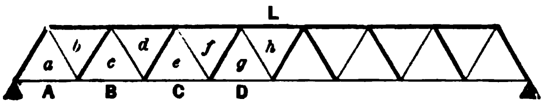
where P_2 is the load in the right panel, including the wheel at the end, P_1 is the load in the left panel, l_2 the length of the right, and l_1 the length of the left panel.

The preceding cases cover the entire theory of using our diagram, except for the case of the *cantilever*, which will be discussed at the end of this chapter. We now proceed to give illustrations of the application of the preceding criterions.

APPLICATION TO THE CASES OF CHAPTERS III. AND IV., SECTION II.*

WE can now give the solution of the cases of Chapters III. and IV., Section II., pages 103, 116, by means of a concentrated load system, as specified and explained here and on pages 90 *et seq.* This method, as we have said, is the present practice, and the student should be thoroughly familiar with it. He should prepare a diagram, to a scale of 20 feet to an inch, as directed on page 87 *et seq.*, and refer to it constantly in checking our results.

EXAMPLE 1. WARREN GIRDER.—Let us take the example of page 103, $l = 80$ feet, $d = 10$ feet = depth, $N = 8$, live load on bottom chord, single track, two trusses.



Dead Load Stresses.—Let the panel dead load be 11400 lbs., of which 9000 acts at the lower chord panel points, and 2400 at the upper.

The stresses due to dead load may be found by any of the methods of Chapter III., page 103. The method of moments is perhaps the most convenient. Let us adopt it.

We have, for the reaction, $R = 41100$ lbs. For the top chords, therefore,

$$Lb \times 10 = -41100 \times 10 + 2400 \times 5, \qquad Lb = -39900 \text{ lbs.}$$

$$Ld \times 10 = -41100 \times 20 + 2400(5 + 15) + 9000 \times 10, \qquad Ld = -68400 \text{ ''}$$

$$Lf \times 10 = -41100 \times 30 + 2400(5 + 15 + 25) + 9000(10 + 20), \qquad Lf = -85500 \text{ ''}$$

$$Lh \times 10 = -41100 \times 40 + 2400(5 + 15 + 25 + 35) + 9000(10 + 20 + 30), \quad Lh = -91200 \text{ ''}$$

For the bottom chords,

$$Aa \times 10 = +41100 \times 5, \qquad Aa = +20550 \text{ lbs.}$$

$$Bc \times 10 = +41100 \times 15 - 2400 \times 10 - 9000 \times 5, \qquad Bc = +54750 \text{ ''}$$

$$Ce \times 10 = +41100 \times 25 - 2400(10 + 20) - 9000(5 + 15), \qquad Ce = +77550 \text{ ''}$$

$$Dg \times 10 = +41100 \times 35 - 2400(10 + 20 - 30) + 9000(5 + 15 + 25), \qquad Dg = +88950 \text{ ''}$$

The sec $\theta = 1.117$, and we have for the bracing

$$La = -41100 \times 1.117 = -45909 \text{ lbs.}, \qquad ab = +(41100 - 2400)1.117 = +43228 \text{ lbs.}$$

$$bc = -(41100 - 11400)1.117 = -33175 \text{ lbs.}, \qquad cd = +(41100 - 13800)1.117 = +30494 \text{ ''}$$

$$de = -(41100 - 22800)1.117 = -20441 \text{ ''} \qquad ef = +(41100 - 25200)1.117 = +17760 \text{ ''}$$

$$fg = -(41100 - 34200)1.117 = -7707 \text{ ''} \qquad gh = +(41100 - 36600)1.117 = +5026 \text{ ''}$$

One half these results should be taken for each truss.

Live Load Stresses.—Having prepared a diagram according to the instructions on page 87 *et seq.*, the student should carefully check the following results:

It should be noted that for the braces we multiply the maximum shear, as given by our diagram, by 1.117. We should take *one half* the results as given for one truss, single track. If we had double track the results, as given, would be the correct stresses for one truss, without dividing. Our total results are as follows:

* The student should prepare a diagram like that given on page 243, to a scale of 20 feet to an inch, and have it constantly at hand while reading the following pages.

LIVE LOAD—BRACES.

	Total Load.	M_r.	M_1.	Max. Shear.	
$z = 10$, p_2 at point	345000	13629466	108800	159488	$La = -$ 178148 lbs. $ab = +$ 178148 "
$z = 20$, p_2 "	291200	10305786	128000	116022	$bc = -$ 129596 " $cd = +$ 129596 "
$z = 30$, p_2 "	240000	7728666	128000	83808	$de = -$ 93613 " $ef = +$ 93613 "
$z = 40$, p_2 "	224000	5447066	128000	55288	$fg = -$ 61757 " $gh = +$ 61757 "
$z = 50$, p_2 "	184000	3443733	128000	30246	$fg = +$ 33785 " $gh = -$ 33785 "

All these shears are greater than for uniform train load alone, page 105.

We see that fg and gh must be counterbraced for the difference $33785 - 7707 = 26078$ lbs.

For the unloaded chord we apply the diagram, as directed, page 93, and give our results for the student to check.

LIVE LOAD—TOP CHORD.

	Total Load.	M_r.	M_x.	Max. Moment.	
$z = 10$, p_2 at point	352000	13629466	108800	$-$ 159488	$Lb = -$ 159488 lbs.
$z = 20$, p_{14} "	332400	13293220	579200	$-$ 2744105	$Ld = -$ 274410 "
$z = 30$, p_{18} "	328400	12990420	1538000	$-$ 3333407	$Lf = -$ 333341 "
$z = 40$, p_{18} "	328400	12934286	2965866	$-$ 3501277	$Lh = -$ 350128 "

For the loaded chord we must find the position by the criterion given, page 243. In the present case $\frac{e}{p} = \frac{1}{2}$, and our criterion may be written

$$(P_n + wy_n) \gtreqless \frac{80}{z}\left(P_2 + \frac{P_1}{2}\right).$$

Let us try for the maximum moment at the first upper apex on left of centre, that is, the point of moments for Dg.

Since $z = 35$, $\frac{80}{z} = \frac{16}{7}$.

When p_{14} is at centre of span, we have $y_n = 6.9$, $P_n = 304000$, $P_2 = 96000$, $P_1 = 51200$. Hence $P_n + wy_n = 331600$, and $\frac{16}{7}\left(P_2 + \frac{P_1}{2}\right) = 277943$.

If p_{14} is moved just a little to right, the total load is unchanged, but P_1 becomes 76800. Hence

$$\frac{16}{7}\left(P_2 + \frac{P_1}{2}\right) = 307200.$$

We see that 331600 is greater than both these results, therefore we try for p_{15} at centre.

We have for this position $P_n + wy_n = 328400$, and for the two values of $\frac{16}{7}\left(P_2 + \frac{P_1}{2}\right)$, 290743 and 320000. As 328400 is greater than both these, we try for p_{16} at centre.

For this position $y_n = 15.4$, $P_n + wy_n = 325600$, and the two values of $\frac{16}{7}\left(P_2 + \frac{P_1}{2}\right)$ are

303543 and 332800. Since 325600 is less than the first and greater than the second, this position gives a maximum.

For this position $M_r + 12738853$, $P_2(x_2 + e) = 2011600$, $\frac{P_1}{2}x_2 = 163200$, hence

$$M = -\frac{35}{80}M_r + 2011600 + 163200 = -3398448.$$

In the same way we get the following results:

LIVE LOAD—LOWER CHORD.

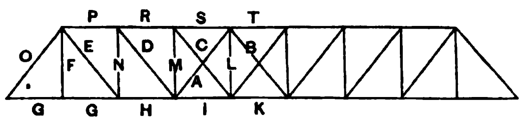
	M_r.	$M_2 + P_2e$.	$\frac{e}{p}M_1$.	Maximum Moment.	
$z = 5, p_{18}$ at 10 ft. from left end	14919666	0	54400	$-$ 878079	$Aa = +$ 87808 lbs.
$z = 15, p_{18}$ at 20 "	13441220	198400	163200	$-$ 2158628	$Bc = +$ 215863 "
$z = 25, p_{18}$ at 30 "	12800320	825600	163200	$-$ 3011300	$Ce = +$ 301130 "
$z = 35, p_{18}$ at 40 "	12738853	2011600	163200	$-$ 3398448	$Dg = +$ 339845 "

All these moments are greater than for uniform train load alone.

EXAMPLE 2. PRATT TRUSS.—As an illustration, let us take the Pratt Truss given in Plate 22, at the end of this work. .

Span $= 153$ feet $= l$, number of panels $N = 9$, panel length $p = 17$ feet, depth $= 26$ feet, $\tan\theta = 0.654$, $\sec\theta = 1.195$.

We adopt for the live load the system of our diagram, instead of that specified on Plate 22, and for dead load 1800 lbs. per foot, of which 500 is for the upper chord and 1300 is for lower chord.



Our dead load panel weights are, then, 8500 lbs. at each upper apex, and 22100 lbs. at each lower apex—total, 30600 lbs. This dead load is greater than that for which the truss was actually designed, but our live load is much larger than that assumed by the Bridge Company, and hence we should have heavier trusses.

Dead Load Stresses.—We give the results of calculation, according to the above data, in order that the student may check them. We shall adopt for the chords the method of coefficients, page 107, as requiring the least work.

We have, then, for the chords,

$$P = -\ 7 \times 30600 \times 0.654 = -\ 140086 \text{ lbs.} \qquad H = +\ 140086 \text{ lbs.}$$

$$R = -\ 9 \times 30600 \times 0.654 = -\ 180111 \text{ "} \qquad I = +\ 180111 \text{ "}$$

$$T = S = -\ 10 \times 30600 \times 0.654 = -\ 200124 \text{ "} \qquad K = +\ 200124 \text{ "}$$

$$G = +\ 4 \times 30600 \times 0.654 = +\ 80049 \text{ "}$$

For the web members

$$O = -4 \times 30600 \times 1.195 = -\ 146268 \text{ lbs.} \qquad E = +3 \times 30600 \times 1.195 = +109701 \text{ lbs.}$$

$$D = +2 \times 30600 \times 1.195 = +\ 73134 \text{ "} \qquad C = + \ 30600 \times 1.195 = +\ 36567 \text{ "}$$

$$F = + \ 22100 \text{ "} \qquad N = -\ 2 \times 30600 + 8500 = -\ 69700 \text{ "}$$

$$M = - \ 30600 - 8500 = -\ 39100 \text{ "} \qquad L = - \ 8500$$

$$A = B = 0.$$

Live Load Stresses.—Applying our diagram, we have the following results:

	M_r	M_1	Shear		
p_4 at $z =$ 17 ft.	50118746	590400	292843	$O = -$ 349947 lbs.	
p_3 at $z =$ 34 ft.	37377086	304800	226364	$E = +$ 270505 "	
p_3 at $z =$ 51 ft.	28509886	304800	168409	$D = +$ 201249 "	$N = -$ 168409 lbs.
p_3 at $z =$ 68 ft.	20798533	304800	118008	$C = +$ 141019 "	$M = -$ 118008 "
p_2 at $z =$ 85 ft.	12774906	12800	75966	$B = +$ 90779 "	$L = -$ 75966 "
p_2 at $z =$ 102 ft.	7968666	12800	44552	$C = -$ 53239 "	$F = +$ 91247 "

Since the dead load stress in C is $-$ 36567, the counter A is strained

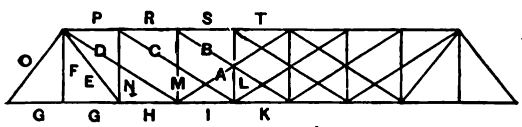
$+ 53239 - 36567 = + 16672$ lbs. Half these values for one truss.

For the chords, we have

	M_r	M_2	M		
p_4 at $z =$ 17 ft.	50118746	590400	$-$ 4978460	$G = +$ 191479 lbs.	
p_{15} at $z =$ 34 ft.	48221353	2206333	$-$ 8509514	$H = +$ 327289 "	$P = -$ 327289 lbs.
p_{15} at $z =$ 51 ft.	47776606	5108186	$-$ 10817349	$I = +$ 416051 "	$R = -$ 416051 "
p_{14} at $z =$ 68 ft.	50017886	10083866	$-$ 12146305	$K = +$ 467165 "	$S = T = -$ 467165 "

Half of these values for a single truss, if there are two trussses.

EXAMPLE 3. DOUBLE INTERSECTION PRATT TRUSS.—Let us take the same span as before, $l = 153$ feet, $N = 9$, $p = 17$ feet, depth $= 26$ feet. For O and E $\tan\theta = 0.654$, $\sec\theta = 1.195$. For the other ties, $\tan\theta = 0.765$, $\sec\theta = 1.7$. Let us take the same dead load as before, *viz.*, 8500 lbs. at each upper apex, and 22100 lbs. at each lower apex. Total, 30600 lbs.



Dead Load Stresses.—We must use for dead load the method of coefficients, page 122. We have for the chords,

$$P = -6 \times 30600 \times 0.654 - 30600 \times 0.765 = -143483 \text{ lbs.} \qquad I = +143483 \text{ lbs.}$$

$$R = S = T = -6 \times 30600 \times 0.654 - 2 \times 30600 \times 0.765 = -166892 \text{ lbs.} \quad K = +166892 \text{ "}$$

$$G = +4 \times 30600 \times 0.654 = +80049 \text{ lbs.}$$

$$H = +6 \times 30600 \times 0.654 = +120074 \text{ lbs.}$$

For the web members,

$A = B = 0$, $\quad F = + 22100$ lbs., $\quad E = + 2 \times 30600 \times 1.195 = + 73134$ lbs.

$O = - 4 \times 30600 \times 1.195 = - 146268$ lbs., $\quad D = + 30600 \times 1.7 = + 52080$ lbs. $= C$.

$L = M = - 8500$ lbs., $\quad N = - 30600 - 8500 = - 39100$ lbs.

Live Load Stresses.—For all multiple systems the stresses are indeterminate, as it is impossible to say how much in practice each system will take. For this reason such systems are usually avoided.

The accurate method of finding the stresses for live load, for any panel or brace, would be to find by diagram the position of the system which gives the maximum moment or shear, and then for this position find the actual loads which take effect at each apex, and find the stress for this loading.

As this is exceedingly tedious, and the indeterminate character of the stresses in practice renders such accuracy delusive, we adopt the following method, as being simpler and sufficiently accurate:

For the Braces.—Find the maximum shear for any brace by our diagram, as usual. Then find that uniform load which would give the same shear at the same point. Divide this load into apex loads, and calculate the brace for this loading.

If w is the uniform moving load per foot, coming on from the right and reaching to a distance z from the left end, then the shear due to this load is $\frac{w\,(l-z)^2}{2l}$. We may take the distance z as extending to the middle of the panel in front of the point.

If the maximum shear determined by diagram is S, then we have for w,

$$w = \frac{2lS}{(l-z)^2}.$$

If p is the panel length, we have the apex load

$$P = \frac{2plS}{(l-z)^2}.$$

Taking this apex load at each apex from right end up to the brace, we find the stress in the brace for this loading.

In the present case we have the following values:

		S	z	P		
p_4 at	17 feet from left,	292843 lbs.	8.5 feet.	72957 lbs.	$O = -348734$ lbs.	
p_3 "	34 " "	226364 "	25.5 "	72436 "	$E = +153886$ "	
p_3 "	51 " "	168409 "	42.5 "	71748 "	$D = +162631$ "	
p_3 "	68 " "	118008 "	59.5 "	70219 "	$C = +119272$ "	$N = -70219$ lbs.
p_2 "	85 " "	75966 "	76.5 "	65816 "	$B = +74591$ "	$M = -43877$ "
p_2 "	102 " "	44552 "	93.5 "	65464 "	$A = +49461$ "	$L = -29095$ "
						$F = +89153$ "

Thus, for O we have eight panel loads of 72957 lbs., and hence

$$R = 291828, \quad \text{and} \quad O = -291828 \times 1.195 = -348734 \text{ lbs.}$$

For E we have the panel load 72436 lbs., and since four of these loads are on the system to which E belongs, we have

$$R = \left(\frac{7}{9} + \frac{5}{9} + \frac{3}{9} + \frac{1}{9}\right) 72436 = 128775, \quad \text{and} \quad E = -128775 \times 1.195 = -153886 \text{ lbs.}$$

For C we have the panel load 70219 lbs., and since three of these loads act on the system to which C belongs,

$$R = \left(\frac{5}{9} + \frac{3}{9} + \frac{1}{9}\right) 70219 = 70219, \quad \text{and} \quad C = -70219 \times 1.7 = -119272 \text{ lbs.}$$

The shear for C is the compression for N.

For the Chords.—We find by our diagram the maximum moment for any panel, at the nearest

centre of moments. Then find what uniform load over the whole girder would give the same moment at this point. Divide this load into apex loads, and find the **stress** in the panel for this loading, by coefficients, in the usual manner. Each panel is thus found for its **own** equivalent uniform load.

If we denote the uniform load per foot by u, then, if z is the distance from the left end to any centre of moments, the moment at this point is $\frac{uz}{2}(l-z)$. If we denote the maximum moment as found by diagram by M, we have the equivalent uniform load $u = \frac{2M}{z(l-z)}$. If p is the panel length, the apex load is

$$P = \frac{2pM}{z(l-z)}.$$

In the present case we have the following values:

	M.	z	P		
p_4 at 17 feet from left,	-4978460	17	73212 lbs.	$G = +191522$ lbs.	
p_{10} " 34 " "	-8509514	34	71508 "	$H = +280597$ "	
p_{16} " 51 " "	-10817349	51	70701 "	$I = +331517$ "	$P = -331517$ lbs.
p_{14} " 68 " "	-12146305	68	71449 "	$K = +389682$ "	$R = -389682$ "

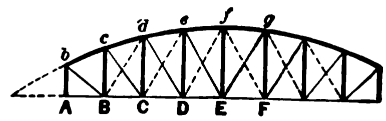
$$R = S = T = -389682 \text{ lbs.}$$

Half of these values to be taken for one truss. The same method applies to lattice truss of three or more systems, or to the Post Truss.

In view of what has preceded, there should now be no difficulty in finding the stresses for concentrated load system for any of the trusses with horizontal chords given in Chapter IV., page 116. We shall not, therefore, give here examples of the Baltimore or of the Kellogg and Fink Trusses. The latter are entirely antiquated, and no more built. For long spans, instead of a double-system Pratt, some modification of the Baltimore Truss is used, generally with inclined chords.

EXAMPLE 4. INCLINED CHORDS.—Let us take as an example of inclined chords the case already given, page 136.

Here the span $l = 120$ feet, number of panels $N = 8$, panel length $p = 15$ feet, bracing vertical and inclined. Height, 20 feet at centre, 10 feet at ends, apices of upper chord in a parabola. We have already found for this case the lever arms of the various members, page 138.



Lower chord,	AB	BC	CD		DE	
lever arms,	10	14.375	17.5		19.375 feet.	
Upper chord,	bc	cd	de		ef	
lever arms,	13.8	17.13	19.22		19.98 feet.	
Inclined braces,	bB	cC	dD		eE	fD
lever arms,	27.33	58.33	117.69		379.53	372 feet.
Vertical braces,	bA	cB	dC	eD	fE	eC
lever arms,	34.285	49.285	84	155	480	110.6 feet.

For the distance d, on the left of A, at which the upper panels intersect the lower chord, we have,

panel,	bc	cd	de	ef
	$d = 34.285$	**54**	**110**	**420 feet.**

Let the dead load be 1150 lbs. per foot on lower chord, and 350 lbs. per foot on upper chord, or 17250 lbs. at each lower apex, and 5250 lbs. at each upper apex ; total, 22500 lbs.

Dead Load Stresses.—Making use of our lever arms, and the method of moments, we have $R = 78750$ lbs.

$AB = 0.$

$BC \times 14.375 = +78750 \times 15$	$BC = +\ 82173$ lbs.
$CD \times 17.5 = +78750 \times 30 - 22500 \times 15,$	$CD = +115714$ "
$DE \times 19.375 = +78750 \times 45 - 22500(15+30),$	$DE = +130645$ "

$bc \times 13.8 = -78750 \times 15,$	$bc = -\ 85600$ lbs.
$cd \times 17.13 = -78750 \times 30 + 22500 \times 15,$	$cd = -118797$ "
$de \times 19.22 = -78750 \times 45 + 22500(15+30),$	$de = -131698$ "
$ef \times 19.98 = -78750 \times 60 + 22500(15+30+45),$	$ef = -135135$ "

$bA = -\ 78750$ lbs.

$bB \times 27.33 = +78750 \times 34.285,$	$bB = +98790$ lbs.
$cC \times 58.33 = +78750 \times 54 - 22500 \times 69,$	$cC = +46288$ "
$dD \times 117.69 = +78750 \times 110 - 22500(125+140),$	$dD = +22941$ "
$eE \times 379.53 = +78750 \times 420 - 22500(435+450+465)$	$eE = +\ 7114$ "

$cB \times 49.285 = -78750 \times 34.285 + 17250 \times 49.285,$	$cB = -\ 37532$ lbs.
$dC \times 84 = -78750 \times 54 + 22500 \times 69 - 17250 \times 84,$	$dC = -\ 14893$ "
$eD \times 155 = -78750 \times 110 + 22500(125+140) - 17250 \times 155,$	$eD = -\ 170$ "
$fE \times 480 = -78750 \times 420 + 22500(435+450+465) - 17250 \times 480 - eE \times 379.53,$	$fE = +\ 6000$ "

Half these results to be taken for a single truss if there are two trusses.

Live Load Stresses.—For the criterion giving the position of load for maximum shear we have, in this case, page 243, since P_2 will be found to be zero, and $l = Np$,

$$\frac{P_n + wy_n}{N} \overset{=}{>} P_1 \frac{d+np}{d},$$

and for the maximum stress in a brace,

$$\text{Brace stress} = \frac{1}{y}\left[\frac{M_r d}{l} - \frac{M_1}{p}(d+z_1)\right],$$

where y is the lever arm for the brace, and d is taken for the brace in question.

For a counter, we find the stress in the corresponding brace on the other side of the centre and take $d + l$ in place of d, and z_2 the distance from the right end to the panel.

We have in the present case the following results:

	M_r	M_1	d	y	z_2	
p_4 at 15 feet from left,	30231280	326400	34.285	27.33	0	$bB = +$ 288740 lbs.
			34.285	34.285	0	$bA = -$ 230166 "
p_2 " 30 " "	21130133	128000	54	58.33	15	$cC = +$ 152920 "
			34.285	49.285	15	$cB = -$ 113960 "
p_2 " 45 " "	15206066	128000	110	117.69	30	$dD = +$ 108286 "
			54	84	30	$dC = -$ 72928 "
p_2 " 60 " "	10305786	128000	420	379.53	45	$eE = +$ 84610 "
			110	155	45	$eD = -$ 52415 "
p_2 " 75 " "	6567066	128000	420	372	45	$fD = +$ 78373 "
p_2 " 90 " "	3480533	128000	110	110.6	30	$eC = +$ 59236 "

We see that the counter stresses for $fD = 78373$ lbs., and for $eC = -59236$ lbs.

For the criterion giving the position of load for maximum moment, we have in this case for both chords

$$\frac{z}{l}(P_n + wy_n) \gtreqless P_m,$$

and for the moment

$$M = \frac{M_r z}{l} - M_m.$$

We find the moments, therefore, precisely as so often illustrated in preceding examples. This moment must be divided by the lever arm of the panel to get the stress

We shall leave the results to be found by the student. If we had inclined bracing we should use the above criterion and formula for M for the unloaded chord only, and for the loaded chord should have the criterion

$$\frac{z}{l}(P_n + wy_n) \gtreqless P_2 + \frac{P_1}{p}e.$$

$$M = \frac{M_r z}{l} - M_2 + P_2 e - \frac{e}{p}M_1.$$

APPLICATION TO SKEW SPAN.—There are two cases of skew spans: 1st, where the end floor beams are supported by both trusses; 2d, where the end floor beams rest on the masonry, or are attached at one end to the end foot of one truss. The skew in no way affects the conditions for finding the positions of the system for maximum shear and moment, but in finding these positions the loading *is to be taken along the centre line.* Since the panels on the centre line are not necessarily equal, we cannot put in general $Np = l$, and hence the condition for maximum shear is to be written when $P_2 =$ zero,

$$\frac{(P_n + wy_n)p}{l} \gtreqless P_1, \text{ or } \gtreqless \frac{d + z_2}{d} P_1,$$

according as the chords are horizontal or inclined, where p is the *length of the panel on the centre line*, for which the shear is required, and l is the length of centre line or span.

The condition for maximum moment for the chords is

$$\frac{(P_n + wy_n)z}{l} \gtreqless P_m, \text{ or } \gtreqless P_2 + \frac{P_1}{p}e,$$

according as the bracing is vertical and inclined, or all inclined, page 219, where we take in like manner $z =$ the distance from *left end of centre line* to the point in question, *on centre line.*

The positions being thus determined, the shear and moment must be found for each truss as follows:

CASE 1. *When the end floor beams are supported by both trusses.*—For the moment at the right end of Truss A, Fig. (f), when the position of the load system on the centre line has been found as directed, we must take the uniform train load wy_n as concentrated at the centre of y_n, and then find its moment with reference to end of Truss A. Its lever arm is, therefore, $y_n - \frac{s}{2} - \frac{y_n}{2} = \frac{y_n - s}{2}$. Hence, for moment at right end of Truss A, we have, if we denote the skew by s,

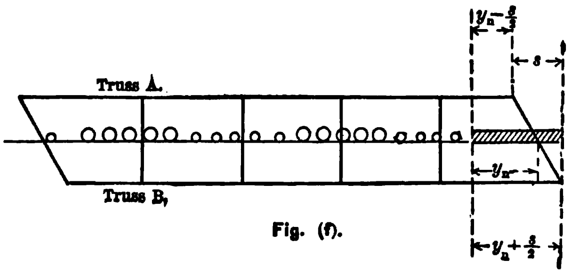


Fig. (f).

$$M_r = M_n + P_n\left(y_n - \frac{s}{2}\right) + wy_n\left(\frac{y_n - s}{2}\right),$$

and in like manner, for Truss B,

$$M_r = M_n + P_n\left(y_n + \frac{s}{2}\right) + wy_n\left(\frac{y_n + s}{2}\right).$$

The shear is now given by

$$S = \frac{M_r}{l} - \frac{M_1}{p}\text{; or generally, when } P_1 \text{ is zero, } \frac{1}{d + z_1 + p}\left[\frac{M_r d}{l} - \frac{M_1(d + z_1)}{p}\right],$$

and the moment by

$$M = -\frac{M_r}{l}z + M_z\text{, or generally } -\frac{M_r}{l}z + \sum_H^A Pc + \frac{e}{p}M_1,$$

where p is the actual panel length of *truss itself*, and z is the distance *on the truss* from left end to point in question.

It must be distinctly remembered, that while $\frac{z}{l}$ is a ratio on the centre line in getting the *position* for maximum moment, it is a ratio *on the truss*, in getting the moment itself. Also, that $\frac{p}{l}$ is always a ratio on the centre line.

CASE 2. *When the end floor beam rests on the masonry, or is attached to the end foot of one truss.*—In this case all loads between a and the right end come directly upon the masonry at b, and have no effect whatever upon Truss A.

We have, therefore, for Truss A,

$$M_r = M_n + P_n\left(y_n - \frac{s}{2}\right) + \frac{w}{2}\left(y_n - \frac{s}{2}\right)^2,$$

while we have for Truss B

$$M_r = M_n + P_n\left(y_n + \frac{s}{2}\right) + wy_n\left(\frac{y_n + s}{2}\right).$$

We see at once that Truss B is exactly the same as in Case 1. But Truss A is the same as a square span of length cd so far as shear and moments are concerned, but in determining *positions* its length is ab, and we simply consider all loads on centre line, between a and right end, as non-existing.

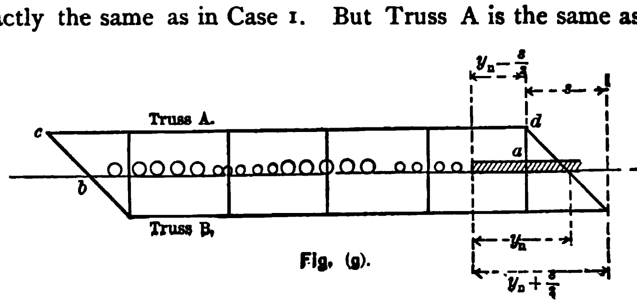


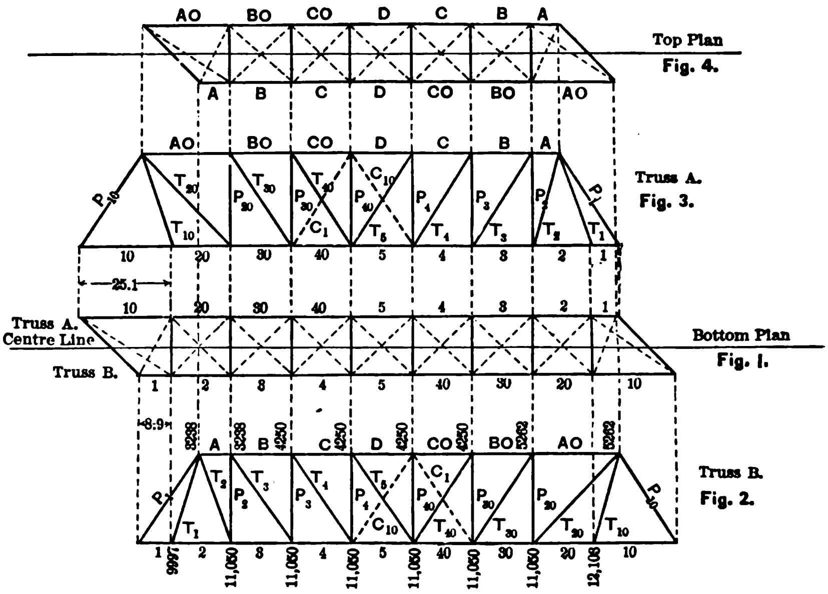
Fig. (g).

Truss B we treat exactly as in Case 1. Since the forward end of Truss A is the same as the rear end of Truss B, we have only to compute one end of each truss.

In case the skew is just one panel length, the stresses at each end of Truss B will be equal for both chords and web.

EXAMPLE 5. SKEW SPAN.—Let us take the span at 153 feet, with 9 panels of 17 feet each on the centre line; two Pratt Trusses, depth 26 feet, width between trusses 16.25 feet. Skew $= 45°$ right end forward, or $s = 16.25$ feet.

In Fig. 1 we have represented the bottom plan with the wind bracing. In Fig. 2 we have the elevation of Truss B, and in Fig. 3 the elevation of Truss A. It will be noted that the end posts of both trusses have the same inclination. In Fig. 4 we have the top plan with wind bracing. At the right end of Truss B and left end of Truss A the post at the end of T_{10} is omitted. This is done to save material. The post at the foot of T_2 is, however, retained in Truss B and A. This is necessary in order that the top wind bracing may be as shown in Fig. 4.



The student should study carefully these different Figs., and observe the notation adopted, members on one side of the centre being denoted by letters and numbers, and on the other side by the same letters and numbers, with o annexed. All posts or struts are denoted by P, all tension braces by T, counters by C, lower flanges by numbers, upper flanges by letters.

Thus B is second upper panel from left end of Truss B, and Bo, the corresponding panel on right end. Truss A is the same as Truss B turned round. The stresses in the left half of Truss A are the same as for the right half of Truss B.

Dead Load Stresses.—Let us take the track at 400 lbs. per lineal foot, and the cross girders, stringers, floor, etc., at 380 lbs. per lineal foot. Then we have $\frac{780}{2} = 390$ lbs. per lineal foot for each truss, *applied along the centre line.*

This gives $17 \times 390 = 6630$ lbs. at every lower apex of Truss B, if the bridge is a through span.

Suppose the weight of the trusses themselves and the wind bracing is 1020 lbs. per lineal foot. This gives 510 lbs. per lineal foot for each truss, of which we assume 250 lbs. for the upper chord and 260 lbs. for the lower chord, *applied along the truss itself.*

We have, then, at the first and second upper apices $(8.5 + 4.45)\ 250 = 3238$ lbs. At the last and next to last upper apex $(8.5 + 12.55)\ 250 = 5262$ lbs. At all the other upper apices $17 \times 250 = 4250$ lbs.

At the first lower apex, we have $(8.5 + 4.45)\ 260 = 3367$ lbs., at the last lower apex $(8.5 + 12.55)\ 260 = 5473$ lbs., at all the other lower apices, $17 \times 260 = 4420$ lbs.

Taking both these loadings, we have the apex loads given in Fig. 2, Truss B, and the dead load stresses are the stresses due to these loads.

For T_1 and T_{10} we have $\tan\theta = 0.312$, $\sec\theta = 1.048$. For T_2 we have $\tan\theta = 0.342$, $\sec\theta = 1.057$. For T_{20}, we have $\tan\theta = 0.965$, $\sec\theta = 1.39$. For all other inclined members $\tan\theta = 0.654$, $\sec\theta = 1.195$.

The left reaction for Truss B is given by

$$R \times 153 = 9997 \times 144.1 + 11050\,(127.1 + 110.1 + 93.1 + 76.1 + 59.1 + 42.1) + 12103 \times 25.1 + 3238\,(136 + 127.1) + 4250\,(110.1 + 93.1 + 76.1 + 59.1) + 5262\,(42.1 + 17).$$

Hence $R = 65062$ lbs. The right-hand reaction is 57338 lbs.

We have therefore, for the dead load stresses,

$P_3 = -65062 \times 1.195 = -77749$ lbs.

$T_1 = +\ 9997 \times 1.048 = +10477$ lbs.

$T_2 = +51827 \times 1.057 = +54781$ "

$T_3 = +37539 \times 1.195 = +44859$ "

$T_4 = +22239 \times 1.195 = +26575$ "

$P_2 = -37539 + 3238 = -40777$ "

$P_5 = -22239 + 4250 = -26489$ "

$T_6 = +\ 6939 \times 1.195 = +\ 8292$ "

$P_4 = -\ 6939 - 4250 = -11189$ "

$P_{40} = -\ 4250$

$T_{40} = +\ 8361 \times 1.195 = +\ 9991$ "

$P_{30} = -\ 8361 + 4250 = -12611$ "

$T_{30} = +23661 \times 1.195 = +28275$ "

$P_{20} = -23661 + 5262 = -28923$ "

$T_{20} = +39973 \times 1.39 = +55562$ "

$T_{10} = +12103 \times 1.048 = +12684$ "

$P_{10} = -57338 \times 1.195 = -68519$ "

The stresses are the same in the members of Truss A, which are denoted by the same letters.

$A \times 26 = -65062 \times 25.9 + 9997 \times 17 + 3238 \times 8.9$ $\qquad A = -57167$ lbs.

$B \times 26 = -65062 \times 42.9 + 9997 \times 34 + 3238 \times 25.9 + 14288 \times 17$ $\qquad B = -81723$ "

$C \times 26 = -65062 \times 59.9 + 9997 \times 51 + 3238 \times 42.9 + 14288 \times 34 + 15300 \times 17$ $\qquad C = -96274$ "

$D \times 26 = -65062 \times 76.9 + 9997 \times 68 + 3238 \times 59.9 + 14288 \times 51 + 15300\,(17 + 34)$ $\qquad D = -100820$ "

$Co \times 26 = -57338 \times 76.1 + 12103 \times 51 + 5262 \times 59.1 + 16312 \times 34 + 15300 \times 17$ $\qquad Co = -100787$ "

$Bo \times 26 = -57338 \times 59.1 + 12103 \times 34 + 5262 \times 42.1 + 16312 \times 17$ $\qquad Bo = -95307$ "

$Ao \times 26 = -57338 \times 42.1 + 12103 \times 17 + 5262 \times 25.1$ $\qquad Ao = -79850$ "

$3 = +57167$ lbs. $\quad 4 = +81723$ lbs. $\quad 5 = +96274$ lbs. $\quad 40 = +95307$ lbs.

$30 = +79850$ "

$1 \times 26 = +65062 \times 17$ $\qquad 1 = +42540$ lbs.

$2 \times 26 = +65062 \times 17 - 9997 \times 8.1$ $\qquad 2 = +39426$ "

$20 \times 26 = +57338 \times 17 + 12103 \times 8.1$ $\qquad 20 = +41261$ "

$10 \times 26 = +57338 \times 17$ $\qquad 10 = +37490$ "

Live Load Stresses.—In the present case $s = 16.25$ feet. For Truss B, we have, page 257,

$$M_r = M_n + P_n\left(y_n + \frac{s}{2}\right) + wy_n\left(\frac{y_n + s}{2}\right),$$

and for the criterion for position for maximum shear, since the chords are horizontal,

$$(P_n + wy_n)\,\frac{p}{l} \underset{>}{=} P_1,$$

where p *is to be taken on the centre line.*

The maximum shear itself is given by

$$S = \frac{M_r}{l} - \frac{M_1}{p},$$

where p is the panel length *on the truss itself.*

Applying our diagram, we have the following results for Truss B. We take, of course, one-half of the shear for one truss.

Position on Centre Line.	y_n	M_r	M_x	$\frac{s}{2}$	
p_4 at 17 feet from left,	48.2	55309226	590400	147580	$P_1 = -$ 176358 lbs.
p_2 at 34 " "	26.9	41879598	304800	127896	$T_2 = +$ 135186 "
p_2 at 51 " "	9.9	32460238	304800	97114	$P_2 = -$ 97114 " $T_3 = +$ 116051 "
p_2 at 68 " "	5.4	24103333	304800	69804	$P_3 = -$ 69804 " $T_4 = +$ 83416 "
p_2 at 85 " "	0.6	15548346	128000	47046	$P_4 = -$ 47046 " $T_5 = +$ 56220 "
p_2 at 102 " "	4.3	9888666	128000	28551	$T_{40} = -$ 34118 "
p_2 at 119 " "	4.8	5706933	128000	14885	$T_{30} = -$ 17887 "

T_{40} must be counterbraced for $34118 - 9991 = 24127$ lbs., and this is, therefore, the tension in C_1.

As the dead load tension in T_{30} is greater than 17787, T_{30} does not need to be counterbraced.

For Truss A, we have

$$M_r = M_n + P_n\left(y_n - \frac{s}{2}\right) + wy_n\left(\frac{y_n - s}{2}\right).$$

Hence, for Truss A, we have

Position on Centre Line.	y_n	M_r	M_x	$\frac{s}{2}$	
p_4 at 17 feet from left,	48.2	44928266	590400	135063	$P_{10} = -$ 161400 lbs.
p_2 at 34 " "	26.9	32871346	304800	98457	$T_{20} = +$ 136855 "
p_2 at 51 " "	9.9	24562306	304800	71304	$P_{20} = -$ 71304 " $T_{20} = +$ 85208 "
p_2 at 68 " "	5.4	17493733	304800	48204	$P_{30} = -$ 48204 " $T_{40} = +$ 57604 "
p_2 at 85 " "	0.6	10001466	12800	28918	$P_{40} = -$ 28918 " $T_5 = -$ 34557 "
p_2 at 102 " "	4.3	6024666	12800	15923	$T_4 = -$ 19028 "

T_5 must be counterbraced for $34557 - 8292 = 26265$ lbs., and this is the tension in C_{10}. As the dead load tension in T_4 is greater than 19028, T_4 does not need to be counterbraced.

Finally, we have, for the greatest load concentration which can come at the foot of T_1 or T_{10} when p_4 is at the foot, 45620 lbs. Hence,

$$T_1 = +\,45620 \times 1.048 = +\,48810 \text{ lbs., and } T_{10} = +\,45620 \times 1.39 = +\,63412 \text{ lbs.}$$

For the chords we can find panels 1 and 10 by simply multiplying the maximum end shears already found by tan θ. Hence, we have

$$1 = +\,147580 \times 0.654 = +\,96517 \text{ lbs.} \qquad 10 = +\,135063 \times 0.654 = +\,88331 \text{ lbs.}$$

For panel 2 we have already found the maximum end shear for p_4 at first lower apex of Truss B, 147580 lbs., and the concentration at this point 45620 lbs. Hence

$$2 \times 26 = +\,147580 \times 17 - 45620 \times 8.1, \text{ or } 2 = +\,82282 \text{ lbs.}$$

For panel 20 we have found the maximum end shear for p_4 at first lower apex of Truss A, 135063 lbs., and the concentration at this point 45620 lbs. Hence

$$20 \times 26 = +135063 \times 17 + 45620 \times 8.1, \quad \text{or} \quad 20 = +102522 \text{ lbs.}$$

For the other panels we have the following results:

Position on centre line.	y_n	M_r	M_x	M	
p_{16} at 34 feet from left,	90.1	54751373	2206333	− 7062033	$A = -135808$ lbs. $3 = +135808$ “
p_{14} “ 34 “ “	85.9	40695413	1633733	− 9564151	$Ao = -183930$ “ $3o = +183930$ “
p_{16} “ 51 “ “	73.1	53759226	5108186	− 9965458	$B = -191643$ “ $4 = +191643$ “
p_{14} “ 51 “ “	68.9	43984866	5556505	− 11433740	$Bo = -219880$ “ $4o = +219880$ “
p_{14} “ 68 “ “	51.9	55209026	10083866	− 11530629	$C = -221740$ “ $5 = +221740$ “
p_{14} “ 68 “ “	51.9	44826746	10083866	− 12212292	$Co = -234850$ “

The case which we have solved is that of Fig. (f), page 257. The student **should find no** difficulty with the case of Fig. (g), page 257, or for the case where the skew is "right forward" and "left back," or *vice versa,* instead of, as in this case, "right and left forward."

SKEW SPAN ON CURVES.—This is perhaps the most complicated case which can arise.

The curve of the track will cause little or no difference in the dead load stresses. As to the live load, our diagram will be practically unaffected, and is to be used as in the preceding case to find positions giving maximum shear and moment at any point. But when these positions are known we must find the wheel load concentration for any position *at each floor beam,* where it is crossed by the centre line of the track, and then find the portion of each such concentration which goes to each truss, *according to the point at which this concentration acts on each floor beam.*

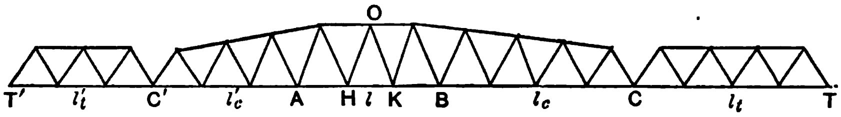
This involves considerable tedious figuring in order to determine maximum positions. But with this explanation and the work of the preceding case fully understood, there should be no difficulty in solving any such example.

THE CANTILEVER.

The principle of the cantilever has been already illustrated on page 60.

It consists in general of a *fixed span,* $AB = l$, Fig. (h), on either side of which project the *cantilever spans,* $BC = l_c$ and $AC' = l'_c$, which need not be equal. Then come on either side the *suspended trusses,* $CT = l_t$ and $C'T' = l'_t$, which also may have different lengths.

Fig. (h).

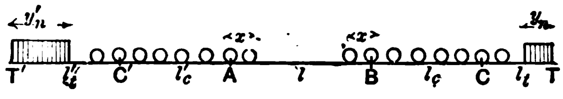


All cantilevers are modifications of this general type. If the spans $T'C'$ and $C'A$ are omitted, so that the left abutment is at A, the fixed span AB becomes an *anchored shore arm,* and is calculated upon the same general principles as the fixed span. If the piers A and B are brought close together so that both A and B rest upon the same pier, we have a *central fixed panel,* or panels,

which are calculated on the same general principles as the fixed span, except that loads between A and B rest directly on the pier, and the stresses in the central panel are due to outside loads only.

FIXED SPAN—MAXIMUM MOMENT.—It is evident from inspection that every load between B and T or between A and T', Fig. (h), causes a positive moment in AB (tension in top chord). Every load between A and B causes a negative moment (compression in top chord).

Maximum Positive Moment—Fixed Span.—The position of loading causing greatest positive moment (tension in top chord) in the fixed span, AB, is found by trial, by our diagram, for some wheel at C, the forward wheel being near B, for a train coming on from right, while *at the same time* we have a train coming on from left, with one wheel at C' and forward wheel near A.



It may happen that a wheel or two, by reason of the system adopted, may lie on the left of B and on the right of A. Let the distance thus covered on either side be x. Then, if we denote by t the distance of any wheel from T, Fig. (h), and by b the distance of any wheel from B, all distances taken without regard to sign, or direction, we have for the moment at B due to the right-hand train,

$$M_B = +\frac{l_o}{l_t}\sum_{T}^{C} Pt + \frac{wy_n^2 l_o}{2l_t} + \sum_{C}^{B} Pb - \sum_{X}^{B} Pb \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

when y_n is the distance covered by the uniform train load w per foot, if any.

For the moment at A, due to the left-hand train, we have

$$M_A = +\frac{l'_o}{l'_t}\sum_{T'}^{C'} Pt' + \frac{wy'^2_n l'_o}{2l'_t} + \sum_{C'}^{A} Pa - \sum_{X}^{A} Pa. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the right-hand train moves a very small distance δx to the left, we have,

$$M_B + \delta M_B = +\frac{l_o}{l_t}\sum_{T}^{C} P(t+\delta x) + \frac{wl_c}{2l_t}(y+\delta x)^2 + \sum_{C}^{B} P(b-\delta x) - \sum_{X}^{B} P(b+\delta x).$$

By subtraction therefore,

$$\frac{\delta M_B}{\delta x} = +\frac{l_o}{l_t}\sum_{T}^{C} P + \frac{l_o}{l_t} wy_n - \sum_{C}^{B} P - \sum_{X}^{B} P.$$

The criterion for maximum positive moment at B for the right-hand train is then,

$$\frac{\sum_{T}^{C} P + wy}{l_t} \underset{>}{=} \frac{\sum_{C}^{B} P + \sum_{X}^{B} P}{l_o}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and for maximum positive moment at A for the left-hand train,

$$\frac{\sum_{T'}^{C'} P + w'y_n}{l'_t} \underset{>}{=} \frac{\sum_{C'}^{A} P + \sum_{X}^{A} P}{l'_o}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By trial with the diagram, with these criterions, we can find the position of the trains, and the moments are then given by (1) and (2).

If no wheels are found on the left of B or right of A, we see at once that the maximum positive moment (tension in top chord) occurs *when the span AB is empty, when trains come on from the right and left, so that a wheel stands at C and C', and when the average load on the suspended tress on either side, including the wheels at C and C', is equal to or just greater than the average load on the cantilever span on that side.*

By trial with the diagram, using this criterion, the position of the train on each side can be found, which gives the maximum positive moments (tension in top chord), M_A and M_B, at A and B. These moments can then be found from (1) and (2).

The reaction or shear at A is then given by (see equation (IV.), page 173),

$$S_A = \frac{M_A - M_B}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If M_{2A} is greater than M_{2B}, S_A will be positive or upward, otherwise negative or downward.

The moment and shear at S being now known, we can find the stresses, as in the case of Fig. 126, page 181, for "exterior loading."

Thus, if z is the distance from the left end A to any point O, Fig. (h), where the maximum positive moment is required, we have the moment at that point,

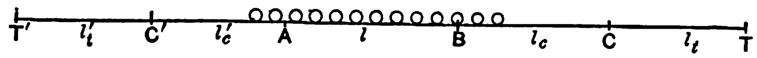
$$M = - S_A z + M_A, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where S_A and M_{2A} are found from (1), (2), and (5), and inserted with their proper signs. A positive moment means tension in top chord.

Central Fixed Panel.—When A and B are close together and both fastened down to the same pier, the stresses in the *central fixed panels* between A and B are given by the maximum positive moment as just found.

All loads between A and B come directly on the pier.

Maximum Negative Moment—Fixed Span.—The position of the loading causing the greatest negative moment (compression in top chord) in the fixed span AB, (Fig. (h), is the same as for a simple span, according to the general criterion already found, page 243, Fig. (a), provided



that there are no wheels upon BC or AC', Fig. (h). The train should approximate as near this as the actual wheel concentrations of the locomotives and tenders will permit. If the uniform train load comes on, it should end at B.

If, however, it happens that wheels are found in BC or AC', we shall have moments at A and B, Fig. (h), given by

$$M_A = + \sum_{C'}^{A} Pa \quad \text{and} \quad M_B = + \sum_{C}^{B} Pb. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

These moments cause a shear at the left end A, given by

$$S_A = \frac{M_A - M_B}{l} = \frac{\sum_{C}^{B} Pb - \sum_{C'}^{A} Pa}{l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which acts down if negative, and up if positive.

The reaction at the left end due to the loading in AB is then increased or diminished by S_A, and we have for the moment at any point, according to the notation of Fig. (a), page 242,

$$M = - \left(\frac{M_r}{l} + S_A\right) z - M_A + M_2 + P_2 e + \frac{e}{p} M_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This may be written

$$M = - \frac{z}{l} \sum_{B}^{A} Pb - \frac{z}{l} \sum_{C'}^{A} Pa + \frac{z}{l} \sum_{C}^{B} Pb + \sum_{C'}^{A} Pa + \sum_{H}^{A} Pc + \frac{e}{p} \sum_{K}^{H} Pk.$$

Moving the train a small distance, δx, to the left, we have $b + \delta n$, $a - \delta x$, $c + \delta x$, $k + \delta x$, in place of b, a, c, and k. Hence we find the criterion

$$\frac{P_n + wy_n - \sum_{C'}^{A} P - \sum_{C}^{B} P}{l} \geqq \frac{P_2 + \frac{eS_1}{p} - \sum_{C'}^{A} P}{z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This reduces to the criterion for simple span (page 243) when there are no wheels on left of A or right of B.

The position once found by this criterion, the moment is given by (9), in which S_A and M_A are to be inserted with their *proper signs*, as given by (7) and (8).

Fixed Span—Locomotive Excess—Maximum Moment.—If the method of *locomotive excess* is used (page 100), we simply cover BT and AT', with locomotive excess at C and C', for maximum positive moment (tension in upper chord) in span AB, (Fig. (h); and for maximum negative moment (compression in upper chord) in span AB, we treat the span exactly as for a simple span.

In the first case equations (1), (2), (5) and (6) still hold.

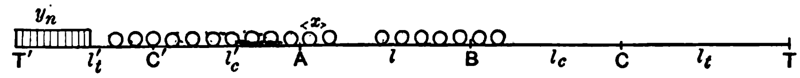
Anchored Shore Span—Maximum Moment.—If the fixed span AB becomes an *anchored shore span* by omission of $T'C'$ and $C'A$, there is no change in the preceding, except that the moment at the shore end, M_A or M_B, as the case may be, is zero in all the equations containing these quantities.

FIXED SPAN—MAXIMUM SHEAR.—We see at once from inspection of Fig. (h,) that for loads in the fixed span AB, the maximum positive shear (upward) at any point O is for the same load position as for a simple span.

We see also from (8) that any load on the left of A causes a positive shear, while any load on the right of B causes a negative shear at A.

Fixed Span—Maximum Positive Shear.—The maximum positive shear (upward) at O, Fig. (h), occurs, therefore, *for a train coming on from the right wholly within the span AB as for a simple span, while at the same time BT is empty and a train coming on from the left covers AT' with a wheel at C'.*

The trains should approximate as near this as the actual wheel concentrations of the locomotives and tenders will permit, and when the uniform train load comes on they can be made to conform exactly.



The criterion for the position of the train coming on from the left, which makes M_A a maximum, is given by (4), already deduced.

The criterion for the position of the train AB in coming on from the right is

$$\frac{P_n + wy_n + \sum\limits_{C}^{B} P}{l} \gtreqless \frac{d + z_2}{d}\frac{P_1}{p} - \frac{P_2}{d} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

where the notation is as in Fig. (b), page 243, and $\sum\limits_{C}^{B} P$ gives the loads on the right of B, if any. If there are none, the criterion reduces to that for a simple span. If the uniform train load comes on, $\sum\limits_{C}^{B} P$ is, of course, zero. If this is not zero, there is no train load.

When the position of the trains is found by diagram and these criterions, the moment at A is given by (2), *viz.:*

$$M_A = +\frac{l'_c}{l'_t}\sum_{T'}^{C} Pt' + \frac{wy'^2_n l'_c}{2l'_t} + \sum_{C'}^{A} Pa - \sum_{X}^{A} Pa, \quad . \; . \; . \; . \; . \; . \; . \quad (12)$$

where $\sum\limits_{X}^{A} Pa$ is the moment with reference to A of all loads on the right of A, if any.

If there are loads on the right of B also, belonging to the train in AB, which comes on from the right, we have for the shear at A, not including that due to wheels in the span AB,

$$S_A = \frac{M_A - M_B}{l} = \frac{\frac{l'_c}{l'_t}\sum\limits_{T'}^{C'} Pt + \frac{wy'^2_n l'_c}{2l'_t} + \sum\limits_{C'}^{A} Pa - \sum\limits_{X}^{A} Pa - \sum\limits_{C}^{B} Pb}{l} \quad . \; . \; . \; . \; . \quad (13)$$

We have acting at A, in addition to S_A, the reaction due to the loading in the span AB. The maximum compression in OH or tension in OK, Fig. (h), is then

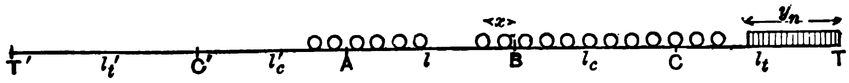
$$\text{Brace stress} = \frac{1}{y}\left[\frac{M_r d}{l} + S_A d - \frac{M_1(d+z_2)}{p} - P_2(d+z_2) + M_2\right], \quad . \quad . \quad . \quad (14)$$

where the notation is the same as in Fig. (b), page 244. We must take y and d for the brace desired. M_r is the moment at the right end B of all loads in the span AB, including the uniform train load, if any, and is equal to $\sum\limits_B^A Pb + \frac{wy_n^2}{2}$. The value of S_A is given by (13), with its proper sign.

For a simple span, M_A and M_B and therefore S_A are zero, and (14) reduces to (3), page 245. For horizontal chords, $d = d + z_2 = \infty$, and $y = \infty \cos \theta$, where θ is the angle of the brace at O with the vertical.

Maximum Negative Shear—Fixed Span.—For the maximum negative shear, if the structure is symmetrical with respect to the centre, we have simply to find the maximum positive shear for the corresponding brace on the other side of the centre. In such case we must put $d + l$ in place of d in (14), and remember that z_2 is now the distance of H' from B, and d is on the right of B, Fig. (b), page 244.

But if there is no symmetry we must take the train in the span AB as coming on from the left, while AT' is empty and the other train coming on from the right reaches from T to B.



The criterion for this second train, coming on from the right, is given by (3), already found.

The criterion for the first train in AB, coming on from the left, is

$$\frac{P_n + wy'_n + \sum\limits_{C'}^{A} P}{l} \gtreqless \frac{d+z_2}{d}\frac{P_1}{p} - \frac{P_2}{d} - \frac{wy'_n}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11')$$

where $\sum\limits_{C'}^{A} P$ is the sum of the wheels on the left of A, if any, and y'_n is the distance on right of A covered by the uniform train load, if any. When there is a uniform train load, $\sum\limits_{C'}^{A} P$ is, of course, zero. When this is not zero, there is no train load.

The moment at B due to the train coming on from the right is,

$$M_B = +\frac{l_c}{l_t}\sum_T^C Pt + \frac{wy''^2 l_c}{2l_t} + \sum_C^B Pb - \sum_X^B Pb \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12')$$

where $\sum\limits_X^B Pb$ is the moment with reference to B of wheels on left of B, if any.

The shear at A, due to this moment and the wheels on left of A, if any, is,

$$S_A = \frac{M_A - M_B}{l} = -\frac{\frac{l_c}{l_t}\sum\limits_T^C Pt + \frac{wy''^2 l_c}{2l_t} + \sum\limits_C^B Pb - \sum\limits_X^B Pb - \sum\limits_{C'}^A Pa}{l}. \quad . \quad . \quad . \quad (13')$$

We have, therefore, for the compression in OK or tension in OH, Fig. (h),

$$\text{Brace stress} = \frac{1}{y}\left[\frac{M_r d}{l} + S_A d - \frac{M_1(d+z_2)}{p} - P_2(d+z_2) + M_2\right], \quad . \quad . \quad . \quad (14')$$

where S_A is given by (13′) with its proper sign, and the notation is the same as in Fig. (b), page 244.

We must take y and d for the brace desired. M_r is the moment at the right end B of all loads in AB, including uniform train load, if any, and is equal to $\sum_B^A Pb + wy'_n\left(l + \frac{y'_n}{2}\right)$.

Locomotive Excess—Maximum Shear—Fixed Span.—If the method by locomotive excess is used (page 100) we have for maximum positive (upward) shear at O, Fig. (h), the loading in the span AB the same as for a simple span, while at the same time BT is empty and AT' is covered, with locomotive excess at C.

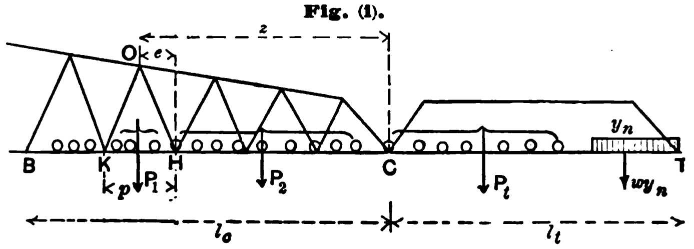
For maximum negative shear (downward), the loading in the span AB extends from A to the right, as for a simple span, while at the same time AT' is empty and BT is covered, with locomotive excess at C.

The values for S_A are given in each case by (13) or (13′) and the brace stresses by (14) and (14′).

Anchored Shore Span—Maximum Shear.—If the fixed span AB becomes an anchored shore span, nothing is changed except the moment at the shore end, M_B or M_A, as the case may be, is zero.

CANTILEVER SPAN—MAXIMUM MOMENT.—For the maximum moment at any point O of the cantilever span, let z be the distance of this point from the end of the cantilever, Fig. (i), and let o be the distance of any wheel from O, and k of any wheel from K, these distances being taken without regard to sign or direction.

Fig. (i).



Then we have for the moment at O,

$$M = +\frac{z}{l_t}M_r + \sum_C^H Po + \frac{e}{p}\sum_K^H Pk, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where M_r is the moment at the right end T of the suspended truss of all loads on the truss CT, including the uniform train load, if any.

We then have from (15),

$$M = +\frac{z}{l_t}\sum_T^C Pt + \frac{z}{l_t}\frac{wy_n^2}{2} + \sum_C^H Po + \frac{e}{p}\sum_K^H Pk.$$

If the train moves a very small distance, δx, to the left, we have $t + \delta x$, $y_n + \delta x$, $o - \delta x$ and $k - \delta x$, in place of t, y_n, o, and k. Hence, by subtraction,

$$\frac{\delta M}{\delta x} = +\frac{z}{l_t}\sum_T^C P + \frac{z}{l_t}wy_n - \sum_C^H P - \frac{e}{p}\sum_K^H P.$$

The criterion for maximum moment at the point O is then,

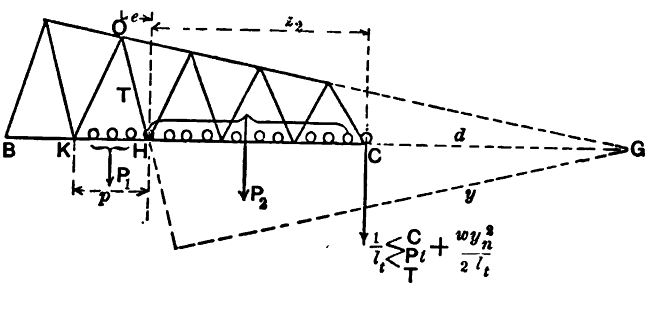
$$\frac{P_t + wy_n}{l_t} \overset{=}{>} \frac{P_2 + \frac{e}{p}P_1}{z}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

That is, the moment at any point O, Fig. (i), of a cantilever span, is a maximum, *when a wheel is at C or H, and when the average load on the suspended truss, including the wheel at C if any, is equal to or just greater than the average load over the distance z, including the wheel at H, if any.*

The position which gives the maximum moment at O is thus easily found by trial with the diagram, and then the maximum moment is given by (15). It is always positive (tension in upper chord). There can be no negative moment in the cantilever span.

CANTILEVER SPAN—MAXIMUM SHEAR.—Let z_2 be the distance from the end of the cantilever C, Fig. (j), to the point H at which the shear is required, let d be the distance CG of the intersection G of the upper and lower chords in the panel $HK = p$ from C, let c be the distance of any wheel from C, and k of any wheel from K, all distances taken without reference to sign or direction.

Fig. (j).



The downward force at C due to the suspended truss is $\frac{1}{l_t}\sum_T^C Pt + \frac{1}{l_t}\frac{wy_n^2}{2}$ if the uniform train load covers the distance y_n from the right end T of the suspended truss.

We have then for the tension in OH, or compression in OK,

$$T = \text{Brace stress} = \frac{1}{y}\left[\frac{d}{l_t}M_r + \sum_C^H P\,(d+c) + \frac{d+z_2}{p}\sum_K^H Pk\right] \quad . \quad . \quad . \quad . \quad (17)$$

where M_r is the moment at the right end T of the suspended truss, of all the loads on the suspended truss, including the uniform train load, if any. We must take y and d for the brace desired.

We can write (17) in the form,

$$T = \frac{1}{y}\left[\frac{d}{l_t}\sum_T^C Pt + \frac{d}{l_t}\frac{wy_n^2}{2} + \sum_C^H P\,(d+c) + \frac{d+z_2}{p}\sum_K^H Pk\right].$$

If the train moves a very small distance to the left, δx, we have $t + \delta x$, $c + \delta x$, $k - \delta x$, $y_n + \delta x$, in place of t, c, k and y_n. Hence,

$$\frac{\delta T}{\delta x} = \frac{1}{y}\left[\frac{d}{l_t}\sum_T^C P + \frac{d}{l_t}wy_n + \sum_C^H P - \frac{d+z_2}{p}\sum_K^H Pk\right].$$

If P_t is the sum of the wheels in the suspended truss, we have then for the criterion giving maximum shear,

$$\frac{P_t + wy_n}{l_t} \overset{=}{>} \frac{d+z_2}{d}\frac{P_1}{p} - \frac{P_2}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

By trial with the diagram, we can find the position which satisfies this criterion, so that when a wheel is at C or H, $\frac{P_t + wy_n}{l_t}$ is equal to or greater than the right-hand member, while if this wheel moves to the left, so as to pass out of the suspended span or into the panel, $\frac{P_t + wy_n}{l_t}$ becomes less than the right-hand member. P_1 and P_2 do not therefore include the wheel at C or H.

Criterion (18) is general. For horizontal chords, $d = d + z = \infty$, and the criterion reduces to

$$\frac{P_t + wy_n}{l_t} \overset{=}{>} \frac{P_1}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

or the shear for horizontal chords, at any point O of a cantilever span, Fig. (j), is a maximum, *when a wheel is at H or C, and the average load on the suspended truss, including the wheel at C, if any, is equal to or just greater than the average load in the panel, not including the wheel at H, if any.*

The position being thus found by (18) or (19), which makes the shear a maximum, the corresponding brace stress is given by (17). The value of y and d must be taken for each brace OH and OK. As the shear at H is always negative (downward), we have always tension in OH and compression in OK.

If the method by locomotive excess is used (page 100) we simply take the uniform load from K to T, and locomotive excess either at H or C. Equations (15) and (17) still hold.

FIXED SPAN—DEAD LOAD—MOMENTS.—Let z be the distance from the left end A of the fixed span AB, to any point O, Fig. (a), at which the moment is required, and let o be the distance of any apex dead load from this point. Then the moment at O is

$$M = -\frac{z}{l} M_r - S_A z - M_A + \sum_O^A Po, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where M_r is the moment at the right end B of all the apex dead loads in the span AB, and S_A and M_A are given by (1), (2), and (5).

FIXED SPAN—DEAD LOAD—SHEAR.—Let a be the distance of any apex dead load from the left end A, and let z_1 be the distance from A to the end of the brace through the point O, *nearest* to A, Fig. (b).

Then the stress in any brace OH or OK is

$$\text{Brace stress} = \frac{1}{y}\left[\left(\frac{M_r}{l} + S_A\right) d - \sum_O^A P\,(d + a)\right], \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where y and d are to be taken for the brace desired, M_r is as in (20), and S_A is given by (1), (2), and (5). If (21) comes out minus it shows negative (downward) shear at O, Fig. (h), or compression in OK and tension in OH.

CENTRAL FIXED PANEL—DEAD LOAD.—If A and B are close together and fastened down to the same pier, loads in AB are disregarded and M_r is zero, and $\sum_O^A P\,(d + a)$ is zero, also M_r and $\sum_O^A Po$ are zero.

ANCHORED SHORE SPAN—DEAD LOAD.—If the fixed span AB becomes an anchored shore span, the moment at the shore end, M_A or M_B, as the case may be, is zero.

CANTILEVER SPAN—DEAD LOAD—MOMENTS.—Taking the notation of Fig. (i), we have for the moment at any point O,

$$M = +\frac{z}{l_t} \sum_T^C Pt + \sum_O^C Po, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where t and o are the distances of any apex dead load from T and O, without regard to sign or direction.

CANTILEVER SPAN—DEAD LOAD—SHEAR.—Taking the notation of Fig. (j), we have

$$\text{Brace stress} = -\frac{1}{y}\left[\frac{d}{l_t} \sum_T^C Pt + \sum_O^C P\right], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where y and d are to be taken for the brace OH or OK desired. The minus sign denotes negative (downward) shear at H, and therefore tension in OH and compression in OK.

BEST PROPORTION FOR CANTILEVER SPAN.—Comparing the results of pages 298 and 303, we see that the material in a cantilever is one half that of a span of a length twice that of the cantilever.

The length of the suspended truss should then be about *equal to the sum of the lengths of the cantilevers on each side, for greatest economy of material.*

WIND STRESSES IN CANTILEVER.—The wind stresses should be calculated as stated in Chapter VI., Part II., for a dead wind load of 30 lbs. per square foot of exposed surface of both trusses, and a live wind load of 300 lbs. per linear foot, or a dead wind load of 50 lbs. per square foot of exposed surface of both trusses, and the greatest stresses in either case taken.

The conditions for maximum stresses are the same as for vertical loading already discussed, and

the computation is simplified by reason of the wind loading being of uniform intensity. There should therefore be no difficulty in applying preceding results to this case.

It is evident that the action of the wind upon a cantilever is of very great importance, and that the conditions of loading are much more varied than for ordinary spans.

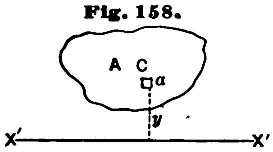
CHAPTER II.

STRENGTH OF MATERIALS AND THEORY OF FLEXURE.

MOMENT OF INERTIA OF AN AREA.—This is a convenient term for a quantity which occurs so often in applications of the theory of flexure that a special name for it is desirable.

The moment of inertia of an area with respect to any axis is a general term *for the algebraic sum of the products obtained by multiplying every elementary area by the square of the distance of that element from the axis.*

Fig. 158.



Thus let A, Fig. 158, be an area and a any indefinitely small elementary area and $X'X'$ any axis. Let y be the distance of the element a from the axis. Then ay^2 is the moment of inertia of the elementary area with respect to the axis $X'X'$, and the moment of inertia of the entire area A with respect to the axis $X'X'$ is

$$\Sigma ay^2,$$

the summation extending to every element of the area A. We denote the moment of inertia of an area with reference to an axis *in its plane through its centre of mass* by I; with reference to an "*eccentric*" axis, i.e., an axis in its plane but *not* through its centre of mass, by I'. In the present case of Fig. 158 we should write, then,

$$I' = \Sigma ay^2,$$

while if the axis passed through the centre of mass C, we should write

$$I = \Sigma ay^2.$$

If the axis is at right angles to the area it is called a *polar axis*, and the moment of inertia is called the *polar moment of inertia*, and denoted by I_p or I_p' according as the polar axis passes through the centre of mass or is eccentric.

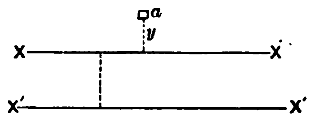
REDUCTION OF MOMENT OF INERTIA OF AN AREA.—If then I denotes the moment of inertia of an area with reference to an axis through the centre of mass, and I' the moment of inertia with reference to a *parallel eccentric axis*, we can easily prove that if d is the distance between these axes, we shall have

$$I' = I + Ad^2,$$

where A is the area.

That is, *the moment of inertia I' of an area A with reference to any eccentric axis is equal to the moment of inertia I with reference to a parallel axis through the centre of mass, plus the product Ad^2 of the area A by the square of the distance d between the axes.*

We can thus always find I' if we know I, A, and d. For this reason, only the moment of inertia I for the centre of mass is given in works on Mechanics.



The proof is as follows: Let XX be the axis through the centre of mass, a any elementary area, and y its distance from XX. Then since XX passes through the centre of mass, we must have $\Sigma ay = 0$.

Let $X'X'$ be a parallel eccentric axis at a distance d. Then we have

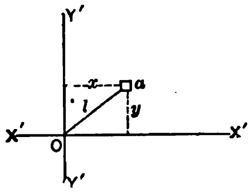
$$I' = \Sigma a(y+d)^2 = \Sigma ay^2 + 2d\Sigma ay + d^2\Sigma a.$$

But we have $I = \Sigma ay^2$, $\Sigma ay = 0$, and $\Sigma a = A$. Hence

$$I' = I + Ad^2.$$

Polar Moment of Inertia of an Area.—Let $X'X'$, $Y'Y'$, be two rectangular axes, and a any elementary area. Then the moment of inertia of a with reference to $X'X'$ is ay^2, and with reference to $Y'Y'$ it is ax^2. But with reference to the polar axis through O it is $al^2 = a(x^2+y^2)$. Hence, *the polar moment of inertia for any axis passing through any point O is equal to the sum of the moments of inertia for any two rectangular axes passing through that point.*

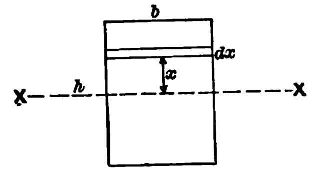
Fig. 159



Radius of Gyration.—If in any case we divide the moment of inertia by the area A, we obtain the square of the distance from the axis to that point at which, if the entire area A were concentrated, the moment of inertia would be the same as for the actual area. This distance is called the *radius of gyration.* We denote it by r We have then, in general,

$$r^2 = \frac{I'}{A}, \quad \text{or} \quad I' = Ar^2.$$

Fig. 160



Determination of Moment of Inertia of Areas.—By the aid of the calculus we can readily determine the moment of inertia for all the most usual areas.

Rectangle.—Let the breadth, Fig. 160, be b, and height h. Suppose a strip at a distance x from the axis XX through the centre of mass. The area is bdx. The moment of inertia of the strip is then

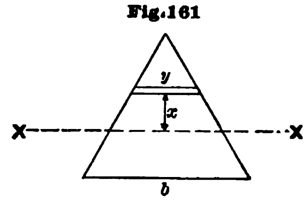
$$bx^2dx.$$

Integrating this between $+\frac{h}{2}$ and $-\frac{h}{2}$ we have

$$I = \int_{-\frac{h}{2}}^{+\frac{h}{2}} b\,x^2\,dx = \frac{bh^3}{12}.$$

Triangle.—Let the base of the triangle, Fig. 161, be b and the height h, and take the axis XX through the centre of gravity, or $\frac{2}{3}h$ below the apex.

Fig. 161



Take a strip at a distance x from the axis. The length of this strip y is from similar triangles, given by the proportion

$$\tfrac{2}{3}h - x : y :: h : b, \quad \text{or } y = \frac{(\frac{2}{3}h - x)\,b}{h}.$$

The area of the strip is, then,

$$y\,dx = \frac{\frac{2}{3}hbdx - bxdx}{h} = \tfrac{2}{3}bdx - \frac{bxdx}{h}.$$

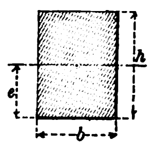
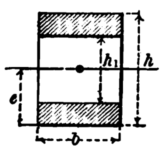
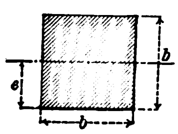
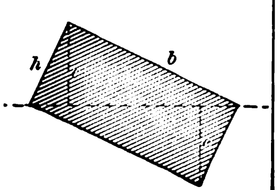
The moment of inertia of the strip is

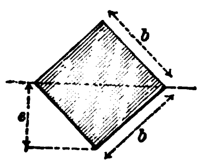
$$yx^2\,dx = \tfrac{2}{3}\,bx^2\,dx - \frac{bx^3\,dx}{h}.$$

Integrating between $+\tfrac{2}{3}h$ and $-\tfrac{1}{3}h$, we have

$$I = \int_{-\frac{1}{3}h}^{+\frac{2}{3}h} \tfrac{2}{3}\,bx^2\,dx - \frac{bx^3\,dx}{h} = \frac{bh^3}{36}.$$

We give below the moment of inertia I for various areas such as are likely to occur in practice for horizontal axis through the centre of mass. The student will do well to check them. We also give the area A, and the distance e of the centre of mass from outer edge.

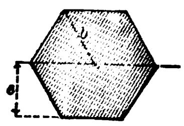
	Axis of x: $A = bh$, $e = \frac{h}{2}$, $I = \frac{1}{12}bh^3$.	Axis of y: $e = \frac{b}{2}$, $I = \frac{1}{12}hb^3$
	Axis of x: $A = b(h - h_1)$, $e = \frac{h}{2}$, $I = \frac{b(h^3 - h_1^3)}{12}$.	Axis of y: $e = \frac{b}{2}$, $I = \frac{(h - h_1)b^3}{12}$.
	$A = b^2$, $e = \frac{b}{2}$, $I = \frac{b^4}{12}$.	
	$A = bh$, $e = \frac{bh}{\sqrt{b^2 + h^2}}$, $I = \frac{b^3h^3}{6(b^2 + h^2)}$.	



$$A = b^2,$$

$$e = \frac{b}{\sqrt{2}},$$

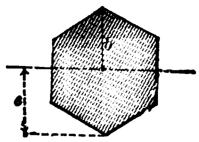
$$I = \frac{b^4}{12}.$$



$$A = 2.598\ b^2 = \frac{3\,b^2\sqrt{3}}{2},$$

$$e = 0.866\ b = \frac{b\sqrt{3}}{2},$$

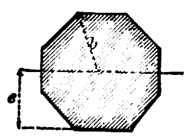
$$I = 0.5413\ b^4 = \frac{5\,b^4\sqrt{3}}{16}.$$



$$A = 2.598\ b^2 = \frac{3\,b^2\sqrt{3}}{2},$$

$$e = b,$$

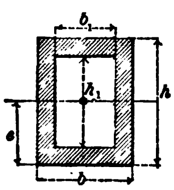
$$I = 0.5413\ b^4 = \frac{5\,b^4\sqrt{3}}{16}.$$



$$A = 2.828\ b^2 = 2\,b^2\sqrt{2}.$$

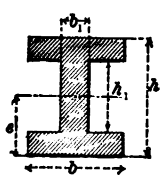
$$e = 0.924\,b = \frac{b}{2}\sqrt{2+\sqrt{2}} = b\cos 22\tfrac{1}{2}^\circ,$$

$$I = 0.638\ b^4 = \frac{b^4}{6}(1 + 2\sqrt{2}).$$



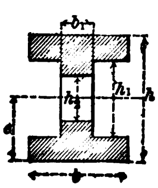
$$A = bh - b_1h_1,$$

Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \frac{1}{12}(bh^3 - b_1h_1^3). \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2} \\ I = \frac{1}{12}(hb^3 - h_1b_1^3). \end{cases}$



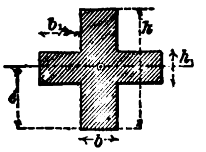
$$A = bh - (b - b_1)\,h_1,$$

Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \frac{1}{12}[bh^3 - (b - b_1)h_1^3]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}[(h - h_1)\,b^3 + h_1b_1^3]. \end{cases}$



$$A = b(h - h_1) + b_1(h_1 - h_2),$$

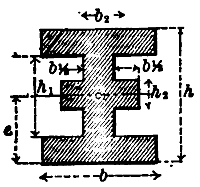
Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \frac{1}{12}[b(h^3 - h_1^3) + b_1(h_1^3 - h_2^3)]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}[(h - h_1)\,b^3 + (h_1 - h_2)\,b_1^3]. \end{cases}$



$$A = bh + b_1h_1,$$

Axis of x: $e = \frac{h}{2}$, $I = \frac{1}{12}(bh^3 + b_1h_1^3)$.

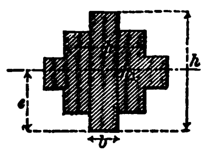
Axis of y: $e = \frac{b+b_1}{2}$, $I = \frac{1}{12}[hb^3 + h_1(b+b_1)^3 - h_1b^3]$.



$$A = bh - (b - b_2)h_1 + b_1h_2,$$

Axis of x: $e = \frac{h}{2}$, $I = \frac{1}{12}[bh^3 - (b-b_2)h_1^3 + b_1h_2^3]$.

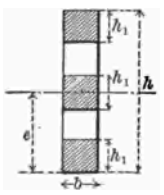
Axis of y: $e = \frac{b}{2}$, $I = \frac{1}{12}[(h-h_1)b^3 + (h_1-h_2)b_2^3 + h_2(b_1+b_2)^3]$.



$$A = bh + (h_1 - b)h_1 + (h - h_1)b,$$

$$e = \frac{h}{2},$$

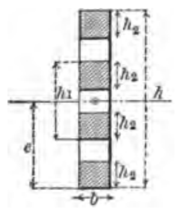
$$I = \frac{1}{12}[bh^3 + (h_1 - b)h_1^3 + (h - h_1)b^3].$$



$$A = 3\,bh_1,$$

Axis of x: $e = \frac{h}{2}$, $I = \frac{b}{12}[9\,h_1^3 + 6\,hh_1(h - 2\,h_1)]$.

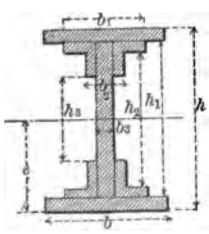
Axis of y: $e = \frac{b}{2}$, $I = \frac{3}{12}h_1b^3$.



$$A = 4\,bh_2,$$

Axis of x: $e = \frac{h}{2}$, $I = \frac{b}{12}[16\,h_2^3 + 6\,hh_2(h - 2\,h_2) + 6\,h_1h_2(h_1 - 2\,h_2)]$.

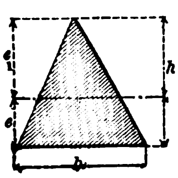
Axis of y: $e = \frac{b}{2}$, $I = \frac{4}{12}h_2b^3$.



$$A = b(h - h_1) + b_1(h_1 - h_2) + b_2(h_2 - h_3) + b_3h_3,$$

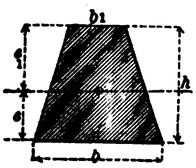
Axis of x: $e = \frac{h}{2}$, $I = \frac{1}{12}[b(h^3 - h_1^3) + b_1(h_1^3 - h_2^3) + b_2(h_2^3 - h_3^3) + b_3h_3^3]$.

Axis of y: $e = \frac{b}{2}$, $I = \frac{1}{12}[(h-h_1)b^3 + (h_1-h_2)b_1^3 + (h_2-h_3)b_2^3 + h_3b_3^3]$.



$$A = \frac{bh}{2},$$

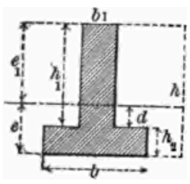
$e_1 = \frac{2}{3}h,\quad e = \frac{1}{3}h,$	$e_1 = h,\quad e = 0,$	$e_1 = 0,\quad e = h,$
$I = \frac{bh^3}{36}.$	$I = \frac{bh^3}{12}.$	$I = \frac{bh^3}{4}.$



$$A = \frac{b + b_1}{2} h,$$

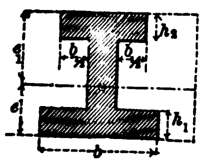
$$e = \frac{b + 2b_1}{b + b_1} \frac{h}{3}, \quad e_1 = \frac{2b + b_1}{b + b_1} \frac{h}{3},$$

$$I = \frac{b^2 + 4bb_1 + b_1^2}{b + b_1} \frac{h^3}{36}.$$



$$A = b_1 h_1 + b h_2,$$

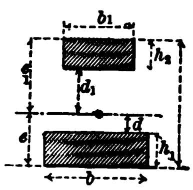
Axis of x $\begin{cases} e = \dfrac{bh_2^2 + b_1 h_1 (h + h_2)}{2[bh - (b - b_1) h_1]}, \\ I = \frac{1}{3}[b(e^3 - d^3) + b_1 (d^3 + e_1^3)]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}(h_1 b_1^3 + h_2 b^3). \end{cases}$



$$A = bh_1 + b_1 h_2 + \delta h,$$

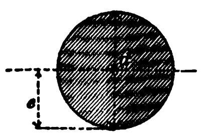
Axis of x $\begin{cases} e = \dfrac{\delta h^2 + 2b_1 h_2 h + bh_1^2 - b_1 h_2^2}{2(\delta h + bh_1 + b_1 h_2)}, \\ I = \frac{1}{3}[(b + \delta) e^3 - b(e - h_1)^3 + (b_1 + \delta) e_1^3 - b_1 (e_1 - h_2)^3]. \end{cases}$

Axis of y $\begin{cases} e = \dfrac{b + \delta}{2}, \\ I = \frac{1}{12}[h_2 (b_1 + \delta)^3 + (h - h_1 - h_2)\delta^3 + h_1 (b + \delta)^3]. \end{cases}$



$$A = bh_1 + b_1 h_2,$$

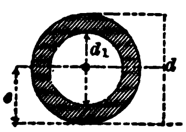
Axis of x $\begin{cases} e = \dfrac{b_1 h_2 (2h - h_2) + bh_1^2}{2(bh_1 + b_1 h_2)}, \\ I = \dfrac{b}{3}[e^3 - d^3] + \dfrac{b_1}{3}[e_1^3 - d_1^3]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}(h_2 b_1^3 + h_1 b^3). \end{cases}$



$$A = \frac{\pi}{4} d^2, \quad \pi = 3.1416,$$

$$e = e_1 = \frac{d}{2},$$

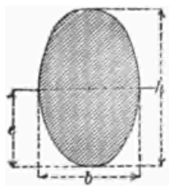
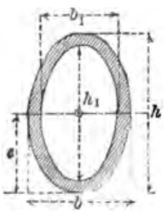
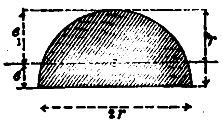
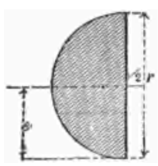
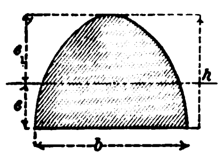
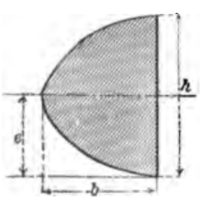
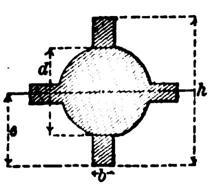
$$I = \frac{\pi}{64} d^4 = 0.0491\, d^4.$$

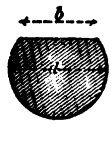


$$A = \frac{\pi}{4}(d^2 - d_1^2),$$

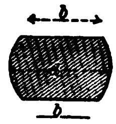
$$e = e_1 = \frac{d}{2},$$

$$I = 0.0491\,(d^4 - d_1^4).$$

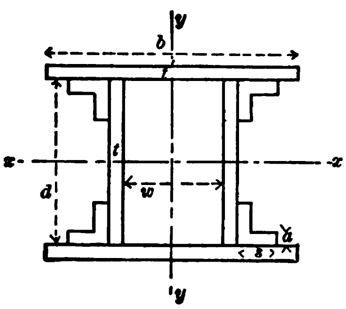
	$A = \frac{\pi}{4} bh,$ $e = e_1 = \frac{h}{2},$ $I = \frac{\pi}{64} bh^3 = 0.0491\ bh^3.$
	$A = \frac{\pi}{4} (bh - b_1h_1),$ $e = e_1 = \frac{h}{2},$ $I = 0.0491\ (bh^3 - b_1h_1^3).$
	$A = \frac{\pi r^2}{2},$ $e_1 = 0.5765\ r, \quad e = 0.4244\ r,$ $I = 0.1098\ r^4.$
	$A = \frac{\pi r^2}{2},$ $e = e_1 = r,$ $I = 0.3927\ r^4.$
	$A = \frac{2}{3} bh,$ $e = \frac{2}{5} h, \quad e_1 = \frac{3}{5} h,$ $I = \frac{8}{175} bh^3 = \frac{12}{175} Fh.$
	$A = \frac{2}{3} bh,$ $e = e_1 = \frac{h}{2},$ $I = \frac{1}{30} bh^3 = \frac{1}{20} Fh^2.$
	$A = \frac{\pi}{4} d^2 + 2b(h - d),$ $e = e_1 = \frac{h}{2},$ $I = \frac{1}{12}\left[\frac{3\pi}{16} d^4 + b(h^2 - d^2) + b^3(h - d)\right].$



$$\begin{cases} b = \dfrac{d}{3}, \quad e = 0.476\,d, \quad A = 0.779\,d^2, \\ I = 0.048\,d^4, \\ b = \dfrac{d}{2}, \quad e = 0.447\,d, \quad A = 0.763\,d^2, \\ I = 0.044\,d^4. \end{cases}$$



$$\begin{cases} b = \dfrac{d}{3}, \quad e = e_1 = 0.471\,d_1, \quad A = 0.714\,d^2, \\ I = 0.047\,d^4, \\ b = \dfrac{d}{2}, \quad e = e_1 = 0.433\,d, \quad A = 0.711\,d^2, \\ I = 0.043\,d^4. \end{cases}$$



$$A = 2\,bt' + 4\,(2\,sa - a^2) + 2\,dt,$$

Axis of x
$$\begin{cases} e = d + t', \\ I = \dfrac{bt'^3}{6} + bt'\dfrac{(d+t')^2}{2} + \dfrac{(s+t)\,d^3}{6} - \left[\dfrac{(s-a)(d-2a)^3 + a(d-2s)^3}{6}\right]. \end{cases}$$

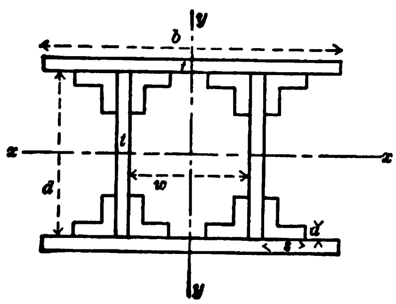
If one plate is replaced by latticing, $I = \dfrac{bt'^3}{12} + bt'\dfrac{(d+t')^2}{4} + \text{etc.}, \quad e = d + \dfrac{t'}{2},$

$$A = bt' + \text{etc.}$$

Axis of y
$$\begin{cases} e = \dfrac{b}{2}, \\ I = \dfrac{t'b^3}{6} + \dfrac{a(w+2t+2s)^3}{6} + \dfrac{(s-a)(w+2t+2a)^3}{6} + \dfrac{(d-2s)(w+2t)^3}{12} \\ \qquad - \dfrac{dw^3}{12}. \end{cases}$$

If one plate is replaced by latticing, $I = \dfrac{t'b^3}{12} + \text{etc} \quad A = bt' + \text{etc.}$

If both plates are replaced by latticing, $t' = 0$.



$$A = 2bt' + 8\,(2sa - a^2) + 2dt,$$

Axis of x
$$\begin{cases} e = d + t', \\ I = \dfrac{bt'^3}{6} + bt'\dfrac{(d+t')^2}{2} + \dfrac{(2s+t)d^3}{6} - \left[\dfrac{(s-a)(d-2a)^3 + a(d-2s)^3}{3}\right]. \end{cases}$$

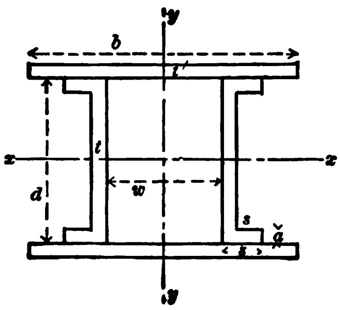
If one plate is replaced by latticing, $I = \dfrac{bt'^3}{12} + bt'\dfrac{(d+t')^2}{4} + \text{etc.} \quad e = d + \dfrac{t'}{2},$

$$A = bt' + \text{etc.}$$

Axis of y
$$\begin{cases} e = \dfrac{b}{2}, \\ I = \dfrac{t'b^3}{6} + \dfrac{a[(w+2t+2s)^3 - (w-2s)^3]}{6} \\ \qquad + \dfrac{(s-a)\,[(w+2t+2a)^3 - (w-2a)^3]}{6} + \dfrac{(d-2s)\,[(w+2t)^3 - w^3]}{12}. \end{cases}$$

If one plate is replaced by latticing, $I = \dfrac{t'b^3}{12} + \text{etc.} \quad A = bt + \text{etc.}$

If both plates are replaced by latticing, $t' = 0$.



$$A = 2bt' + 4sa + 2dt,$$

Axis of x $\begin{cases} e = d + t', \\ I = \dfrac{bt'^3}{6} + bt'\dfrac{(d+t')^2}{2} + \dfrac{(s+t)d^3 - s(d-2a)^3}{6}. \end{cases}$

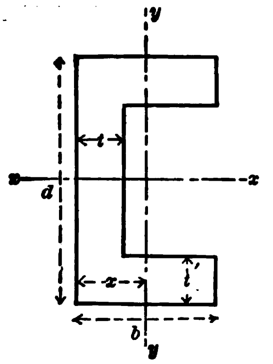
If one plate is replaced by latticing, $I = \dfrac{bt'^3}{12} + bt'\dfrac{(d+t')^2}{4} +$ etc. $\quad e = d + \dfrac{t'}{2}.$

$$A = bt' + \text{etc.}$$

Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \dfrac{t'b^3}{6} + \dfrac{2a(w + 2t + 2s)^3 + (d - 2a)(w + 2t)^3 - dw^3}{12}. \end{cases}$

If one plate is replaced by latticing, $I = \dfrac{t'b^3}{12} +$ etc. $\quad A = bt' +$ etc.

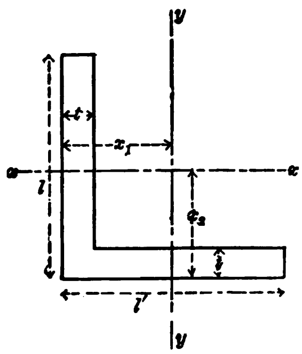
If both plates are replaced by latticing, $t' = 0$.



$$A = dt + 2(b - t)t',$$

Axis of x $\begin{cases} e = \dfrac{d}{2}, \\ I = \dfrac{bd^3 - (b - t)(d - 2t')^3}{12}. \end{cases}$

Axis of y $\begin{cases} e = b - x, \quad x = \dfrac{b^2d - (b^2 - t^2)(d - 2t')}{2A}, \\ I = \dfrac{2t'(b - x)^3 + dx^3 - (d - 2t')(x - t)^3}{3}. \end{cases}$

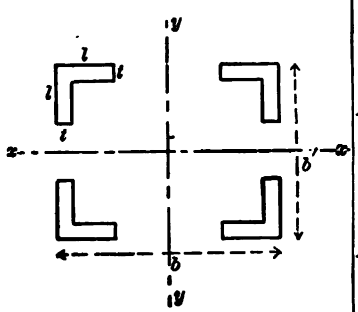


$$A = (l + l' - t)t,$$

$$x_1 = \frac{ll'^2 - (l'^2 - t^2)(l - t)}{2A}, \qquad x_2 = \frac{l^2l' - (l' - t)(l^2 - t^2)}{2A},$$

Axis of x $\begin{cases} e = l - x_2, \\ I = \dfrac{t(l - x_2)^3 + l'x_2^3 - (l' - t)(x_2 - t)^3}{3}, \end{cases}$

Axis of y $\begin{cases} e = l' - x_1, \\ I = \dfrac{t(l' - x_1)^3 + lx_1^3 - (l - t)(x_1 - t)^3}{3}, \end{cases}$

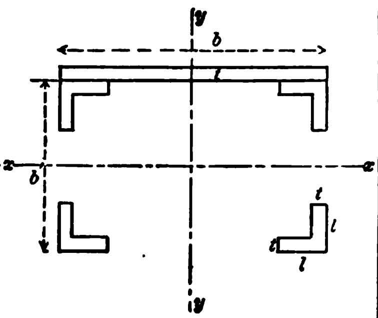


$$A = 4(2lt - t^2), \qquad x_1 = \frac{l^3 - (l^2 - t^2)(l - t)}{2A} = x_2 = x,$$

Axis of x $\begin{cases} e = \dfrac{b'}{2}, \\ I_1 = \dfrac{4[t(l - x)^3 + lx^3 - (l - t)(x - t)^3]}{3} + A\left(\dfrac{b'}{2} - x\right)^2. \end{cases}$

Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I_2 = \dfrac{4[t(l - x)^3 + lx^3 - (l - t)(x - t)^3]}{3} + A\left(\dfrac{b}{2} - x\right)^2. \end{cases}$

The angles are connected by latticing.



$A = 4(2lt - t^2) + bt.$ I_1 and I_2 as above.

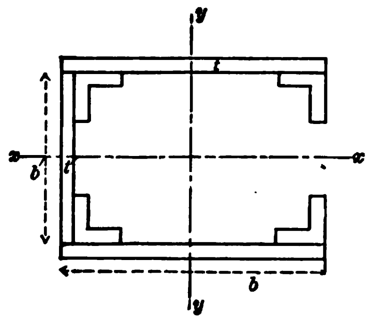
Axis of x $\begin{cases} e = \dfrac{b'}{2} + t, \\ I = I_1 + \dfrac{bt}{4}\left[\dfrac{t^2}{3} + (b' + t)^2\right]. \end{cases}$

If there are two plates, above and below, $I_3 = I_1 + \dfrac{bt}{2}\left[\dfrac{t^2}{3} + (b' + t)^2\right].$

$A = 4(2lt - t^2) + 2bt.$

Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = I_2 + \dfrac{tb^3}{12}. \end{cases}$

If there are two plates, $I_4 = I_2 + \dfrac{tb^3}{6}.$ $A = 4(2lt - t^2) + 2bt.$



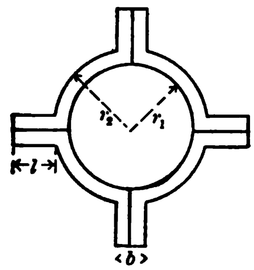
$A = 4(2lt - t^2) + 2bt + b't'.$ I_3 and I_4 as above.

Axis of x $\begin{cases} e = \dfrac{b'}{2} + t, \\ I = I_3 + \dfrac{t'(b' + 2t)^3}{12}. \end{cases}$

For four plates, $I = I_3 + \dfrac{t'(b' + 2t)^3}{6}.$ $A = 4(2lt - t^2) + 2(bt + b't').$

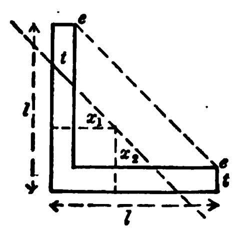
Axis of y $\begin{cases} e = \dfrac{b}{2} + t, \\ I = I_4 + \dfrac{t'(b' + 2t)}{4}\left[\dfrac{t'^2}{3} + (b + t')^2\right]. \end{cases}$

For four plates, $I = I_4 + \dfrac{t'(b' + 2t)}{2}\left[\dfrac{t'^2}{3} + (b + t')^2\right].$



$$A = \pi(r_2^2 - r_1^2) + 4bl.$$

$$I = \frac{\pi(r_2^4 - r_1^4)}{4} + 2bl\left(r_2 + \frac{l}{2}\right)^2.$$

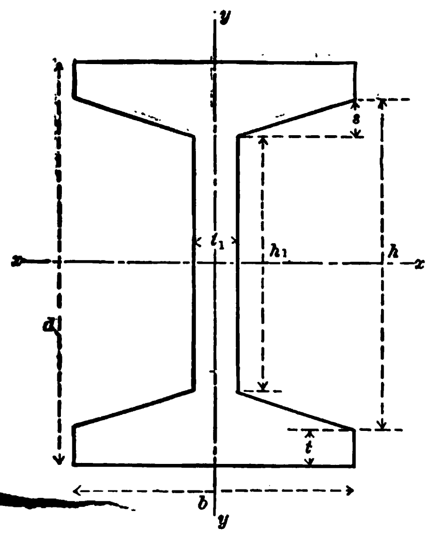


Axis through centre of gravity, parallel to ee.

$$x_1 = \frac{ll'^2 - (l'^2 - t^2)(l - t)}{2A}, \quad x_2 = \frac{l^2 l' - (l' - t)(l^2 - t^2)}{2A},$$

$$A = (l + l' - t)\,t.$$

$$I = \frac{2x_2^4 - 2(x_2 - t)^4 + t\left[l' - \left(2x_2 - \dfrac{t}{2}\right)\right]^3}{3}.$$

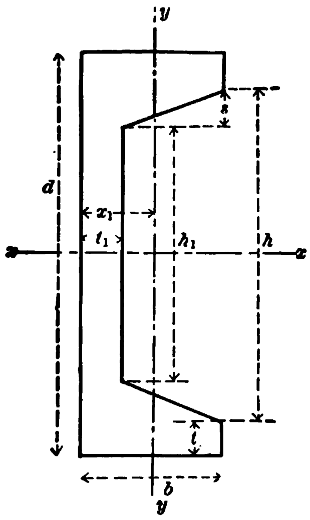


$$A = 2bt + ht_1 + s(b - t_1).$$

Let r = batter $= \dfrac{2s}{b - t_1}$,

Axis x, $I = \dfrac{bd^3 - \dfrac{1}{4r}(h^4 - h_1^4)}{12}$.

Axis y, $I = \dfrac{2tb^3 + h_1 t_1^3}{12} + \dfrac{r(b^4 - t_1^4)}{48}$.

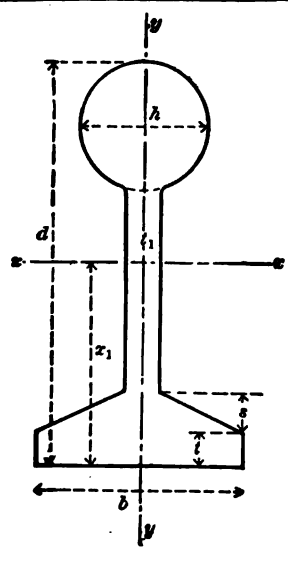


$$A = 2bt + ht_1 + s(b - t_1), \qquad r = \text{batter} = \frac{s}{b - t_1},$$

$$x_1 = \frac{b^2 t + \frac{1}{2} h t_1^2 + \frac{1}{3} s (b - t_1)(b + 2t_1)}{A}.$$

Axis x, $I = \dfrac{bd^3 - \dfrac{1}{8r}(h^4 - h_1^4)}{12}$.

Axis y, $I = \dfrac{2tb^3 + h_1 t_1^3 + \frac{1}{2} r(b^4 - t_1^4)}{3} - Ax_1^2$.



Let area of circular head $= a$,

$$A = a + (d - h)\, t_1 + (b - t_1)(t + \tfrac{1}{2}s),$$

$$x_1 = \frac{a(2d - h) + t_1 (d - h)^2 + bt^2 + s(b - t_1)(t + \frac{1}{3}s)}{2A}.$$

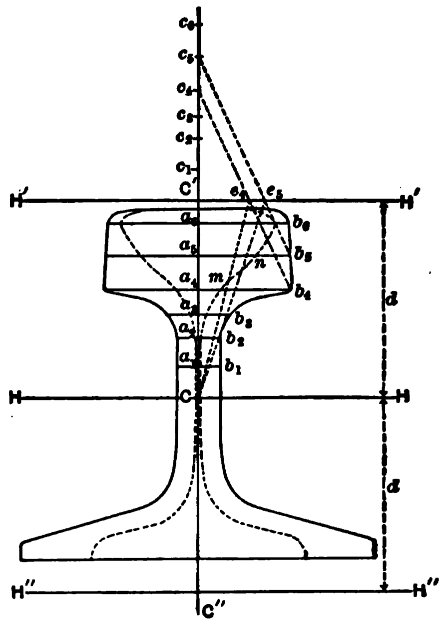
Let $r = \dfrac{2s}{b - t_1}$,

Axis x, $I = a\left[\dfrac{h^2}{16} + \left(d - \dfrac{h}{2}\right)^2\right] + \dfrac{t_1(d - h)^3}{3} + \dfrac{(t + s)^4 - t^4}{6r} - Ax_1^2$.

Axis y, $I = \dfrac{\frac{3}{4} a h^2 + t_1^3 (d - h - t - s) + tb^3 + \dfrac{r}{8}(b^4 - t_1^4)}{12}$.

DETERMINATION OF MOMENT OF INERTIA.—The preceding Table comprises cross-sections of such shape that the moment of inertia can be readily calculated. For more complex cross-sections we may proceed as follows :

First. Graphically.—Draw the cross-section accurately on a piece of cardboard or stiff manilla paper. Then cut it out and balance it on a knife-edge, first along one axis and then along another. The intersection of these two axes will give the centre of mass, C in the accompanying figure.



We can now find the moment of inertia with reference to the axis CH graphically, as follows : Divide the figure into elementary areas by the lines a_1b_1, a_2b_2, etc. Draw $C''CC'$ and $C'H'$, $C''H''$ parallel to CH at any distance d from CH. Lay off $C'c_1$, $C'c_2$, etc., so that $C'c_1 = Ca_1$, $C'c_2 = Ca_2$, etc.

Then for any point b_4 draw b_4c_4 intersecting $C'H'$ at e_4; then Ce_4 intersecting a_4b_4 at m. In the same way for b_5, draw b_5c_5 intersecting $C'H'$ at e_5; then Ce_5 intersecting a_5b_5 at n.*

We thus obtain points m, n, etc., above and below CH, giving the area indicated by a dotted line. Measure this area A by the planimeter. Then we have for the moment of inertia I with reference to CH

$$I = Ad^2.$$

Proof.—We have by construction, for any line a_5b_5,

$$d : Ca_5 :: C'e_5 : a_5n, \quad \text{or} \quad \frac{d}{Ca_5} = \frac{C'e_5}{a_5n}.$$

We have also

$$d : Ca_5 :: a_5b_5 : C'e_5, \quad \text{or} \quad \frac{d}{Ca_5} = \frac{a_5b_5}{C'e_5}.$$

Multiplying, we have

$$\frac{d^2}{\overline{Ca_5}^2} = \frac{a_5b_5}{a_5n}, \quad \text{or} \quad a_5n = a_5b_5 \cdot \frac{\overline{Ca_5}^2}{d^2}.$$

Hence

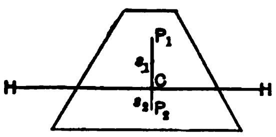
$$a_5n \times a_5a_4 = a_5b_5 \times a_5a_4 \cdot \frac{\overline{Ca_5}^2}{d^2},$$

or

$$d^2 \times a_5n \times a_5a_4 = a_5b_5 \times a_5a_4 \times \overline{Ca_5}^2.$$

But $a_5b_5 \times a_5a_4$ is the area of the slice if the lines of division are close together, and $a_5b_5 \times a_5a_4 \times \overline{Ca_5}^2$ is the moment of inertia of the slice with reference to CH. Also, $a_5n \times a_5a_4$ is the area bounded by the broken line. For all the slices, then, the moment of inertia is the area A bounded by the broken line multiplied by d^2.

Second. By Experiment.—Let C be the centre of mass, and let P_1, P_2 be two points in the same straight line with the centre of mass.



Let the distance of the centre of mass from P_1 be s_1, and from P_2 be s_2. Suspend the body from P_1 like a pendulum, and note the time of vibration. Then suspend it from P_2, and again note the time of vibration. *If these two times are equal*, the radius of gyration r will be given by

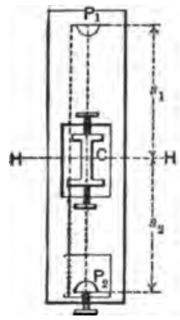
$$r^2 = s_1s_2,$$

or the radius of gyration is a mean proportional between s_1 and s_2.

---

*Note that m is the intersection of Ce_4 with a_4b_4; n the intersection of Ce_5 with a_5b_5, etc. The figure is incorrectly drawn.

If, then, we know the area A, we have the moment of inertia *for the horizontal axis HH* through the centre of mass C in the plane of the cross-section given by



$$I = As_1s_2.$$

In order to suspend the body, we may make use of an apparatus like the following:

Let a graduated prismatic rod be arranged so that it can be swung on knife-edges at P_1 and P_2. The bearing at P_2 is made adjustable with a tangent-screw and vernier, so that the distance s_2 can be accurately measured and changed. The rod has a slot in the centre in which the cross-section can be clamped by adjusting screws.

Take the rod with slot empty. Weigh it and determine its mass M. Balance it and determine the axis HH and the centre of mass C_1 and the distance s_1.

Swing the rod from P_1 and note the time of vibration. Then swing from P_2, and by means of the tangent-screw raise or lower the bearing until the time of vibration is the same as before. Then we have the moment of inertia of the rod with reference to HH.

$$I_r = Ms_1s_2.$$

Now take the cross-section. Weigh it and determine its mass m. Balance it and determine the axis HH and its centre of mass C. Place it in the slot and adjust it by the screws, so that *the axes HH and centres of mass C of cross-section and rod coincide.*

Now swing the entire apparatus from P_1 and P_2 as before, and determine the new values s_1' and s_2'.

We have then the *combined moment of inertia*

$$I_0 = (M + m)s_1's_2'.$$

Subtract from this I_r already found, and we have the moment of inertia I of the cross-section Divide this by the mass m of the cross-section, and we have the radius of gyration r of the cross-section, given by

$$r^2 = \frac{I}{m}.$$

The area of the cross-section can be determined by dividing it into parallelograms, trapezoids, triangles, etc., and finding the area for each. Or we can measnre the area of a sheet of paper and weigh it carefully. Then draw and cut out the cross-section and weigh it. The area of the cross-section will be to the area of the sheet as the weight of the cross-section is to the weight of the sheet. We have then

$$I = Ar^2.$$

Experimental Laws.—Experiments made upon materials have established the following laws:

1. *Set.*—When a small stress, either tensile, compressive or shearing or twisting, is applied to a body, a small corresponding strain is produced.

On removal of the stress, if the body is perfectly elastic and the stress does not exceed a certain amount, the body returns to its original dimensions. If the body is not perfectly elastic, or if the stress exceeds a certain amount, which varies according to the material and character of the stress, the body does not return to its original dimensions. The portion of the strain which thus remains permanent is called the *set.*

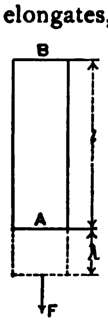
As no body is perfectly elastic, there is probably a small set for every stress, however small. The stress for which the set first becomes noticeable by experiment we may call the *limiting stress for set.*

2. *Elastic Limit.*—So long as the stress does not exceed a certain amount (usually greater than the limiting stress for set), we find that *the strain is proportional to the stress.* The limiting stress up to which, in any case, this law of proportionality of stress to strain is found to practically hold, is called the *elastic limit stress.* No material should be strained beyond this limit. In practice the actual stress is always far within this limiting stress.

The theory of flexure is based upon the assumption that this limiting stress is not exceeded.

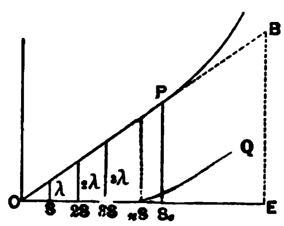
The *unit stress* of the elastic limit is called the elastic limit unit stress, or simply the *elastic limit.* We denote it by S_e.

DETERMINATION OF THE ELASTIC LIMIT.—The limiting unit stress, up to which, in any case, the law of proportionality of stress to strain is found to practically hold, is thus the *elastic limit* S_e. We say "practically," because the precise limit, like that for set, is difficult to determine, if indeed it really exists. In practice, however, it is not difficult to fix by experiment that point beyond which the strain sensibly deviates from the law of proportionality.



Thus let a bar AB of uniform cross-section A have a force F applied to it, which elongates, compresses, shears, twists, or in general *strains* it. In the Figure we suppose a strain of elongation. Let this strain be λ, and the original length be l. The unit stress is then $\frac{F}{A}$.

Now according to the law of proportionality of stress to strain, if $\frac{F}{A}$ is small and well within the elastic limit, if we double $\frac{F}{A}$ we shall observe a double strain 2λ. If we apply a unit stress of $\frac{3F}{A}$ we shall observe a strain 3λ, and so on.



If then we lay off the unit stresses, $S = \frac{F}{A}$, $2S = \frac{2F}{A}$, $3S = \frac{3F}{A}$, etc., to scale along a horizontal line, and lay off the corresponding observed strains λ, 2λ, 3λ, etc., as ordinates, we shall obtain, so long as the unit stress S does not exceed the elastic limit S_e, *a straight line OP.*

By thus carefully plotting the results of experiment, we can locate more or less precisely the point P, at which deviation from the straight line begins. The corresponding unit stress S_e is the elastic limit.

When the unit stress exceeds S_e, we no longer have a straight line, but the strain increases more rapidly than the unit stress, until rupture occurs, and we have from P a curve convex to the horizontal.

Also, if we observe the *set* in each experiment, wehave a similar curve represented by $n\ S - Q$ in the Figure, the ordinates to which give the set for any unit stress greater than nS, which is therefore the limiting unit stress for set.

As we see from the Figure, nS and S_e *are in general not the same.*

COEFFICIENT OF ELASTICITY.—We see at once from the Figure preceding, that within the elastic limit S_e we have, if we denote by λ the strain for any unit stress $\frac{F}{A}$,

$$\frac{F}{A\lambda} = \text{a constant.}$$

If then l is the original length, we have

$$\frac{Fl}{A\lambda} = \text{a constant.}$$

This latter constant is called the *coefficient of elasticity*, and is denoted by E. We have then

$$E = \frac{Fl}{A\lambda}. \qquad \text{(I)}$$

But $\frac{F}{A}$ is the unit stress, or stress per unit of area, and $\frac{\lambda}{l}$ is the unit strain, or strain per unit of length.

We can therefore define the coefficient of elasticity in general as the *unit stress divided by the unit strain.*

Also we can say, that since the unit stress $\frac{F}{A}$ causes the strain λ, then if the law of proportionality of stress to strain held good without limit, it would require as many times this unit stress to cause a strain l as λ is contained in l. Or from the Figure preceding, if we prolong OP until $BE = l$, we have

$$\lambda : \frac{F}{A} :: l : E, \text{ or } E = \frac{Fl}{A\lambda}.$$

We may therefore define the *coefficient of elasticity as that theoretic unit stress which would cause a strain equal to the original length, provided the law of proportionality of stress to strain held good without limit.*

The first definition—*unit stress divided by unit strain*—is, however, the best, most general, and most easily retained in memory.

The value of E thus determined by experiments within the elastic limit is an accurate measure of the elasticity of any material, since, other things being the same, it depends upon the strain caused by a given stress. It varies of course with different materials, and even somewhat with the same material, owing to processes of manufacture, etc. Thus E for iron varies with the kind, whether wrought or cast, and with the shape, whether in bars, rods or wire, etc., owing to difference of treatment in the manufacture.

In any particular case, however, we may consider it as constant. Thus batches of iron produced at the same establishment, from the same ore, by the same processes, ought to be identical in properties. It is therefore assumed in the Theory of Flexure as a constant. Experimental values for E for different materials are given on page 292.

Considering E then as a constant, known in any case, we have from (I)

$$\lambda = \frac{Fl}{AE}. \qquad \text{(2.)}$$

From (2) we can compute the strain due to a given stress when the dimensions are known. Or inversely, knowing the strain, we can compute the stress.

WORK OF STRAINING.—If the stress F is gradually applied, increasing from zero up to F, the average stress is $\frac{F}{2}$ and the work done in straining is, from (2),

$$\text{Work} = \frac{F}{2}\lambda = \frac{F^2 l}{2AE}. \qquad \text{(II)}$$

The work of straining is then, in general, *one half the product of the stress and strain.*

WORK AND COEFFICIENT OF RESILIENCE.—We see from (II) that if S_e is the elastic limit unit stress, the work done in straining the body *up to the elastic limit* is

$$\frac{S_e^2}{2E}Al = \frac{S_e^2}{2E}V,$$

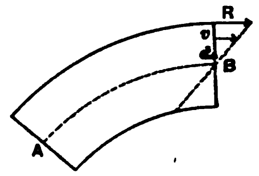
where $V = Al$ is the volume of the body. This is the work which the strained body would perform in coming back to its original dimension. It is therefore called the *work of resilience*. The coefficient $\frac{S_e^2}{2E}$ is called the *coefficient of resilience*.

The work of resilience is then *the work which a body can do in returning to its original dimensions when it has been strained to the elastic limit.*

The coefficient of resilience *is the work per unit of volume under the same circumstances.*

The work of resilience measures the ability of the material to withstand shock or suddenly applied stress. It is therefore a valuable criterion of the value of the material for purposes of construction.

NEUTRAL AXIS OF A BEAM.—When a beam is bent, as shown in the accompanying Figure, the upper fibres are extended and the lower fibres compressed. Between the upper and lower fibres there must then be a horizontal plane AB, the fibres in which are not strained by bending. This is the *neutral plane*. The intersection of this plane by a vertical plane through the axis of the beam is the *neutral axis.*



Above and below this axis the fibre forces of extension and compression are directly proportional to their distance.

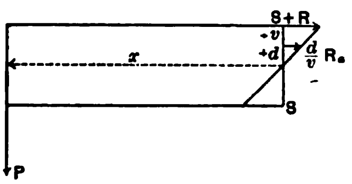
Let R be the fibre unit stress in the most remote fibre, at a distance v from the neutral axis. If a is the area of cross-section of the fibre, then Ra is the stress in the most remote fibre. The stress in any other fibre at a distance d from the neutral axis, positive above and negative below, is then $\frac{d}{v}Ra$. But for equilibrium the sum of all the fibre stresses must be zero, or

$$\Sigma\frac{d}{v}Ra = \frac{R}{v}\Sigma ad = 0.$$

But $\Sigma ad = 0$ is the condition for an axis through the centre of mass.

The neutral axis therefore passes through the centre of mass at each cross-section.

BENDING MOMENT AND RESISTING MOMENT.—The algebraic sum of the moments at any cross-section of all the external forces on the right or left of that section tends to bend the beam. It is therefore called the *bending moment.* We denote it by M. In any case M is known when we know the external forces and their points of application. In taking the algebraic sum, counter clockwise rotation is positive and clockwise rotation negative.



Thus, in the figure, if we have the load P at the end of a beam, the bending moment for any section SS is $M = +Px$ if P is downward and on the left of the section. If P were upward and on left of section, we should have $M = -Px$. For P on right of section we have $M = -Px$ for P downward, and $M = +Px$ for P upward.

The moment M at the section SS must be resisted by the algebraic sum of the moments of the fibre stresses in that section. This is called the *resisting moment.*

Any fibre stress at a distance d from the neutral axis is, as we have seen, given by

$$S = \frac{d}{v}Ra,$$

where a is the area of cross-section of the fibre and R is the stress on the most remote fibre at a distance v. The moment of this fibre stress is then

$$-\frac{d^2}{v}Ra,$$

where R is positive when acting towards the right, negative towards the left, and v is positive above and negative below the neutral axis. Thus in the figure, where R and v are positive, the moment of the fibre stress is negative.

The resisting moment for the entire cross-section is then

$$\Sigma - \frac{d^2Ra}{v} = -\frac{R}{v}\Sigma ad^2.$$

But Σad^2 is the moment of inertia I of the cross-section with reference to an axis in the plane of the section, passing through the centre of mass at right angles to the neutral axis. Hence the resisting moment is

$$-\frac{RI}{v}.$$

For equilibrium we must have the algebraic sum of the bending and resisting moments equal to zero, or

$$M - \frac{RI}{v} = 0 \quad \text{or} \quad M = \frac{RI}{v}. \qquad \text{(III)}$$

In (III) R is positive when acting towards the right, negative towards the left, and v is positive above and negative below the neutral axis. Thus in our figure R and v are positive and M is positive.

WORK OF BENDING A BEAM.—If M is the bending moment at any point of the neutral axis of a beam, whether straight or curved, and s is the length of the neutral axis, then ds is the distance between two consecutive sections at this point, and we have from (2), for the strain in any fibre between two consecutive parallel cross-sections at the point, since $l = ds$ and $M = \frac{FI}{Ad}$

$$\lambda = \frac{Ma \,.\, ds}{EI}, \qquad (3)$$

where d is the distance of the fibre from the neutral axis.

Also, from (III), the stress in the fibre is

$$S = \frac{Mad}{I}. \qquad (4)$$

The work on the fibre is, then,

$$\tfrac{1}{2}S\lambda = \frac{M^2ad^2 \,.\, ds}{2EI^2}.$$

The work on all the fibres of the cross-section is, then, since $\Sigma ad^2 = I$,

$$\frac{M^2\Sigma ad^2 . ds}{2EI^2} = \frac{M^2ds}{2EI},$$

and for all the cross-sections the work is

$$\text{work} = \int_0^s \frac{M^2 ds}{2EI}, \quad \ldots \ldots \ldots \ldots \ldots \quad \text{(IV)}$$

Equation (IV) is general whatever the shape of the beam. If the beam is straight we can put the length l for s, and dx for ds, and have

$$\text{work} = \int_0^l \frac{M^2 dx}{2EI}. \quad \ldots \ldots \ldots \ldots \ldots \quad \text{(IV')}$$

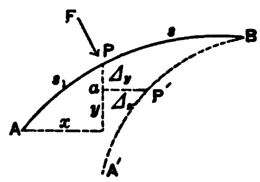
DEFLECTION OF A BEAM.*—Let M be the actual bending moment at any point of the neutral axis. Then as before, from (3), the strain in any fibre between two consecutive parallel cross-sections at the point, due to the actual loading is

$$\lambda = \frac{Md \,.\, ds}{EI},$$

where d is the distance of the fibre from the neutral axis.

Let AB, Fig. 163, be the neutral axis before deflection and $A'B$ that after. Take the origin at A, and let x, y, be the co-ordinates of any point P. At this point suppose a force F to act, and let its moment with reference to any point between P and the end B be m.

Fig. 163.



Then from (4) the stress *due to this force F* in any fibre is

$$S = \frac{mad}{I}.$$

The work *of this force F* on any fibre is then

$$\tfrac{1}{2}S\lambda = \frac{Mmad^2 . ds}{2EI^2}.$$

On all the fibres of a cross-section, since $\Sigma ad^2 = I$, it is

$$\frac{Mmds}{2EI},$$

and for all the cross-sections between B and P, if $AB = s$ and $AP = s_1$, it is

$$\text{work} = \int_{s_1}^{s} \frac{Mmds}{2EI}. \quad \ldots \ldots \ldots \ldots \ldots \quad \textbf{(5)}$$

This equation is general whatever the direction of F.

Suppose F, in Fig. 163, to be *vertical* and let it cause a moment m at any point between P and B in *the same direction* as M. Let $\bar{x}$ be the abscissa of that point.

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* For application of this method to a *framed* beam, see Chap. VI., page 153.

Then we have

$$m = F(\bar{x} - x),$$

where m has the same sign always as M. From (5) we have for the work in this case of F, from B to P.

$$\text{work} = \int_{s_1}^{s} \frac{MF(\bar{x} - x)ds}{2EI},$$

which is always positive, since the product mM is always positive.

Let Δ_y be the vertical deflection Pa of the point P, positive upwards and negative downwards. Then the work of F is also

$$\text{work} = \pm \frac{F\Delta_y}{2},$$

where the (+) sign is taken when Δ_y is upwards and the (−) sign when Δ_y is downwards, so that the work is always positive. Equating these two values of the work of F and dividing both sides by F, we have

$$EI\Delta_y = \int_{s_1}^{s} \pm M(\bar{x} - x)ds. \quad \text{(V)}$$

If the beam is straight, we have $y = 0$, $ds = d\bar{x}$; and if l is the length of beam, putting y for Δ_y, we have

$$EIy = \int_{x}^{l} \pm M(\bar{x} - x)d\bar{x}, \quad \text{(V}')$$

where y is the *deflection* at any point P of a straight beam, given by x.

Since the deflection must have the same sign as M when M is taken for all forces on the right, and the opposite sign from M when M is taken for all forces on the left, we take in (V) and (V′) the (+) sign in the first case and the (−) sign in the second.

Again, suppose F in Fig. 163 to be *horizontal*. Let $\bar{y}$ be the ordinate of any point between P and B. Then we have

$$m = F(\bar{y} - y),$$

and from (5) the work in this case of F from B to P is

$$\text{work} = \int_{s_1}^{s} \frac{MF(\bar{y} - y)ds}{2EI}.$$

Let Δ_x be the horizontal deflection aP' of the point P positive to the right, negative to the left. Then the work of F is

$$\text{work} = \pm \frac{F\Delta_x}{2}.$$

Equating these two values of the work of F, and cancelling F, we have

$$EI\Delta_x = \int_{s_1}^{s} \pm M(\bar{y} - y)ds, \quad \text{(VI)}$$

where the (+) and (−) signs are taken as before.

For a straight beam, $y = 0$, $\bar{y} = 0$, and the horizontal deflection is zero.

We can write (V') in the form

$$EIy = \int_x^l \pm M\bar{x}d\bar{x} - x\int_x^l \pm Md\bar{x}.$$

In this form we can replace $\bar{x}$ by x, and $d\bar{x}$ by dx, and have

$$EIy = \int_x^l \pm Mxdx - x\int_x^l \pm Mdx.$$

If we differentiate this, we have

$$EIdy = \pm Mxdx \mp Mxdx - dx\int_x^l \pm Mdx,$$

or,

$$EI\frac{dy}{dx} = \int_l^x \pm Mdx, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VII.)}$$

where the (+) and (−) signs are taken as before.

Equation (VII.) gives for a straight beam the tangent $\frac{dy}{dx}$ of the angle which the tangent to the curve of the deflected neutral axis at any point makes with the axis of x.

If we differentiate (VII.) we have

$$EI\frac{d^2y}{dx^2} = \pm M, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VIII.)}$$

where the (+) and (−) signs are taken as before.

Equation (VIII.) is the differential equation of the curve of the deflected neutral axis, for a straight beam. If we integrate it once, we obtain (VII.). If we integrate it again, we obtain (VI.).

The radius of curvature of a curve ρ, is by calculus, approximately

$$\frac{1}{\rho} = \pm \frac{d^2y}{dx^2}.$$

Hence we have

$$M = \frac{EI}{\rho}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.)}$$

where M is taken without regard to sign.

RECAPITULATION.—For convenience of reference we group together the preceding fundamental equations.

Coefficient of elasticity E is equal to unit stress divided by unit strain. If S is the stress, A the area of cross-section, λ the strain, and L the original length,

$$E = \frac{SL}{A\lambda}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I.)}$$

Work of straining is one-half the product of stress and strain, or

$$\text{work} = \tfrac{1}{2}S\lambda = \frac{S^2L}{2AE} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

If M is the bending moment at any point of the neutral axis, and R the unit stress in the most remote fibre at a distance v, then

$$M = \frac{RI}{v}, \quad \dots \dots \dots \dots \quad \text{(III.)}$$

where I is the moment of inertia of the cross-section at the point, with reference to a horizontal axis in the plane of the cross-section through its centre of mass, at right angles to the neutral axis. The values of I for use in (III.) are given in the table, page 272.

Only the absolute values of R, v, and M are required in (III.), without reference to sign.

If S is the length of the neutral axis of a beam, the work of bending is

$$\text{work} = \int_o^s \frac{M^2 ds}{2EI} \quad \dots \dots \dots \dots \quad \text{(IV.)}$$

This holds for any shape of neutral axis.

For a *straight* beam of length l

$$\text{work} = \int_o^l \frac{M^2 dx}{2EI} \cdot \quad \dots \dots \dots \dots \quad \text{(IV'.)}$$

For the vertical deflection Δ_y of any point of the neutral axis of a beam of any shape, we have

$$EI\Delta_y = \int_{s_1}^{s} \pm M(\bar{x} - x)ds, \quad \dots \dots \dots \dots \quad \text{(V.)}$$

where s is the length of neutral axis, s_1 the length to the point from the origin, $\bar{x}$ the abscissa of any point, and x the abscissa of the point at which the deflection is required.

We take the $(+)$ sign when M is taken for all forces on the right, the $(-)$ sign when M is taken for all forces on the left of the point. Δ_y is positive upwards, negative downwards.

For a *straight* beam, we have the deflection y given by

$$EIy = \int_x^l \pm M(\bar{x} - x)d\bar{x}, \quad \dots \dots \dots \dots \quad \text{(V'.)}$$

where l is the length, $(+)$ and $(-)$ signs as in (V).

For the horizontal deflection Δ_x of any point of the neutral axis of a beam of any shape we have

$$EI\Delta_x = \int_{s_1}^{s} \pm M(\bar{y} - y)ds, \quad \dots \dots \dots \dots \quad \text{(VI.)}$$

where $\bar{y}$ is the ordinate of any point and y the ordinate of the point at which the deflection is required; $(+)$ and $(-)$ signs as in (V).

The tangent $\frac{dy}{dx}$ of the angle which the tangent to the curve of the deflected neutral axis of a straight beam makes at any point with the axis of x is given by

$$EI\frac{dy}{dx} = \int_l^x \pm M dx, \quad \dots \dots \dots \dots \quad \text{(VII.)}$$

The differential equation of the curve is

$$EI\frac{d^2y}{dx^2} = \pm M. \quad \dots \dots \dots \dots \quad \text{(VIII.)}$$

In all equations the $(+)$ sign is taken when M is taken for all forces on the right and the $(-)$ sign when M is for all forces on the left. M is always taken with its proper sign, $(+)$ for counter clockwise and $(-)$ for clockwise rotation.

We have also for the radius of curvature ρ

$$M = \frac{EI}{\rho}, \quad \ldots \ldots \ldots \ldots \ldots \quad \text{(IX.)}$$

where only the absolute value of M is required without reference to sign.

These are the fundamental equations of the Theory of Flexure, so far as beams are concerned. It only remains to give their application.

Assumptions upon which the Theory of Flexure is Based.—A close examination of the foregoing will reveal the assumptions which lie at the bottom of the theory. Thus we have assumed, first, that the coefficient of elasticity is constant. Second: That fibres at equal distances above and below the neutral axis are equally strained, and hence the neutral axis passes through the centre of mass of the cross-section. Third: That the deflection is small compared to the length. Fourth: That any two plane sections remain plane after flexure. Fifth: That the elastic limit is not exceeded.

Upon these assumptions the theory rests. The comparison of its results with experiment shows them to be correct, *so long as the elastic limit is not exceeded.*

Crippling or Limit Load—Breaking Load.—Let max. M be the maximum moment at any point, and let S_e be the unit stress of the most remote fibre at the elastic limit. Then we have, from (III.),

$$max\, M = \frac{S_e I}{v}. \quad \ldots \ldots \ldots \ldots \ldots \quad (1)$$

This equation gives us at once, in any case, the load which will strain a beam to its elastic limit. We call this load the *crippling load*, or *limit load.* It marks the limit beyond which the beam should not be loaded. Beyond this limit the Theory of Flexure does not hold.

Take for instance a rectangular beam of constant cross-section, of breadth b and depth d, and length l, fixed horizontally at one end and loaded with P at the free end. Then $I = \frac{1}{12} bd^3$, $v = \frac{d}{2}$, the maximum moment will be Pl at the fixed end, and the most remote fibres will be most strained at this end. We have then for the crippling or limit load

$$Pl = \frac{S_e bd^3}{12\frac{d}{2}} = \frac{S_e bd^2}{6} \quad \text{or} \quad P = \frac{S_e bd}{6l}.$$

If now we know S_e we can find P.

But S_e is in general not the same for compression and tension. The beam will therefore fail in either the compressive or tensile outer fibres, according as S_e is *least* for compression or tension. Moreover, the value of S_e is not the same for pure tension or pure compression as it is for flexure. It is also difficult to determine S_e for a beam by experiment, and no such experiments are at hand.

In these circumstances, the best we can do is to take for S_e the *least* of the two values for pure tension and pure compression as given by experiment, and *for v the distance to the outer fibre in which this least S_e occurs.*

The customary method of estimating the strength of a beam is by loading a beam *to the point of rupture* and, from equation (III), determining the value of R. The value of R being thus known by direct experiment, we can find the *breaking weight* by

$$\max. M = \frac{RI}{v}. \quad \ldots \ldots \ldots \ldots \quad (2)$$

We can then adopt a factor of safety, and thus arrive at the safe load.

This use of equation (III) is employing the Theory of Flexure *beyond the elastic limit.*

Equation (2), then, is a purely empirical formula, whose *form only* is given by theory.

If experiments are not at hand for the value of R at the breaking point, we can replace R by the tensile strength T per square inch, or the compressive strength C per square inch, *whichever is the least*, and *take for v the distance to the outer fibre in which this least stress occurs.*

We have, then, for crippling or limit load

$$\max. M = \frac{S_e I}{v}, \quad \ldots \ldots \ldots \ldots \quad (X.)$$

where S_e is the *least* elastic limit unit stress, either for pure tension or pure compression, and *v is the distance to the outer fibre in which this least S_e occurs.*

For the breaking load we have

$$\max. M = \frac{RI}{v}, \quad \text{or} \quad \frac{(T \text{ or } C)I}{v}, \quad \ldots \ldots \ldots \quad (XI.)$$

where R is the unit stress as determined by experiment, and *v the distance to the most remote fibre ;* or, in the lack of experiments, we take the tensile strength T or compressive strength C for pure tension or compression, whichever is the least, and *for v the distance to the outer fibre in which this least stress occurs.*

SHEARING STRESS.—The algebraic sum of the components parallel to a section at any point of all the external forces *on the left* of that section we call the *shearing stress* for that section.

It is the force which tends to make one section slide upon the next consecutive section on the right.

In the case of a horizontal beam acted upon by vertical forces only, the algebraic sum of all the forces *on the left* of any vertical cross-section is the shearing stress for that section. Upward forces are taken as positive, downward forces as negative, in taking the algebraic sum.

We give in the following Table the values of C, T, R and E, for all materials of usual occurrence, in pounds per square inch. We also give the value of the shearing strength S and the average weight per cubic foot.

The authority quoted is given in the second column, and the name of the experimenter, when known, is indicated by one of the following abbreviations : B = Barlow, Bv = Bevan, C = Clark, D = Denison, F = Fairbairn, G = Grant, H = Hodgkinson, Hl = Hill, K = Kirkaldy, K C = Keystone Bridge Co., M = Moore, Mu = Muschenbroeck, Re = Rennie, Ro = Rondelet, T = Tredgold, Wd = Wade, Wi = Wilkinson. The table is an extension of that given by J. D. Crehore, C. E., "Mechanics of the Girder,"—Wiley & Sons, 1886.

MATERIAL.	Authority	C Lbs. per sq. in. compressive strength.	T Lbs. per sq. in. tensile strength.	R Lbs. per sq. in. cross breaking strength by rupture.	S Lbs. per sq. in. shearing strength.	E Lbs. per sq. in. coefficient of elasticity.	Weight in lbs. per cubic ft.
CAST IRON.							
Average	Wood	96,000	16,000	36,000	....	17,000,000	
Cannon specimens	Lanza	84,500 to 175,000 Wd	20,148 to 28,805 Wd	....	....	....	
Mean of 9 specimens	Stoney	105,945 H	16,720 H	37,695 H	....	....	
Mean of 16 specimens	Stoney	86.284 H	15.298 H	....	....	12,000,000	
Bars less 1 inch wide	Stoney	....	....	45,696 C	....	....	
Bars 3 inches wide	Stoney	....	....	30,240 C	....	....	

MATERIAL.	Authority	C Lbs. per sq. in. compressive strength.	T Lbs. per sq. in. tensile strength.	R Lbs. per sq. in. cross-breaking strength by rupture.	S Lbs. per sq. in. shearing strength.	E Lbs. per sq. in. coefficient of elasticity.	Weight in lbs. per cubic foot.
CAST IRON (*Cont.*).							
Bars small round	Stoney			26,880 C			450
Circular tubes	Stoney			38,304 C			
Square tubes	Stoney			45,965 C			
Various qualities	Lanza	82,000 to 145,000	13,400 to 29,000	30,000 to 43,500	16,000 to 24,740	14 to 29,000,000	
Average	Bovey	100,000	15,000		18,000	17,000,000	
Average market value	Bovey	76,000	12,000		18,000		
Very good	Bovey		22,000 to 27,000				
Average	Weisbach		18,500			14,220,000	
Average	Rankine	112,000	16,500	38,250		17,000,000	
WROUGHT IRON.							
Bars rolled	Wood		57,557				
Angle iron	Wood	30,000	54,729	33,000		24,000,000	
Plates, lengthways	Wood		50,737				
Plates, crossways	Wood		46,171				
Bars, new	Stoney			51,341 C			
Bars, previously strain'd	Stoney			74,995 C			480
Bars, new, round	Stoney			30,240 C			
Boiler tubes, welded	Stoney			70,291 C			
Circular tubes, riveted	Stoney			43,814 C			
Rolled I beams	Stoney			61,824 C			
T iron, flange up	Stoney			53,760 C			
T iron, flange down	Stoney			51,475 C			
Average	Stoney	40,320	57,555 K	52,567 C		24,000,000	
Bars and Bolts	Rankine	36,000	60,000				
Bars and Bolts	Rankine	40,000	70,000			29,000,000	
Plates	Rankine		51,000				
Plates, double riveted	Rankine		35,700				
Plates, single riveted	Rankine		28,600				
Hoops, best-best	Rankine		64,000				480
Wire	Rankine		70,000				A bar one square inch in cross section and 3 feet long weighs 10 lbs.
Wire	Rankine		100,000			25,300,000	
Wire ropes	Rankine		90,000			15,000,000	
Plate beams	Rankine			42,000			
Mean of 113 tests	Lovett		50,915				
Mean of 27 tests	Lovett					27,300,000	
Low average	Bovey	32,000			40,000		
Bar average	Bovey	26,000 to 66,000	40,000 to 52,000	33,000 to 58,000	29,000 to 42,000	29,000,000	
Market bars, full size	Bovey		41,000 to 44,000				
Market bars, prepared	Bovey		44,000 to 46,000				
L, T, and other sections	Bovey		44,766				
Plate, average	Bovey		41,000 to 44,733				
Plate, prepared	Bovey		42,000				
Plates, punched	Bovey				45,000 to 54,000		
Iron wire	Bovey		62,000 to 89,000			25,300,000	
STEEL.							
Bessemer, hammered	Stoney	225,568 F	83,391 F	128,083 K		31,000,000	
Bessemer, rolled	Stoney		71,658 K	115,181 K			
Crucible, hammered	Stoney		85,546 K	147,840 K			
Crucible rolled	Stoney		68,589 K	118,272 K			
Cast, not hardened	Stoney	198,914 Wd					
Cast, low temper	Stoney	354,544 Wd					
Cast, mean temper	Stoney	391,985 Wd					490
Cast, high temper	Stoney	372,598 Wd					
6 eye bars ¾″ round	Lanza		73,150 K C			28,210,000	
6 rolled and annealed	Lanza		69,470 K C			29,210,000	
Bars	Rankine		100,000			29,000,000	
Bars	Rankine		130,000			42,000,000	
Plates, average	Rankine		80,000				
Plates	Lanza		77,840 to 86,330 Hl				
Plates, L and T bars	Bovey	60,000 to 80,000	60,000 to 80,000				
Bessemer, average	Bovey		56,000	80,000 to 129,000	48,000	30,000,000	
WOOD.							
Alder	Stoney	6,831 H	13,900 Mu				50
Apple	Bovey		17,600	5,300 to 7,000			50
Ash	Stoney	9,363 H	16,700 Bv	12,156 B	1,250	1,525,000	43 to 53
Ash	Rankine	9,000	17,000 B	13,000		1,600,000	47
Beech	Rankine	11,500	9,360	10,500		1,350,000	43 to 53
Beech	Stoney	9,363 H	11,500 B	9,366 B			
Beech	Stoney		17,300 Mu				
Birch, American	Stoney	11,663 H		12,366 B		1,645,000	45 to 49
Birch, English	Stoney	6,402 H	15,000 Bv	11,568 B			
Box	Stoney	8,000	20,000 B	14,670 T		1,800,000	64
Box	Rankine	10,300	20,000				
Cedar, American	Stoney	5,000	10,000	4,596 D		486,000	35 to 47
Cedar, Lebanon	Rankine	5,860	11,400	7,400		486,000	
Chestnut, Spanish	Stoney	5,060	13,300 Ro		616	1,140,000	35 to 41
Chestnut	Rankine	5,350	11,500	10,660		1,140,000	
Deal, Christiana	Stoney		12,900 Bv	9,372 B			43
Deal, red	Stoney	6,586 H					
Deal, white	Stoney	7,293 H					

MATERIAL.	Authority	*C* Lbs. per sq. in. compressive strength.	*T* Lbs. per sq. in. tensile strength.	*R* Lbs. per sq. in cross-breaking strength by rupture.	*S* Lbs. per sq. in. shearing strength.	*E* Lbs. per sq. in. coefficient of elasticity.	Weight in lbs. per cubic foot.
WOOD (*Cont.*).							
Elm	Rankine	10,300	14,000	7,850	1,250	1,020,000	34 to 37
Elm	Stoney	10,331 H	14,400 Bv				
Elm, English	Stoney			4,692 B			
Fir, spruce	Stoney	6,819 H	9,000	8,076 M	420	1,800,000	29 to 32
Fir, red pine	Rankine	5,375	12,000	7,100		1,460,000	
Fir, red pine	Rankine	6,200	14,000	9,540		1,900,000	
Fir, larch	Rankine	5,570	9,000	5,000		900,000	
Fir, larch	Rankine		10,000	10,000		1,360,000	
Hemlock	Stoney			6,852 D	480		47
Larch	Stoney	5,568 H	10,220 Ro	8,010 B	860 to 1,520	1,360,000	32 to 38
Lignum Vitae	Rankine	8,920	11,800	12,000		1,000,000	41 to 83
Locust	Rankine	4,500	16,000	11,200	1,070		58
Locust	Stoney		20,100 Mu	20,580 B			
Mahogany	Rankine	6,600	8,000	7,600		1,255,000	53
Mahogany	Rankine	8,200	21,800	11,500			
Mahogany	Stoney	8,198 H	8,000 B			3,000,000	
Mahogany	Stoney		16,500 Bv	10,314 M			
Maple	Stoney		17,400 Bv	10,164 D			49
Maple	Rankine	8,150	10,600				
Oak, European	Rankine	7,700	10,000	8,700	2,680 to 4,460	1,200,000	49 to 58
Oak, European	Rankine	10,000	19,800	13,600	6,960	1,750,000	
Oak, American red	Rankine	6,000	10,250	10,600		2,150,000	61
Oak, English	Stoney	10,058 H	10,000 B	10,164 B			49 to 58
Oak, English	Stoney	5,780 to 8,980	19,800 Bv				
Oak, French	Stoney		13,950 Ro	8,898 M			
Oak, Quebec	Stoney	5,982 H					
Oak, American red	Stoney			10,122 D			61
Oak, American white	Stoney			10,458 B			
Pine, American red	Stoney	7,518 H	2,400 to 7,200	9,162 B	440 to 720	1,960,000	34
Pine, American pitch	Stoney	6,000	7,650 Mu	10,362 B		1,252,000	41 to 58
Pine, American white	Stoney		2,600 to 6,600	7,374 D	440	2,300,000	36
Pine, American yellow	Stoney	5,445 H	4,400 to 10,600	7,110 B	454	1,600,000	32
Pine, Norway	Stoney		14,300 Bv			3,000,000	
Pine, Norway	Stoney		7,287 Bv			2,350,000	
Poplar	Bovey	2,760 to 4,560	5,360 to 6,400			763,000	23 to 26
Sycamore	Rankine	6,320	13,000	9,600		1,040,000	36 to 43
Sycamore	Stoney	7,082 H	13,000 Bv				
Teak	Stoney	12,101 H	15,000 Bv	12,648 B		2,100,000	41 to 52
Teak, Indian	Rankine	12,000	15,000	12,000 to 19,000		2,400,000	
Walnut	Stoney	7,227 H	8,130 Mu	8,000			38 to 57
Walnut	Stoney	6,400	7,800 Bv				
Willow	Stoney	6,128 H	14,000 Bv	3,300 to 4,700		1,400,000	24 to 35
Willow	Rankine	5,400 to 2,600	9,000 to 12,500	6,600			
STONE.							
Granite	Stoney	3,173 to 13,440 Wi		456 to 2,442 Wi			168
Granite	Rankine	4,000 to 11,000					
Limestone	Stoney	3,050 F to 18,043 Wi	670 to 2,800	1,698 to 2,484 Wi			96
Marble	Stoney	200,160 Wi to 3,216 Re	551 H to 722 Bv	1,252 H to 2,697 H			96
Sandstone	Stoney	2,185 to 7,884	1,054 to 1,261	2,010 to 5,142 Re			150
Sandstone	Rankine	2,200 to 5,500		1,100 to 2,360			
Slate	Rankine	17,344	9,600 to 12,800	5,000 to 7,370		1,300,000 to 1,600,000	175
Bricks, pale red	Stoney	562 Re					150
Bricks, red	Stoney	808 Re					
Bricks, fire	Stoney	1,717 Re					
Bricks, Gault clay	Stoney	2,240 G					
Bricks, ordinary	Rankine		280 to 300				125
Lime, mortar	Stoney	618 Ro	51				100
Portland cement	Stoney	5,984 G	358 G				80
Plaster of Paris	Stoney		71 Ro				144
Roman cement, 2 years	Stoney		546 G				80
Roman cement, 3 years	Stoney		604 G				
Roman cement, 4 years	Stoney		632 G				
Roman cement, 5 years	Stoney		627 G				
Roman cement, 6 years	Stoney		666 G				
Roman cement, 7 years	Stoney		709 G				

In using our formulas, all dimensions should be in inches, if T, C, R, E are in lbs. or tons per *square inch*, and the result P will then be in lbs. or tons. If the dimensions are all taken in feet, T, C, R and E must be taken in lbs. or tons per *square foot*.

APPLICATION OF THEORY TO BEAMS.*

We can now apply our fundamental equations to the various cases of beams which occur in practice.

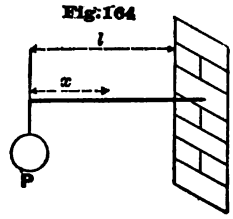
The complete discussion consists in finding the change of shape of the beam, its deflection at any point, and the breaking weight or the load it will carry before breaking; both for constant

* For curved beams see page 214.

cross-section and for uniform strength, as well as the proper shape for uniform strength. A beam is said to be of uniform strength when it is so proportioned that we have the same unit stress at all points.

CASE I. BEAM FIXED AT ONE END AND LOADED AT THE OTHER—CONSTANT CROSS-SECTION.—A moment is always positive when it causes counter-clockwise rotation, negative when its rotation is clockwise. Therefore a positive moment *on the left* of any cross-section causes tension in the upper fibres and compression in the lower fibres.

Fig. 164



(*a*) *Deflection and Change of Shape.*—In Fig. 164, take the origin at the free end; let the length be l. Then for any point of the neutral axis at a distance x from the origin the moment is

$$M = +Px.$$

We have, then, from (VIII.), page 290, since M is taken for all forces on the left,

$$EI\frac{d^2y}{dx^2} = -M = -Px. \quad (1)$$

Integrating once, we have

$$EI\frac{dy}{dx} = -\frac{Px^2}{2} + C. \quad (2)$$

Since the beam is fixed horizontally at the right end, the tangent to the curve of deflection must be horizontal at that end. Hence when $x = l$, $\frac{dy}{dx} = 0$, and the constant is $C = +\frac{Pl^2}{2}$. We have, then, from (2),

$$EI\frac{dy}{dx} = -\frac{Px^2}{2} + \frac{Pl^2}{2}. \quad (3)$$

We should obtain the same result directly from (VII.), page 290.

Integrating again,

$$EIy = -\frac{Px^3}{6} + \frac{Pl^2x}{2} + C. \quad (4)$$

Since the deflection at the fixed end is zero, for $x=l$, $y=0$, and hence $C = \frac{Pl^3}{6} - \frac{Pl^3}{2} = -\frac{Pl^3}{3}$.

We have, then, from (4),

$$EIy = -\frac{Px^3}{6} + \frac{Pl^2x}{2} - \frac{Pl^3}{3}. \quad (5)$$

We should obtain the same result directly from (V.′), page 290.

This equation gives the deflection at any point. The deflection at the free end is evidently the greatest. Making, then, $x = 0$, we have the maximum deflection

$$\Delta = -\frac{Pl^3}{3EI}. \quad (6)$$

The minus sign shows that the deflection is downwards.

If the cross-section is rectangular,

$$I = \tfrac{1}{12}bh^3,$$

and we have

$$\Delta = -\frac{4Pl^3}{Ebh^3}.$$

The student should solve this and other cases by taking the origin at different places, such as the right end, at the free end *after deflection*, etc. He should also reverse Fig. 164, so as to have the left end fixed, and take the origin in different places as before.

(*b*) *Breaking Load.*—In order to find the breaking load, we have from (XI.), page 292,

$$Pl = \text{max. } M = \frac{RI}{v}, \text{ or } \frac{(T \text{ or } C)I}{v}, \text{ or } P = \frac{RI}{vl}, \text{ or } \frac{(T \text{ or } C)I}{vl}$$

where R is the most remote fibre stress as determined by experiments at the breaking point, or if R is not known, we take the tensile strength T or the compressive strength C, whichever is the least, and for v *the distance to the outer fibre in which this least stress occurs.*

For rectangular cross-section, $I = \frac{1}{12}bh^3$, $v = \frac{h}{2}$, and we have breaking load given by

$$Pl = \frac{Rbh^2}{6}, \quad \text{or} \quad P = \frac{Rbh^2}{6l}, \quad \text{or} \quad \frac{(T \text{ or } C)bh^2}{6l}.$$

CASE 2. BEAM FIXED AT ONE END AND LOADED AT THE OTHER—UNIFORM STRENGTH.—Suppose the cross-section or I is not constant as before, but varies in such a manner that at every point of every cross-section the unit stress is constant. Then we have from (XI.), page 292,

$$Px = \frac{RI}{v}, \quad \text{or} \quad R = \frac{Pvx}{I}.$$

For a rectangular cross-section, for instance, $v = \frac{h}{2}$, $I = \frac{1}{12}bh^3$, and

$$R = \frac{6Px}{bh^2}.$$

This, then, gives the value of R at any point distant x from the end. Suppose the breadth and height at the fixed end are denoted by b_1 and h_1. Then,

$$R = \frac{6\,Pl}{b_1h_1^2}.$$

Now since R is required to be constant, we have,

$$\frac{6\,Px}{bh^2} = \frac{6\,Pl}{b_1h_1^2}, \quad \text{or} \quad \frac{bh^2}{b_1h_1^2} = \frac{x}{l} \quad \ldots\ldots\ldots\ldots \quad (9)$$

If the height is constant, then $h = h_1$, and we have the breadth at any point $b = b_1\frac{x}{l}$. That is, the breadth varies as the ordinates to a straight line, as shown in Fig. 165. If, on the other hand, the breadth is constant, $b = b_1$, and we have $h^2 = h_1^2\frac{x}{l}$. That is, the height varies as the ordinates to a parabola, as shown in Fig. 166.

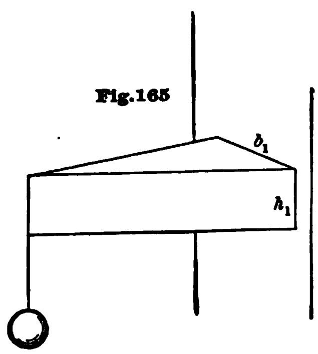


Fig.165

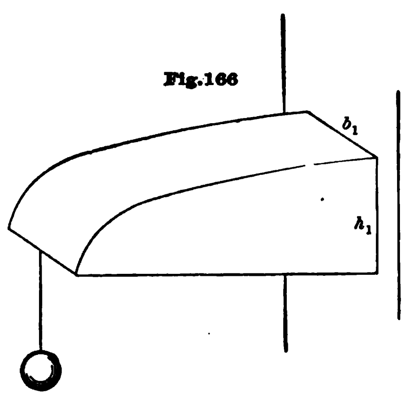


Fig.166

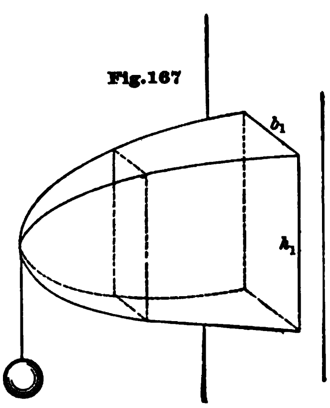


Fig.167

If both b and h vary, but the cross section at all points is similar, we have,

$$\frac{b_1}{h_1} = \frac{b}{h}, \quad \text{or}\; b = \frac{b_1h}{h_1},$$

and hence substituting in (9), $h^3 = h_1^3 \frac{x}{l}$, which is the equation of a cubic parabola. The breadth varies according to the same law, as shown in Fig. 167.

(*a*). *Deflection and Change of Shape.*—Since I is no longer constant, we have in the present case, from (VIII),

$$\frac{d^2y}{dx^2} = -\frac{Px}{EI} = -\frac{Px}{E \times \frac{bh^3}{12}},$$

where b and h are variable, as we have just seen. If, as in Fig. 165, the height is constant and always equal to h_1, then, as we have seen, $b = b_1 \frac{x}{l}$.

Hence for rectangular cross section,

$$\frac{d^2y}{dx^2} = -\frac{12\,Pl}{E h_1^3 b_1}.$$

Integrating this, since for $x = l$, $\frac{dy}{dx} = 0$, we have

$$\frac{dy}{dx} = -\frac{12\,Plx}{E\,h_1^3 b_1} + \frac{12\,Pl^2}{E\,h_1^3 b_1}.$$

Integrating again, since for $x = l$, $y = 0$, we have

$$y = -\frac{6\,Plx^2}{Eh_1^3 b_1} + \frac{12\,Pl^2x}{Eh_1^3 b_1} - \frac{6\,Pl^3}{Eh_1^3 b_1} \quad \text{. . . . . (10)}$$

This equation gives the deflection at any point for a beam, as shown in Fig. 165.

The greatest deflection will be at the end, and is equal to

$$\Delta = -\frac{6\,Pl^3}{E\,h_1^3 b_1}.$$

The deflection for a beam of the same length with constant cross-section, we have already found to be $-\frac{4\,Pl^3}{E\,b_1 h_1^3}$ for rectangular cross-section. We see, then, that, other things being the same, the beam of uniform strength deflects $\frac{3}{2}$ as much as the beam of constant cross-section.

In similar manner we find for constant breadth, Fig. 166,

$$y = -2\,\Delta_0\left[1 - 3\frac{x}{l} + 2\sqrt{\left(\frac{x}{l}\right)^3}\right]. \quad \text{. . . . . (11)}$$

$$\Delta = 2\,\Delta_0 = -\frac{8\,Pl^3}{E\,b_1 h_1^3},$$

where Δ_0 stands for the deflection of the beam of constant cross-section, or $\frac{4\,Pl^3}{E\,b_1 h_1^3}$.

For similar cross-sections, Fig. 167, we have

$$y = -\frac{6}{5}\,\Delta_0\left[1 - \frac{5}{2}\frac{x}{l} + \frac{3}{2}\sqrt[3]{\left(\frac{x}{l}\right)^5}\right]. \quad \text{. . . . . (12)}$$

$$\Delta = \frac{6}{5}\,\Delta_0 = -\frac{24}{5}\,\frac{Pl^3}{E\,b_1 h_1^3}.$$

If we call the volume of the beam of constant cross section V, then in the first case, Fig. 165, the volume $V_1 = \frac{1}{2} V$; in the second, Fig. 166, $V_2 = \frac{2}{3} V$; in the third, Fig. 167, $V_3 = \frac{3}{5} V$, or

$$V : V_2 : V_3 : V_1 = 30 : 20 : 18 : 15.$$

The maximum deflections, as we see, are as

$$2\,\Delta_0, \quad \tfrac{9}{5}\,\Delta_0, \quad \tfrac{3}{2}\,\Delta_0, \quad \text{or as } 20, 18 \text{ and } 15.$$

That is, the deflections at the ends for a beam of uniform strength in the three cases *are as the volumes*.

(*b*) *Breaking Strength.*—We have, just as in the case of constant cross-section,

$$P = \frac{RI_1}{vl} \quad \text{or} \quad \frac{(T \text{ or } C)I_1}{vl}$$

where I_1 is the moment of inertia of the cross-section at the fixed end $= \frac{1}{12} b_1 h_1^3$ for rectangular cross section.

The breaking weight is evidently the same as for beam of constant cross-section, if the weight of beam itself be disregarded. The only difference is, that more material is required in the latter case.

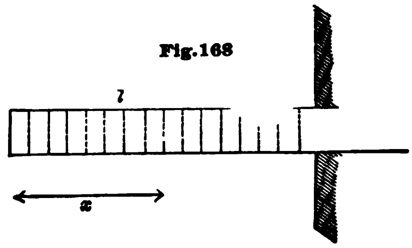


Fig.168

Case 3.—Beam as Before, Fixed at One End—Uniform Load—Constant Cross-section.—If p is the load per unit of length, we have for the moment at any point distant x from the free end, Fig. 168, from (VIII.),

$$EI\frac{d^2y}{dx^2} = -px \times \frac{x}{2} = -\frac{px^2}{2}.$$

Integrating once, since for $x = l$, $\frac{dy}{dx} = 0$, we have

$$EI\frac{dy}{dx} = -\frac{px^3}{6} + \frac{pl^3}{6}.$$

Integrating again, since for $x = l$, $y = 0$, we have

$$EIy = -\frac{px^4}{24} + \frac{pl^3x}{6} - \frac{pl^4}{8}. \quad \ldots\ldots\ldots\ldots \quad (13)$$

The deflection at the end, then, is

$$\Delta = -\frac{pl^4}{8\,EI},$$

or only $\frac{3}{8}$ as great as for an equal load at the end.

For the breaking weight, we have, since the greatest moment is at the fixed end and equal to $\frac{pl^2}{2}$, from (XI.),

$$\frac{pl^2}{2} = \frac{RI}{v} \quad \text{or} \quad \frac{(T \text{ or } C)I}{v}; \quad \text{hence} \quad pl = \frac{2\,RI}{vl} \quad \text{or} \quad \frac{2\,(T \text{ or } C)I}{vl},$$

taking always whichever value of T or C is the least, or twice as much as for an equal weight at the end.

Case 4.—Beam Fixed at One End—Uniform Load—Constant Strength.—We have the moment at any point $\frac{px^2}{2}$. Putting this equal to $\frac{RI}{v}$, we find $R = \frac{pvx^2}{2\,I}$, or for rectangular cross-section $R = \frac{3\,px^2}{bh^2}$. If b_1 and h_1 are the breadth and height at the fixed end, then since R must be constant,

$$\frac{3px^2}{bh^2} = \frac{3\,pl^2}{b_1h_1^2}, \quad \text{or} \quad \frac{bh^2}{b_1h_1^2} = \frac{x^2}{l^2}. \quad \ldots\ldots\ldots \quad (14)$$

If the height is constant $h = h_1$, and $b = b_1 \left(\frac{x}{l}\right)^2$. This is the equation of a parabola, as

Fig.169

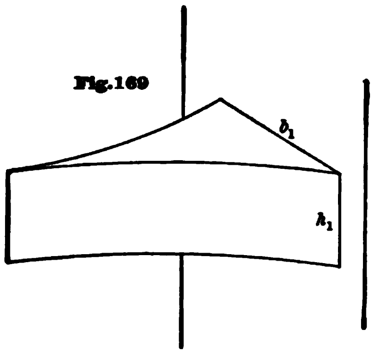
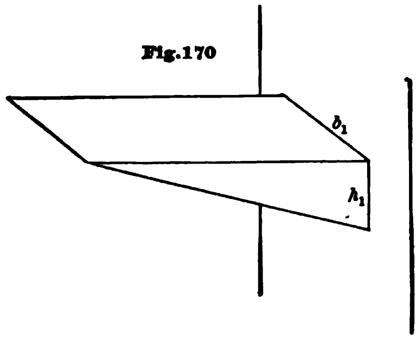


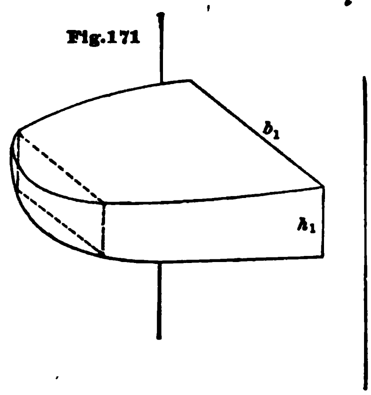
Fig.170



shown in Fig. 169. If the breadth is constant, $b = b_1$, and (14) becomes $h = h_1 \frac{x}{l}$. This is the equation of a straight line, as shown in Fig. 170.

For similar cross sections we have $\frac{b_1}{h_1} = \frac{b}{h}$, or $b = \frac{b_1 h}{h_1}$. Hence (14) becomes $h^3 = h_1^3 \frac{x^2}{l^2}$. This is the equation of a cubic parabola. The shape of the beam is, therefore, as shown in Fig. 171.

Fig.171



CHANGE OF SHAPE.—We have from (VIII.),

$$\frac{d^2y}{dx^2} = -\frac{px^2}{2EI}$$

or for rectangular cross-section,

$$\frac{d^2y}{dx^2} = -\frac{6px^2}{Ebh^3}.$$

For constant height we have, as we have seen, $b = b_1 \frac{x^2}{l^2}$, and $h = h_1$. Hence

$$\frac{d^2y}{dx^2} = -\frac{6pl^2}{Eb_1h_1^3}.$$

Integrating, since for $x = l$, $\frac{dy}{dx} = 0$, we have

$$\frac{dy}{dx} = -\frac{6pl^2x}{Eb_1h_1^3} + \frac{6pl^3}{Eb_1h_1^3}.$$

Integrating again, since for $x = l$, $y = 0$, we have

$$y = -\frac{3pl^2x^2}{Eb_1h_1^3} + \frac{6pl^3x}{Eb_1h_1^3} - \frac{3pl^4}{Eb_1h_1^3}. \quad \dots\dots\dots\dots \quad (15)$$

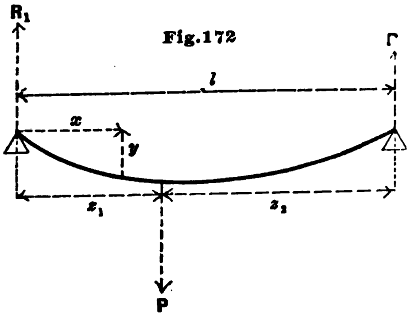
The deflection at the end is, then,

$$\Delta = -\frac{3pl^4}{Eb_1h_1^3},$$

or twice as much as for a beam of constant cross-section. In a similar manner we can easily find the deflection in the cases of Figs. 170 and 171.

CASE 5.—BEAM SUPPORTED AT BOTH ENDS—CONSTANT CROSS-SECTION—CONCENTRATED LOAD.—Let the weight P be distant from the left end, Fig. 172, by a distance z_1 and from the right end by a distance z_2. Let the distance of any point from the left end be x.

Fig. 172



The upward reaction at the left support is by moments $R_1 \times l = P \times z_2$, or $R_1 = \frac{Pz_2}{l}$. The moment at any point between the left end and the weight, or when $x < z_1$,

$$M = -R_1 x = -\frac{Pz_2 x}{l}.$$

For any point to the right of P, or when $x > z_1$,

$$M = -R_1 x + P(x - z_1) = -\frac{Pz_2 x}{l} + P(x - z_1).$$

The greatest moment is evidently at the point of application of the load, or when $x = z_1$. Hence the maximum moment is $= -\frac{Pz_1 z_2}{l}$.

(*a.*) *Breaking Weight.*—From (XI.) we have

$$\max M = \frac{Pz_1 z_2}{l} = \frac{RI}{v}, \quad \text{or} \quad P = \frac{RIl}{vz_1 z_2}, \quad \text{or} \quad \frac{(T \text{ or } C)Il}{vz_1 z_2},$$

where we must use R when known by experiment, or that value of T or C which is the smallest.

For rectangular cross-section $I = \frac{bh^3}{12}$, and hence $P = \frac{Rbh^2 l}{6\, z_1 z_2}$. For a load in the middle $z_1 = z_2 = \frac{1}{2} l$, and $P = \frac{4\, RI}{vl}$, or 4 times as great as for a beam of the same length fixed at one end and free at the other end.

(*b.*) *Change of Shape.*—From (VIII.) we have

$$\text{when } x < z_1, \quad EI\frac{d^2y}{dx^2} = \frac{Pz_2 x}{l}; \qquad \text{when } x > z_1, \quad EI\frac{d^2y}{dx^2} = \frac{Pz_1}{l}(l - x).$$

Integrating, we have

$$EI\frac{dy}{ax} = \frac{Pz_2 x^2}{2\,l} + C_1; \qquad EI\frac{dy}{dx} = \frac{Pz_1}{l}\left(lx - \frac{x^2}{2}\right) + C_2.$$

For $x = z_1$ these two values of $\frac{dy}{dx}$ are equal, and hence, since $z_2 = l - z_1$, we have $C_2 = C_1 - \frac{Pz_1^2}{2}$.

We thus have the two equations

$$EI\frac{dy}{dx} = \frac{Pz_2 x^2}{2\,l} + C_1; \quad \text{and} \quad EI\frac{dy}{dx} = \frac{Pz_1}{l}\left(lx - \frac{x^2}{2}\right) - \frac{Pz_1^2}{2} + C_1,$$

both containing the same constant C_1.

Integrating again we have

when $x < z_1$, $EIy = \frac{Pz_2x^3}{6l} + C_1x + C_2$; when $x > z_1$, $EIy = \frac{Pz_1}{2l}\left(lx - \frac{x^3}{3}\right) - \frac{Pz_1^2x}{2} + C_1x + C_4$.

In the first of these equations, when $x = 0, y = 0$; hence $C_2 = 0$. When $x = z_1$, y in one equals in the other, hence $C_4 = \frac{Pz_1^3}{6}$. For $x = l$, y in the second equation is zero, hence $C_1 = -\frac{Pz_1z_2}{6l}(2l - z_1)$.

Substituting these constants, we have, when

$$x < z_1, \quad y = \frac{Pz_2x}{6EIl}(x^2 - 2lz_1 + z_1^2); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

$$\text{when } x > z_1, \quad y = \frac{Pz_1(l - x)}{6EIl}(z_1^2 - 2lx + x^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

The deflection at the load is, therefore, for $x = z_1$,

$$y = -\frac{Pz_1^2z_2^2}{3EIl}.$$

If we insert the value of C_1 in the value for $\frac{dy}{dx}$ and place $\frac{dy}{dx} = 0$, we find for the value of x *which makes the deflection a maximum*,

$$x = \sqrt{\tfrac{1}{3}(2l - z_1)z_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

The greatest deflection is not at the weight, therefore, except when the weight is in the middle. Inserting this value of x in the value for y, we have the for maximum deflection

$$\Delta = -\frac{Pz_1z_2(2l - z_1)}{27EIl}\sqrt{3z_1(2l - z_1)}.$$

If the load is in the middle of the beam, we have $z_1 = z_2 = \frac{1}{2}l$, and the equation of the curve of deflection is

$$y = -\frac{Px}{48EI}(3l^2 - 4x^2).$$

The deflection at the weight in this case is found by making $x = \frac{1}{2}l$, or

$$\Delta = -\frac{Pl^3}{48EI},$$

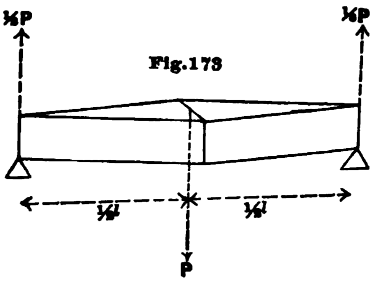
or only $\frac{1}{16}$th as much as for a beam of the same length fixed at one end and loaded at the other end

(*c*.) *Uniform Strength.*—The change of shape and form for uniform strength may be easily found, precisely as on page 296, for a beam fixed at one end and loaded at the other end.

If the weight, for instance, is at the centre of the beam, the deflection is greatest at the weight. Each half of the beam may then be considered as a beam of the length $\frac{1}{2}l$, fixed horizontally at one end and with an upward force $\frac{P}{2}$ at the other. Each half of the beam should then have the shape of Figs. 165, 166, or 167, according as the height or breadth is constant, or the cross-sections are similar.

Thus, Fig. 173 shows the shape of a beam of uniform strength, for constant height, weignt in the middle.

Fig. 174. for constant breadth, weight in the middle.



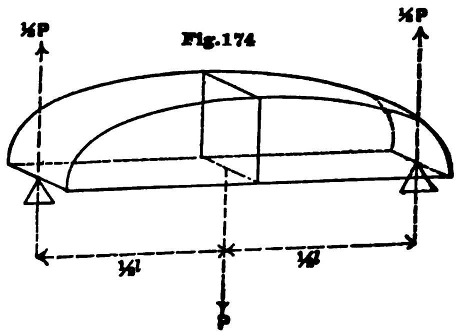
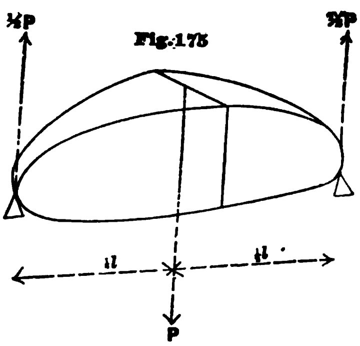
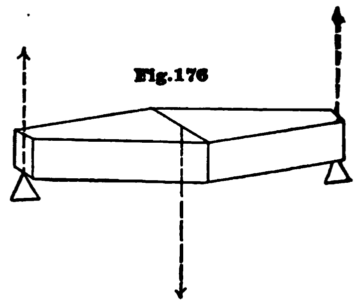


Fig. 175, for similar cross sections, weight in the middle.

In each of these cases, the deflection is the same as for a beam whose length is $\frac{1}{2}l$, fixed at one end horizontally and with an upward force of $\frac{P}{2}$ at the other. The deflection in each case is given by (10), (11) and (12), where for P we must insert $\frac{P}{2}$, and for l, $\frac{1}{2}l$.

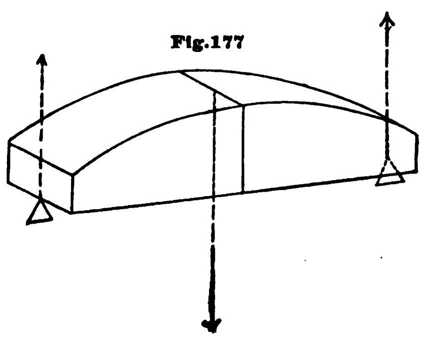
When the weight P is placed at any point, we have only to find the point at which the deflection is greatest, or that point for which $\frac{dy}{dx} = 0$. This point we may consider as the fixed end of a beam, whose length is the distance to each of the other ends, the force at the extremity being the reaction.

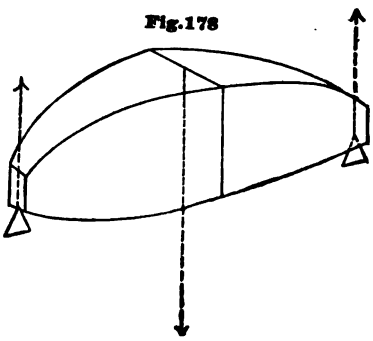




Equations (10), (11) and (12), will then give the deflection, when we put for l the length of each portion, and for P the reaction at the end.

The method of page 296 must be followed in each case. Owing to the shear, Figs. 173, 174

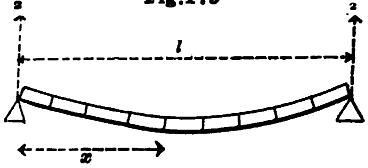




and 175 cannot end in a line as shown, but cross-section enough should be allowed at the ends to resist the shear at those points, as shown in Figs. 176, 177, and 178.

CASE 6.—BEAM SUPPORTED AT BOTH ENDS—CONSTANT CROSS-SECTION—UNIFORM LOAD.—For a load p per unit of length, the entire load is pl, Fig. 179. The reaction at each end is $\frac{pl}{2}$. The moment at any point is

Fig. 179



$$M = -\frac{plx}{2} + \frac{px^2}{2}.$$

This is evidently greatest at the centre, or when $x = \frac{1}{2}l$. Hence

$$\max M = -\frac{pl^2}{8}$$

For the breaking weight then, from (XI.),

$$\frac{pl^2}{8} = \frac{RI}{v}, \quad \text{or } pl = \frac{8\,RI}{vl}, \qquad (19)$$

or four times as much as for a beam of the same length loaded uniformly and fixed at one end.

For the change of shape, we have from (VIII.),

$$EI\frac{d^2y}{dx^2} = \frac{plx}{2} - \frac{px^2}{2}.$$

Integrating once, since for $x = \frac{1}{2}l$, $\frac{dy}{dx} = 0$, we have

$$EI\frac{dy}{dx} = \frac{plx^2}{4} - \frac{px^3}{6} - \frac{pl^3}{24}.$$

Integrating again, since for $x = 0, y = 0$,

$$EIy = \frac{plx^3}{12} - \frac{px^4}{24} - \frac{pl^3x}{24},$$

or

$$y = \frac{px}{24\,EI}(2\,lx^2 - x^3 - l^3). \qquad (20)$$

This is greatest at the centre, or for $x = \frac{1}{2}l$. Hence the maximum deflection is

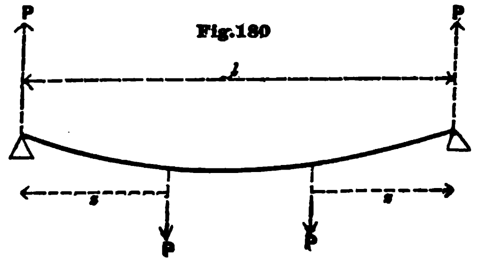
$$\Delta = -\frac{5\,pl^4}{384\,EI},$$

or only $\frac{5}{48}$ of a beam of the same length fixed at one end and uniformly loaded.

For uniform strength, since the deflection is greatest at the centre, we can consider each half of the beam as a beam fixed horizontally at one end and with an upward force at the other equal to $\frac{pl}{2}$.

For rectangular cross section each half will then be as shown in Figs. 173, 174 and 175. The deflection in each case may be found as in equation (15). The same method applies easily to any other form of cross section.

Fig. 180



CASE 7.—BEAM SUPPORTED AT BOTH ENDS—CONSTANT CROSS SECTIONS—WITH TWO EQUAL AND SYMMETRICALLY PLACED LOADS.—Let the beam, Fig. 180, support two weights P, P, placed at equal distances z from each end. The reaction at each support is then P, and the greatest moment is evidently at the centre and equal to Pz.

For the breaking weight we have, then,

$$Pz = \frac{RI}{v}, \quad \text{or } P = \frac{RI}{vz}, \quad \text{or } \frac{(T \text{ or } C)I}{vz}.$$

For rectangular cross-section, $I = \frac{1}{12}bh^3$, and $v = \frac{h}{2}$, hence

$$P = \frac{Rbh^2}{6z}.$$

For change of shape, we have, from (VIII.),

$$\text{when } x < z, \quad EI\frac{d^2y}{dx^2} = Px, \qquad \text{when } x > z, \quad EI\frac{d^2y}{dx^2} = Pz.$$

Integrating, we have,

$$EI\frac{dy}{dx} = \frac{Px^2}{2} + C_1, \qquad EI\frac{dy}{dx} = Pzx + C_2.$$

In the second of these equations, when $x = \frac{1}{2}l$, $\frac{dy}{dx} = 0$; hence $C_2 = -\frac{Pzl}{2}$. When $x = z$, $\frac{dy}{dx}$ in the first is the same as $\frac{dy}{dx}$ in the second, hence $C_1 = \frac{Pz^2}{2} - \frac{Pzl}{2}$. Hence

$$EIy\frac{dy}{dx} = \frac{Px^2}{2} + \frac{Pz^2}{2} - \frac{Pzl}{2}, \quad EI\frac{dy}{dx} = Pzx - \frac{Pzl}{2}.$$

Integrating again, since for $x = 0$ in the first of these equations $y = 0$, we have

$$EIy = \frac{Px^3}{6} + \frac{Pz^2x}{2} - \frac{Pzlx}{2}, \qquad EIy = \frac{Pzx^2}{2} - \frac{Pzlx}{2} + C_3,$$

when $x = z$, y in the first is the same as y in the second, hence $C_3 = \frac{Pz^3}{6}$.

The deflection for any point on the left of the first weight is given by

$$y = \frac{Px}{6EI}(x^2 + 3z^2 - 3zl),$$

and for any point between the weights,

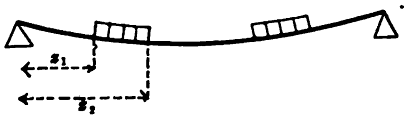
$$y = \frac{Pz}{6EI}(z^2 + 3x^2 - 3xl). \quad \text{(21)}$$

The maximum deflection is at the centre and equal to

$$\Delta = \frac{Pz}{24EI}(4z^2 - 3l^2). \quad \text{(22)}$$

If the loads are uniformly distributed, instead of being concentrated as shown in Fig. 181, we can put pdz in the place of P. Equation (22) then becomes

Fig. 181



$$\Delta = \int \frac{pzdz}{24EI}(4z^2 - 3l^2).$$

If we integrate this between the limits z_2 and z_1, we have for the deflection at the centre,

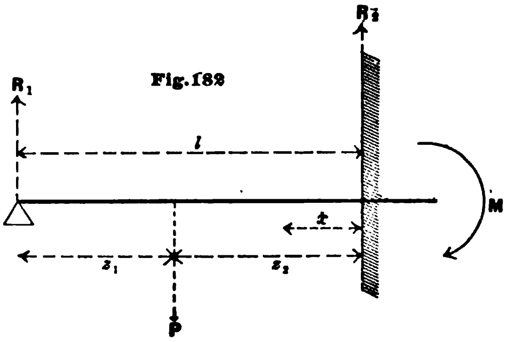
$$\varDelta = \frac{p}{96\,EI}[4(z_2^4 - z_1^4) - 6l^2(z_2^2 - z_1^2)]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

When the load covers the whole beam, $z_2 = \frac{1}{2}l$, and $z_1 = 0$, and

$$\varDelta = -\frac{5pl^4}{384\,EI},$$

as already found.

CASE 8.—BEAM SUPPORTED AT ONE END AND FIXED AT THE OTHER—CONSTANT CROSS-SECTION—CONCENTRATED LOAD.—Let the beam be fixed horizontally at the right end, Fig. 182. At this end, then, we have not only a vertical reaction R_2, but also a negative moment M, which causes the beam to be horizontal. At the left end we have only the reaction R_1. Let the weight P be distant from the left end by a distance z_1, and from the right end by a distance z_2. Then from (VIII.), taking *x from the fixed end*,



$$\text{when } x > z_2, \quad EI\frac{d^2y}{dx^2} = R_1(l - x);$$

$$\text{when } x < z_2, \quad EI\frac{d^2y}{dx^2} = R_1(l - x) - P(z_2 - x).$$

Integrating, we have

$$EI\frac{dy}{dx} = R_1lx - \frac{R_1x^2}{2} + C_1, \quad EI\frac{dy}{dx} = R_1lx - \frac{R_1x^2}{2} - Pz_2x + \frac{Px^2}{2} + C_2. \quad . \quad . \quad (24a)$$

When $x = 0$, $\frac{dy}{dx}$ in the second equation is zero, and hence $C_2 = 0$. When $x = z_2$, $\frac{dy}{dx}$ is the same in both. Hence $C_1 = -\frac{Pz_2^2}{2}$. Inserting these values of C_1 and C_2, and integrating again, we have

$$EIy = \frac{R_1lx^2}{2} - \frac{R_1x^3}{6} - \frac{Pz_2^2x}{2} + C_3, \qquad EIy = \frac{R_1lx^2}{2} - \frac{R_1x^3}{6} - \frac{Pz_2x^2}{2} + \frac{Px^3}{6} + C_4. \quad (24b)$$

When $x = 0$ in the second equation of (24b), $y = 0$, and hence $C_4 = 0$. When $x = z_2, y$ is equal in both; hence $C_3 = \frac{Pz_2^3}{6}$.

When $x = l$ in the first, $y = 0$. Hence

$$R_1 = \frac{Pz_2^2(3l - z_2)}{2l^3}.$$

If we put the value of $\frac{dy}{dx}$ in (24a) equal to zero, and insert the values of C_1, C_2, and R_1, we have for the point at which the deflection is a maximum,

$$\text{when } x > z_2, \quad x = l - l\sqrt{\frac{l - z_2}{3l - z_2}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25a)$$

$$\text{when } x < z_2, \quad x = \frac{2lz_2(2l - z_2)}{2l^2 + z_2(2l - z_2)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25b)$$

When $x = z_2$ in these equations the maximum deflection will be at the load and will be the greatest possible. Placing therefore $x = z_2$, we obtain from both these equations the condition

$$z_2 = l(2 - \sqrt{2}) = 0.58578l.$$

That is, the greatest maximum deflection is at the load when the load is at a distance of $2 - \sqrt{2} = 0.58578$ of the span from the fixed end. For any other position of the load the maximum deflection is between the load and the supported end when $z_2 < l(2 - \sqrt{2})$, and between the load and the fixed end when $z_2 > l(2 - \sqrt{2})$.

If we substitute the values of x in (25*a*) and (25*b*) in the values for y in (24*b*) and insert the values of C_2, C_4 and R_1, we obtain for the maximum deflection,

$$\text{when } x > z_2, \quad \varDelta = -\frac{Pz_2^2}{6EI}(l - z_2)\sqrt{\frac{l - z_2}{3l - z_2}}; \quad \ldots \ldots \quad (26a)$$

$$\text{when } x < z_2, \quad \varDelta = -\frac{Pz_2^2(l - z_2)(2l - z_2)^2}{3EI\,[l^2 + z_2(2l - z_2)]^2}, \quad \ldots \ldots \quad (26b)$$

Both of these are equal and have their greatest value for $z_2 = l(2 - \sqrt{2})$.

Inserting this value of z_2, we have for the greatest maximum deflection at the load,

$$\text{when } z_2 = l(2 - \sqrt{2}), \quad \varDelta = -\frac{5888\,Pl^3}{600000\,EI},$$

or only about $\frac{47}{100}$ as much as for a beam supported at the ends. When, then, $z_2 > l(2 - \sqrt{2})$ the greatest deflection is between the load and the fixed end, and x and $\varDelta$ are given by (25*b*) and (26*b*). When $z_2 < l(2 - \sqrt{2})$ the greatest deflection is between the load and the supported end, and x and $\varDelta$ are given by (25*a*) and (26*a*).

If the load is in the middle

$$R_1 = \frac{5}{16}P,$$

and since $z_2 = l$ is less than $l(2 - \sqrt{2})$ we use the values of x and $\varDelta$ given by (25*a*) and (26*a*), and obtain the maximum deflection at a distance x from the fixed end given by

$$x = l\left(1 - \frac{1}{\sqrt{5}}\right) = 0.55l,$$

and for the maximum deflection itself in this case

$$\varDelta = -\frac{Pl^3}{48EI} \times \frac{1}{\sqrt{5}},$$

or $\frac{1}{16\sqrt{5}}$ as much as for a beam of the same length fixed at one end and loaded at the other, and $\frac{1}{\sqrt{5}}$ as much as for a beam of the same length supported at the ends.

Determination of R_1 by the principle of least work.—The moment at any point for

$$x > z_2 \text{ is} \qquad M = -R_1(l - x),$$

and for

$$x < z_2 \qquad M = -R_1(l - x) + P(z_2 - x).$$

From (IV.′) we have then for the work of bending,

$$\text{work} = \int_{z_2}^{l}[-R_1(l - x)]^2\frac{dx}{2EI} + \int_0^{z_2}[-R_1(l - x) + P(z_2 - x)]^2\frac{dx}{2EI}.$$

If we differentiate this with respect to R_1 and put $\frac{d(\text{work})}{dR_1} = 0$, we have for the value of R_1, which gives the work of bending a minimum,

$$\int_{z_2}^{l}R_1(l - x)^2dx + \int_0^{z_2}[R_1(l - x)^2dx - P(l - x)(z_2 - x)dx] = 0.$$

Performing the integrations we obtain

$$R_1 = \frac{Pz_2^2(3l - z_2)}{2l^3},$$

just as already obtained.

Breaking Weight.—Since we know R_1, we can find the moment at any point. Rupture will occur where the moment is greatest, that is, either at the fixed end or at the load. The moment at the load is $-R_1(l - z_2)$ and at the fixed end $-R_1 l + Pz_2$. The first is always negative and the second always positive, hence $R_1 l$ is less than Pz_2. If we subtract the first from the second we have $Pz_2 - R_1 z_2$ which is positive, since P is greater than R_1. The moment at the fixed end is then the greatest and equal to $-R_1 l + Pz_2$, or

$$\max M = Pz_2 - \frac{P_2z^2(3l - z_2)}{2l^2}.$$

This is greatest for $z_2 = l(1 - \sqrt{\tfrac{1}{3}}) = 0.4226l$. That is the greatest maximum moment at the fixed end is when the load is distant 0.4226 of the span from that end.

The value of this greatest maximum moment is then

$$\frac{Pl}{3\sqrt{3}}.$$

Hence from (XI.),

$$\frac{Pl}{3\sqrt{3}} = \frac{RI}{v} \text{ or } \qquad P = \frac{3\sqrt{3}RI}{vl} \text{ or } \frac{3\sqrt{3}(T \text{ or } C)I}{vl}.$$

That is, the breaking weight is $\frac{3\sqrt{3}}{4} = 1.3$ times as great as for a beam supported at the ends.

If the load is in the middle, we have the moment at the fixed end $\frac{3}{16}Pl$, and

$$P = \frac{16RI}{3vl} \text{ or } \frac{16(T \text{ or } C)I}{3vl}$$

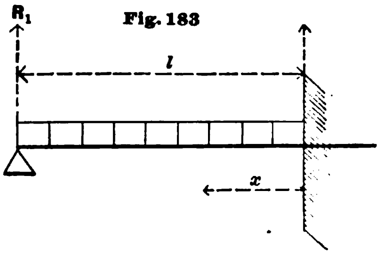
or $\frac{4}{3}$ as much as for the same beam supported at the ends.

The breaking load for a load anywhere is given by

$$P = \frac{2RIl^2}{vz_2z_1(2l - z_2)} \text{ or } \frac{2(T \text{ or } C)Il^2}{vz_1z_2(2l - z_2)}. \qquad (27)$$

CASE 9.—BEAM FIXED AT ONE END AND SUPPORTED AT THE OTHER—CONSTANT CROSS-SECTION—UNIFORM LOAD.—In this case, Fig. 183, the moment at any point is

Fig. 183



$$EI\frac{d^2y}{dx^2} = R_1(l - x) - \frac{p(l - x)^2}{2}.$$

Integrating twice and determining the constants by the conditions that for $x = 0$, $\frac{dy}{dx} = 0$, and $y = 0$, we easily obtain $R_1 = \frac{3}{8}pl$, and

$$\frac{dy}{dx} = \frac{px}{48EI}(6l^2 - 15lx + 8x^2); \qquad (28)$$

$$y = -\frac{px^2}{48EI}(l - x)(3l - 2x). \qquad (29)$$

Putting (28) equal to zero, we find for the point at which the deflection is a maximum,

$$x = \frac{15 - \sqrt{33}}{16}\,l, \quad \text{or} \quad x = 0.5785\,l.$$

The maximum deflection itself is then

$$\Delta = -\frac{39 + 55\sqrt{33}}{16^4}\frac{pl^4}{EI}.$$

For the breaking weight we have, since the greatest moment is at the fixed end and equal to $\frac{pl^2}{8}$,

$$\frac{pl^2}{8} = \frac{RI}{v}, \quad \text{or} \quad pl = \frac{8\,RI}{vl}, \quad \text{or} \quad \frac{8\,(T \text{ or } C)I}{vl}.$$

The strength is then $\frac{4}{3}$ as great as for the same load in the middle, but no greater than for beam of same length and load supported at both ends.

Determination of R_1 by the Principle of Least Work.—The moment at any point is

$$M = -R_1(l - x) + \frac{p(l - x)^2}{2}.$$

From (IV′) we have, then, for the work of bending,

$$\text{work} = \int_0^l \left[-R_1(l - x) + \frac{p(l - x)^2}{2}\right]^2 \frac{dx}{2EI}.$$

If we differentiate this with respect to R_1, and put $\frac{d\,(\text{work})}{dR_1} = 0$, we have for the value of R_1, which gives the work of bending a minimum,

$$\int_0^l \left[R_1(l - x)^2dx - \frac{p(l - x)^3}{2}dx\right] = 0.$$

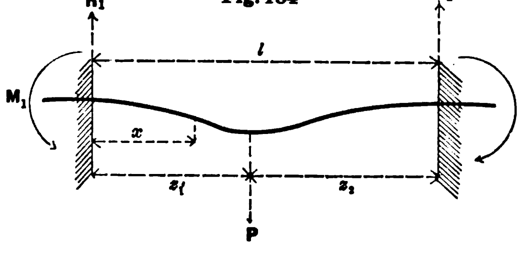
Performing the integrations, we obtain

$$R_1 = \tfrac{3}{8}pl,$$

just as already obtained.

CASE 10.—BEAM FIXED AT BOTH ENDS—CONSTANT CROSS-SECTION—CONCENTRATED LOAD.—Let z_1 be the distance from the left end to the weight, Fig. 184, and z_2 the distance from the right end to the weight. Let the reaction at the left end be R_1 and the moment at the left end M_1. Let x be measured from the left end.

Fig. 184



Then we have from (VIII),

$$\text{when } x < z_1, \quad EI\frac{d^2y}{dx^2} = R_1x - M_1;$$

$$\text{when } x > z_1, \quad EI\frac{d^2y}{dx^2} = R_1x - P(x - z_1) - M_1.$$

Integrating, we have

$$EI\frac{dy}{dx} = R_1\frac{x^2}{2} - M_1x + C_1; \qquad EI\frac{dy}{dx} = R_1\frac{x^2}{2} - P\frac{x^2}{2} + Pz_1x - M_1x + C_2.$$

If $x = 0$, $\frac{dy}{dx}$ in the first equation equals zero, and $C_1 = 0$. For $x = z_1$, $\frac{dy}{dx}$ is the same in both equations, and hence $C_2 = -\frac{Pz_1^2}{2}$. For $x = l$, $\frac{dy}{dx}$ in the second equation is zero, and hence

$$-2M_1 l = Pz_1^2 - 2Pz_1 l - R_1 l^2 + Pl^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Integrating again, after substituting the values of C_1 and C_2,

$$EIy = R_1\frac{x^3}{6} - M_1\frac{x^2}{2} + C_3; \qquad EIy = R_1\frac{x^3}{6} - \frac{Px^3}{6} + \frac{Pz_1x^2}{2} - M_1\frac{x^2}{2} - \frac{Pz_1^2x}{2} + C_4.$$

For $x = 0$, y in the first equation is zero, and hence $C_3 = 0$.

For $x = z_1$, y in both equations is the same, hence $C_4 = \frac{Pz_1^3}{6}$.

For $x = l$, $y = 0$ in the second equation, and hence

$$-3M_1l^2 = 3Pz_1^2l - 3Pz_1l^2 - R_1l^3 + Pl^3 - Pz_1^3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Equations (30) and (31) contain two unknown quantities, M_1 and R_1. Eliminating M_1 we have

$$R_1 = \frac{Pl^3 + 2Pz_1^3 - 3Pz_1^2l}{l^3},$$

or

$$R_1 = P\frac{z_2^2(3z_1 + z_2)}{l^3}, \quad \text{and} \quad R_2 = P\frac{z_1^2(3z_2 + z_1)}{l^3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Eliminating R_1, we have

$$M_1 = +P\frac{z_1z_2^2}{l^2}, \qquad \text{and} \quad M_2 = +P\frac{z_2z_1^2}{l^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

Substituting these values, we have,

$$\text{when } x < z_1, \quad \frac{dy}{dx} = -\frac{Pz_2^2x}{2EIl^3}[2lz_1 - (3z_1 + z_2)x], \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$y = -\frac{Pz_2^2x^2}{6EIl^3}[3lz_1 - (3z_1 + z_2)x]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

The point at which the deflection is a maximum is always between the load and the farthest end, or,

$$\text{when } z_1 > \tfrac{1}{2}l, \quad x = \frac{2lz_1}{3z_1 + z_2};$$

and from the other end we have,

$$\text{when } z_1 < \tfrac{1}{2}l, \quad x = \frac{2lz_2}{3z_2 + z_1}.$$

For the maximum deflection we have,

$$\text{when } z_1 > \tfrac{1}{2}l, \quad \Delta = -\frac{2Pz_1^3z_2^2}{3EI(3z_1 + z_2)^2};$$

$$\text{when } z_1 < \tfrac{1}{2}l, \quad \Delta = -\frac{2Pz_2^3z_1^2}{3EI(3z_2 + z_1)^2}.$$

This will be greatest when $z_1 = z_2$ or $z_1 = \frac{1}{2}l$. That is, the greatest deflection is at the weight when the weight is in the middle. This deflection is

$$\Delta = -\frac{Pl^3}{192EI},$$

or only $\frac{1}{4}$ as much as for beam supported at the ends.

Determination of R_1 and M_1 by the Principle of Least Work.—The moment at any point is, for

$$x < z_1, \quad M = - R_1x + M_1;$$

and for

$$x > z_1, \quad M = - R_1x + P(x - z_1) + M_1.$$

From (IV′) we have then for the work of bending

$$\text{work} = \int_0^{z_1} (M_1 - R_1x)^2 \frac{dx}{2\,EI} + \int_{z_1}^{l} [(M_1 - R_1x) + P(x - z_1)]^2 \frac{dx}{2\,EI}.$$

If we differentiate this with respect to R_1 and with respect to M_1, and put $\dfrac{d\,(\text{work})}{dR_1} = 0$ and $\dfrac{d\,(\text{work})}{dM_1} = 0$, we have for the values of R_1 and M_1 which make the work of bending a minimum

$$\int_0^{l} [R_1x^2 - M_1x]dx + \int_{z_1}^{l} - Px(x - z_1)dx = 0;$$

$$\int_0^{l} [M_1 - R_1x]dx + \int_{z_1}^{l} P(x - z_1)dx = 0.$$

Performing the integrations, we have

$$2\,R_1l^3 - 3\,M_1l^2 = 2\,Pl^3 - 3\,Pz_1l^2 + Pz_1^3;$$

$$R_1l^2 - 2\,M_1l \;=\; Pl^2 - 2\,Pz_1l \;+ Pz_1^2.$$

From these two equations we obtain

$$M_1 = + \frac{Pz_1z_2^2}{l^2}, \qquad R_1 = \frac{Pz_2^2(3\,z_1+z_2)}{l^3},$$

just as already obtained.

Breaking Weight.—The greatest moment is easily shown to be at the nearest end, and equal to

$$\frac{Pz_1z_2^2}{l^2} \quad \text{or} \quad \frac{Pz_2z_1^2}{l^2}.$$

This is a maximum for $z_1 = \frac{1}{3}l$. That is, the greatest moment at the end occurs when the load is distant one third of the length from that end.

The value of this greatest moment is $\dfrac{4Pl}{27}$. Hence, from (XI),

$$\frac{4Pl}{27} = \frac{RI}{v}, \quad \text{or} \quad P = \frac{27RI}{4vl}, \quad \text{or} \quad = \frac{27(T \text{ or } C)I}{4vl},$$

or $\frac{27}{16}$ times as great as for a beam supported at the ends. If the weight is in the middle, we have

$$\frac{Pl}{8} = \frac{RI}{v}, \quad \text{or} \quad P = \frac{8RI}{vl}, \quad \text{or} \quad = \frac{8(T \text{ or } C)I}{vl},$$

or twice as much as the same beam simply supported at the ends.

Case 11.—Beam Fixed at Both Ends—Constant Cross-section—Uniform Load.—In this case, Fig. 185, the reaction at each end is $\dfrac{pl}{2}$.

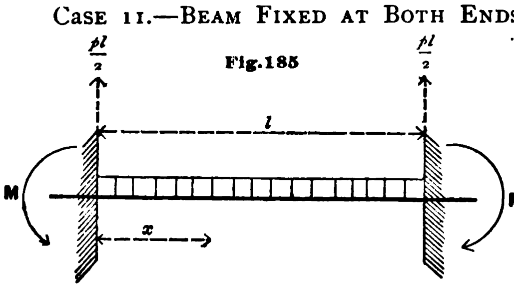


Fig.185

We have then, from (VIII),

$$EI\frac{d^2y}{dx^2} = \frac{plx}{2} - \frac{px^2}{2} - M.$$

Integrating, since, for $x = 0$, $\dfrac{dy}{dx} = 0$,

$$EI\frac{dy}{dx} = \frac{plx^2}{4} - Mx - \frac{px^3}{6}.$$

When $x = l$, $\frac{dy}{dx}$ also equals zero, hence $M = +\frac{pl^2}{12}$.

Inserting this value of M and integrating again,

$$EIy = \frac{plx^3}{12} - \frac{px^4}{24} - \frac{pl^2x^2}{24}.$$

Since for $x = 0, y = 0$, the constant is zero.
The deflection at any point is then

$$y = \frac{px^2}{24EI}(2lx - x^2 - l^2). \quad \ldots \ldots \ldots \ldots \quad (36)$$

This is greatest at the centre, or for $x = \frac{l}{2}$. The greatest deflection is then

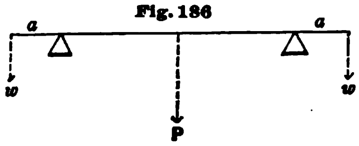
$$\Delta = -\frac{pl^4}{384EI}.$$

The greatest moment is easily proved to be at the end. Hence the breaking weight

$$\frac{pl^2}{12} = \frac{RI}{v}, \quad \text{or} \quad pl = \frac{12RI}{vl}, \quad \text{or} \quad \frac{12(T \text{ or } C)I}{vl}.$$

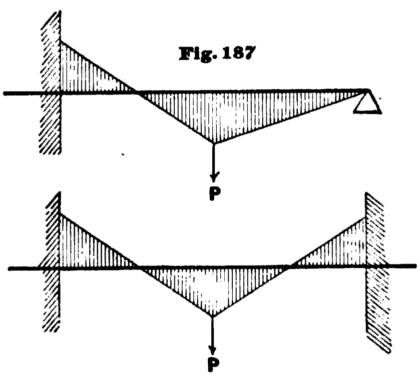
The beam may be fixed either by letting it into the wall or by prolonging it beyond the support and suspending a weight from the end, as shown in Fig. 186. In this case, the moment at the end being found as above, we can easily find the weight w, if the prolongation a is given, or the prolongation a if the weight w is given. Thus wa must equal the moment at the end.

Fig. 186



From the fixed end the moment decreases to a point where the moment is zero. Past this point the moment becomes negative, and, in the case of the beam supported at one end, increases gradually to a maximum and then decreases to zero at the supported end. In the beam fixed at both ends, it increases to a maximum, then decreases to zero, then changes sign and becomes positive again and increases to the other end, as shown in Fig. 187. These points at which the moments become zero are *points of inflection*, because here the moment changes sign, *i.e.*, the curvature changes from convex to concave or the reverse. They can be easily found by finding the values of x which make the expression for the moments zero.

Fig. 187



Thus for a beam fixed at one end and supported at the other, uniform load, the inflection point is at a distance from the fixed end of $x = \frac{1}{4}$ the length. For both ends fixed, we make

$$M_x = -\frac{plx}{2} + \frac{px^2}{2} + \frac{pl^2}{12}$$

equal to zero, and find $x = 0.21131l$ and $0.7887l$, where l is the length.

The curve of moments in any case may be determined graphically according to the principles of Chap. IV., page 32, or by a discussion of the equation of moments.*

* Examples for practice illustrative of the foregoing will be found at the end of this chapter, and the student is earnestly recommended to solve them.

Determination of M_1 by the Principle of Least Work.—The moment at any point is

$$M = -\frac{plx}{2} + \frac{px^2}{2} + M_1 .$$

From (IV′) we have for the work of bending

$$\text{work} = \int_0^l \left[-\frac{plx}{2} + \frac{px^2}{2} + M_1 \right]^2 \frac{dx}{2EI}.$$

If we differentiate this with respect to M_1 and put $\frac{d(\text{work})}{dM_1} = 0$, we have for the value of M_1 which makes the work of bending a minimum

$$\int_0^l \left[M_1 - \left(\frac{plx}{2} - \frac{px^2}{2} \right) \right] dx = 0.$$

Performing the integrations, we have

$$M_1 = \frac{pl^2}{12},$$

just as already obtained.

Combined Tension and Flexure.—A beam may sometimes be subjected to flexure and at the same time to tension. Thus, for instance, a lower chord panel of a bridge truss may be in tension, and at the same time it may sustain loads applied by means of cross-ties between the panel points.

In such a case let S be the tensile stress and A the area of cross-section. Then $\frac{S}{A}$ is the unit tensile stress. From (III) we have for the unit stress R in the most remote fibre, at a distance v from the neutral axis, due to flexure,

$$R = \frac{MI}{v},$$

where M is the moment at any cross-section.

The combined unit stress on the outer fibres will then be

$$R + \frac{S}{A}$$

on the tension side and

$$R - \frac{S}{A}$$

on the compression side.

The neutral axis is now no longer at the centre of mass of the cross-section, and a strict discussion leads to results of great complexity. If, however, we neglect the deflection, as in all practical cases we may safely do, we can proceed as follows:

Let σ be the allowable unit stress which must not be exceeded. Then

$$\sigma = R + \frac{S}{A}, \quad \text{or} \quad R = \sigma - \frac{S}{A}.$$

We have then, from (XI),

$$\text{max. } M = \frac{\left(\sigma - \frac{S}{A}\right)I}{v}, \qquad \text{(XII)}$$

where max. M is the maximum moment due to the loading, I the moment of inertia of the cross-section with reference to a horizontal axis through the centre of mass of the cross-section, at right angles to the neutral axis, and v is the distance from the neutral axis to the most *remote fibre on the tensile side.* Putting for I its value Ar_2, where r is the radius of gyration for the cross-section, we have

$$A = \frac{Mv}{\sigma r_2} + \frac{S}{\sigma}.$$

That is, the required area is that due to flexure alone plus that due to the tensile stress.

From these equations we can find the dimensions required in any practical case for a member subjected to flexure and tension simultaneously.

EXAMPLE.—*A rectangular iron bar which forms the lower panel of a bridge is* 12 *feet long,* 2 *inches wide, and has a longitudinal tension of* 20000 *lbs. If it supports in addition a load of* 5000 *lbs. at the centre, what should be the depth in order that the unit stress shall not exceed* 10000 *lbs. per square inch?*

Here $M = 2500 \times 6 \times 12 = 180000$ inch-lbs., $I = \frac{1}{12}ba^3 = \frac{d^3}{6}$, $v = \frac{d}{2}$, $\sigma = 10000$, $\frac{S}{A} = \frac{20000}{2d}$. Hence,

$$180000 = \left(10000 - \frac{10000}{d}\right)\frac{d^2}{3}, \quad \text{or} \quad d^2 - d = 54, \therefore \; d = 7.86 \text{ inches.}$$

COMBINED COMPRESSION AND FLEXURE.—This case is exactly similar to the preceding, except that for the allowable unit stress σ we must take the value given by one of the long column formulas, as given in Chapter IV., page 332.

SECONDARY STRESSES.—All the members of a framed structure which meet at an apex should be loaded in their axes, and these axes should meet in a point. If these conditions are not complied with, we have a secondary stress due to bending, as well as the direct stress in the members.

If the members are not loaded in their axes, we have a bending moment M due to eccentric load, which is equal to the stress on the member multiplied by the perpendicular distance between the point of application of the stress and the centre of cross-section of the member.

From (XII) we can then find the unit stress σ,

$$\sigma = \frac{S}{A} + \frac{Mv}{I}.$$

If the axes of the members do not meet in a point, we have from equation (VII), for each member,

$$EI\frac{dy}{dx} = \int_l^x \pm Mdx.$$

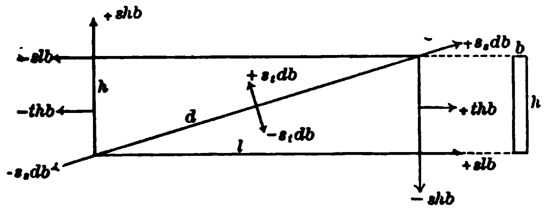
Hence we see that M for each member is proportional to

$$E\frac{dy}{dx}\cdot\frac{I}{l}$$

Since E and $\frac{dy}{dx}$ are the same for each member, we have simply to divide the total moment at the apex among the several members in the proportion of $\frac{I}{l}$ for each member. The moment for each member thus found, we have from (XII) the unit stress σ.

Combined Tension and Shear.—If a body whose cross-section at any point is A, is subjected to a direct tension T, the direct tensile unit stress is $t = \frac{T}{A}$. Suppose at the same time a direct vertical shear S; then the direct shearing unit stress is $s = \frac{S}{A}$. It is required to find the combined shearing unit stress s_s; and the combined tensile unit stress s_t.

Take any element of very small height h, length l, and breadth b.



Then we have acting on this element the two equal and opposite tensile stresses $+ thb$ and $- thb$, which are in equilibrium. We have also the shearing stresses $+ shb$ and $- shb$, forming a couple. This can only be held in equilibrium by the opposite couple $- slb$ and $+ slb$.

Let d be the diagonal and α the angle of the diagonal with the side l. Then

$$\sin\alpha = \frac{h}{d}, \qquad \cos\alpha = \frac{l}{d}.$$

We have then the algebraic sum of the components parallel to the diagonal giving the combined shearing stress $s_s db$, and the algebraic sum of the components perpendicular to the diagonal giving the combined tensile stress $s_t db$.

We have then

$$s_s db = thb\cos\alpha + slb\cos\alpha - shb\sin\alpha,$$
$$s_t db = thb\sin\alpha + slb\sin\alpha + shb\cos\alpha;$$

or, substituting the values of $\sin\alpha$ and $\cos\alpha$,

$$s_s = t\sin\alpha\cos\alpha + s\cos^2\alpha - s\sin^2\alpha = \frac{t}{2}\sin 2\alpha + s\cos 2\alpha,$$

$$s_t = t\sin^2\alpha + 2s\sin\alpha\cos\alpha = \frac{t}{2} - \frac{t}{2}\cos 2\alpha + \sin 2\alpha.$$

Differentiating, and putting $\frac{ds_s}{dt} = 0$ and $\frac{ds_t}{dt} = 0$, we have, when s_s is a maximum,

$$\tan 2\alpha = \frac{t}{2s}, \text{ or } \sin 2\alpha = \frac{t}{\sqrt{4s^2+t^2}}, \quad \cos 2\alpha = \frac{2s}{\sqrt{4s^2+t^2}};$$

when s_t is a maximum we have

$$\tan 2\alpha = -\frac{2s}{t}, \text{ or } \sin 2\alpha = -\frac{2s}{\sqrt{4s^2+t^2}}, \quad \cos 2\alpha = \frac{t}{\sqrt{4s^2+t^2}}.$$

Substituting, we have

$$\max s_s = \sqrt{s^2 + \frac{t^2}{4}}, \quad \ldots \ldots \ldots \quad (1)$$

$$\max s_t = \frac{t}{2} + \sqrt{s^2 + \frac{t^2}{4}}. \quad \ldots \ldots \ldots \quad (2)$$

Equation (1) gives the combined shearing unit stress when we have given the direct tensile and shearing stresses t and s. Equation (2) gives the combined tensile unit stress when t and s are given.

COMBINED COMPRESSION AND SHEAR.—Let the direct compressive unit stress be c. Then just as before, we have for the combined shearing unit stress

$$s = \sqrt{s^2 + \frac{c^2}{4}}, \quad \ldots \ldots \ldots \quad (1)$$

and for the combined compressive unit stress

$$s_c = \frac{c}{2} + \sqrt{s^2 + \frac{c^2}{4}}. \quad \ldots \ldots \ldots \quad (2)$$

EXAMPLE.—*A beam* 4 *inches wide*, 12 *inches aeep, and* 8 *feet long carries a loaa of* 500 *lbs. at the centre. Fina the maximum combined unit shear.*

Ans.—From (III) we have

$$t \text{ or } c = \frac{Mv}{I} = \frac{M\frac{h}{2}}{\frac{1}{12}bh^3} = \frac{6M}{bh^2} = \frac{M}{96}.$$

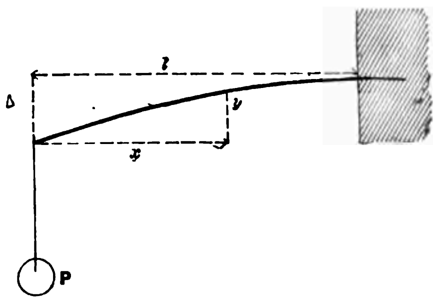
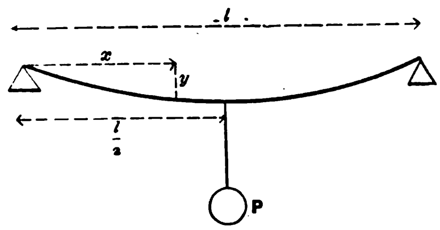
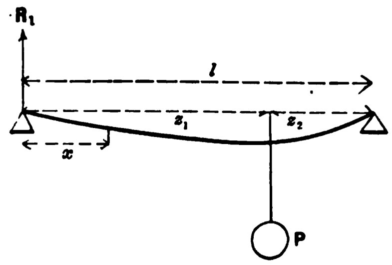
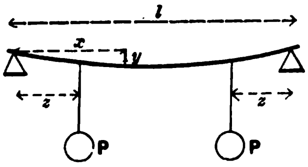
The maximum moment is $M = \frac{500}{2} \times 4 \times 12 = 12000$ in.-lbs. Hence t or $c = 125$ lbs. per sq. inch.

The maximum direct shear is 250 lbs. Hence $s = \frac{250}{48}$ lbs. per sq. inch. We have then at the centre the combined unit shear.

$$s_s = \sqrt{s^2 + \frac{t^2}{4}} = 62.7 \text{ lbs. per sq. in.}$$

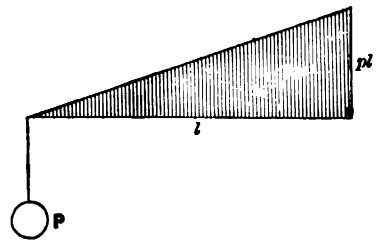
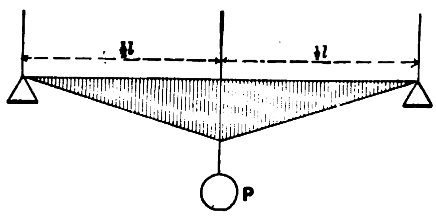
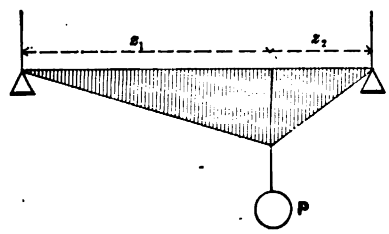
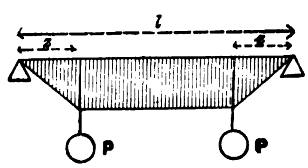
We give below a recapitulation of our results, as well as some

• BEAMS OF CONSTANT

CASE.	MOMENT.	EQUATION OF ELASTIC LINE.
	$M_x = + Px$, max. $M = + Pl$.	$y = \frac{P}{6EI}(3l^2x - x^3)$, $\Delta = \frac{Pl^3}{3EI}$ at end.
	when $x < \frac{1}{2}l$, $M_x = -\frac{P}{2}x$, max. $M = -\frac{Pl}{4}$.	$y = -\frac{Px}{48EI}[3l^2 - 4x^2]$, $\Delta = -\frac{Pl^3}{48EI}$ at centre.
	$x < z_1$, $M_x = -R_1x = -\frac{Pz_2x}{l}$, $x > z_1$, $M_x = -\frac{Pz_2x}{l} + P(x - z_1)$ max. $M = -\frac{Pz_1z_2}{l}$.	$x < z_1$, $y = -\frac{Pz_2x}{6EIl}[2lz_1 - z_1^2 - x^2]$, $x > z_1$, $y = -\frac{Pz_1(2l - x)}{6EIl}[2lx - x^2 - z_1^2]$, $\Delta = -\frac{Pz_1z_2(2l - z_1)}{27EIl}\sqrt{3z_1(2l - z_1)}$, max. deflection occurs at $x = \sqrt{\frac{1}{3}(2l - z_1)z_1}$.
	$x < z$, $M_x = -Px$, $x > z$, $M_x = -Pz = \text{max. } M$.	$x < z$, $y = -\frac{Px}{6EI}[3lz - 3z^2 - x^2]$, $x > z$, $y = -\frac{Pz}{6EI}[3lx - 3x^2 - z^2]$, $\Delta = -\frac{Pz}{24EI}[3l^2 - 4z^2]$ at centre.

others which the student can now readily demonstrate.

CROSS-SECTION.

BREAKING WEIGHT.	RELATIVE STRENGTH.	GRAPHIC ILLUSTRATION OF MOMENTS.
$P=\frac{RI}{vl}$, or $\frac{(T \text{ or } C)I}{vl}$.	1	
$P=\frac{4RI}{vl}$, or $\frac{4(T \text{ or } C)I}{vl}$.	4	
$P=\frac{RIl}{vz_1z_2}$, or $\frac{(T \text{ or } C)Il}{vz_1z_2}$. In general, either T or C, whichever is the least, is to be put for R in all formulas for breaking weight, and v is then the distance from the neutral axis to the outer fibre at which this least value occurs.	$\frac{l^2}{z_1z_2}$.	
$P=\frac{RI}{vz}$, or $\frac{(T \text{ or } C)I}{vz}$.	$\frac{l}{z}$.	

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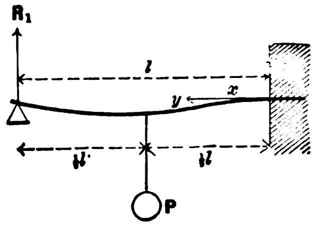
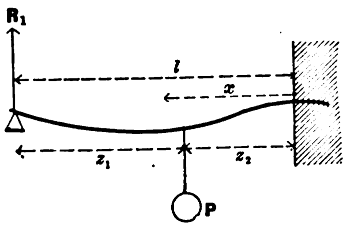
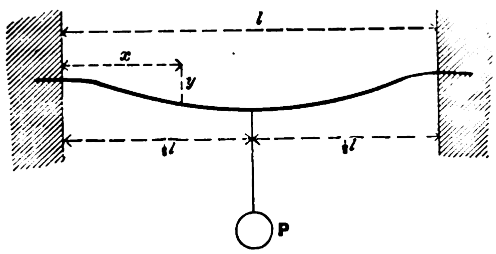
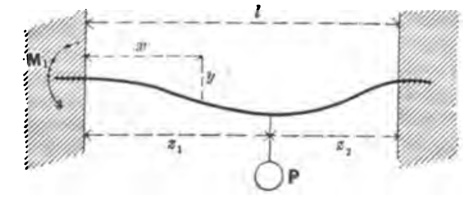
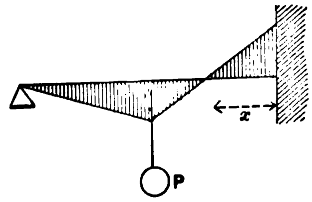
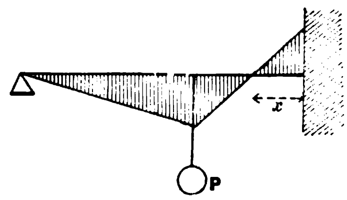
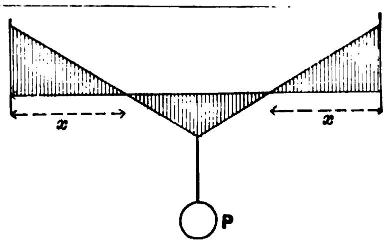
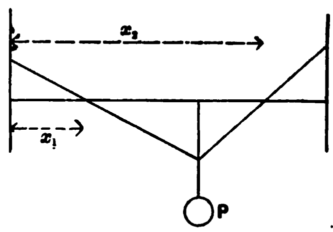
BEAMS OF CONSTANT

CASE.	MOMENT.	EQUATION OF ELASTIC LINE.
	$R_1 = \frac{5}{16}P$, $x > \frac{1}{2}l$, $M_x = -\frac{5}{16}P(l-x)$, $x < \frac{1}{2}l$, $M_x = -\frac{P}{16}(11x - 3l)$, max. $M = -\frac{5}{32}Pl$.	$x < \frac{1}{2}l$, $y = -\frac{Px^2}{96EI}[9l - 11x]$, $x > l$, $y = -\frac{P}{96EI}[5x^3 - 15lx^2 + 12l^2x - 2l^3]$, $\Delta = -\frac{1}{48\sqrt{5}}\frac{Pl^3}{EI}$, Max. deflection occurs at $x = l\left(1 - \frac{1}{\sqrt{5}}\right)$.
	$R_1 = P\frac{3z_2^2l - z_2^3}{2l^3}$, $x < z_2$, $M_x = -R_1(l-x)$ $+ P(z_2 - x)$, $x > z_2$, $M_x = -R_1(l-x)$, max. M, see page 307.	$x < z_2$, $y = -\frac{1}{6EI}[R_1x^3 - 3R_1lx^2$ $+ 3Pz_2x^2 - Px^3]$, $x > z_2$, $y = -\frac{1}{6EI}[R_1x^3 - 3R_1lx^2$ $+ 3Pz_2^2x - Pz_2^3]$, $\Delta = -\frac{Pz_2^3}{6EI}(l - z_2)\sqrt{\frac{l - z_2}{3l - z_2}}$, Max. deflection, see page 305.
	$x < \frac{1}{4}l$, $M_x = -\frac{P}{8}(4x - l)$, $x > \frac{1}{4}l$, $M_x = -\frac{P}{8}(3l - 4x)$, max. $M = +\frac{Pl}{8}$.	$x < \frac{1}{2}l$, $y = -\frac{Px^2}{48EI}[3l - 4x]$, $x > \frac{1}{2}l$, $y = -\frac{P}{48EI}[4x^3 + 6l^2x - l^3 - 9lx^2]$, $\Delta = -\frac{Pl^3}{192EI}$.
	$R_1 = P\frac{z_2^2(3z_1 + z_2)}{l^3}$, $M_1 = +P\frac{z_1z_2^2}{l^2}$, $x < z_1$, $M_x = -R_1x + M_1$, $x > z_1$, $M_x = -R_1x$ $+ P(x - z_1) + M_1$, max. $M = \frac{Pz_1z_2^2}{l^2}$ at end.	$x < z_1$, $y = -\frac{Px^2z_2^2}{6EIl^3}[3z_1l - (3z_1 + z_2)x]$, $x > z_1$, $y = -\frac{Px^2z_2^2}{6EIl^3}\left[\frac{l^3(x - z_1)^3}{x^2z_2^2}\right.$ $\left.+ 3z_1l - (3z_1 + z_2)x\right]$. $\Delta = -\frac{2Pz_1^3z_2^2}{3EI(3z_1 + z_2)^2}$, when $x < z_1$ and $z_1 > l/2$, Max. deflection occurs at $x = \frac{2lz_1}{3z_1 + z_2}$, when $z_1 > l/2$.

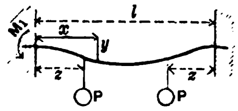
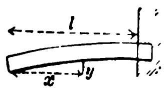
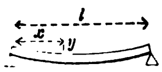
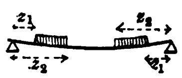
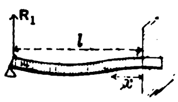
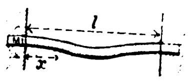
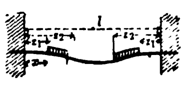
others, which the student can now readily demonstrate.—*Continued.*

CROSS SECTION.

BREAKING WEIGHT.	RELATIVE STRENGTH.	GRAPHIC ILLUSTRATION OF MOMENTS.
$P = \frac{16RI}{3vl}$ or $\frac{16(T \text{ or } C)I}{3vl}$.	$\frac{16}{3}$.	 Distance of point of inflection $x = \frac{3}{11}l$.
$P = \frac{3\sqrt{3}RI}{vl}$ or $\frac{3\sqrt{3}(T \text{ or } C)I}{vl}$. $P = \frac{2RIl^2}{vz_2z_1(2l - z_2)}$ or $\frac{2(T \text{ or } C)Il^2}{vz_2z_1(2l - z_2)}$.	$3\sqrt{3}$.	 Distance of point of inflection $x = \frac{Pz_2 - R_1 l}{P - R_1}$.
$P = \frac{8RI}{vl}$ or $\frac{8(T \text{ or } C)I}{vl}$.	8.	 Distance to point of inflection $x = \frac{l}{4}$.
$P = \frac{27RI}{4vl}$ or $\frac{27(T \text{ or } C)I}{4vl}$.	6.75.	 Distance to point of inflection $x_1 = \frac{z_1}{3z_{\cdot} + z_2}l$.

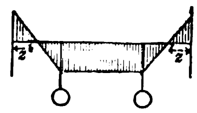
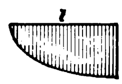
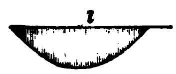
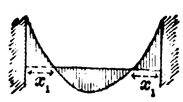
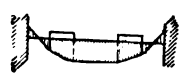
We give below a recapitulation of our results, as well as some

BEAMS OF CONSTANT

CASE.	MOMENT.	EQUATION OF ELASTIC LINE.
	$x < z$, $M_x = -Px - \frac{Pz^2}{l} + Pz$, $x > z$, $M_x = -\frac{Pz^2}{l}$ = max. M.	$x < z$, $y = -\frac{Px^2}{6EIl}[3lz - 3z^2 - xl]$, $x > z$, $y = -\frac{Pz^2}{6EIl}[3lx - 3x^2 - zl]$, $\Delta = \frac{Pz^2}{24EI}[3l - 4z]$ at centre.
	$M_x = +\frac{px^2}{2}$, max. $M = +\frac{pl^2}{2}$.	$y = \frac{p}{24EI}[4l^3x - x^4]$, $\Delta = \frac{pl^4}{8EI}$.
	$M_x = -\frac{plx}{2} + \frac{px^2}{2}$, max. $M = -\frac{pl^2}{8}$.	$y = -\frac{px}{24EI}[x^3 - 2lx^2 + l^3]$.
	$x < z_1$, $M_x = -p(z_2 - z_1)x$, $x > z_2$, $M_x = -\frac{p(z_2^2 - z_1^2)}{2}$.	$x < z_1$, $y = -\frac{px}{6EI}[x^2(z_2 - z_1) + (z_2^3 - z_1^3)$ $- \frac{3l}{2}(z_2^2 - z_1^2)]$, $x > z_2$, $y = -\frac{p}{24EI}[(z_2^2 - z_1^2)(6xl - 6x^2)$ $- (z_2^4 - z_1^4)]$.
	$R_1 = \frac{3}{8}pl$, $M_x = -\frac{p}{8}(4x - l)$ $+ (l - x)$, max. $M = +\frac{pl^2}{8}$.	$y = -\frac{px^2}{48EI}(l - x)(3l - 2x)$, $\Delta = -\frac{39 + 55\sqrt{33}\,pl^4}{16^4 EI}$, max. deflection occurs at $x = 0.5785\,l$.
	$M_1 = +\frac{pl^2}{12}$, $M_x = -\frac{plx}{2} + \frac{px^2}{2} + \frac{pl^2}{12}$ max. $M = +\frac{pl^2}{12}$.	$y = -\frac{px^2}{24EI}[l^2 + x^2 - 2lx]$, $\Delta = -\frac{pl^4}{384EI}$ at centre.
	$x < z_1$, $M_x = -px(z_2 - z_1)$ $- \frac{p}{3l}(z_2^3 - z_1^3)$ $+ \frac{p}{2}(z_2^2 - z_1^2)$, $x > z_2$, $M_x = -\frac{p}{3l}(z_2^3 - z_1^3)$.	$x < z_1$, $y = -\frac{px^2}{6EIl}\Big[3\frac{l}{2}(z_2^2 - z_1^2) - (z_2^3 - z_1^3)$ $- lx(z_2 - z_1)$. $x > z_2$, $y = -\frac{p}{12EIl}[(2lx - 2x^2)(z_2^3 - z_1^3)$ $- \frac{l}{2}(z_2^4 - z_1^4)\Big]$.

others, which the student can now readily demonstrate.—*Continued.*

CROSS SECTION.

BREAKING WEIGHT.	RELATIVE STRENGTH.	GRAPHIC ILLUSTRATION OF MOMENTS.
$P = \frac{RIl}{vz^2}$ or $\frac{(T \text{ or } C)\,Il}{vz^2}$.	$\frac{l^2}{z^2}$.	 $x_1 = z - \frac{z^2}{l}$.
$P = \frac{2\,RI}{vl}$ or $\frac{2\,(T \text{ or } C)I}{vl}$.	2.	 Curve of moments a parabola.
$P = \frac{8\,RI}{vl}$ or $\frac{8\,(T \text{ or } C)\,I}{vl}$.	8.	 Curve of moments a parabola.
$P = \frac{4\,RI}{v\,(z_2 + z_1)}$ or $\frac{4\,(T \text{ or } C)\,I}{v\,(z_2 + z_1)}$.	$4\frac{l}{z_2 + z_1}$.	
$P = \frac{8\,RI}{vl}$ or $\frac{8\,(T \text{ or } C)\,I}{vl}$.	8.	 $x_1 = \frac{1}{4}\,l$. Curve of moments a parabola.
$P = \frac{12\,RI}{vl}$ or $\frac{12\,(T \text{ or } C)\,I}{vl}$.	12.	 $x_1 = 0.42262\,l$.
$P = \frac{6\,RIl}{v\,(z_2^2 + z_2 z_1 + z_1^2)}$.	$\frac{6\,l^2}{z_2^2 + z_2 z_1 + z_1^2}$.	

EXAMPLES.*

The student who has carefully studied this work, should be able to solve easily and accurately the following examples:

1. A wrought iron tie-rod, 30 feet long and 4 sq. ins. in area of cross section, is subjected to 40000 lbs. tension. What is the unit stress? If the coefficient of elasticity is 30000000 lbs. per sq. in., what is the elongation?

Unit stress = 10000 lbs. per sq. in. Elongation = 0.01 ft.

2. An iron bar, 10 ft. in length, stretches .012 ft. under a unit stress of 25000 lbs. per sq. in. What is E?

E = 20833333 lbs. per sq. in.

3. A rectangular timber tie is 12 ins. deep and 40 ft. long. If E = 1200000 lbs. per sq. inch, find the proper thickness of the tie, so that its elongation under a pull of 270000 lbs. may not exceed 1.2 ins.

Thickness = 7.5 ins.

4. A roof tie-rod, 142 feet in length and 4 sq. ins. in sectional area, is subjected to a stress of 80000 lbs. If E = 30000000 lbs. find the elongation of the rod.

Elongation = 1.136 ins.

5. The length of a cast iron pillar is diminished from 20 ft. to 19.97 ft. under a given load. Find the compressive unit stress, E being 17000000 lbs. per sq. in.

Unit stress = 25500 lbs. per sq. in.

6. A wrought iron bar, 2 sq. ins. sectional area, has its ends fixed between two immovable blocks when the temperature is at 60° F. Taking the coefficient of expansion at 0.000006944 per unit of length, for one degree, what pressure will be exerted upon the blocks when the temperature is 100° F.?

Pressure = 0.00055552 E.
If E = 30000000 lbs. per sq. in., Pressure = 16665.6 lbs.

7. The dead load of a bridge is 5 tons, and the live load 10 tons per panel, the corresponding factors of safety being 3 and 6. Find the *compound* factor of safety.

Factor = 5.

8. The dead load upon a short hollow cast iron pillar, with a sectional area of 20 sq. ins., is 50 tons. If the compression is not to exceed 0.0015 of the length, find the greatest live load to which the pillar can be subjected, E being 17000000 lbs. per sq. in.

Live load = 410000 lbs. = 205 tons.

9. A steel suspension rod, 30 ft. long and ½ sq. in. sectional area, carries 3500 lbs. of the roadway and 3000 lbs. of the live load. Determine the *gross* load and also the extension of the rod, E being 35000000 lbs.

Gross load = 6500 lbs. Extension = 0.133 inch.

10. A beam 40 ft. long carries a load of 20000 lbs. Find the shearing force at 15 ft. from one end, and also the maximum bending moment of the beam:—

(*a*) When the beam is supported at the ends and loaded in the middle.
(*b*) When it is supported at the ends and loaded uniformly.
(*c*) When it is fixed at one end and loaded at the other.
(*d*) When it is fixed at one end and loaded uniformly.

(*a*) Shear = 10000 lbs. Maximum moment = 200000 ft. lbs. at middle.
(*b*) Shear = 2500 lbs. Maximum moment = 100000 ft. lbs. at middle.
(*c*) Shear = 20000 lbs. Maximum moment = 800000 ft. lbs. at end.
(*d*) Shear = 7500 lbs. Maximum moment = 400000 ft. lbs. at end.

Draw the curves of shearing force and bending moment.

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* These examples have been compiled from Prof. Bovey's "Applied Mechanics," Stoney's "Theory of Strains," Wood's "Strength of Materials," and Weisbach's "Mechanics of Engineering."

11. Discuss the effect produced in each of the cases of Question (10): *first*, when a single weight of 2000 lbs. passes over the beam ; *second*, when a train weighing 2000 lbs. per lineal ft. moves across the beam.

Draw the curves of shearing force and bending moment.

12. A beam 20 ft. in length rests upon two supports and carries a weight of 10 tons at 5 ft. from one end. Find the maximum bending moment.

Maximum moment at weight = 37.5 ft. tons.

Draw the curves of shearing force and bending moment.

13. A uniform rigid bar weighs W lbs., and is supported by two strings attached to its ends. Find the tensions in the strings and the inclination of the bar when the strings are inclined to the vertical at angles of 60° and 30° respectively.

Tensions = 0.5 W and 0.866 W.
Inclination of bar with horizontal = 30°.
Compression in bar = 0.5 W.
Vertical components of string tensions = 0.25 W and 0.75 W.
Horizontal component of string tensions = 0.433 W.

Solve by diagram and calculation.

14. A car of weight W for a 4 ft. 8½ in. gauge, is 33 ft. long, 6 ft. deep, and its bottom is 2 ft. 6 ins. above the rails. Find the additional weight thrown upon the leeward rails, when the wind blows upon the side of the car with a pressure of 20 lbs. per sq. ft. Find the minimum wind pressure that will blow the car over.

Additional weight = 4625.84 lbs.
Minimum pressure = 0.428 W.

15. What is the breadth and depth of the strongest rectangular beam which can be cut from a cylindrical log of diameter D?

Breadth $= D\sqrt{\tfrac{1}{3}}$. Depth $= D\sqrt{\tfrac{2}{3}}$.

16. A round beam and a square beam are equal in length and equally loaded. Find the ratio of the diameter to the side of the square, so that the two beams may be of equal strength.

$$\frac{\text{diameter}}{\text{side}} = 2\sqrt[3]{\frac{2}{3\pi}}.$$

17. Compare the relative strengths of a cylindrical beam and the strongest rectangular and square beams that can be cut from it.

$$\frac{\text{Strength of cylindrical}}{\text{Strongest rectangular}} = \frac{9\pi\sqrt{3}}{32} = 1.53. \qquad \frac{\text{Strength of cylindrical}}{\text{Strongest square}} = \frac{3\pi\sqrt{2}}{8} = 1.66$$

18. Compare the relative strengths of a solid square beam to that of the solid inscribed cylinder.

$$\frac{\text{Strength of square}}{\text{Strength of cylinder}} = \frac{16}{3\pi} = 1.7.$$

19. Compare the strength of a square beam with its sides vertical, to that of the same beam with one diagonal vertical.

$$\frac{\text{Strength side vertical}}{\text{Strength diagonal vertical}} = \sqrt{2} = 1.414.$$

20. A beam of yellow pine, 14 ins. wide, 15 ins. deep, and resting upon supports 10 ft. 9 ins. apart, was just able to bear a weight of 34 tons at the centre. What weight will a beam of the same material, 3 ft. 9 ins. between the supports and 5 ins. square bear?

3.86 tons.

21. Determine the form of a beam of uniform strength, for constant depth and for constant breadth.

(1) When the beam rests upon two supports and is uniformly loaded.
(2) When the beam rests upon two supports and is loaded at the centre.
(3) When the beam is fixed at one end and loaded at the other.
(4) When the beam is fixed at one end and uniformly loaded.
(5) When the beam in cases (1) and (4) carries an additional weight at the centre and end respectively.

22. Compare the strengths of two rectangular beams of equal length, the breadth and depth of one, being respectively equal to the depth and breadth of the other.

The strengths are directly as the breadths, and inversely as the depths.

23. A cast iron beam 4 ins. square rests upon supports 6 ft. apart. Determine the breaking weight at the centre, taking R = 30000 lbs. per sq. in.

Breaking weight = 17777⅞ lbs.

24. A yellow pine beam, 14 ins. wide, 15 ins. deep, and resting upon supports 10 ft. 6 ins. apart, broke down under a uniformly distributed load of 60.97 tons. Find the coefficient of rupture R.

$R = 3658.2$ lbs.

25. A cast iron rectangular girder rests upon supports 12 ft. apart, and carries a weight of 2000 lbs. at the centre. If the breadth is one-half the depth, find the sectional area of the girder, so that the inch stress in the metal may nowhere exceed 4000 lbs.

Area = 18 sq. ins., depth = 6 ins., breadth = 3 ins.

26. A wrought iron bar, 4 ins. deep, $\frac{3}{4}$ in. wide, and rigidly fixed at one end, gave way when loaded with 1568 lbs. at the free end, at a point 2 ft. 8 ins. from the load. Find R.

$R = 25088$ lbs.

27. A wrought iron bar, 2 ins. wide and 4 ins. deep, rests upon supports 12 ft. apart. Determine the uniformly distributed load which the bar will safely carry in addition to its own weight, if $R = 50000$ lbs. and factor of safety is 4. A bar of iron 3 ft. long and one square inch in cross section is assumed to weigh 10 lbs.

Weight = 3384 lbs.

28. Find the length of a beam of ash 6 ins. square, which would break of its own weight when supported at the ends, the weight of the timber being 30 lbs. per cubic ft. and $R = 7000$ lbs. per sq. in.

Length = $149\frac{2}{3}$ ft.

29. A railway girder 50 ft. in the clear and 6 ft. deep, carries a uniformly distributed load of 50 tons. Find the maximum shearing stress at 20 ft. from one end, when a train weighing $1\frac{1}{4}$ tons per lineal foot crosses the girder.

Also, find the minimum theoretic thickness of the web, 4 tons being the safe shearing inch stress of the metal.

Shear = 16.25 tons. Thickness = 0.056 in.

30. A cast iron semi-girder, 8 ft. long and 12 ins. deep, carries a uniformly distributed load of 16000 lbs. Find the area of the top flange at the fixed end, neglecting the web, so that the inch stress may not exceed 3000 lbs.

Area = 21.3 sq. inches.

31. A cast iron girder, $27\frac{1}{2}$ ins. deep, rests upon supports 26 ft. apart. Its bottom flange is 16 ins. wide and 3 ins. deep. Neglecting the web, find the breaking weight at the centre, the tearing inch stress of cast iron being 15000 lbs.

Weight = 253846 lbs.

32. The lattice bridge at the Boyne Viaduct is in three spans, continuous. Each side span is 140 ft. 11 ins. long, and 22 ft. 3 ins. deep. The permanent load supported by one main girder of a side span is 0.68 ton per running foot, and the sectional area of its lower flange over the centre pier is 127 sq. ins. On one occasion an extraordinary load in the centre span depressed it to such an extent as to raise the ends of the side spans off the abutments, thus forming each side span into a semi-girder. What was the compressive inch stress in the lower flange at the pier?

Inch stress = 2.4 tons.

33. A semi-girder, 44.7 ft. long, and 22 25 ft. deep, supports a uniformly distributed load of 1.82 tons per foot, and a weight of 161.6 tons in addition at the extremity. What is the inch stress on the net section of the tension flange at the point of support, neglecting the web, the gross area being 132.6 ins., but reduced by rivet holes to the extent of $\frac{2}{9}$ths?

Inch stress = 3.94 tons.

34. A girder, 50 ft. long and 4 ft. deep, supports a uniformly distributed load of 32 tons. Find the stress in either flange at 9 feet from one end, neglecting the web.

Stress = 29.5 tons.

35. A piece of teak, 2 ins. deep, and $1\frac{15}{16}$ ins. wide, is fixed at one extremity. Find the weight which if hung at 2 ft. from the point of attachment will break it by crushing the fibres of the lower side, assuming that the crushing strength for teak is considerably less than its tearing strength, and equal to 12000 lbs. per square inch.

Weight = 646 lbs.

36. The effective length and depth of a cast iron girder were $27\frac{1}{3}$ ft. and 18 ins. respectively, and its bottom flange was 10 ins. wide and $1\frac{1}{2}$ ins. deep. The girder failed under a weight of $29\frac{1}{2}$ tons at the centre. Find the maximum inch stress in the bottom flange, neglecting the web.

Stress = 8.96 tons.

37. A cylindrical beam 2 ins. in diameter, 60 inches long, and weighing $\frac{1}{4}$ lb. per cubic inch, deflects $\frac{3}{8}$ in. under a weight of 3000 lbs. at the centre. Find E.

$E = 28929144$.

38. A rectangular beam, 5 ft. long, 3 ins. wide, and 3 ins. deep, is deflected $\frac{1}{10}$ in. by a weight of 3000 lbs. applied at the middle. Find E.

$E = 20000000$.

39. A joist, whose length is 16 ft., width 2 ins., depth 12 ins., and coefficient of elasticity 1600000 lbs., is deflected $\frac{1}{2}$ in. by a weight in the middle. Find the weight, neglecting the weight of the beam.

Weight = 1562 lbs.

40. An iron rectangular beam, whose length is 12 ft., breadth $1\frac{1}{2}$ ins., coefficient of elasticity 24000000 lbs., has a weight of 10000 lbs. suspended at the middle. Find the depth in order that the deflection may be $\frac{1}{480}$th of the length.

Depth = 8.8 in.

41. A rectangular wooden beam, 6 ins. wide and 30 ft. long, is supported at its ends. The coefficient of elasticity is 1800000 lbs. The weight of a cubic foot of the beam is 50 lbs. Find the depth that it may deflect 1 inch from its own weight.

Depth = 6.5 ins.

How deep must it be to deflect $\frac{1}{400}$th of its length?

Depth = 6.8 ins.

42. Required the depth of a rectangular beam which is supported at its ends, and so loaded at the middle that the elongation of the lowest fibre shall equal $\frac{1}{1400}$th of its original length.

$$\text{Depth} = \sqrt{\frac{2100\ Pl}{Eb}}.$$

43. Required the radius of curvature at the middle point of a wooden beam, when the load is 3000 lbs., the length 10 ft., breadth 4 ins., depth 8 ins., and $E = 1000000$ lbs.

Radius = 1896 inches.

44. Let the beam be of iron, supported at its ends. Let the breadth be 1 in., depth 2 ins., length 8 ft., and E 25000000 lbs. Required the radius of curvature at the middle when the deflection is $\frac{1}{6}$th of an inch.

Radius = 3840 inches.

45. A beam whose depth is 8 ins., and length 8 feet, is supported at its ends, and sustains 500 lbs. per foot. Find its breadth so that it shall have a factor of safety of $\frac{1}{10}$th, R being 14000 lbs.

Breadth = $3\frac{3}{14}$ ins.

46. A beam, whose length is 12 ft., breadth 2 ins., and depth 5 ins., is supported at its ends. Find the weight uniformly distributed, it will sustain, the coefficient of safety being $\frac{1}{4}$ and $R = 80000$ lbs.

Weight = 9259 lbs.

47. A wooden beam, whose length is 12 ft., is supported at its ends. Find its breadth and depth so that it shall sustain one ton uniformly distributed over its whole length, R being 15000 lbs., the coefficient of safety $\frac{1}{10}$th, and the depth 4 times the breadth.

Breadth = 2.08 ins.
Depth = 8.32 ins.

48. A wrought iron beam 12 ft. long, 2 ins. wide, and 4 ins. deep, is supported at its ends. The material weighs $\frac{1}{4}$ lb. per cubic inch. Taking R at 54000 lbs., find what weight uniformly distributed it will sustain.

Without the weight of the beam, 16000 lbs.
With the weight of the beam, 15712 lbs.

49. A beam is fixed at one end. Length 20 ft., breadth $1\frac{1}{2}$ ins. $R = 40000$ lbs. If the weight of the material is $\frac{1}{4}$ lb. per cubic inch, find the depth so that it may sustain its own weight and 500 lbs. at the free end.

Depth = 4.05 inches.

50. The breadth of a beam is 3 ins., depth 8 ins., weight of a cubic ft. 50 lbs., $R = 12000$ lbs. Find the length so that it will break from its own weight when supported at the ends.

Length = 175.27 feet.

51. If a beam 6 ft. long, $1\frac{1}{2}$ ins. wide and 4 ins. deep is supported at its ends, and loaded at the middle so as to produce a deflection of $\frac{3}{4}$ inch, find the greatest inch stress on the fibres, taking $E = 25000000$ lbs. Also find the load.

Stress = 86805 lbs.
Load = 19290 lbs.

52. For the same beam, if the greatest fibre stress is 12000 lbs. per sq. in. find the greatest deflection.

Deflection = 0.103 inch.

53. What should be the size of a square wooden beam of 12 feet span, which sustains a load of 300 lbs. at the centre, and has at the same time a longitudinal tension of 2000 lbs.; the maximum working unit stress being taken at 1000 lbs. per square inch.

Size = 4.02 inches.

54. A rectangular oak beam 1 foot deep and ½ ft. wide, and 15 feet long, is fixed horizontally at one end and is free at the other end. Let the weight of the beam itself be 54 pounds per cubic foot. Suppose it sustains a uniform load of 100 pounds per foot of length extending over only 4 feet of the beam, beginning at 5 feet from the fixed end; also a weight of 100 pounds placed at 11 feet from the fixed end. Let $E = 2000000$ lbs. per square inch. What is the total deflection at the free end?

Deflection due to weight of beam = 0.17086 inch. Deflection due to the weight = 0.0684 inch.
" " uniform load = 0.12627 " Total deflection = 0.36553 inch.

55. If the same beam is loaded with 5 equal weights of 100 lbs. each, at intervals of 3 feet, what is the deflection at the free end, and at the third loaded point from the fixed end?

Total deflection at the free end = 0.27 inch.
Total deflection at the third point = 0.12555 inch.

56. Same beam of oak, supported at the two ends. What is the central deflection due to its own weight?

Deflection = 0.001483 foot.

57. A beam of pine weighing 40 lbs. per cubic foot, 18½ inches deep, 15 inches wide, 12½ feet long, is supported at the ends, and has a weight of 17935 lbs. placed at 48 inches from one end. What is the deflection at centre and point of application of weight? $E = 1680000$ lbs. per sq. in.

Deflection at centre due weight of beam = 0.0032 inch.
Deflection at centre for weight added = 0.078617 inch.

Deflection at 48 inches due weight of beam = 0.0027 inch.
Deflection at 48 inches due weight added = 0.07185 inch.

58. A wrought iron 15 inch I beam, whose moment of inertia is 691, has a length of 30 feet. $E = 24000000$. If supported at the ends, and a uniform load of 75 lbs. per inch covers the first 10 feet, what is the deflection at the end of the load?

Deflection = 0.23444 inch.

What is the deflection at the centre of the beam?

Deflection = 0.24421 inch.

What is the deflection 10 feet from the unloaded end?

Deflection = 0.19537 inch.

Where is the point of greatest deflection? and what is the greatest deflection?

At 13.1676 feet. Greatest deflection = 0.24847 inch.

If the beam's own weight is 5.573 lbs. per inch, what is the deflection at centre?

Deflection = 0.07349 inch.

If the same 10 foot load is moved along to the centre, what is the deflection at the centre?

Deflection = 0.50063 inch.

If the uniform load of 75 lbs. per inch covers the whole span, what is the central deflection?

Deflection = 0.98905 inch.

If the same beam is half loaded with 75 pounds per inch, what is the deflection at centre? What is the maximum deflection? and at what point is it?

Deflection = 0.494525 inch.
Max. deflection = 0.49855 inch.
Within the loaded part and 14.48 inches from centre of beam.

If the same beam has 3 weights of 4500 lbs. each, placed at intervals of 60 inches, beginning at one end, what is the deflection at the centre?

Deflection = 0.6154 inch.

If there are 8 weights, each equal to 3000 lbs., at intervals of 40 inches, what is the central deflection?

Deflection = 0.97926 inch.

59. Suppose the same beam as in 58 to be fixed horizontally at both ends, and loaded uniformly with 75 lbs. per inch. What is the deflection 10 feet from either end? At the centre?

Deflection = 0.1563 inch.
At centre = 0.19781 inch.

If only one end is fixed, the other supported, what is the deflection at 10 feet? At centre? At 20 feet? What is the maximum deflection? Where is it?

Deflection at 10 feet = 0.39074 inch.
Deflection at centre = 0.39563 inch.
Deflection at 20 feet = 0.27352 inch.
Maximum deflection = 0.41018 inch.
At 151.7524 inches from supported end.

60. Same beam, fixed horizontally at both ends, with a concentrated load of 27000 lbs. If the load is in the centre, what is the deflection at half way between the centre and either end? What is centre deflection? Where are the points of contrary flexure?

Deflection = 0.19781 inch.
Centre deflection = 0.39562 inch.
At 90 inches from each end.

If the load is 7.5 feet from the left end, where and what is the maximum deflection?

Maximum deflection = 0.2136 inch.
At 12 feet from left end.

If only the right end is fixed and the other supported, and the load of 27000 lbs. is at the centre, what are the deflections at the quarter points? The centre? And what is the maximum deflection?

At the quarter points, deflection = 0.5316 inch and 0.3091 inch.
Central deflection = 0.69234 inch.
Maximum deflection = 0.70732 inch at $2l\sqrt{\frac{1}{5}}$ from supported ends.

61. Same beam as in 58, fixed horizontally at both ends, has 3 weights of 4500 lbs. each, placed at intervals of 60 inches, beginning at the left end. What is centre deflection?

Deflection = 0.13187 inch.

If 2 other equal weights of 4500 lbs. each are added at the same interval of 60 inches, what is the central deflection due to these last two weights?

Deflection = 0.06594 inch.

Suppose the fifth weight removed, what is the deflection at the fourth weight? At the third weight? And second weight?

Fourth weight, deflection = 0.13748 inch.
Third weight, " = 0.18072 inch.
Second weight, " = 0.1458 inch.

What are the end moments due to these four weights? and where are the points of contrary flexure?

$M = -750000$ inch-pounds.
$M_1 = -600000$ inch-pounds.
74.806 and 275.294 inches.

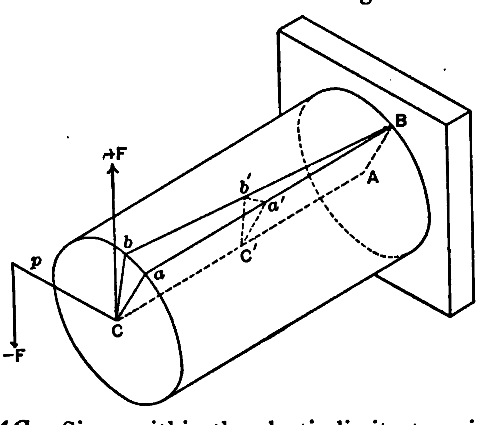
CHAPTER III.

TORSION.

In the preceding we have given the application of the Theory of Flexure to Beams. For the sake of completeness we give here its application to shafts subjected to torsion.

TORSION.—Torsion occurs when the external forces acting upon a body tend to twist it, so that each cross-section turns on the next adjacent, about a common axis at right angles to the plane of the section.

Let a horizontal shaft of length l be fixed at one end and let a force couple $+F, -F$ act at the free end, whose moment about the axis is Fp.



The shaft will be twisted about the axis AC, so that any radial line as aC moves to bC through the angle of twist $aCb = \theta$.

If the elastic limit is not exceeded, any longitudinal plane $aBAc$ before twisting remains plane after, as $bBAC$. Also the angle of twist aCb is proportional to the distance $AC = l$. Thus if θ is the angle aCB at the distance l from the fixed end, the angle $a'C'b'$ at the distance x from the fixed end is $\frac{x}{l}\theta$.

NEUTRAL AXIS.—Consider the shaft to be made up of an indefinitely great number of fibres parallel to AC. Since within the elastic limit, stress is proportional to strain, as one cross-section turns about the axis and slides upon the adjacent cross-section, the strain and therefore the shearing stress on the end of each fibre is proportional to its distance from the axis AC. For the fibre at the axis, there is then no shearing stress. The axis AC is then the *neutral axis*. (Compare page 285.)

POSITION OF THE NEUTRAL AXIS FOR TORSION.—Let a be the cross-section of any fibre, and R the shearing unit stress within the elastic limit for the most remote fibre at a distance v from the neutral axis. Then the shear for the most remote fibre is Ra, and for any other fibre in the same cross-section at the distance d it is $\frac{d}{v}Ra$. The sum of all the fibre stresses is then $\frac{R}{v}\Sigma da$. But the sum of the external forces $+F, -F$ is zero. Hence for equilibrium we have

$$\frac{R}{v}\Sigma da = 0.$$

But $\Sigma da = 0$ only when the neutral axis passes through the centre of mass of the cross-section. (Compare page 285.)

TWISTING MOMENT AND RESISTING MOMENT.—The twisting moment is $M = Fp$. (See Figure preceding.) This moment is the same at every point of the neutral axis. For equilibrium, there must be between any two adjacent cross-sections an equal and opposite resisting moment due to the shearing stress between these cross-sections.

Since for any cross-section the shearing stress for any fibre at a distance d from the neutral axis is, as we have seen,

$$\frac{d}{v}Ra,$$

the moment of this stress about the neutral axis is

$$\frac{R}{v}ad^2.$$

The sum of the moments of all the stresses for any cross-section about the axis is then

$$\frac{R}{v}\Sigma ad^2.$$

But Σad^2 is the *polar moment of inertia* I_z *of the cross-section* with reference to the axis through the centre of mass. (See page 270.) We have then for equilibrium

$$M = \frac{RI_z}{v}, \qquad \text{(XII.)}$$

This, it will be seen, is just the same as equation (IV.) for bending, except that R is now the unit *shear* in the most remote fibre at the distance v, and I_z is the polar moment of inertia.

COEFFICIENT OF ELASTICITY FOR TORSION.—The coefficient of elasticity E is always equal to unit stress divided by unit strain (page 284). The unit stress in any fibre at a distance d from the neutral axis is

$$\frac{d}{v}R.$$

If θ is the angle of twist in radians, $d\theta$ is the strain; and if l is the distance from the fixed end, the unit strain is

$$\frac{d}{l}\theta.$$

We have then

$$E = \frac{d}{v}R \div \frac{d}{l}\theta = \frac{Rl}{v\theta}.$$

But from (XII.),

$$R = \frac{Mv}{I_z}.$$

Hence we have

$$E = \frac{Ml}{\theta I_z}, \qquad \text{(XIII.)}$$

where E is the coefficient of elasticity for torsion, and θ is the angle of twist in radians.

WORK OF TORSION.—If θ is the angle of torsion in radians, for any cross-section, the strain of any fibre in that cross-section at a distance d from the neutral axis is $d\theta$, and the stress is $\frac{d}{v}Ra$, where a is the area of cross-section of the fibre and R is the unit stress in the most remote fibre at a distance v. The work on the fibre is then one half the product of the stress and strain (page 284), or $\frac{R\theta}{2v}ad^2$. The work on all the fibres is, then,

$$\frac{R\theta}{2v}\Sigma ad^2;$$

or, since $\Sigma ad^2 = I_z =$ the polar moment of inertia of the cross-section with reference to the axis through the centre of mass, we have for the work, from (XII.) and (XIII.),

$$\text{work} = \frac{R\theta I_z}{2v} = \frac{M\theta}{2} = \frac{E\theta^2 I_z}{2l} = \frac{M^2 l}{2EI_z}, \quad \ldots \ldots \quad \text{(XIV.)}$$

where θ is in radians.

TRANSMISSION OF POWER BY SHAFTS.—Work is the product of the force by the distance through which it acts. Power is rate of work. A horse-power is 33,000 ft.-lbs. per minute. If a shaft makes n revolutions per minute, and the twisting force is F with a lever arm of p, then $2\pi pn$ is the distance, and $2\pi pnF$ is the work per minute. If p is in inches, the horse-power is

$$\text{HP} = \frac{2\pi pnF}{33{,}000 \times 12}.$$

But $Fp = M = \frac{RI_z}{v}$. Hence

$$\text{HP} = \frac{\pi nRI_z}{198{,}000v}, \quad \ldots \ldots \quad \text{(XV.)}$$

where n is the number of revolutions per minute, HP the horse-power transmitted, and I_z and v must be taken in inches and R in pounds per square inch.

COMBINED FLEXURE AND TORSION.—Let R_f be the unit stress *due to flexure* in the most remote fibre at a distance v from the neutral axis. Then from equation (III.), if M_f is the bending-moment, we have

$$R_f = \frac{M_f v}{I}.$$

Let R_t be the unit stress *due to torsion*. Then from equation (XII.), we have, if M_t is the twisting moment,

$$R_t = \frac{M_t v}{I_z}.$$

Then, as we have seen (page 315), we have for the combined shearing unit stress

$$s_s = \sqrt{R_t^2 + \frac{R_f^2}{4}},$$

and for the combined tensile or compressive unit stress

$$s_t \text{ or } s_c = \frac{R_f}{2} + \sqrt{R_t^2 + \frac{R_f^2}{2}}.$$

EXAMPLES.—(1) *A circular shaft* 2 *ft. long is twisted through an angle of* 7 *degrees by a couple of* ± 200 *lbs. with a lever arm of* 6 *inches. Find the angle for a shaft of the same size and material* 4 *ft. long when twisted by a couple of* 500 *lbs. with a lever arm of* 18 *inches.* Ans. 105 degrees.

(2) *A circular shaft when twisted by a couple of* ± 90 *lbs. with a lever arm of* 27 *inches has a shearing unit stress of* 2000 *lbs. per square inch. If the same shaft is twisted by a couple of* ± 40 *lbs. with a lever arm of* 57 *inches, find the shearing unit stress.*

Ans. 1877 lbs. per sq. inch.

(3) *An iron shaft* 5 *ft. long and* 2 *inches diameter is twisted through an angle of* 7 *degrees by a couple of* ± 5000 *lbs. with a lever arm of* 6 *inches, and on removal of the couple springs back to its original position. Find the value of E for shearing.* Ans. 9,390,000 lbs. per sq. inch.

(4) *What is the couple which acting with a lever arm of* 12 *inches will cripple a steel shaft* 1.4 *inches in diameter, the value of R for rupture being* 75,000 *lbs. per sq. inch.*

Ans. ± 1683 lbs.

(5) *Compare the strength of a square shaft with that of a circular shaft of equal area of cross-section.*

Ans. $\frac{\sqrt{2\pi}}{3}$.

(6) *Find the combined unit stresses for a wrought-iron shaft* 3 *inches in diameter and* 12 *feet long, resting on bearings at each end, which transmits* 40 *horse-power while making* 120 *revolutions per minute, upon which a load of* 800 *lbs. is brought by a belt and pulley at the middle.*

Ans. The unit stress for flexure is

$$R_f = \frac{M_f v}{I} = \frac{wl}{\pi r^3} = 10{,}800 \text{ lbs. per sq. inch.}$$

The unit stress for torsion is

$$R_t = \frac{198{,}000 \times 40 \times r}{n\pi I_s} = 4000 \text{ lbs. per sq. inch.}$$

The maximum combined stresses are then :

For tension or compression, $5400 + \sqrt{4000^2 + 5400^2} = 12{,}100$ lbs. per sq. in. ; for shear, 6700 lbs. per sq. in.

(7) *A vertical shaft weighing with its load* 6000 *lbs. is subjected to a twisting moment by a force of* 300 *lbs. with a lever arm of* 4 *feet. If the shaft is of wrought iron* 4 *feet long and* 2 *inches diameter, find its maximum unit stress provided the shaft is so supported that it cannot bend sideways.*

Ans. Compressive unit stress = 10170 lbs. per sq. inch.
Shearing " " = 9215 " " " "

(8) *Find the diameter of a short vertical steel shaft to carry a load of* 6000 *lbs. when twisted by a force of* 300 *lbs. with a leverage of* 4 *feet, taking the unit stress for shear at* 7000 *lbs. and for compression at* 10,000 *lbs. per sq. in.*

Ans. About 2.5 inches.

CHAPTER IV.

COLUMN FORMULAS.

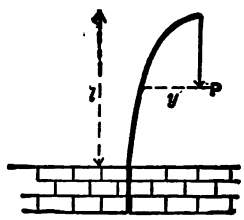
THE IDEAL COLUMN.—The "ideal column" is supposed to be perfectly homogeneous; to have a constant coefficient of elasticity E, a constant cross-section A, a perfectly straight axis; and to have a load P applied exactly in that axis. A load not so applied we call an eccentric load.

THEORY OF THE IDEAL COLUMN.—Such an ideal column, ideally loaded, has no tendency to bend in any direction. It can therefore be loaded up to its elastic limit S_e, so that $\frac{P}{A} = S_e$. But if while loaded with any load P below the elastic limit it is bent very slightly to one side by a lateral force, and then that lateral force is removed, there is a certain value of P for which the column *will not spring back*. For P less than this, the column will spring back. For P greater than this, the column will bend to failure. This value of P is then the *crippling load* for the ideal column.

PRICHARD'S FORMULA.*—Suppose then an ideal column, whose original length is l, to be compressed by the load p in its axis. The new length l_1 of the column, by the theory of flexure (page 284), is

$$l_1 = l\left(1 - \frac{P}{AE}\right). \quad \ldots \ldots \ldots \ldots \quad (1)$$

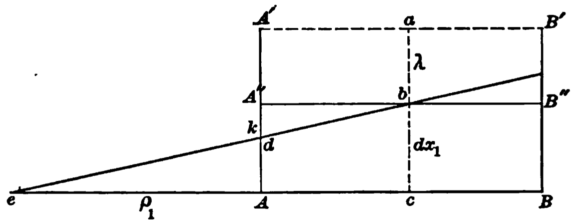
If now the column is very slightly bent by a lateral force, and then that lateral force removed, if the column does not spring entirely back, it remains deflected, as shown in the figure. Let y be the deflection at any point x_1 from the free end. Then the moment M at any point x_1 is $M = Py$. Let dx be the original distance ac, along the axis, between two consecutive sections $A'B'$ and AB of the ideal column, before the load P is applied. Then when the load P is applied the unit stress is $\frac{P}{A}$, and the shortening of the axis $ab = \lambda$ is by the theory of flexure (page 284),



$$\lambda = \frac{Pdx}{AE}.$$

The section $A'B'$ has now the new position $A''B''$, so that the distance $bc = dx_1$.

After the ideal column is thus directly compressed by the load P in the axis, let it be deflected very slightly to one side by a lateral force, and then let this lateral force be removed. If the column remains deflected, there will be equilibrium, and at the point c of the axis a moment $M = Py$ (see first of preceding figures).



Let $v = A''b$ be the distance of the extreme fibre on the concave side from the axis. The unit stress on this fibre due to flexure is by the theory

* This investigation of the ideal column was communicated to the author by H. S. Prichard, M. Am. Soc. C. E. The formula to which it leads is, so far as the author knows, new and original. We therefore call it the *Prichara formula*.

of flexure (page 286) $\frac{Mv}{I}$, where I is the moment of inertia of the cross-section for an axis through c at right angles to the plane of bending. The total unit stress on this fibre is then $\frac{P}{A} + \frac{Mv}{I}$, and its total shortening $\kappa_1 = A'd$ is by the theory of flexure (page 284)

$$\kappa_1 = \frac{\frac{P}{A} + \frac{Mv}{I}}{E} dx = \left(\frac{P}{AE} + \frac{Mv}{EI}\right) dx.$$

Let the distance $A''d = \kappa$. Then we have $\kappa = \kappa_1 - \lambda = \frac{Mvdx}{EI}$.

Now let $\rho_1 = ec$ be the radius of curvature of the deflected axis. Then by similar triangles

$$\rho_1 : dx_1 :: v : \kappa, \quad \text{or} \quad \frac{1}{\rho_1} = \frac{Mdx}{EIdx_1}.$$

Now $M = Py$, $\frac{dx}{dx_1} = \frac{l}{l_1}$, and by Calculus $\frac{1}{\rho_1} = -\frac{d^2y}{dx_1^2}$.

We have then for equilibrium

$$\frac{d^2y}{dx_1^2} = -\frac{lPy}{l_1EI}. \quad \ldots \ldots \ldots \quad (2)$$

Multiply both sides of (2) by $2dy$ and integrate, and we obtain $EI\frac{dy^2}{dx_1^2} = -\frac{lPy^2}{l_1} + C$.

But when $y = \Delta =$ the maximum deflection, $\frac{dy}{dx} = 0$. Hence $C = \frac{lP\Delta^2}{l_1}$, and we have

$$dx_1 = \sqrt{\frac{l_1EI}{lP}} \frac{dy}{\sqrt{\Delta^2 - y^2}}.$$

Integrating again, we obtain $\quad x_1 = \sqrt{\frac{l_1EI}{lP}} \text{ arc sin } \frac{y}{\Delta} + C'$.

When $y = 0$, $x_1 = 0$, hence $C' = 0$, and we obtain

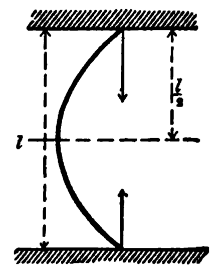
$$y = \Delta \sin x_1\sqrt{\frac{lP}{l_1EI}}. \quad \ldots \ldots \ldots \quad (3)$$

(*a*) *Column Fixed at One End, Free at the Other.*—For a column fixed at one end and free at the other, as in figure, page 332, we have from (3), $y = \Delta$ when $x_1 = l_1$; hence

$$l_1\sqrt{\frac{lP}{l_1EI}} = \frac{\pi}{2},$$

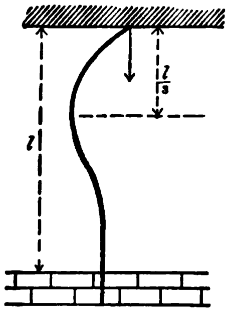
or since $I = Ar^2$, where r is the radius of gyration of the cross-section in the plane of bending, we obtain

$$\frac{P}{A} = \frac{\pi^2Er^2}{4ll_1}.$$



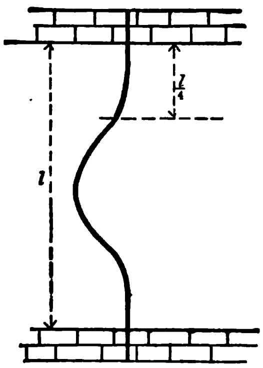
(*b*) *Column with Two Pin Ends.*—In this case we have only to make $x_1 = \frac{l_1}{2}$ in equation (3), when $y = \Delta$. We thus obtain

$$\frac{P}{A} = \frac{\pi^2Er^2}{ll_1}.$$



(*c*) *Column Fixed at One End, Pin at the Other.*—In this case we must make $x_1 = \frac{l_1}{3}$ in equation (3), for $y = \varDelta$. We thus have

$$\frac{P}{A} = \frac{9\pi^2 E r^2}{4 l l_1}.$$



(*d*) *Column Fixed at Both Ends.*—In this case we must make $x_1 = \frac{l_1}{4}$ in equation (3) for $y = \varDelta$. We then have

$$\frac{P}{A} = \frac{4\pi^2 E r^2}{l l_1}.$$

Now all these equations are of the form

$$\frac{P}{A} = \frac{n^2 E r^2}{l l_1}, \quad \ldots \ldots \ldots \ldots \quad (4)$$

where we have for n the values

One fixed, one free end.	Two pin ends.	One fixed, one pin end.	Two fixed ends.
$n = \frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π

If in (4) we substitute the value of l_1 from (1), we obtain the general equation

$$\frac{P}{A} = \frac{P^2}{A^2 E} + \frac{n^2 E r^2}{l^2}. \quad \ldots \ldots \ldots \ldots \quad (P)$$

Equation (P) is *Prichard's* formula. It gives that value of the load P which, when the ideal column is very slightly bent to one side by a lateral force, will hold the column deflected when that lateral force is removed.

EULER'S FORMULA.—We see from (1) that

$$\frac{P}{AE} = \frac{l - l_1}{l}; \quad \therefore \frac{P^2}{A^2 E} = \frac{P}{A}\left(\frac{l - l_1}{l}\right).$$

Now in all practical cases the compression of the axis $l - l_1$ is very small compared to the original length l. Hence the term $\frac{P^2}{A^2 E}$ in Prichard's formula can be practically disregarded. We have then approximately and with practical exactness,

$$\frac{P}{A} = \frac{n^2 E r^2}{l^2}. \quad \ldots \ldots \ldots \ldots \quad (E)$$

Equation (E) is known as *Euler's* formula.

We see then that Euler's formula neglects the effect $(l - l_1)$ of direct compression. Also that such neglect is admissible. Euler's formula gives then with all desirable exactness that value of the load P which, when the ideal column is very slightly bent to one side by a lateral force, will just hold the column deflected when that lateral force is removed.

DEPORTMENT OF THE IDEAL COLUMN.—We have then the following conditions :

1st. So long as the load P is less than given by (E) there will be no deflection of the ideal column. If the column is very slightly deflected by a lateral force and then this force removed, the column will *spring back.*

Since we cannot load the column above the elastic limit unit stress S_e without ultimate failure, if we put $\frac{P}{A} = S_e$ in equation (E) we obtain

$$\frac{l}{r} = \sqrt{\frac{n^2 E}{S_e}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (L)$$

For any value of l *less* than given by (L) we can then load the column up to the elastic limit S_e.

2d. If the load P is equal to the value given by (E), and we deflect the column very slightly, it will not spring back, nor will it bend further. It stays in equilibrium whatever the deflection, provided this deflection is very small.

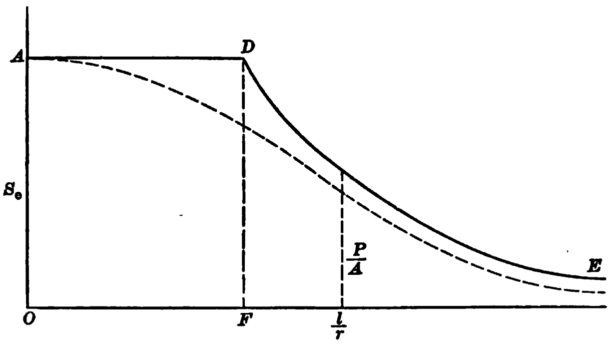
3d. So long as there is no outside lateral force, the ideal column, whatever its length, can be loaded up to the elastic limit. But if it is very slightly deflected, and P is greater than given by (E), the column will bend to failure.

Thus equation (E) *gives the breaking load* for the plane of bending, and the *least* breaking load will be when r is the *least radius of gyration* of the cross-section.

These conclusions have all been verified by experiment.*

THE IDEAL CURVE.—If we lay off $\frac{l}{r}$ along the axis of x and take $\frac{P}{A}$ as ordinate, we obtain then for the "ideal curve" the accompanying figure.

Up to $OF = \sqrt{\frac{n^2 E}{S_e}}$, we have $\frac{P}{A} = S_e$, and the curve is a straight line from A to D. Beyond OF we have the curve DE given by Euler's formula (E).



THE ACTUAL CURVE.—Ordinarily the conclusions and deportment given for the ideal column are not found to be in accord with fact, because the ideal conditions of the ideal column are not realized. Thus no column is perfectly homogeneous, has a perfectly straight axis, nor has the load exactly centered.

Lack of ideality in any of these conditions will cause a column of any length to deflect when loaded, and this is in accord with common experience.

We can then only load the actual column up to S_e when it is very short—theoretically only when the length is zero. For any finite length, $\frac{P}{A}$ must always be less than given by the preceding figure and must decrease as the length increases.

The actual curve, then, for any actual column will be some curve as represented by the broken curve in the preceding figure, which is tangent at A to the line AD, and at E at an infinite distance, to the curve DE.

We see at once that any such curve which should give the actual values of $\frac{P}{A}$ for any one actual column must really depend upon the actual eccentricity of the load and upon all other deviations from ideal conditions.

As all such deviations can never be identical for any two actual columns, the actual curve *must be a different one for each column.*

* See "*A Practical Treatise on Bridge Construction,*" by T. Claxton Fidler (London, Charles Griffin & Co., 1887), page 158, for an experimental apparatus by which these conclusions are practically verified.

It is then obvious that any one curve which gives the *average* experimental values of $\frac{P}{A}$ for any number of actual columns must rest at bottom upon the *average* deviations from ideal conditions, whether the equation of the curve explicitly contains them or not. Such a curve must then be based entirely upon *average* experimental results, and to attempt to deduce theoretically any single curve which shall give actual results for all columns must ever be to attempt the impossible.

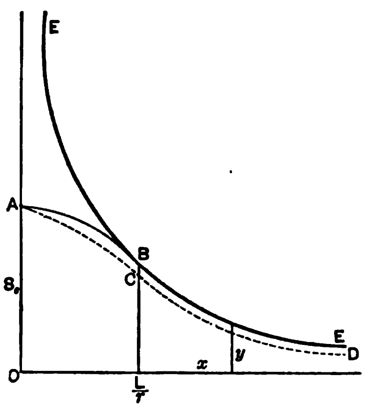
PRACTICAL VALUES FOR n.—The theoretic values of n given on page 334 disregard friction. Also the ends in practice cannot be perfectly "fixed." We have to do practically with two pin ends with friction, or one pin end with friction and one flat end, or two flat ends. A flat end is not perfectly "fixed." Hence the practical values of n are different from the theoretic values given on page 334.

Experiments show that Euler's formula (E) gives the average crippling unit stress $\frac{P}{A}$ with very good accuracy for very long columns, i.e., when $\frac{l}{r}$ is very great, if we take for n the following values:

Two pin ends.	One pin, one flat end.	Two flat ends.
$n = \pi\sqrt{\frac{5}{3}}$	$\frac{5\pi}{2\sqrt{3}}$	$\pi\sqrt{\frac{5}{2}}$

These values of n should be used when Euler's formula is used.

PRACTICAL FORMULAS FOR LONG COLUMNS.—Experiment also shows that actual results are so scattered, that almost any curve through A and tangent to Euler's curve EE gives very satisfactory average results for $\frac{P}{A}$. On this fact all our practical column formulas are based.



STRAIGHT-LINE FORMULA. — Of these practical formulas, one of the best known is the "straight-line formula," first given by Thomas H. Johnson, C.E. (*Trans. Am. Soc. C. E.*, July, 1886). It consists in drawing a straight line through A tangent to Euler's curve EE. The point of tangency B is at a distance from O (preceding figure) given by $\frac{L}{r}$. Both curve EE and straight line AB have then a common ordinate at the point B.

The equation of a straight-line through A is

$$y = S_e + bx. \quad \ldots \quad (1)$$

The equation of Euler's curve EE is

$$y = \frac{n^2E}{x^2}. \quad \ldots \quad (2)$$

If we differentiate (1) and (2), and equate the values of $\frac{dy}{dx}$, making $x = \frac{L}{r}$, we have for the condition of a common tangent at B

$$b = -\frac{2n^2Er^3}{L^3}. \quad \ldots \quad (3)$$

If we equate (1) and (2), making $x = \frac{L}{r}$, we have for the condition of a common ordinate at B

$$S_e + \frac{bL}{r} = \frac{n^2Er^2}{L^2}. \quad \ldots \quad (4)$$

From (3) and (4) we find for the limiting value of $\frac{L}{r}$

$$\frac{L}{r} = n\sqrt{\frac{3E}{S_e}}, \quad \text{and hence} \quad b = -\frac{2S_e\sqrt{S_e}}{3n\sqrt{3E}}.$$

Inserting this value of b in (1), and putting $y = \frac{P}{A}$ and $x = \frac{l}{r}$, we have for the straight-line formula,

$$\text{when } \frac{l}{r} < n\sqrt{\frac{3E}{S_e}}, \qquad \frac{P}{A} = S_e\left[1 - \frac{2\sqrt{S_e}}{3n\sqrt{3E}} \cdot \frac{l}{r}\right], \quad \text{(S)}$$

where, as always, r is the *least radius of gyration of the cross-section.*

This formula holds for any value of $\frac{l}{r}$ so long as

$$\frac{l}{r} < n\sqrt{\frac{3E}{S_e}}.$$

Beyond this limit we use Euler's formula, and have,

$$\text{when } \frac{l}{r} > n\sqrt{\frac{3E}{S_e}}, \qquad \frac{P}{A} = \frac{n^2Er^2}{l^2}.$$

Values for E and S_e will be given in Part II.

The straight-line formula is simple and easily applied, and contains no experimental constants except S_e, E, and n.

It gives values for $\frac{P}{A}$ for small values of $\frac{l}{r}$ considerably less than the average of experiments, owing to the fact that the tangent at A (figure, page 336) is not horizontal.

PARABOLA FORMULA.—This formula is given by Prof. J. B. Johnson (Theory and Practice of Modern Framed Structures—Wiley & Sons). The curve AB (figure, page 336) is assumed as a parabola tangent to Euler's curve at B. We have then

$$y = S_e + bx^2, \quad (1)$$

where b must be determined by the condition of tangency.

This equation gives $y = S_e$ for $x = 0$, and the tangent at A is horizontal.

From Euler's formula we have

$$y = \frac{n^2E}{x^2} \quad (2)$$

Differentiating (1) and (2), and proceeding as before, we have the parabola formula,

$$\text{when } \frac{l}{r} < n\sqrt{\frac{2E}{S_e}}, \qquad \frac{P}{A} = S_e\left[1 - \frac{S_e}{4n^2E} \cdot \frac{l^2}{r^2}\right], \quad \text{(P)}$$

where, as always, r is the least radius of gyration of the cross-section.

This formula holds for any value of $\frac{l}{r}$ so long as

$$\frac{l}{r} < n\sqrt{\frac{2E}{S_e}}.$$

Beyond this limit we use Euler's formula, and have,

$$\text{when } \frac{l}{r} > n\sqrt{\frac{2E}{S_e}}, \qquad \frac{P}{A} = \frac{n^2Er^2}{l^2}.$$

Values for E and S_e will be given in Part II.

The parabola formula is as simple and easily applied as the straight-line formula. It also

contains no experimental constants except S_e, E, and n. It gives on the whole better average values for $\frac{P}{A}$, owing to the fact that the tangent at A (figure, page 336) is horizontal.

REMARKS ON THESE FORMULAS.—Both the straight-line and the parabola formulas are lines tangent to Euler's curve EE (preceding figure). This means, in the light of our remarks, page 335, that both implicitly *assume ideal conditions for all columns at and beyond a certain length L, which is a different length for each formula.*

Such an assumption is of course incorrect. There is no one length, to say nothing of two different lengths, at which ideal conditions can be considered as always existing. Experiments, however, show that average values of $\frac{P}{A}$ approach at and beyond both these lengths very closely to Euler's curve EE (preceding figure), being always, however, slightly below; and hence the assumption is practically justified.

The actual average curve, however, as we have seen (page 336) should run through A as shown by the dotted curve in the preceding figure, should have a horizontal tangent at A, and should then run as shown, somewhat below Euler's curve EE, and be tangent to it at E at an infinite distance.

RANKINE'S FORMULA.—Such a curve is "Rankine's formula." Let Δ be the maximum deflection. Then the maximum moment is $P\Delta$.

From the theory of flexure (page 286) we have for the unit stress R, due to flexure in the most compressed fibre, at a distance v from the axis in the plane of bending,

$$P\Delta = \frac{RI}{v}, \quad \text{or} \quad R = \frac{P\Delta v}{I} = \frac{P\Delta v}{Ar^2}.$$

We have, in addition, a direct compressive unit stress $\frac{P}{A}$.

If, then, S_e is the elastic limit unit stress, we have for the crippling unit stress

$$\frac{P}{A} + \frac{P\Delta v}{Av^2} = S_e,$$

or

$$\frac{P}{A} = \frac{S_e}{1 + \frac{\Delta v}{r^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Equation (1) is rational in form, and if we knew Δ it would give accurately the crippling unit stress $\frac{P}{A}$.

If we suppose for small deflections the curve of deflection to be practically a circle of radius of curvature ρ, we should have

$$\Delta : l :: l : \rho - \Delta;$$

or, since Δ is small compared to ρ,

$$\Delta = \frac{l^2}{\rho}.$$

In general, whatever the curve of deflection may be, we can assume Δ to be some function of l^2, and to vary inversely as v. We can then write

$$\Delta = \frac{cl^2}{v}, \quad \text{or} \quad \frac{\Delta v}{r^2} = c\frac{l^2}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Inserting this value of $\frac{\Delta v}{r^2}$ in (1), we have

$$\frac{P}{A} = \frac{S_e}{1 + c\frac{l^2}{r^2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (R)$$

where, as always, r is the least radius of gyration of the cross-section, and c is a constant to be determined by experiment, depending upon the material and the end conditions. Since the column bends easiest in the direction of its least dimension, *we take for r the least radius of gyration.*

Equation (R) is Rankine's formula for long columns. It holds for all values of $\frac{l}{r}$. Values for S_e and c will be given in Part II.

We see that Rankine's formula gives $\frac{P}{A} = S_e$ for $\frac{l}{r} = 0$. The tangent at A (figure, page 336) is horizontal, and we have $\frac{P}{A} = 0$ for $\frac{l}{r} = \infty$. It therefore complies with the conditions for the average actual curve given on page 335.

It is not so simple or easily applied as the straight-line or parabola formulas, and the experimental constant c must be determined before it can be used in any case.

GORDON'S FORMULA.—Since r is a function of the *least dimension* d of the cross-section, we may also write for the crippling unit stress

$$\frac{P}{A} = \frac{S_e}{1 + c\frac{l^2}{d^2}}, \quad \text{(G)}$$

where c is again a constant, to be determined by experiment. Equation (G) is known as Gordon's formula for long struts. It also holds for all values of $\frac{l}{r}$, and the same remarks apply as for Rankine's formula.

MERRIMAN'S FORMULA.—The equation of the curve AB (figure, page 334) has been assumed by Prof. Merriman (*Engineering News*, July 19, 1894) as identical in form with Rankine's formula. We have, then,

$$y = \frac{S_e}{1 + bx^2}.$$

Instead, however, of regarding b as an experimental constant, Prof. Merriman determines b precisely as in the case of the straight-line and parabola formulas, by the condition of tangency.

We thus obtain

$$\frac{P}{A} = \frac{S_e}{1 + \frac{S_e l^2}{n^2 E r^2}}, \quad \text{(M)}$$

where, as always, r is the *least radius of gyration of the cross-section.*

Values for S_e and E will be given in Part II.

Equation (M) is Merriman's formula for long columns. Like Rankine's formula, it complies with the conditions of the average actual curve given on page 335. It is preferable to Rankine's in that it contains no experimental constant. It is therefore probably nearer the true curve for an average actual column than any of the formulas thus far given.

ALLOWABLE UNIT STRESS.—The preceding formulas give the crippling unit stress for long columns. To find the allowable or safe unit stress σ we must divide the crippling unit stress by the factor of safety adopted.

COMBINED COMPRESSION AND FLEXURE.—The formula for this case has already been given, page 313.

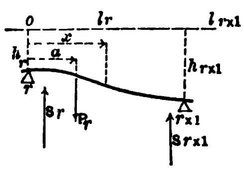
CHAPTER V.

CONTINUOUS GIRDER.

In the following we shall give the complete development of the general formulas of Chapter VIII, page 171. As these formulas include, as we have seen, all the others as special cases, it is sufficient to show how they are obtained in order to enable the reader to deduce all the others. The notation adopted is the same as that given on page 173.

CONDITIONS OF EQUILIBRIUM.—In the rth span of a continuous girder, whose length is l_r, Fig. 188, take a point o vertically above the rth support as the origin of co-ordinates, and the horizontal through o as the axis of abscissas. At a distance x from the left support, conceive a vertical section, and between the support and this section let there be a concentrated load P_r, whose distance from the left support is a.

Fig. 188



Now, if the girder is continuous over any number of supports, we have at the support r a moment M_r, and just to the right of support r a shear S_r.

For any point of the girder the necessary conditions of equilibrium are,

1st. The algebraic sum of all the horizontal forces must be zero.

2d. The algebraic sum of all the vertical forces must be zero.

3d. The algebraic sum of the moments of all the forces must be zero.

Thus for any section x we have from the third condition for the moment M_x at the section x

$$M_x = M_r - S_r x + P_r (x - a). \quad \ldots \ldots \ldots \quad (1)$$

If in this we make $x = l_r$, M_x becomes M_{r+1}, and we have

$$M_{r+1} = M_r - S_r l_r + P_r (l_r - a).$$

From this we obtain the shear S_r in terms of the moments at the two supports, or

$$S_r = \frac{M_r - M_{r+1}}{l_r} + \frac{P_r}{l_r}(l_r - a). \quad \ldots \ldots \ldots \quad (2)$$

This is the same as equation (III.a), page 173.

For an unloaded span the weight P disappears, and we have

$$S_m = \frac{M_m - M_{m+1}}{l_m}.$$

This is equation (IV.), given on page 173.

For the shear just to the left of the right support of loaded span

$$S'_{r+1} = P - S_r = \frac{M_{r+1} - M_r}{l} + \frac{P_r a}{l_r}.$$

This is the same as equation (III.b,) given on page 173. For unloaded span, the weight P disappears, and

$$S'_m = \frac{M_m - M_{m-1}}{l_{m-1}}.$$

S'_m is the shear on the left of any support m, and S_m that on the right. The reaction at any support is

$$R_m = S'_m + S_m.$$

These are the formulas already given in Chapter VIII, page 173.

EQUATION OF THE ELASTIC LINE.—We can now easily deduce the equation of the elastic line for the continuous girder of constant cross-section, or constant moment of inertia.

The differential equation of the elastic line is (page 289)

$$EI\frac{d^2y}{dx^2} = -M_x, \quad \ldots \ldots \ldots \ldots \quad (3)$$

where E is the coefficient of elasticity, and I is the moment of inertia of the cross-section.

Inserting the value of M_x as given by (1) we have

$$\frac{d^2y}{dx^2} = \frac{S_rx - P_r(x-a) - M_r}{EI}.$$

We can integrate this expression between the limits $x = 0$, and x, upon the condition that x is always greater than a, that is, *the point considered is always on the right of the weight.* When, therefore, $x = 0$, a must be zero also, and hence $(x - a) = 0$. We must, therefore, take the integral of $P_r(x - a)$ simultaneously between the limits $x = a$, and x, or treat $(x - a)$ as a variable which becomes zero when $x = 0$.

We have, then, integrating once,

$$\frac{dy}{dx} = \frac{S_rx^2 - P_r(x-a)^2 - 2M_rx}{2EI} + C,$$

where the constant of integration $C = \frac{dy}{dx} = t_r$ = the tangent of the angle which the tangent at the support r to the curve of deflection makes with the horizontal. Hence

$$\frac{dy}{dx} = t_r + \frac{S_rx^2 - P_r(x-a)^2 - 2M_rx}{2EI}. \quad \ldots \ldots \ldots \quad (3a)$$

If we take the origin at a distance h_r above the support r, Fig. 188, and integrate again, the constant will be $-h_r$, and hence

$$y = -h_r + t_rx + \frac{S_rx^3 - P_r(x-a)^3 - 3M_rx^2}{6EI}, \quad \ldots \ldots \ldots \quad (4)$$

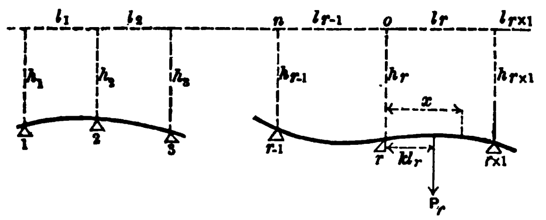
which is the general equation of the elastic curve. If in this we make $x = l_r$, y becomes $-h_{r+1}$. If also we put $\frac{a}{l_r} = k$, or $a = kl_r$, and insert for S_r its value as given by (2), we have for t_r

$$t_r = -\frac{h_{r+1} - h_r}{l_r} + \frac{1}{6EI}[2M_rl_r + M_{r+1}l_r - P_rl_r^2(2k - 3k^2 + k^3)]. \quad \ldots \quad (5)$$

We see, therefore, that the equation of the curve of deflection is completely determined when we know M_r and M_{r+1}, the moments at the two supports of the loaded span.

THEOREM OF THREE MOMENTS.—These moments are readily found by applying the "theorem of three moments," which we shall now deduce.

Fig. 189



In Fig. 189 we have represented a portion of a continuous girder, the spans being l_1, l_2, etc., l_r, and the supports 1, 2 . . . . r.

The equation of the elastic line between P_r and the $r+1$th support is given by (4), and the tangent of the angle which the curve makes with the horizontal is given by (3 a). If in (3 a) we substitute for S_r its value as given by (2), and for t_r its value from (5), and make at the same time $x = l_r$, then $\frac{dy}{dx}$ becomes t_{r+1}, or the tangent at $r+1$, and we have

$$t_{r+1} = -\frac{h_{r+1} - h_r}{l_r} - \frac{1}{6EI}\left[M_r l_r + 2 M_{r+1} l_r - P_r l_r^2 (k - k^3)\right] \quad . \quad . \quad . \quad . \quad (6)$$

Equation (6) gives the tangent of the angle which the tangent to the curve, at the support $r+1$, makes with the horizontal.

If we were to suppose a weight P_{r-1} in the span l_{r-1} at a distance kl_{r-1} from the support $r-1$, the origin being taken at n, Fig. 189, instead of at o, and were to find in a similar manner t_r, we should evidently obtain precisely the same equation as (6), only each of the subscripts would be diminished by unity. Hence we can write down at once

$$t_r = -\frac{h_r - h_{r-1}}{l_{r-1}} - \frac{1}{6EI}\left[M_{r-1} l_{r-1} + 2 M_r l_{r-1} - P_{r-1} l_{r-1}^2 (k - k^3)\right] \quad . \quad . \quad . \quad (7)$$

If there is no weight in the span l_{r-1}, equation (7) still holds good, only P_{r-1} is zero.

But equation (5) gives us t_r for a weight P_r in the span l_r. If there is no weight in that span P_r is zero. Equating these two values of t_r, we have, generally,

$$M_{r-1} l_{r-1} + 2 M_r (l_{r-1} + l_r) + M_{r+1} l_r = + 6 EI\left[\frac{h_{r-1} - h_r}{l_{r-1}} + \frac{h_{r+1} - h_r}{l_r}\right]$$

$$+ P_{r-1} l_{r-1}^2 (k - k^3) + P_r l_r^2 (2k - 3k^2 + k^3) \quad . \quad . \quad . \quad . \quad (8)$$

This is the general form of the theorem of three moments for a girder of constant cross section. It gives the relation between the moments at three consecutive supports, in terms of the spans, the load in the spans and the height of the supports.

The moments at the end supports are of course zero, when the girder is merely supported at the ends. For each of the piers, then, we can write an equation like the above, and thus we have as many equations as there are unknown moments.

DETERMINATION OF MOMENTS—UNIFORM LOAD—SUPPORTS ALL ON A LEVEL—SPANS ALL EQUAL.—When all the supports are in the same horizontal line, the ordinates h_1, h_2, h_3, etc., are all equal. Hence the term involving EI disappears, and we have simply

$$M_{r-1} l_{r-1} + 2 M_r (l_{r-1} + l_r) + M_{r+1} l_r = + P_{r-1} l_{r-1}^2 (k - k^3) + P_r l_r^2 (2k - 3k^2 + k^3). \quad . \quad (9)$$

If the spans are all equal, we have

$$M_{r-1} + 4 M_r + M_{r+1} = + P_{r-1} l (k - k^3) + P_r l (2k - 3k^2 + k^3). \quad . \quad . \quad (10)$$

If we have the girder loaded from end to end with the load u per unit of length, then $u\,da = P$

Substituting this value of P and remembering that $k = \frac{a}{l}$, and integrating between $a = 0$ and $a = l$, we have

$$P_{r-1} l_{r-1}^2 \,(k - k^3) = \frac{u l_{r-1}^3}{4}, \text{ and } P_r l^2 \,(2k - 3k^2 + k^3) = \frac{u l_r^3}{4}.$$

Hence for level supports, spans all equal, and uniform load, our theorem reduces to

$$M_{r-1} + 4 M_r + M_{r+1} = + \frac{u l^2}{2} \quad \ldots\ldots\ldots\ldots \quad (11)$$

Let s be the number of spans. Then, applying equation (11) and remembering that M_1 and M_{s+1} are both zero, we can write down the following equations:

$$\left.\begin{array}{lr}
c_2 & 4 M_2 + M_3 = + \frac{u l^2}{2}, \\
c_3 & M_2 + 4 M_3 + M_4 = + \frac{u l^2}{2}, \\
c_4 & M_3 + 4 M_4 + M_5 = + \frac{u l^2}{2}, \\
c_5 & M_4 + 4 M_5 + M_6 = + \frac{u l^2}{2}, \\
c_6 & M_5 + 4 M_6 + M_7 = + \frac{u l^2}{2}, \\
 & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
 & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
 & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
c_{s-1} & M_{s-2} + 4 M_{s-1} + M_s = + \frac{u l^2}{2}, \\
c_s & M_{s-1} + 4 M_s = + \frac{u l^2}{2}.
\end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (12)$$

The solution of these equations can be best effected by the method of indeterminate coefficients. Thus we multiply the first equation by a number c_2, whose value we shall hereafter determine, so as to satisfy desired conditions. The second we multiply by c_3, the third by c_4, the r th by c_{r+1}, etc., the index of c corresponding always to that of M in the middle term. Having performed these multiplications, add the equations and arrange according to the coefficients of M_2, M_3, etc. We thus obtain the equation

$$\left.\begin{array}{l}
(4 c_2 + c_3) M_2 + (c_2 + 4 c_3 + c_4) M_3 + (c_3 + 4 c_4 + c_5) M_4 + \ldots\ldots\ldots \\
+ (c_{s-2} + 4 c_{s-1} + c_s) M_{s-1} + (c_{s-1} + 4 c_s) M_s = + \frac{u l^2}{2} (c_2 + c_3 + \ldots . c_s)
\end{array}\right\} \quad \ldots \quad (13)$$

Now suppose we wish to determine M_s. We have only to require that such relations shall exist among the multipliers c, that all the terms except the last in the above equation shall disappear. We have, then, for the conditions which these multipliers must satisfy,

$$4c_2 + c_3 = 0, \quad c_3 + 4c_4 + c_5 = 0, \quad c_{s-2} + 4c_{s-1} + c_s = 0,$$
$$c_2 + 4c_3 + c_4 = 0, \quad c_4 + 4c_5 + c_6 = 0, \text{ etc.}$$

Assuming $c_2 = 1$, we find, therefore,

$$c_3 = -4, \quad c_4 = +15, \quad c_5 = -56, \quad c_6 = +209, \quad c_7 = -780, \quad c_8 = +2{,}911, \text{ etc.}$$

The numbers, as we see, change sign alternately, and each is equal to four times the preceding, minus the one next preceding it.

From equations (12) we see now that

$$M_3 = c_3 M_2 + \frac{ul^2}{2},$$

$$M_4 = -4M_3 - M_2 + \frac{ul^2}{2} = -4c_3 M_2 - 2ul^2 - M_2 + \frac{ul^2}{2} = c_4 M_2 - \frac{3ul^2}{2},$$

$$M_5 = -4M_4 - M_3 + \frac{ul^2}{2} = -4c_4 M_2 + 6ul^2 - c_3 M_2 - \frac{ul^2}{2} + \frac{ul^2}{2} = c_5 M_2 + \frac{12\,ul^2}{2}.$$

In similar manner,

$$M_6 = c_6 M_2 - \frac{44\,ul^2}{2}, \quad M_7 = c_7 M_2 + \frac{165\,ul^2}{2}, \text{etc.}$$

But $1, -3, +12, -44, +165$, etc., are the algebraic sums of c_2, $c_2 + c_3$, $c_2 + c_3 + c_4$, $c_2 + c_3 + c_4 + c_5$, etc., respectively. Hence we have in general for the moment at any support,

$$M_m = c_m M_2 + \frac{ul^2}{2}(c_2 + \ldots c_{m-1}). \quad \ldots \ldots \quad (14)$$

Now from equation (13), since all the terms except the one containing M_s are zero, we have

$$M_s = + \frac{\frac{ul^2}{2}(c_2 + c_3 + \ldots c_s)}{c_{s-1} + 4c_s}.$$

But since, according to the law of the numbers denoted by c, $c_{s-1} + 4c_s + c_{s+1} = 0$, we have $c_{s-1} + 4c_s = -c_{s+1}$. Hence

$$M_s = -\frac{ul^2(c_2 + c_3 + \ldots c_s)}{2c_{s+1}}.$$

If the spans are all equal, the supports horizontal, and the load uniform over the whole girder, the moment at the support s must be the same as the moment at the support 2, or $M_s = M_2$. Hence

$$M_2 = -\frac{ul^2(c_2 + c_3 + \ldots\ldots c_s)}{2c_{s+1}}. \quad \ldots \ldots \quad (15)$$

Equation (14) then becomes

$$M_m = -c_m\frac{ul^2(c_2 + c_3 + \ldots c_s)}{2c_{s+1}} + \frac{ul^2}{2}(c_2 + \ldots\ldots c_{m-1}). \quad \ldots \ldots \quad (16)$$

If we write down the values

$$
\begin{aligned}
c_2 &= 1,\\
4\,c_2 + c_3 &= 0,\\
c_2 + 4\,c_3 + c_4 &= 0,\\
c_3 + 4\,c_4 + c_5 &= 0, \text{ etc.,}
\end{aligned}
$$

we see that the sum is in general

$$6\,(c_2 + c_3 + \ldots\, c_m) + 5\,c_{m+1} + c_{m+2} = 1.$$

Hence the sum of the first m numbers is

$$(c_2 + c_3 + \ldots\ldots\, c_m) = \tfrac{1}{6}\,(1 - 5\,c_{m+1} - c_{m+2}) \quad \ldots\ldots \quad (17)$$

Applying this formula for the sum of the numbers to equation (16), we have

$$(c_2 + \ldots\, c_s) = \tfrac{1}{6}\,(1 - 5\,c_{s+1} - c_{s+2}),$$

$$(c_2 + \ldots\, c_{m-1}) = \tfrac{1}{6}\,(1 - 5\,c_m - c_{m+1}).$$

Hence

$$M_m = -\frac{ul^2}{12\,c_{s+1}}\left[c_m\,(1 - 5\,c_{s+1} - c_{s+2}) - c_{s+1}\,(1 - 5\,c_m - c_{m+1})\right],$$

or, after reducing,

$$M_m = -\frac{ul^2}{12\,c_{s+1}}\left[c_m\,(1 - c_{s+2}) - c_{s+1}\,(1 - c_{m+1})\right] \quad \ldots\ldots\ldots\ldots \quad (18)$$

This is the formula given on page 179.

DETERMINATION OF THE MOMENTS—SUPPORTS ALL ON LEVEL—CONCENTRATED LOAD—SPANS ALL DIFFERENT.—When all the supports are on a level, the term involving EI in the theorem of three moments disappears, and we have

$$M_{r-1}l_{r-1} + 2\,M_r\,(l_{r-1} + l_r) + M_{r+1}l_r = + P_{r-1}l_{r-1}^2\,(k - k^3) + P_r\,l_r^2\,(2\,k - 3\,k^2 + k^3).$$

Now let s be the number of spans, and let a single load P be placed in the rth span.

From the above theorem, since M_1 and M_{s+1} are zero, we may write down the following equations:

$$
\left.
\begin{aligned}
2\,M_2\,(l_1 + l_2) + M_3\,l_2 &= 0,\\
M_2\,l_2 + 2\,M_3\,(l_2 + l_3) + M_4\,l_3 &= 0,\\
\cdots\cdots\cdots\cdots&\cdots\\
\cdots\cdots\cdots\cdots&\cdots\\
M_{r-1}l_{r-1} + 2\,M_r\,(l_{r-1} + l_r) + M_{r+1}l_r &= -\,P_r l_r^2\,(2\,k - 3\,k^2 + k^3) = +\,A,\\
M_r\,l_r + 2\,M_{r+1}\,(l_r + l_{r+1}) + M_{r+2}l_{r+1} &= -\,P_r\,l_r^2\,(k - k^3) = +\,B,\\
\cdots\cdots\cdots\cdots&\cdots\\
\cdots\cdots\cdots\cdots&\cdots\\
M_{s-2}l_{s-2} + 2\,M_{s-1}\,(l_{s-2} + l_{s-1}) + M_s\,l_{s-1} &= 0,\\
M_{s-1}l_{s-1} + 2\,M_s\,(l_{s-1} + l_s) &= 0,
\end{aligned}
\right\} \quad \ldots\ldots \quad (19)
$$

We can best solve these equations by the method of indeterminate coefficients.

Thus we multiply the first equation by c_2, the second by c_3, and so on, the index of c corresponding always to that of the middle term. Having performed these multiplications, add the equations and arrange according to the coefficients of M_2, M_3, etc. We thus have the equation

$$\begin{aligned}
&[2\,c_2\,(l_1 + l_2) + c_3\,l_2]\,M_2 + [c_2\,l_2 + 2\,c_3\,(l_2 + l_3) + c_4\,l_3]\,M_3 + \ldots, \\
&+ [c_{r-1}l_{r-1} + 2\,c_r\,(l_{r-1} + l_r) + c_{r+1}l_r]\,M_r + \ldots, \\
&+ [c_{s-2}l_{s-2} + 2\,c_{s-1}\,(l_{s-2} + l_{s-1}) + c_s\,l_{s-1}]\,M_{s-1}, \\
&+ [c_{s-1}l_{s-1} + 2\,c_s\,(l_{s-1} + l_s)]\,M_s = + Ac_r + Bc_{r+1} \quad \ldots \quad (20)
\end{aligned}$$

Now suppose we wish to determine M_s. We have only to impose such conditions upon the multipliers that all the terms in the first member of the above equation, except the last, shall disappear. We have then evidently, for the conditions which the multipliers must satisfy,

$$\begin{aligned}
2\,c_2\,(l_1 + l_2) + c_3l_2 &= 0, \\
c_2l_2 + 2\,c_3\,(l_2 + l_3) + c_4l_3 &= 0, \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
c_{r-1}l_{r-1} + 2\,c_r\,(l_{r-1} + l_r) + c_{r+1}l_r &= 0, \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
c_{s-2}l_{s-2} + 2\,c_{s-1}\,(l_{s-2} + l_{s-1}) + c_sl_{s-1} &= 0, \text{ etc.,}
\end{aligned}$$

while for M_s we have at once

$$M_s = + \frac{Ac_r + Bc_{r+1}}{c_{s-1}l_{s-1} + 2\,c_s\,(l_{s-1} + l_s)} = - \frac{Ac_r + Bc_{r+1}}{c_{s+1}l_s} \quad \ldots \quad (21)$$

If in similar manner we multiply the *last* of equations (19) by the number d_2, the last but one by d_3, the rth by d_{s-r+1}, etc., then add, and make all terms except that containing M_2 equal to zero, we should have the conditions

$$\begin{aligned}
2\,d_2\,(l_s + l_{s-1}) + d_3l_{s-1} &= 0, \\
d_2l_{s-1} + 2\,d_3\,(l_{s-1} + l_{s-2}) + d_4l_{s-2} &= 0, \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
d_{s-r+1}l_r + 2\,d_{s-r+2}(l_r + l_{r-1}) + d_{s-r+3}l_{r-1} &= 0, \\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\
d_{s-2}l_3 + 2\,d_{s-1}\,(l_3 + l_2) + d_sl_2 &= 0,
\end{aligned}$$

while for the moment M_2 we have

$$M_2 = + \frac{Ad_{s-r+2} + Bd_{s-r+1}}{d_{s-1}l_2 + 2\,d_s\,(l_2 + l_1)} = - \frac{Ad_{s-r+2} + Bd_{s-r+1}}{d_{s+1}l_1} \quad \ldots \quad (22)$$

The values of M_2 and M_s are thus given in terms of the quantities A and B and c and d.

A and B depend simply upon the load and its position in the rth span. Thus $A_r = Pl_r^2\,(2\,k - 3\,k^2 + k^3)$, $B_r = Pl_r^2\,(k - k^3)$, as for the multipliers c and d, they depend only upon the lengths of the

spans, and need only satisfy the conditions above. Hence assuming $c_1 = 0$, $c_2 = 1$, and $d_1 = 0$, $d_2 = 1$, we can deduce the proper values for all the others. Thus

$$c_1 = 0, \qquad\qquad d_1 = 0,$$

$$c_2 = 1, \qquad\qquad d_2 = 1,$$

$$c_3 = -2\frac{l_1 + l_2}{l_2}, \qquad\qquad d_3 = -2\frac{l_s + l_{s-1}}{l_{s-1}},$$

$$c_4 = -2c_3\frac{l_2 + l_3}{l_3} - c_2\frac{l_2}{l_3}, \qquad\qquad d_4 = -2d_3\frac{l_{s-1} + l_{s-2}}{l_{s-2}} - d_2\frac{l_{s-1}}{l_{s-2}},$$

$$c_5 = -2c_4\frac{l_3 + l_4}{l_4} - c_3\frac{l_3}{l_4}, \qquad\qquad d_5 = -2d_4\frac{l_{s-2} + l_{s-3}}{l_{s-3}} - d_3\frac{l_{s-2}}{l_{s-3}},$$

$$c_6 = -2c_5\frac{l_4 + l_5}{l_5} - c_4\frac{l_4}{l_5}, \text{ etc.,} \qquad\qquad d_6 = -2d_5\frac{l_{s-3} + l_{s-4}}{l_{s-4}} - d_4\frac{l_{s-3}}{l_{s-4}}, \text{ etc.}$$

Now from equations (19) we see at once by examination, that $M_3 = c_3M_2$, $M_4 = c_4M_2$, $M_5 = c_5M_2$, etc., or generally when $m < r + 1$

$$M_m = c_mM_2 = -\frac{c_m}{d_{s+1}l_1}(A_rd_{s-r+2} + B_rd_{s-r+1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Also taking the same equations in reverse order, $M_{s-1} = d_3M_s$, $M_{s-2} = d_4M_s$, etc., or generally, when $m > r$,

$$M_m = d_{s-m+2}M_s = -\frac{d_{s-m+2}}{c_{s+1}l_s}(A_rc_r + B_rc_{r+1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

These are the equations I. and II., given on page 173.

Uniform Load.—The above equations (23) and (24), give the moment at any support for a concentrated load in any span. For a uniform load over the whole of any one span, we have only to give a different value to A and B.

Thus, for several concentrated loads, we should have

$$A_r = \Sigma P_rl_r^2\,(2k - 3k^2 + k^3). \qquad\qquad B_r = \Sigma P_rl_r^2\,(k_r - k_r^3).$$

For a uniform load over the whole span l_r, let w be the load per unit of length, then

$$\Sigma P = \int_0^{l_r} w\,da, \quad \text{or since } a = kl_r,\ \Sigma P = \int_0^1 wl_r\,dk.$$

Inserting this in place of ΣP, and integrating, we have

$$A = \tfrac{1}{4}\,wl_r^3.$$

In similar way we find

$$B = \tfrac{1}{4}\,wl_r^3.$$

These are the values of A and B given on page 174.

Equations (23) and (24) hold good, therefore, both for uniform and concentrated loading, for any number of spans of any lengths, provided only the supports are all on a level, and only one span is loaded.

UNIFORM LOAD OVER ENTIRE LENGTH OF GIRDER.—Let the uniform load u, per unit of length extend over the entire girder, covering all the spans. Then the general theorem of three moments (page 342) becomes

$$M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r = +\frac{u}{4}(l^3_{r-1} + l_r^3).$$

Applying this theorem and remembering that M_1 and M_{s+1} are zero, we have the following equations:

$$\left.\begin{aligned}
2M_2(l_1 + l_2) + M_3 l_2 &= +\frac{u}{4}(l_1^3 + l_2^3),\\
M_2 l_2 + 2M_3(l_2 + l_3) + M_4 l_3 &= +\frac{u}{4}(l_2^3 + l_3^3),\\
M_3 l_3 + 2M_4(l_3 + l_4) + M_5 l_4 &= +\frac{u}{4}(l_3^3 + l_4^3),\\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot\\
M_{s-2}l_{s-2} + 2M_{s-1}(l_{s-2} + l_{s-1}) + M_s l_{s-1} &= +\frac{u}{4}(l^3_{s-2} + l^3_{s-1}),\\
N_{s-1}l_{s-1} + 2N_s(l_{s-1} + l_s) &= +\frac{u}{4}(l^3_{s-1} + l_s^3).
\end{aligned}\right\} \quad \ldots \ldots (1)$$

Let us solve these equations precisely as we have equations (19) on page 345. Thus if we multiply the first by c_2, the second by c_3, and so on, and then add and arrange according to the coefficients of M_2, M_3, etc., we have

$$[2c_2(l_1 + l_2) + c_3 l_2]M_2 + [c_2 l_2 + 2c_3(l_2 + l_3) + c_4 l_3]M_3$$
$$+ \ldots + [c_{s-2}l_{s-2} + 2c_{s-1}(l_{s-2} + l_{s-1}) + c_s l_{s-1}]M_{s-1}$$
$$+ [c_{s-1}l_{s-1} + 2c_s(l_{s-1} + l_s)]M_s = -\frac{u}{4}[(l_1^3 + l_2^3)c_2 + (l_2^3 + l_3^3)c_3 + \ldots (l^3_{s-1} + l_s^3)c_s].$$

If we give these numbers c, such values as shall make each term in the left of this equation, except the one containing M_s, equal to zero, we shall have evidently the same values for c as given on page 347.

If, in similar manner, we multiply the last of equations (1) by d_2, the next by d_3, and so on, and add and arrange as before, we should have

$$[2d_2(l_s + l_{s-1}) + d_3 l_{s-1}]M_s + [d_2 l_{s-1} + 2d_3(l_{s-1} + l_{s-2}) + d_4 l_{s-2}]M_{s-1}$$
$$+ \ldots + [d_{s-2}l_3 + 2d_{s-1}(l_3 + l_2) + d_s l_2]M_3$$
$$+ [d_{s-1}l_2 + 2d_s(l_2 + l_1)]M_2 = -\frac{u}{4}[(l^3_{s-1} + l_s^3)d_2 + (l^3_{s-2} + l^3_{s-1})d_3 + \ldots (l_1^3 + l_2^3)d_s].$$

If we give these numbers d, such values as shall make all the terms on the left except the one

containing M_2, equal to zero, we shall have evidently the same values for d as on page 346. If then we take these values for d, we have

$$M_2 = + \frac{\frac{u}{4}[(l_{s-1}^2 + l_s^2)\, d_2 + \ldots\ldots (l_1^2 + l_2^2)\, d_s]}{d_{s-1} l_2 + 2\, d_s(l_2 + l_1)} \quad \ldots\ldots\ldots \quad (2)$$

But from equation (1) we see that

$$M_3 = + \frac{u}{4 l_2}(l_1^2 + l_2^2) + c_3 M_2,$$

or

$$M_3 = \frac{u}{4} b_3 + c_3 M_2, \qquad \text{where } b_3 = + \frac{l_1^2 + l_2^2}{l_2}.$$

In similar manner,

$$M_4 = \frac{u}{4} b_4 + c_4 M_2, \qquad \text{where } b_4 = + \frac{l_2^2 + l_3^2}{l_3} + 2\, b_3 \frac{l_2 + l_3}{l_3},$$

$$M_5 = \frac{u}{4} b_5 + c_5 M_2, \qquad \text{where } b_5 = + \frac{l_3^2 + l_4^2}{l_4} + 2\, b_4 \frac{l_3 + l_4}{l_4} + b_3 \frac{l_3}{l_4},$$

$$M_6 = \frac{u}{4} b_6 + c_6 M_2, \qquad \text{where } b_6 = + \frac{l_4^2 + l_5^2}{l_5} + 2\, b_5 \frac{l_4 + l_5}{l_5} + b_4 \frac{l_4}{l_5},$$

or generally

$$M_m = \frac{u}{4} b_m + c_m M_2, \qquad \text{where } b_m = + \frac{l_{m-2}^2 + l_{m-1}^2}{l_{m-1}} + 2\, b_{m-1} \frac{l_{m-2} + l_{m-1}}{l_{m-1}} + b_{m-2} \frac{l_{m-2}}{l_{m-1}},$$

Inserting the value of M_2, as given by equation (2), we have

$$M_m = \frac{u}{4}\left[b_m + \frac{c_m[(l_{s-1}^2 + l_s^2)\, d_2 + (l_{s-2}^2 + l_{s-1}^2)\, d_3 + \ldots\ldots (l_1^2 + l_2^2)\, d_s]}{d_{s-1} l_2 + 2\, d_s\, (l_2 + l_1)} \right].$$

This is the formula given on page 179 of the Text.

General Formula; all Spans Different, all Supports out of Level; Constant Moment of Inertia.*—The general theorem of three moments, already deduced, is, Eq. (8),

$$M_{r-1} l_{r-1} + 2 M_r (l_{r-1} + l_r) + M_{r+1} l_r = + Y_r + A_r + B_{r-1}.$$

For all spans loaded and all supports out of level, we have the series of equations

$$\left.\begin{aligned}
2 M_2 (l_1 + l_2) + M_3 l_2 &= + Y_2 + A_2 + B_1, \\
M_2 l_2 + 2 M_3 (l_2 + l_3) + M_4 l_3 &= + Y_3 + A_3 + B_2, \\
M_3 l_3 + 2 M_4 (l_3 + l_4) + M_5 l_4 &= + Y_4 + A_4 + B_3, \\
\ldots\ldots\ldots\ldots\ldots\ldots & \\
M_{m-2} l_{m-2} + 2 M_{m-1} (l_{m-2} + l_{m-1}) + M_m l_{m-1} &= + Y_{m-1} + A_{m-1} + B_{m-2}, \\
M_{m-1} l_{m-1} + 2 M_m (l_{m-1} + l_m) + M_{m+1} l_m &= + Y_m + A_m + B_{m-1}, \\
M_m l_m + 2 M_{m+1} (l_m + l_{m+1}) + M_{m+2} l_{m+1} &= + Y_{m+1} + A_{m+1} + B_m, \\
\ldots\ldots\ldots\ldots\ldots\ldots & \\
M_{s-2} l_{s-2} + 2 M_{s-1} (l_{s-2} + l_{s-1}) + M_s l_{s-1} &= + Y_{s-1} + A_{s-1} + B_{s-2}, \\
M_{s-1} l_{s-1} + 2 M_s (l_{s-1} + l_s) &= + Y_s + A_s + B_{s-1}.
\end{aligned}\right\} \quad \ldots \quad (a)$$

* The method of demonstration here given was first used by C. H. Lindenberger, *Journal of Franklin Institute*, December, 1888.

Multiply the first of equations (a) by b_2, the next by b_3, etc. Arrange the products according to the coefficients of M_2, M_3, etc., and add the resulting equations, and we have

$$[2b_2(l_1 + l_2) + b_3 l_2]M_2 + [b_2 l_2 + 2b_3(l_2 + l_3) + b_4 l_3]M_3 + \ldots$$
$$+ [b_{m-1}l_{m-1} + 2b_m(l_{m-1} + l_m) + b_{m+1}l_m]M_m + \ldots$$
$$+ [b_{s-1}l_{s-1} + 2b_s(l_{s-1} + l_s)]M_s = + \sum_{r=s+1}^{r=1}(Y_r + A_r + B_{r-1})b_r$$

If we give such values to the multipliers b, that the coefficient of every M shall be zero, except the coefficient of M_m, we shall have

$$M_m = + \frac{\sum_{r=s+1}^{r=1}(Y_r + A_r + B_{r-1})\, b_r}{Z_m}; \quad \ldots \ldots \ldots \quad (2)$$

where

$$Z_m = b_{m-1}l_{m-1} + 2b_m(l_{m-1} + l_m) + b_{m+1}l_m. \quad \ldots \ldots \ldots \quad (3)$$

It is required to find the values of b. The coefficient of any M in general is

$$Z_n = b_{n-1}l_{n-1} + 2b_n(l_{n-1} + l_n) + b_{n+1}l_n. \quad \ldots \ldots \ldots \quad (4)$$

For all values of n less than m, the values of c, already found, page 295, will make the coefficient of M_n equal to zero. We have, then, for $n < m$ or $= m$, $b_n = c_n$.

Now the values of d, already found, page 347, are counted from the bottom, and d_{s-n+2} corresponds, therefore, to the same place in the series as b_n. These values of d make the coefficients of M zero also. But d_2 has been assumed equal to 1. If it were taken equal to 2, all the d's would be twice as great; if 3, three times, etc. If then we take it equal to b_s, every d will be b_s times greater, and instead of d_{s-n+2}, we shall have $b_s d_{s-n+2}$. Hence $b_n = b_s d_{s-n+2}$, for $n = m$, or $> m$. But when $n = m$, b_n and c_m are identical. Hence $c_m = b_s d_{s-m+2}$, or $b_s = \frac{c_m}{d_{s-m+2}}$. Substituting this value of b_s, we have, when $n > m$,

$$b_n = \frac{c_m}{d_{s-m+2}} d_{s-n+2}.$$

We have, therefore, from (3),

$$Z_m = c_{m-1}l_{m-1} + 2c_m(l_{m-1} + l_m) + \frac{c_m}{d_{s-m+2}} d_{s-m+1}l_m. \quad \ldots \ldots \quad (5)$$

From the law of the multipliers c and d, page 346, we have also,

$$\left.\begin{array}{r} c_{m-1}l_{m-1} + 2c_m(l_{m-1} + l_m) + c_{m+1}l_m = 0 \\ d_{s-m+1}l_{m-1} + 2d_{s-m+2}(l_{m-1} + l_m) + d_{s-m+3}l_m = 0 \end{array}\right\} \quad \ldots \ldots \quad (6)$$

Subtract the first of these from (5), and we have

$$\frac{Z_m d_{s-m+2}}{l_m} = c_m d_{s-m+1} - c_{m+1}d_{s-m+2} \quad \ldots \ldots \ldots \quad (7)$$

Multiply the first of equations (6) by d_{s-m+2}, and the second by c_m, and subtract, and we have

$$l_{m-1}(c_{m-1}d_{s-m+2} - c_m d_{s-m+3}) + l_m(c_{m+1}d_{s-m+2} - c_m d_{s-m+1}) = 0. \quad \ldots \quad (8)$$

Comparing this with (7) we see that the first term is equal to $Z_{m-1}d_{s-m+3}$, and the second term is $- Z_m d_{s-m+2}$. We have, therefore, the general relation

$$Z_m d_{s-m+2} = Z_{m-1}d_{s-m+3} \quad \ldots \ldots \ldots \quad (9)$$

Since this relation holds generally, we may write

$$Z_m d_{s-m+2} = Z_{m+1} d_{s-m+1} = Z_{m+2} d_{s-m} = \text{etc.} = Z_s d_2 = Z_s.$$

Hence, we have

$$Z_m d_{s-m+2} = Z_s \quad \ldots\ldots\ldots\ldots \quad (10)$$

From (10) and (7) we have at once,

$$Z_s = c_m d_{s-m+1} l_m - c_{m+1} d_{s-m+2} l_m \quad \ldots\ldots\ldots\ldots \quad (11)$$

and this value of Z_s holds good generally *for any value of m.*

If we make in (11) $m = s - 1$, we have $Z_s = c_{s-1} l_{s-1} - c_s d_3 l_{s-1}$. Since we have

$$d_3 = -\frac{2(l_{s-1} + l_s)}{l_{s-1}}, \text{ we have by substitution, } Z_s = c_{s-1} l_{s-1} + 2c_s (l_{s-1} + l_s).$$

Again, if we make $m = 2$ in (11), we have $Z_s = d_{s-1} l_2 - c_3 d_s l_2$. Since we have

$$c_3 = -\frac{2(l_1 + l_2)}{l_2}, \text{ we have by substitution, } Z_s = d_{s-1} l_2 + 2d_s (l_1 + l_2).$$

Again, if we make $m = s$ in (11), we have $Z_s = - c_{s+1} l_s$. If we make $m = 1$ in (11), we have $Z_s = - d_{s+1} l_1$. Any of these values of Z_s may be used.

If, now, we insert in (2) the value of Z_m given by (10), we have for the moment at any support,

$$M_m = + \frac{d_{s-m+2} \sum\limits_{r=s+1}^{r=1} (Y_r + A_r + B_{r-1}) b_r}{Z_s},$$

where $b_r = c_r$ as long as r is less than m, or equal to m, and $b_r = \frac{c}{d_{s-m+2}} d_{s-r+2}$, where r is greater than m.

We can therefore write finally for the moment at any support in general,

$$M_m = + \frac{d_{s-m+2} \sum\limits_{r=m}^{r=1} (Y_r + A_r + B_{r-1}) c_r}{Z_s} + \frac{c_m \sum\limits_{r=s+1}^{r=m+1} (Y_r + A_r + B_{r-1}) d_{s-r+2}}{Z_s} \quad . \; . \quad (A)$$

where, in general, $Z_s = c_m d_{s-m+1} l_m - c_{m+1} d_{s-m+2} l_m$. The values most convenient for use* are $Z_s = c_{s-1} l_{s-1} + 2c_s (l_{s-1} + l_s) = - c_{s+1} l_s = d_{s-1} l_2 + 2d_s (l_1 + l_2) = - d_{s+1} l_1$.

Any of these values of Z_s may be used. The values of Y_r, A_r, and B_r have already been given, as also the values of c and d. We repeat them here for convenience of reference.

For concentrated loads, $A_r = \Sigma P_r l_r^2 (2k_r - 3k_r^2 + k_r^3)$; $B_r = \Sigma P_r l_r^2 (k_r - k_r^3)$.

For uniform loading, $A_r = B_r = \frac{1}{4} w_r l_r^3$.

$$Y_r = 6EI \left[\frac{h_{r-1} - h_r}{l_{r-1}} + \frac{h_{r+1} - h_r}{l_r} \right].$$

$$c_1 = 0, \; c_2 = 1, \; c_3 = -2\frac{l_1 + l_2}{l_2}, \; c_4 = -2c_3 \frac{l_2 + l_3}{l_3} - c_2 \frac{l_2}{l_3}, \; c_n = -2c_{n-1} \frac{l_{n-2} + l_{n-1}}{l_{n-1}} - c_{n-2} \frac{l_{n-2}}{l_{n-1}},$$

$$d_1 = 0, \; d_2 = 1, \; d_3 = -2\frac{l_s + l_{s-1}}{l_{s-1}}, \; d_4 = -2d_3 \frac{l_{s-1} + l_{s-2}}{l_{s-2}} - d_2 \frac{l_{s-1}}{l_{s-2}},$$

$$d_n = -2d_{n-1} \frac{l_{s-n+3} + l_{s-n+2}}{l_{s-n+2}} - d_{n-2} \frac{l_{s-n+3}}{l_{s-n+2}}.$$

For the shear at the left support of a loaded span,

* The equality of these values was first pointed out by C. H. Lindenberger, *Jour. of Franklin Inst.*, Dec., 1888.

$$S_r = \frac{M_r - M_{r+1}}{l_r} + q_r;$$

at the right support of a loaded span,

$$S'_{r+1} = \frac{M_{r+1} - M_r}{l_r} + q'_r,$$

where, for concentrated loads, $q_r = \Sigma P_r(1 - k_r)$, $q'_r = \Sigma P_r k_r$,

and for uniform loading, $q_r = q'_r = \frac{1}{2} w_r l_r$.

For unloaded spans,

$$S_m = \frac{M_m - M_{m+1}}{l_m}, \quad S'_m = \frac{M_m - M_{m-1}}{l_{m-1}}.$$

The formulas of page 351 are all that are needed for the complete solution of any case of continuous girder for constant moment of inertia.

We give in the following pages a series of examples illustrating the use of the general formula (A), which includes all the special cases hitherto discussed.

1. *Let all the spans be equal and unloaded. Find the moment at any support n, due to a change of level of that support.*

Here the numbers c and d are identical, B and A are zero, also $Y_{n-1} = Y_{n+1} = -\frac{Y_n}{2}$, and all other Y's are zero.

From (A) we have for the moment at any support m, on the left of n, or when n is greater than m,

$$M_m = -\frac{c_m}{d_{s+1}l}[Y_{n-1}d_{s-n+3} + Y_n d_{s-n+2} + Y_{n+1}d_{s-n+1}]$$

$$= -\frac{c_m Y_n}{2c_{s+1}l}[-c_{s-n+3} + 2c_{s-n+2} - c_{s-n+1}].$$

Since, for all spans equal, we have $c_{s-n+3} = -4c_{s-n+2} - c_{s-n+1}$, we have, after inserting this value of c_{s-n+3}, and the value of $Y_n = 6EI\left[\frac{2(h_{n-1} - h_n)}{l}\right]$,

when $n > m$,

$$M_m = -\frac{36EI(h_{n-1} - h_n)c_m c_{s-n+2}}{c_{s+1}l^2},$$

which is the equation given on page 184.

For the moment at the support n itself, we have, from (A),

$$M_n = -\frac{d_{s-n+2}}{c_{s+1}l}(Y_{n-1}c_{n-1} + Y_n c_n) - \frac{c_n}{c_{s+1}l}Y_{n+1}d_{s-n+1}$$

$$= -\frac{c_n}{c_{s+1}l}(Y_{n+1}d_{s-n+1} + Y_n d_{s-n+2} + Y_{n-1}d_{s-n+3}) + \frac{Y_{n-1}}{c_{s+1}l}(c_n d_{s-n+3} - c_{n-1}d_{s-n+2})$$

$$= -\frac{Y_n c_n}{2c_{s+1}l}(-c_{s-n+1} + 2c_{s-n+2} - c_{s-n+3}) - \frac{Y_n}{2c_{s+1}l}(c_n d_{s-n+3} - c_{n-1}d_{s-n-2}).$$

But from (11) we have $Z_s = -c_{s+1}l = c_{n-1}d_{s-n+2}l - c_n d_{s-n+3}l$. The second term, therefore, reduces to $\frac{Y_n}{2l}$. Reducing as before, we have,

when $m = n$,

$$M_m = \frac{6EI(h_n - h_{n-1})}{l^2} + \frac{36EI(h_n - h_{n-1})c_n c_{s-n+2}}{c_{s+1}l^2}.$$

For the moment at any support on the right of n, we have $m > n$, and from (A),

$$M_m = -\frac{d_{s-m+2}}{c_{s+1}l}(Y_{n-1}c_{n-1} + Y_n c_n + Y_{n+1}c_{n+1})$$

$$= \frac{Y_n c_{s-m+2}}{2c_{s+1}l}(+c_{n-1} - 2c_n + c_{n+1}).$$

Since $c_{n+1} = -4c_n - c_{n-1}$, we have, after inserting this value of c_{n+1}, and the value of Y_n, when $n < m$,

$$M_m = \frac{36EI(h_n - h_{n-1})c_n c_{s-m+2}}{c_{s+1}l^2}.$$

This equation is given on page 184.

If the spans are all different we have, from (A), for the moment at any support on the left of n,

$$M_m = -\frac{c_m}{d_{s+1}l_1}[Y_{n-1}d_{s-n+3} + Y_n d_{s-n+2} + Y_{n+1}d_{s-n+1}].$$

Putting for Y_n, Y_{n-1}, Y_{n+1}, their values, and remembering that $h_{n-1} - h_n = h_{n+1} - h_n$, we have when $n > m$,

$$M_m = \frac{6EIc_m(h_n - h_{n-1})}{d_{s+1}l_1}\left[\frac{d_{s-n+2} - d_{s-n+3}}{l_{n-1}} + \frac{d_{s-n+2} - d_{s-n+1}}{l_n}\right].$$

For the moment at the support n we have,
when $n = m$,

$$M_n = \frac{6EI(h_n - h_{n-1})}{l^2_{n-1}} + \frac{6EIc_n(h_n - h_{n-1})}{d_{s+1}l_1}\left[\frac{d_{s-n+2} - d_{s-n+3}}{l_{n-1}} + \frac{d_{s-n+2} - d_{s-n+1}}{l_n}\right].$$

For the moment at any support on the right of n we have,
when $n < m$,

$$M_m = \frac{6EId_{s-m+2}(h_n - h_{n-1})}{c_{s+1}l_s}\left[\frac{c_n - c_{n-1}}{l_{n-1}} + \frac{c_n - c_{n+1}}{l_n}\right].$$

These formulas are given on page 184.

2. *Find the general formulas for a continuous beam of two spans.*
Here $s = 2$, and we have, from (A),

$$M_1 = 0,\ M_3 = 0,\ M_2 = +\frac{Y_2 + A_2 + B_1}{2(l_1 + l_2)},\ S_1 = -\frac{M_2}{l_1} + q_1,\ S'_2 = +\frac{M_2}{l_1} + q'_1,\ S_2 = +\frac{M_2}{l_2} + q_2,\ S'_3 = -\frac{M_2}{l_2} + q'_2.$$

For concentrated loading, $q_1 = \Sigma P_1(1 - k_1)$, $q'_1 = \Sigma P_1 k_1$, $q_2 = \Sigma P_2(1 - k_2)$, $q'_2 = \Sigma P_2 k_2$.
For uniform loading, $q_1 = q'_1 = \frac{1}{2}w_1 l_1$, $q^2 = q'^2 = \frac{1}{2}w^2 l^2$.

These formulas will solve any case of two spans. For example:

A plate girder is continuous over three supports, $l_1 = 30$ feet, $l_2 = 50$ feet, the supports being all on level. The uniform load per foot in the first span is $w_1 = 3000$ lbs., in the second $w_2 = 350$ lbs. What are the moments and reactions?

Since the supports are all on level, $Y_2 = 0 = Y_1 = Y_3$, and we have,

$$M_1 = 0, M_3 = 0, M_2 = +\frac{A_2 + B_1}{2(l_1 + l_2)}.$$

In the present case

$$A_2 = \frac{w_2 l_2^3}{4},\ B_1 = \frac{w_1 l^3}{4}.$$

Hence,

$$M_2 = +\frac{w_1 l_1^3 + w_2 l_2^3}{8(l_1 + l_2)} = +\frac{3000 \times 30^3 + 350 \times 50^3}{8(30 + 50)} = +194921.875 \text{ foot lbs.}$$

We have, therefore,

$$R_1 = S_1 = -\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = -\frac{194921.875}{30} + \frac{3000 \times 30}{2} = +38502.6 \text{ lbs.}$$

$$R_3 = S_2' = -\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = -\frac{194921.875}{50} + \frac{350 \times 50}{2} = +4851.5625 \text{ lbs.}$$

$$S'_2 = +\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = +51497.39 \text{ lbs.} \qquad S_2 = +\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = +12648.44 \text{ lbs.}$$

$$R_2 = S'_2 + S_2 = +64145.8 \text{ lbs.}$$

3. *If the centre support is lowered 3 feet below the level of the others, what are the moments and reactions? Let $E = 24000000$ lbs. per sq. in. and $I = 53400$ for dimensions in inches.*

Since $s = 2$, we have, from (A), $M_1 = 0$, $M_3 = 0$.

$$M_2 = -\frac{Y_2 + A_2 + B_1}{c_2 l_2} = +\frac{\frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4} + 6EI\left[\frac{h_1 - h_2}{l_1} + \frac{h_3 - h_2}{l_2}\right]}{2(l_1 + l_2)}.$$

Since the centre support is lowered, h_2 is *greater* than h_1 and h_3, and we have $h_1 - h_2 = h_3 - h_2 = -3$ feet. If we take the span in feet and w_1 and w_2 in lbs. per foot, we must take I in feet and E in lbs. per sq. foot. We must therefore divide the value of I given, by 12^4, and multiply the value of E by 144, or divide the value of EI, as given by 144. We have, therefore,

$$M_2 = +\frac{\frac{3000 \times 30^3}{4} + \frac{350 \times 50^3}{4} - \frac{6 \times 24000000 \times 53400}{144}\left(\frac{3}{30} + \frac{3}{50}\right)}{2(30 + 50)} = -53205078.125 \text{ ft. lbs.}$$

$$R_1 = S_1 = -\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = +1818502.6 \text{ lbs.} \qquad R_3 = S'_3 = -\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = +1072851.56 \text{ lbs.}$$

$$S'_2 = +\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = -1728502.6 \text{ lbs.} \qquad S_2 = +\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = -1055351.56 \text{ lbs.}$$

$$R_2 = S'_2 + S_2 = -2783854.16 \text{ lbs.}$$

If the second support were 3 feet below the first and 3 feet above the third, we would have $h_1 - h_2 = -3$, and $h_3 - h_2 = +3$. So for any differences of level.

4. *How far must the second support be lowered in order that the moment may be zero?*

Since supports 1 and 3 are on level, $h_1 - h_2 = h_3 - h_2$. Since $M_2 = 0$, we have from (A),

$$M_2 = +\frac{\frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4} + 6\,EI\left[\frac{h_1 - h_2}{l_1} + \frac{h_3 - h_2}{l_2}\right]}{2(l_1 + l_2)} = 0,$$

or,

$$\frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4} + 6\,EI\left[\frac{h_1 - h_2}{l_1} + \frac{h_3 - h_2}{l_2}\right] = 0. \quad \therefore h_1 - h_2 = -\frac{w_1 l_1^4 l_2 + w_2 l_2^4 l_1}{24\,EI(l_1 + l_2)}.$$

Inserting numerical values

$$h_1 - h_2 = -\frac{3000 \times 30^4 \times 50 + 350 \times 50^4 \times 30}{\frac{24 \times 24000000 \times 53400}{144}(30 + 50)} = -0.0045 \text{ ft.} = -0.054 \text{ inch.}$$

Therefore a sinking of the second support of only 0.05 inch is sufficient to make M_2 zero.

In this case we have

$$R_1 = S_1 = \frac{w_1 l_1}{2} = +45000 \text{ lbs.} \qquad R_3 = S'_3 = +\frac{w_2 l_2}{2} = +8750 \text{ lbs.}$$

$$S'_2 = \frac{w_1 l_1}{2} = +45000 \text{ lbs.} \qquad S_2 = \frac{w_2 l_2}{2} = +8750 \text{ lbs.} \qquad R_2 = S'_2 + S_2 = +53750 \text{ lbs.}$$

5. *How far must the second support be lowered in order that the pressure on the second support may be zero?*

Here we have

$$S'_2 + S_2 = R_2 = 0, \text{ or } + \frac{M_2}{l_1} + \frac{w_1 l_1}{2} + \frac{M_2}{l_2} + \frac{w_2 l_2}{2} = 0,$$

or,

$$M_2 = -\frac{w_1 l_1^2 l_2 + w_2 l_1 l_2^2}{2(l_1 + l_2)} = -1007812.5 \text{ ft. lbs.}$$

From the general value of M_2 in the preceding case we have

$$-w_1 l_1^2 l_2 - w_2 l_1 l_2^2 = \frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4} + 6EI\left[\frac{h_1 - h_2}{l_1} + \frac{h_3 - h_2}{l_2}\right].$$

Hence, since $h_1 - h_2 = h_3 - h_2$,

$$h_1 - h_2 = -\frac{w_1 l_1^4 l_2 + w_2 l_1 l_2^4 + 4w_1 l_1^3 l_2^2 + 4w_2 l_1^2 l_2^3}{24EI(l_1 + l_2)} = -0.0611 \text{ ft.} = -0.73 \text{ inch.}$$

Therefore, a sinking of the second support of only 0.7 inch is sufficient to convert the two spans into one long span.

$$R_1 = S_1 = -\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = + 78593.75 \text{ lbs.} \qquad R_3 = S'_3 = -\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = + 28906.25 \text{ lbs.}$$

$$S'_2 = +\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = + 11406.25 \text{ lbs.} \qquad S_2 = +\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = - 11406.25 \text{ lbs.}$$

$$R_2 = S'_2 + S_2 = 0.$$

If the spans l_1 and l_2 were equal, and the loading w_1 and w_2 equal, we would have at once $h_1 - h_2 = -\frac{5wl^4}{24EI}$, or the deflection at the centre of a span $2l$, as should be, and $M_2 = -\frac{wl^2}{2}$, as should be.

6. *If we have a concentrated load $P_1 = 90{,}000$ lbs. in the first span, at a distance $\frac{1}{4}l_1$ from the left end, and P_2 18000 lbs. at a distance $\frac{1}{2}l_2$, what are the moments and reactions?*

This case is precisely like example 2, except in the values of A and B. We have now $k_1 = \frac{1}{4}$, $k_2 = \frac{1}{2}$, $A_2 = P_2 l_2^2 (2k_2 - 3k_2^2 + k_2^3) = \frac{3}{8} P_2 l_2^2$, $B_1 = P_1 l_1^2 (k_1 - k_1^3) = \frac{15}{64} P_1 l_1^2$.

From (A) we find, as in example 2, $M_1 = 0$, $M_3 = 0$, and

$$M_2 = + \frac{A_2 + B_1}{2(l_1 + l_2)}.$$

Inserting the values of A_2 and B_1, we find easily,

$$M_2 = + 224121.094 \text{ ft. lbs.} \qquad R_1 = S_1 = -\frac{M_2}{l} + P_1(1 - k) = + 60029.3 \text{ lbs.}$$

$$R_3 = S'_3 = -\frac{M_2}{l_2} + P_2 k_2 = + 4517.58 \text{ lbs.} \qquad S'_2 = +\frac{M_2}{l_1} + P_1 k_1 = + 29970.7 \text{ lbs}$$

$$S_2 = +\frac{M_2}{l_2} + P_2(1 - k_2) = + 13482.42 \text{ lbs.} \qquad R_2 = S'_2 + S_2 = + 43453.12 \text{ lbs.}$$

7. *If the second support is lowered 3 feet in this case,* we have simply to use the values of A_2 and B_1 for this case, in the formulas of example 3, and we find

$M_2 = -53175878.9$ ft. lbs. $R_1 = S_1 = +1840029.29$ lbs. $R_3 = S'_3 = +1072517.57$ lbs.

$S'_2 = -1750029.29$ lbs. $S_2 = -1054517.57$ lbs. $R_2 = S'_2 + S_2 = -2804546.85$ lbs.

8. *To find the distance the second support must be lowered in order that M_2 may be zero,* we proceed as in example 4, and place $Y_2 + A_2 + B_1 = 0$. Inserting the new values for A_2 and B_1, we find $h_1 - h_2 = -0.01259$ ft. $= -0.1511$ inch.

9. *To find how far the second support must be lowered in order that the pressure on the second support may be zero.*

We have, as in example 5,

$$+\frac{M_2}{l_1} + P_1k_1 + \frac{M_2}{l_2} + P_2(1 - k_2) = 0. \quad \therefore M_2 = -\frac{P_1k_1l_1l_2 + P_2(1 - k_2)l_1l_2}{l_1 + l_2} = -590625 \text{ ft. lbs.}$$

We have also from (A),

$$M_2 = +\frac{A_2 + B_1 + Y_2}{2(l_1 + l_2)}, \text{ and hence } A_2 + B_1 + Y_2 = -2P_1k_1l_1l_2 - 2P_2(1 - k_2)l_1l_2.$$

Putting for Y_2 its value $Y_2 = 6EI\left[\frac{h_1 - h_2}{l_1} + \frac{h_3 - h_2}{l_2}\right]$, and remembering that $h_1 - h_2 = h_3 - h_2$,

we have, after inserting numerical values, $h_1 - h_2 = -0.04577$ ft. $= -0.55$ in.
$R_1 = S_1 = +87187.5$ lbs. $R_3 = S'_3 = +20812.5$. $S'_2 = +2812.5$. $S_2 = -2812.5$. $R_2 = 0$.

10. *How much must the second support be raised or lowered in order that the reaction at the first support may be any required amount?*

We have, in general, $M_2 = R_1l_1 - q_1l_1$ where $q_1 = P_1(1 - k_1)$ for concentrated load, and $q_1 = \frac{w_1l_1}{2}$ for uniform loading.

We have, also,

$$M_2 = +\frac{A_2 + B_1 + Y_2}{2(l_1 + l_2)}, \text{ hence } Y_2 = -A_2 - B_1 - 2(l_1 + l_2)(R_1l_1 - q_1l_1).$$

Inserting the value of Y_2, and remembering that $h_1 - h_2 = h_3 - h_2$, we have

$$h_1 - h_2 = -\frac{[A_2 + B_1 + 2(l_1 + l_2)(R_1l_1 - q_1l_1)]\,l_1l_2}{6EI(l_1 + l_2)}.$$

This is a general formula for two spans, whatever the loading.

If we take R_1 zero, we have the amount of elevation of the second support necessary to just lift the left end. For concentrated load in each span, we have

$$h_1 - h_2 = -\frac{2(l_1 + l_2)l_1^2l_2[R_1 - P_1(1 - k_1)] + P_2(2k_2 - 3k_2^2 + k_2^3)l_2^2l_1 + P_1(k_1 - k_1^3)l_1^2l_2}{6EI(l_1 + l_2)},$$

Inserting numerical values, $k_1 = \frac{1}{4}$, $k_2 = \frac{1}{2}$, as in example 6, and making $R_1 = 0$, we have $h_1 - h_2 = +0.10117$ ft. $= +1.214$ in. The second support must therefore be *raised* 1.2 in., in order that left end may just touch.

In this case, we have $M_2 = + P_1(1 - k_1)l_1 = +$ 2025000 ft. lbs. $R_1 = 0$.

$R_3 = S'_3 = - \frac{M_2}{l_2} + P_2k_2 = -$ 31500 lbs. $S'_2 = + \frac{M_2}{l_1} + P_1k_1 = +$ 90000 lbs.

$S_2 = + \frac{M_2}{l_2} + P_2(1 - k_2) = +$ 49500 lbs. $R_2 = S'_2 + S_2 = +$ 139500 lbs.

11. *Let a beam of two equal spans have a load, P_1 in the first span and P_2 in the second span, each load being at the centre of its span. Let the second support be lowered by an amount. $h_1 - h_2 = - \frac{(P_1 + P_2)l^3}{48EI}$. What are the moments, shears, and reactions?*

In this case, $k_1 = k_2 = \frac{1}{2}, A_2 = \frac{3}{8} P_2l^2, B_1 = \frac{3}{8} P_1l^2,$

$$Y_2 = 6EI\left[\frac{2(h_1 - h_2)}{l}\right] = - \frac{1}{4}(P_1 + P_2)l^2, \text{ and}$$

$$M_2 = + \frac{Y_2 + A_2 + B_1}{4l} = + \frac{3l}{32}(P_1 + P_2) - \frac{l}{16}(P_1 + P_2) = +(P_1 + P_2)\frac{l}{32},$$

$$R_1 = S_1 = - \frac{M_2}{l} + \frac{P_1}{2} = \frac{15P_1 - P_2}{32}. \qquad R_3 = S'_3 = - \frac{M_2}{l} + \frac{P_2}{2} = \frac{15P_2 - P_1}{32}.$$

$$S'_2 = + \frac{M_2}{l} + \frac{P_1}{2} = \frac{17P_1 + P_2}{32}. \qquad S_2 = + \frac{M_2}{l} + \frac{P_2}{2} = \frac{P_1 + 17P_2}{32}.$$

$$R_2 = S'_2 + S_2 = \frac{18(P_1 + P_2)}{32}.$$

12. *A beam of one span is fixed horizontally at the right end. What are the shears and moments?*

Here we have $s = 2, l_2 = 0, c_1 = 0, c_2 = 1, d_1 = 0, d_2 = 1, d_3 = -2, h_3 - h_2 = 0, A_2 = 0 = B_2,$ $Y_2 = 6EI\left[\frac{h_1 - h_2}{l}\right]$.

From (A), $M_1 = 0,\ M_2 = + \frac{B_1 + Y_2}{2l},\ M_3 = 0,\ S_1 = - \frac{M_2}{l} + q_1,\ S'_2 = + \frac{M_2}{l} + q'_1.$

For ends on level and uniform loading,

$$Y_2 = 0, \quad B_1 = \frac{1}{4}wl^2, \quad M_2 = + \frac{wl^2}{8}, \quad R_1 = S_1 = \frac{3}{8}wl, \quad S'_2 = \frac{5}{8}wl.$$

For ends on level and load P anywhere in the span,

$M_2 = + \frac{Pl}{2}(k - k^3)$. For P in centre, $k = \frac{1}{2}$, and $M_2 = + \frac{3Pl}{16},\ S_1 = \frac{5}{16}P,\ S'_2 = \frac{11}{16}P.$

How far must the right end sink in order that the moment may be zero?

Here we have $M_2 = + \frac{B_1 + Y_2}{2l} = 0$, or $Y_2 = - B_1 = 6EI\left[\frac{h_1 - h_2}{l}\right]$.

Hence $h_1 - h_2 = - \frac{B_1l}{6EI},\ S_1 = q_1,\ S'_2 = q'_1.$

If the loading is uniform, $S_1 = \frac{wl}{2} = S'_2.\quad h_1 - h_2 = - \frac{wl^4}{24EI}.$

For concentrated load $S_1 = P(1-k)$ $S'_2 = Pk$. $\quad h_1 - h_2 = -\frac{Pl^3(k-k^3)}{6EI}$.

For load in centre, $k = \frac{1}{2}$, and $S_1 = \frac{P}{2} = S'_2$. $\quad h_1 - h_2 = -\frac{Pl^3}{16EI}$.

How far must the right end rise in order that S_1 may be zero?

Here $-\frac{M_2}{l} + q_1 = 0$, or $-\frac{B_1 + Y_2}{2l^2} + q_1 = 0$, or $Y_2 = -B_1 + 2q_1 l^2 = 6EI\left[\frac{h_1 - h_2}{l}\right]$.

Hence $h_1 - h_2 = \frac{2q_1l^3 - B_1 l}{6EI}$. $\quad M_2 = +q_1 l$, $S'_2 = q_1 + q'_1$.

If load is uniform, $q'_1 = q_1 = \frac{wl}{2}$, $B_1 = \frac{wl^3}{4}$, $h_1 - h_2 = \frac{wl^4}{8EI}$, $M_2 = +\frac{wl^2}{2}$, $S'_2 = wl$.

If load is concentrated, $q'_1 = Pk, q_1 = P(1-k), B_1 = Pl^2(k-k^3), h_1 - h_2 = \frac{Pl^3(2-3k+k^3)}{6EI}$, $M_2 = +Pl(1-k)$, $S'_2 = P$.

13. *Find the general formulas for a continuous beam of three spans.*

Here $s = 3$, and we have, from (A), $M_1 = 0 = M_4$.

$$M_2 = -\frac{d_3(Y_2 + A_2 + B_1)}{d_4 l_1} - \frac{Y_3 + A_3 + B_2}{d_4 l_1}. \quad M_3 = -\frac{Y_3 + A_3 + B_2 + c_3(Y_2 + A_2 + B_1)}{d_4 l_1}.$$

$$S_1 = -\frac{M_2}{l_1} + q_1,\ S'_2 = +\frac{M_2}{l_1} + q'_1,\ S_2 = \frac{M_3 - M_2}{l_2} + q_2,\ S'_3 = \frac{M_2 - M_3}{l_2} + q'_2,\ S_3 = +\frac{M_3}{l_3} + q_3,$$

$$S'_4 = -\frac{M_3}{l_3} + q'_3.$$

For concentrated loads, $q_1 = \Sigma P_1(1-k_1)$, $q'_1 = \Sigma P_1 k_1$, $q_2 = \Sigma P_2(1-k_2)$, $q'_2 = \Sigma P_2 k_2$, $q_3 = \Sigma P_3(1-k_3)$, $q'_3 = \Sigma P_3 k_3$.

For uniform loading, $q_1 = q'_1 = \frac{1}{2} w_1 l_1$, $q_2 = q'_2 = \frac{1}{2} w_2 l_2$, $q_3 = q'_3 = \frac{1}{2} w_3 l_3$.

These general formulas will solve any case of three spans.

14. *A continuous beam of four equal spans, all supports on level, has the second span uniformly loaded. What are the moments and shears?*

Here we have $Y_1 = Y_2 = Y_3 = Y_4 = 0$, $A_1 = A_3 = A_4 = 0 = B_1 = B_3 = B_4$, $A_2 = B_2 = \frac{wl^3}{4}$,

Also, $c_1 = d_1 = 0$, $c_2 = d_2 = 1$, $c_3 = d_3 = -4$, $c_4 = d_4 = +15$, $c_5 = d_5 = -56$, $s = 4$, $r = 2$

We have, therefore, from (A),

$$M_1 = 0,\ M_5 = 0,\ M_2 = -\frac{c_2}{d_5 l}[A_2 d_4 + B_2 d_3] = +\frac{11wl^2}{224},\ M_3 = -\frac{d_3}{d_5 l}[A_2 c_3 + B_2 c_2] = +\frac{12wl^2}{224}.$$

$$M_4 = -\frac{d_2}{d_5 l}[A_2 c_3 + B_2 c_2] = -\frac{3wl^2}{224}.$$

$$R_1 = S_1 = -\frac{M_2}{l} = -\frac{11wl}{224},\ S'_2 = +\frac{M_2}{l} = +\frac{11wl}{224},\ S_2 = \frac{M_2 - M_3}{l} + \frac{wl}{2} = +\frac{111wl}{224},$$

$$S'_3 = \frac{M_3 - M_2}{l} + \frac{wl}{2} = +\frac{113wl}{224}, S_3 = \frac{M_3 - M_4}{l} = +\frac{15wl}{224}, S'_4 = \frac{M_4 - M_3}{l} = -\frac{15wl}{242},$$

$$S_4 = +\frac{M_4}{l} = -\frac{3wl}{224}, R_5 = S'_5 = -\frac{M_4}{l} = +\frac{3wl}{224}.$$

15. *How much should the supports be lowered in order to make all the moments zero?*

Here we have the conditions $(Y_2 + A_2)d_4 + B_2 d_3 + Y_3 d_3 + Y_4 d_2 = 0$, $(Y_2 + A_2)c_3 + B_2 c_2 + Y_3 d_2 + Y_4 d_2 = 0$, $(Y_2 + A_2)c_2 + B_2 c_1 + Y_3 c_3 + Y_4 c_4 = 0$.

Substituting the values of c and d, we find $Y_2 = -A_2$, $Y_3 = -B_3$, $Y_4 = 0$; that is, the fourth support is on level with the third and fifth, and we must have $h_3 = h_4 = h_5$. We have, then, $Y_3 = 6EI\left[\frac{h_2 - h_3}{l} + \frac{h_4 - h_3}{l}\right] = 6EI\left[\frac{h_2 - h_3}{l}\right]$, and hence $h_2 - h_3 = -\frac{B_3 l}{6EI}$. The minus sign shows that the third support is below the second.

We also have $Y_2 = 6EI\left[\frac{h_1 - h_2}{l} + \frac{h_3 - h_2}{l}\right] = -A_2$. Inserting the value of $h_2 - h_3$, we have $h_1 - h_2 = -\frac{(A_2 + B_3)\,l}{6EI}$, and the second support is below the first.

Let $l = 50$ feet, $E = 24000000$ lbs. per square inch, $I = 53400$ for dimensions in inches. Then, if we take dimensions in feet, $EI = \frac{24000000 \times 53400}{144} = 8900000000$. Take $w_2 = 3000$ lbs. per foot.

Then $A_2 = \frac{w_2 l^3}{4} = 118750000 = B_3$, and $h_1 - h_2 = -0.226$ feet $= -2.7$ inches, $h_2 - h_3 = -0.113$ feet $= -1.35$ inches.

In order to make the moment at the second support only equal to zero, we have

$$(Y_2 + A_2)\,d_4 + B_2 d_3 + Y_3 d_3 = 0, \text{ and } Y_2 = 6EI\left[\frac{2\,(h_1 - h_2)}{l}\right],\quad Y_3 = -6EI\left[\frac{h_1 - h_2}{l}\right].$$

Hence,

$$h_1 - h_2 = -\frac{11wl^4}{816EI} = -0.284 \text{ feet} = -3.4 \text{ inches}.$$

16. *A beam continuous over seven spans has a load in every span. Find the moment and shear at the fourth support.*

We have from (A), since $s = 7$,

$$M_4 = -\frac{d_5}{d_8 l_1}[(Y_2 + A_2 + B_1)\,c_2 + (Y_3 + A_3 + B_2)\,c_3 + (Y_4 + A_4 + B_3)\,c_4],$$

$$-\frac{c_4}{d_8 l_1}[(Y_5 + A_5 + B_4)\,d_4 + (Y_6 + A_6 + B_5)\,d_3 + (Y_7 + A_7 + B_6)\,d_2],$$

$$M_5 = -\frac{d_4}{d_8 l_1}[(Y_2 + A_2 + B_1)\,c_2 + (Y_3 + A_3 + B_2)\,c_3 + (Y_4 + A_4 + B_3)\,c_4 + (Y_5 + A_5 + B_4)\,c_5],$$

$$-\frac{c_5}{d_8 l_1}[(Y_6 + A_6 + B_5)\,d_3 + (Y_7 + A_7 + B_6)\,d_2].$$

$S_4 = \frac{M_4 - M_5}{l_4} + q_4$, $q_4 = \Sigma P_4(1 - k_4)$ for concentrated loads. $q_4 = \frac{w_4 l_4}{2}$ for uniform load.

Suppose the supports are all on level, all spans equal, $l = 80$ feet, and only the first, third, and sixth spans are uniformly loaded, with a load $w = 2$ tons per foot.

Then $A_2 = A_4 = A_5 = A_7 = 0$, $B_2 = B_4 = B_5 = B_7 = 0$, $A_1 = A_3 = A_6 = \frac{wl^2}{4} = B_1 = B_3 = B_6$,

$c_1 = d_1 = 0$, $c_2 = d_2 = 1$, $c_3 = d_3 = -4$, $c_4 = d_4 = +15$, $c_5 = d_5 = -56$, $c_6 = d_6 = +209$
$c_7 = d_7 = -780$, $c_8 = d_8 = +2911$, and every Y is zero.

$$M_4 = -\frac{d_5}{d_8 l}[B_1 c_2 + A_3 c_3 + B_3 c_4] - \frac{c_4}{d_8 l}[A_6 d_3 + B_6 d_2] = +\frac{717wl^2}{11644} = +788.18 \text{ ft. tons}.$$

$$M_5 = -\frac{d_4}{d_8 l}[B_1c_2 + A_2c_3 + B_3c_4] - \frac{c_5}{d_8 l}[A_6d_3 + B_6d_2] = -\frac{348wl^2}{11644} = -382.55 \text{ ft. tons.}$$

$S_4 = +14.63$ tons.

Suppose the supports are all on level, all spans equal, $l = 80$ feet, and only the second, fifth, and seventh spans are uniformly loaded, with a load $w = 2$ tons per foot.

Then $A_1 = A_3 = A_4 = A_6 = 0 = B_1 = B_3 = B_4 = B_6$, $A_2 = A_5 = A_7 = \dfrac{wl^2}{4} = B_2 = B_5 = B_7$,

$$M_4 = -\frac{d_5}{d_8 l}[A_2c_3 + B_2c_3] - \frac{c_4}{d_8 l}[A_5d_4 + B_5d_3 + A_7d_2] = -\frac{348wl^2}{11644} = -382.55.$$

$$M_5 = -\frac{d_4}{d_8 l}[A_2c_2 + B_2c_3 + A_5c_5] - \frac{c_5}{d_8 l}[B_5d_3 + A_7d_2] = +\frac{717wl^2}{11644} = +788.18 \text{ ft. tons.}$$

$S_4 = -14.63$ tons.

Suppose a load $P_4 = 20$ tons in the fourth span only.
Here all Y's are zero, and all A's and B's are zero, except

$A_4 = P_4l^2(2k - 3k^2 + k^3)$, $B_4 = P_4l^2(k - k^3)$, and we have

$$M_4 = -\frac{d_5}{d_8 l}[A_4c_4] - \frac{c_4}{d_8 l}[B_4d_4] = +\frac{15Pl}{2911}(97k - 168k^2 + 71k^3).$$

$$M_5 = -\frac{d_4}{d_8 l}[A_4c_4 + B_4c_4] = +\frac{15Pl}{2911}(26k + 45k^2 - 71k^3).$$

$$S_4 = \frac{15P}{2911}(71k - 213k^2 + 142k^3) + P(1 - k).$$

Suppose a uniform load w per foot over the whole girder.
Then

$$M_4 = -\frac{d_5}{d_8 l}[(A_2 + B_1)c_2 + (A_3 + B_2)c_3 + (A_4 + B_3)c_4] - \frac{c_4}{d_8 l}[(A_5 + B_4)d_4$$
$$- (A_6 + B_5)d_3 - (A_7 + B_6)d_2];$$

or

$$M_4 = -\frac{d_5A}{d_8 l}[2c_2 + 2c_3 + 2c_4] - \frac{c_4A}{d_8 l}[2d_4 + 2d_3 + 2d_2] = +\frac{12}{142}wl^2$$

$$M_5 = -\frac{d_4}{d_8 l}[(A_2 + B_1)c_2 + (A_3 + B_2)c_3 + (A_4 + B_3)c_4 + (A_5 + B_4)c_5]$$
$$- \frac{c_5}{d_8 l}[(A_6 + B_5)d_3 + (A_7 + B_6)d_2];$$

or,

$$M_5 = -\frac{d_4A}{d_8 l}[2c_2 + 2c_3 + 2c_4 + 2c_5] - \frac{c_5A}{d_8 l}[2d_3 + 2d_2] = +\frac{12}{142}wl^2. \qquad S_4 = +\frac{wl}{2}.$$

If the spans are all equal, $l = 80$ feet, uniform load $w = 4000$ lbs. per ft. over the whole girder, how far must the fourth support be lowered below the level of the others in order that the moment at the fourth support may be zero?

Here, we have $M_4 = 0$, $h_3 - h_4 = h_5 - h_4$, all the B's and A's are equal to $\dfrac{wl^2}{4} = A$;

$$Y_1 = 0, \quad Y_2 = 0, \quad Y_3 = 6EI\left[\frac{h_4 - h_3}{l}\right], \quad Y_4 = 6EI\left[\frac{2(h_3 - h_4)}{l}\right] = -2Y_3,$$

$$Y_5 = 6EI\left[\frac{h_4 - h_3}{l}\right] = -Y_3, \quad Y_6 = 0.$$

Also, since $M_4 = 0$, we have

$$d_5\,[\,Y_3c_3 - 2\,Y_3c_4 + A\,(2c_3 + 2c_2 + 2c_4)] + c_4\,[-\,Y_3d_4 + A\,(\,+\,2d_4 + 2d_3 + 2d_2)] = 0.$$

Hence $Y_3 = 6EI\left[\dfrac{h_3 - h_4}{l}\right] = \dfrac{A\,(2c_3 + 2c_2 + 2c_4) + A\,(2d_4 + 2d_3 + 2d_2)}{2c_4d_5 + c_4d_4 - c_3d_5} = -\,\dfrac{246wl^3}{1395}$, and

$$h_3 - h_4 = -\,\frac{41wl^4}{1395EI}.$$

If $E = 24000000$ lbs. per square inch, and $I = 53400$ for dimensions in inches, $h_3 - h_4 = -0.541$ ft. $= -\,6.5$ inches.

17. *A beam of one span is fixed horizontally at the ends. What are the end moments and shears?*

Here $s = 3$, $l_1 = 0$, $l_3 = 0$, $B_1 = A_1 = B_3 = A_3 = 0$. $c_1 = d_1 = 0$, $c_2 = d_2 = 1$, $c_3 = d_3 = -\,2$.

We have, from (A),

$$M_2 = +\,\frac{c_2}{l(d_2 + 2d_3)}[(Y_2 + A_2)d_3 + B_2d_2 + Y_3] = \frac{+\,2(Y_2 + A_2) - B_2 - Y_3}{3l};$$

$$M_3 = -\,\frac{Y_2 + A_2 - 2B_2 - 2\,Y_3}{3l}.$$

If the ends are on level, $Y_2 = Y_3 = 0$, and

$$M_2 = \frac{+\,2A_2 - B_2}{3l}, \qquad M_3 = -\,\frac{A_2 - 2B_2}{3l}.$$

For concentrated load and ends level,

$$M_2 = +\,Pl(k - 2k^2 + k^3), \qquad M_3 = +\,Pl(k^2 - k^3), \qquad S_2 = P(1 - 3k^2 + 2k^3).$$

For uniform load and ends level,

$$M_2 = M_3 = +\,\frac{wl^2}{12}, \qquad S_2 = \frac{wl}{2}.$$

For uniform load and ends out of level,

$$M_2 = +\,\frac{2\,Y_2}{3l} + \frac{wl^2}{12} - \frac{Y_3}{3l}, \qquad M_3 = -\,\frac{Y_2}{3l} + \frac{wl^2}{12} + \frac{2\,Y_3}{3l}, \qquad S_2 = \frac{Y_2 - Y_3}{l^2} + \frac{wl}{2}.$$

How much must the left end be lowered to make S_2 zero?

Here, we have,

$$\frac{Y_2 - Y_3}{l^2} + \frac{wl}{2} = 0, \quad \text{or} \quad Y_2 - Y_3 = -\,\frac{wl^3}{2}.$$

Since $Y_3 = -\,Y_2$, we have

$$Y_2 = -\,\frac{wl^3}{4} = 6EI\left[\frac{h_1 - h_2}{l}\right]. \quad \text{Hence, } h_1 - h_2 = -\,\frac{wl^4}{24EI};$$

$$M_2 = -\,\frac{wl^2}{4} + \frac{wl^2}{12} = -\,\frac{wl^2}{6}, \quad M_3 = +\,\frac{wl^2}{4} + \frac{wl^2}{12} = +\,\frac{wl^2}{3}, \quad S_3' = \frac{M_2 - M_3}{l} + \frac{wl}{2} = wl.$$

How much must the left end be lowered to make $M_2 = 0$?

Here $-\,\dfrac{2\,Y_2}{3l} - \dfrac{wl^2}{12} + \dfrac{Y_3}{3l} = 0$, $Y_3 = -\,Y_2$, hence, $Y_2 = -\,\dfrac{wl^3}{12} = 6EI\left[\dfrac{h_1 - h_2}{l}\right]$,

and $h_1 - h_2 = -\,\dfrac{wl^4}{72EI}$. $\quad M_3 = +\,\dfrac{wl^2}{6}$. $\quad S_2 = +\,\dfrac{wl}{3}$. $\quad S_3' = +\,\dfrac{2wl}{3}$.

PART II.

DETERMINATION OF DIMENSIONS AND DESIGNING OF DETAILS.

II. DETERMINATION OF DIMENSIONS.

CHAPTER I.

ULTIMATE STRENGTH.—ELASTIC LIMIT.—OLD AND NEW METHODS OF DIMENSIONING.

IN Part I. we have learned how to find the stresses in the various members of any framed structure due to the action of assumed outer forces. In Chapter VIII. of this Part we shall see how to estimate the intensity of these outer forces, viz.: snow and wind load, live and dead load.

It is evident that having then properly assumed our outer forces, and then having calculated the resulting stresses in the members, as directed in Part I., only one-half of our problem is solved. The other half is to properly determine the cross-section of any member in order that it may resist the stress that comes upon it. This is, in fact, the most important part of our problem, as upon it depends the safety and efficiency of the structure.*

Its proper solution requires a thorough knowledge of the strength of materials. This is in itself a subject for special treatises. That which is necessary to be known has been given in the Appendix, Part I., page 270. We shall content ourselves, therefore, in the present Chapter, with giving the results of the best modern practice as regards wood, iron, and steel, referring the student to other works which treat of the subject specially for fuller information. This part of our problem is still in process of development, as our knowledge of materials is continually being increased by experiment, and the student will therefore bear in mind that the practice of to-day may be modified by future knowledge.

ULTIMATE STRENGTH AND ELASTIC LIMIT.—The smallest quiescent load per square inch which causes rupture of a member, we call the *breaking load*, or the *ultimate strength* of the material.

It is found by experiment that if a member of any material be subjected to pure tension or pure compression, the change of length is, within certain limits, very nearly proportional to the load. That is, a double load causes a double elongation or compression, three times the original load causes three times the original elongation or compression, and so on.

* In the words of Theodore Cooper, "A successful bridge engineer, from the American point of view, must be something more than a mere calculator of stresses. That is the most elementary part of the duty, and does not come within the province of designing. After the selection of the skeleton form and relative proportion of panels, depths, and widths of spans a very moderate knowledge of mechanical mathematics would enable any one to determine the stresses in an American bridge. He must, in addition to his knowledge as to the effects of varying forms and proportions, have a full knowledge of the capacity of his forms and their connections, and also of the practical processes of manufacture and erection. He must know how his design can be made and put together, and whether it is so harmonized in all its parts and connections that each part may do its full duty under all possible conditions of service.

"In addition to knowing all the elements that make up a perfect design, he must have the instinct of designing or the power of adapting his knowledge to any individual case, in order to obtain the best or desired result.

"Then experience, observation, and a sharp competition with men of like knowledge and instinct, will give him his position as a bridge engineer."—*Trans. Am. Soc. C. E.*, July, 1889.

This law is not exactly true, but within certain limits is approximately so. Thus, for any material, the curve denoting the relation between change of length and acting load, is within these limits approximately a straight line. This limit is called the "*elastic limit.*" We may, therefore, define the elastic limit as that point at which the law of proportionality of change of length to acting force ceases to hold good. (See Part I., p. 283.) The load corresponding to this point will evidently be much less than the breaking load.

We give in the following Table a few mean values of the ultimate strength and elastic limit for wood, iron, and steel. These values will of course vary considerably with the quality of the material, mode of manufacture, etc. In any special case the only reliable knowledge for the engineer to build upon is actual experiment. Such values as we give are useful only for preliminary calculations. Much more detailed knowledge may be found in those works which treat specially of the strength of the materials, as well as in the Appendix, Part I., page 270, and the student should read and constantly refer to the specifications at the end of this work.

TABLE OF ULTIMATE STRENGTH AND ELASTIC LIMIT IN POUNDS PER SQUARE INCH.

	ULTIMATE STRENGTH.			LIMIT OF ELASTICITY.			COEFF. OF ELASTICITY
	Comp.	*Tens.*	*Shear.*	*Comp.*	*Tens.*	*Shear.*	
WOOD,							
Oak, parallel to fibre	10,000	11,400	1,140	2,570	3,000	300	1,070,000
" transverse to fibre	5,000	700	2,300	......	......	......	....
Pine, parallel to fibre	8,600	10,000	860	......	3,000	300	....
" transverse to fibre	3,000	640	1,860	......	......	......	....
Beech, parallel to fibre	9,400	14,300	940	......	2,300	......	....
" transverse to fibre	5,000	1,000	......	......	......	......	....
IRON,							
Cast iron	100,000	18,600	15,000	21,400	10,700	8,600	14,000,000
Wrought iron	60,000	57,000	45,700	20,000	20,000	16,000	28,700,000
Plate iron	43,000	47,000	37,000	20,000	20,000	16,000	26,000,000
Wire	......	86,000	......	......	31,400	......	31,300,000
STEEL,							
Soft steel	80,000	71,400	57,000	28,600	28,600	23,000	29,000,000
Plate	70,000	71,400	57,000	36,000	36,000	28,600	....
Wire	......	130,000	......	......	64,300	......	....
Hard	107,000	107,000	85,700	38,600	38,600	30,860	32,000,000
Cast steel—soft	143,000	114,000	91,400	71,400	53,600	40,000	34,000,000
" " hard	......	143,000	114,300	......	95,000	76,140	....
" " wire	......	160,000	......	......	......	......	43,000,000

ALLOWABLE STRESS PER SQUARE INCH—FACTOR OF SAFETY.—The elastic limit marks the point beyond which the material should never be strained. In practice the working stress should be well within this limit, say ½ or ⅔ds of it at most for quiescent loads.

When this limit is not known, it is sometimes customary to take a certain fraction of the ultimate strength as the safe load as, for instance, ⅕th or ⅙th. In such case we call 5 or 6 the "*factor of safety*," that is, it will take five or six times the working load to break the member. Evidently the ultimate load divided by the factor of safety ought to give a result well within the limit of elasticity. Also we may evidently take this factor less for quiescent loads than for intermittent and oft-repeated loading accompanied by shock.

If σ is the allowable stress per square inch, and μ is the ultimate strength, and n the factor of safety, then we have

$$\sigma = \frac{\mu}{n}.$$

We give in the following Table the factor of safety n according to good practice:

TABLE OF FACTOR OF SAFETY.

MATERIAL.	TEMPORARY CONSTRUCTIONS.	BUILDINGS IN GENERAL.	BRIDGE AND ROOF CONSTRUCTIONS.	MACHINES AND STRUCTURES SUBJECT TO SHOCK.
Wood	6	9	10	15
Cast iron	....	6	7	10
Wrought iron	3	4		
Iron plate	....	4		
Ordinary steel	....	....	5 to 6	7 to 8
Bessemer steel	....	....		
Cast steel	....	....		
Stone	10	20	30	35

We have accordingly for the allowable stress in pounds per square inch for average materials, $\sigma = \frac{\mu}{n}$, the following Table:

TABLE OF ALLOWABLE STRESS IN POUNDS PER SQUARE INCH.

MATERIAL.	TEMPORARY CONSTRUCTION.			BUILDINGS IN GENERAL.			BRIDGE AND ROOF CONSTRUCTIONS.			MACHINES AND STRUCTURES SUBJECT TO SHOCK.		
	Tens.	Comp.	Shear.	Tens.	Comp.	Shear.	Tens.	Comp.	Shear.	Tens.	Comp.	Shear.
Oak } Direction of fibre	1,860	1,710	210	1,300	1,143	143	1.143	1,000	114	860	714	86
Pine } Direction of fibre	1,710	1 430	140	1,143	1,000	100	1,000	860	86	714	600	60
Cast iron	4,300	10,700	3,400	3,600	8,600	2,860	2,860	7,140	2,140	1,860	4,300	1,430
Wrought Iron	17,100	17,100	14,300	14,300	14,300	11,430	11,430	11,430	9,140	7,140	7,140	5,710
Iron plate	....	....	....	11,430	11,430	8,600	10,000	10,000	8,000	4,300	4,300	3,430
Iron wire	....	....	....	....	....	....	17,140	....	....	11 430	....	....
Ordinary steel (soft)	....	....	....	....	...	....	14,300	14,300	11,430	10,000	10,000	8,000
Steel (hard)	....	....	....	....	....	....	21,430	21.430	17,140	14,300	14,300	11,430
Cast steel	....	....	....	....	....	....	28,600	28,600	23,000	20,000	20,000	15,71C

Under temporary constructions we include scaffoldings, arch centreings, etc., as well as trusses for quiescent loading. Under constructions in general, such structures as are subjected to but little shock and whose load can be exactly determined.

From *A Manual of Rules, Tables and Data for Mechanical Engineers*, by D. K. Clark, London, 1877, page 625, we extract the following:

"The elastic strength of materials, cast iron excepted, is, in general terms, half of its ultimate or breaking strength. For cast iron, though there is no already defined elastic limit, the same measure may be adopted. If a working load of half the elastic strength, or one-fourth of the ultimate strength, be accepted, equal range for fluctuation within the elastic limit is provided. But, as bodies of the same material are not uniform in strength, it is necessary to observe a lower limit than a fourth where the material is exposed to great or to sudden variations of load."

CAST IRON.—Stoney recommends one-fourth of the ultimate tensile strength, for dead weights; one-sixth for cast-iron bridge girders; one-eighth for frame posts and machinery. In compression, free from flexure, according to Stoney, cast iron will bear 8 tons per square inch; for cast-iron arches, 3 tons per square inch; for cast-iron pillars, supporting dead loads, one-sixth of the ultimate strength; for pillars subjected to vibration from machinery, one-eighth; and for pillars subjected to shocks from heavy loaded wagons and the like, one-tenth, or even less where the strength is exerted in resistance to flexure.

WROUGHT IRON.—For bars and plates, 5 tons per square inch of net section is taken as the safe working tensile stress; for bar iron of extra quality 6 tons. In compression, where flexure is prevented, 4 tons is the safe limit; in small sizes, 3 tons. For wrought-iron columns, subjected to shocks, Stoney allows a sixth of the calculated breaking weight; with quiescent loads, one-fourth. For machinery, an eighth to a tenth is usually practised; and for steam boilers, a fourth to an eighth.

Mr. Roebling says, "Long experience has proved, beyond the shadow of a doubt, that good iron, exposed to a tensile stress not above one-fifth of the ultimate strength, and not subjected to strong vibration or torsion, may be depended upon for a thousand years.*

STEEL.—A committee of the British Association recommended a maximum working tensile stress of 9 tons per square inch. Mr. Stoney recommends, for mild steel, a fourth of the ultimate strength, or 8 tons per square inch. The limit for compression must be regulated very much by the nature of the steel, and whether it be unannealed or annealed. Probably a limit of 9 tons per square inch, the same as the limit for tension, would be the safe maximum for general purposes. In the absence of experience, Mr. Stoney recommends that, for steel pillars, an addition not exceeding 50 per cent. should be made to the safe load for wrought-iron pillars of the same dimensions.

TIMBER.—One-tenth of the ultimate stress is an accepted limit. Timber piles have, in some situations, borne permanently one-fifth of their ultimate compressive strength.

FOUNDATIONS.—According to Professor Rankine, the maximum pressure on foundations in firm earth is from 17 lbs. to 23 lbs. per square inch; and he says that, on rock, it should not exceed one-eighth of the crushing load.

MASON WORK.—Mr. Stoney says that the working load on rubble masonry, brickwork or concrete, rarely exceeds one-sixth of the crushing weight of the aggregate mass; and that this seems to be a safe limit. In an arch, the calculated pressure should not exceed one-twentieth of the crushing pressure of the stone.

ROPES.—For round ropes, the working load should not exceed a seventh of the ultimate strength, and for flat ropes, one-ninth.

Professor Rankine gives the following data as factors of strength:

	Dead Load.	Live Load.
Factors of safety for perfect materials and workmanship	2	4
For good ordinary materials and workmanship:		
Metals	3	6
Timber	4 to 5	8 to 10
Masonry	4	8

A *dead load* on a structure is one that is put on by imperceptible degrees, and that remains steady; such as the weight of the structure itself.

A *live load* is one that is put on suddenly, or is accompanied with vibration; such as a swift train travelling over a railway bridge, or a force exerted in a moving machine."

ALLOWABLE STRESS FOR WROUGHT-IRON BRIDGE MEMBERS.—Evidently, the allowable stress per square inch, even for the same material, must be varied according to the mode of action of the stress, whether quiescent, or intermittent, etc.

In bridge construction the quality of the iron used is carefully covered by specifications stating in detail the tests it must satisfy. We refer the student to the specifications at the end of this work for information as to current practice on this point.

For wrought iron, which shows an ultimate strength of 52,000 lbs. per square inch and stretches 18 per cent. in a distance of 8 inches, the allowable *tensile* stresses adopted by our leading railroads are about as follows: †

* *Engineering*, August, 1867.

† Specifications vary in these values. In any case the designer must be governed by the specifications adopted.

TENSILE WORKING STRESSES FOR WROUGHT-IRON BRIDGE MEMBERS.

	σ Lbs. per square inch.
On lateral bracing	15,000
On solid rolled beams, used as floor beams and stringers	10,000
On bottom chords and main diagonals	10,000
On counter rods and long verticals	8–9,000
On bottom flanges of riveted floor beams, net section	8,000
On bottom flanges of riveted longitudinal plate girders, over 20 feet long	8,000
On bottom flanges of riveted longitudinal plate girders, under 20 feet long	7,000
On floor beam hangers and other members liable to sudden loading	5–6,000

The allowable *compressive* stresses are as follows:

	σ Lbs. per square inch.
On rolled beams used as floor beams and stringers	10,000
On riveted plate girders used as floor beams	6,000
On riveted longitudinal plate girders over 20 feet	6,000
On riveted longitudinal plate girders not over 20 feet	5,000

For Steel, see Specifications at the end of this work.

The formula for BEAMS will be found on page 295, *et seq.*

LONG MEMBERS IN COMPRESSION.—In general, when the length of a member is more than ten times its least dimension, it is called a "long member." When such a long member is in compression, it is subject to flexure, and requires more material than would be necessary for the compressive stress alone. The formula in general use for finding the ultimate or crippling load in pounds per square inch, is Rankine's, as deduced in the Appendix, Part I., page 337:

$$\frac{P}{A} = \frac{S_e}{1 + c\dfrac{l^2}{r^2}} \left\{ \begin{array}{c} \textit{For all cross-sections in general} \\ \textit{except hollow round.} \end{array} \right.$$

where P is the crippling load in lbs., A the area of the cross-section in sq. inches, $l =$ length of strut in inches, and $r =$ *least radius of gyration of the cross-section* in inches. This formula is a modification of that known as "*Gordon's* formula," as deduced from Hodgkinson's experiments upon long struts, and is intended to apply in general to all forms of cross-section *except hollow round.* For hollow round cross-sections we put the *exterior diameter d in place of r.*

The value of the elastic limit unit stress S_e depends upon the material, and the value of c upon the end conditions of the strut.*

			Flat ends.	*Both ends pinned.*	*One end flat, one end pinned.*
Thus for WROUGHT IRON AND STEEL,		$S_e = 40000$,	$c = \frac{1}{36000}$,	$\frac{2}{36000}$,	$\frac{1}{24000}$

For CAST IRON, the crippling strength may be taken at twice as much as for wrought iron.

For *hollow cylindrical struts*					
of WROUGHT IRON,	$r = d$,	$S_e = 40000$,	$c = \frac{1}{4500}$,	$\frac{1}{2250}$,	$\frac{1}{3000}$
of CAST IRON,	$r = d$,	$S_e = 80000$,	$c = \frac{1}{800}$,	$\frac{1}{400}$,	$\frac{3}{1600}$
For rectangular struts					
of WOOD,	$r = d$,	$S_e = 5600$,	$c = \frac{1}{550}$,	$\frac{1}{275}$,	$\frac{1.5}{550}$

* Other values will be found in general specifications, end of this work. The values given here are recommended,

FACTOR OF SAFETY FOR LONG STRUTS.—The preceding formulas will enable us to find the "crippling strength" in pounds per square inch, for struts of any cross-section and length, of wood or iron.

In practice, only a portion of the crippling strength is taken as the "*safe stress.*" This portion is called the "*factor of safety.*" For *quiescent loads* (buildings, etc.), this factor is taken at 4 for wrought iron and steel and 6 for cast iron and wood struts.

For *variable loads* (bridges, etc.), a sliding factor of safety is used equal to $4 + \frac{l}{20\,d}$ for all WROUGHT IRON struts of any cross section except hollow round, and $7 + \frac{l}{20\,d}$ for all CAST IRON struts of any cross section except hollow round.

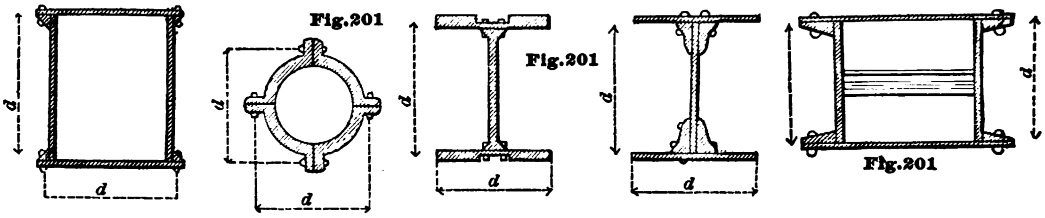
For *hollow round cross sections,* we have $3 + \frac{l}{10\,d}$ for WROUGHT IRON, and $6 + \frac{l}{10\,d}$ for CAST IRON.

For WOOD we have $6 + \frac{l}{10\,d}$.

In all these expressions, $l =$ length in inches and $d =$ least dimension of the rectangle which encloses the given cross section.

SPECIAL FORMS OF CROSS SECTION.—The forms of wrought iron column in general use in American bridge construction are as follows:

A SQUARE COLUMN. B PHŒNIX COLUMN. C AMERICAN COLUMN. D COMMON COLUMN. E COMMON CHORD SECTION.



For these forms, the following special formulas have been recommended by C. Shaler Smith for *wrought iron;* where d is the least dimension of the rectangle enclosing the cross section, and l is the length, both in inches.

	A.	*B.*	*C.*	*D.*	*E.*
Flat ends,	$\dfrac{38500}{1 + \dfrac{l^2}{5820\,d^2}}$,	$\dfrac{42500}{1 + \dfrac{l^2}{4500\,d^2}}$,	$\dfrac{36500}{1 + \dfrac{l^2}{3750\,d^2}}$,	$\dfrac{36500}{1 + \dfrac{l^2}{2700\,d^2}}$,	**The pin being so placed that the moment of inertia is, as near as practicable, equal on both sides of same, use formula for square column.**
One pin end,	$\dfrac{38500}{1 + \dfrac{l^2}{3000\,d^2}}$,	$\dfrac{40000}{1 + \dfrac{l^2}{2250\,d^2}}$,	$\dfrac{36500}{1 + \dfrac{l^2}{2250\,d^2}}$,	$\dfrac{36500}{1 + \dfrac{l^2}{1500\,d^2}}$,	
Two pin end,	$\dfrac{37500}{1 + \dfrac{l^2}{1900\,d^2}}$,	$\dfrac{36600}{1 + \dfrac{l^2}{1500\,d^2}}$,	$\dfrac{36500}{1 + \dfrac{l^2}{1750\,d^2}}$,	$\dfrac{36500}{1 + \dfrac{l^2}{1200\,d^2}}$,	

The safe working stress is found by dividing the "crippling stress," as determined by the above formula, by $4 + \frac{l}{20\,d}$, where l is length in inches, and d is least dimension of enclosing rectangle.

To these we may add, for *open latticed channel struts*, consisting of two channel bars, latticed at sides, the distance between the channels being not less than their depth:

$$P = \underset{\text{Flat ends.}}{\frac{38500}{1 + \frac{l^2}{4880\,d^2}}}, \qquad \underset{\text{One pin end.}}{\frac{38500}{1 + \frac{l^2}{3260\,d^2}}}, \qquad \underset{\text{Two pin end.}}{\frac{38500}{1 + \frac{l^2}{2440\,d^2}}},$$

also, for single I bars,

$$P = \underset{\text{Flat ends.}}{\frac{38500}{1 + \frac{l^2}{1720\,w^2}}}, \qquad \underset{\text{One pin end.}}{\frac{38500}{1 + \frac{l^2}{1150\,w^2}}}, \qquad \underset{\text{Two pin end.}}{\frac{38500}{1 + \frac{l^2}{860\,w^2}}},$$

where w is the width of the flange at top and bottom.

OLD METHOD OF DIMENSIONING.—The method of dimensioning still customary with many engineers is as follows:

Let A be the cross-section of the member, max. S the greatest stress which ever comes upon it, and σ the allowable stress per square inch. Then

$$A = \frac{\text{max. } S}{\sigma}.$$

Max. S is found by calculation of stresses, σ is taken in accordance with the preceding remarks, varying with the action of the stress, whether quiescent or intermittent.

If σ is the nth part of the ultimate strength of the piece it is said to have a factor of safety of n, or n-fold security. This, however, is not really the case except for quiescent loading. For intermittent loading, especially accompanied by shocks, the factor of safety is really less.

The above method gives the cross-section for pure tension or pure compression. If a member is sometimes in compression and sometimes in tension, it is customary to take the area equal to the sum of the area which would be required for each stress separately, or $A = \frac{\text{max. tens.} + \text{max. comp.}}{\sigma}$ provided the member is so short that the compression does not cause flexure as in the case of long struts. In this latter case we have

$$A = \frac{\text{max. tens.}}{\sigma} + \frac{\text{max. comp.}}{\text{column strength}},$$

where "column strength" is to be found from the formula already given for long struts, viz.:

$$\frac{P}{A} = \frac{1}{4 + \frac{1}{20}\frac{l}{d}}\left(\frac{S_t}{1 + c\frac{l^2}{r^2}}\right).$$

For combined flexure and tension or compression, we have the formula deduced in the Appendix, Part I., page 313:

$$A = \frac{Mv}{\sigma r^2} + \frac{S}{\sigma},$$

where M is the maximum moment due to flexure, v the distance to outer fibre from

centre, S the tensile or compressive stress, r the radius of gyration of the cross section, and σ the allowable working stress for tension or compression as given on page 369, or as found from the formula for "column strength" according to whether flexure is to be feared or not.

NEW METHOD.—We have called the method just explained the "old method," not because it is in any sense antiquated, for it is still used by many if not most of our best engineers, but in order to distinguish it from a later method, based upon the experimental results of Wöhler and Spangenberg, and developed mainly by Weyrauch, Launhardt and Winkler. This method we shall therefore call the "new method." It affords a more satisfactory and rational means of allowing for the effect of oft-repeated stress than the "old method," where such allowance is made simply by an arbitrary change in the value of σ, which resembles a *guess*, based upon experience, to be sure, but liable to vary considerably with different engineers. In 1858, Wöhler called attention to the necessity of experiments made with oft-repeated stress, in order to obtain a more rational basis for a method of dimensioning. In the years 1859–1870 he made many very careful experiments, under the appointment of the Prussian Minister of Public Works, upon tension, flexure and torsion. In these experiments, the specimens were rapidly strained and released within fixed limits, by means of an apparatus driven by a steam engine, and the number of deformations registered. In the years 1871–1873, these experiments were continued by Prof. Spangenberg at Berlin.

From these experiments the following conclusions were drawn:

1. Rupture may be caused not only by a stress equal to the so-called "breaking load" once and gradually applied, but by a very much smaller stress than this, if it is often enough repeated.

2. The injurious effect of repeated vibration or change of stress is least near the position of zero strain, and increases as the deformation departs from this position and approaches the allowable limits for quiescent load.

3. When the maximum stress is less than a given amount, depending upon the material, rupture will take place only after an infinite number of repetitions.

4. This given amount is less for alternating stress (alternately compression and tension) than for repeated stress of one kind only, and less for repeated stress of one kind only, than for quiescent stress.

LAUNHARDT'S FORMULA.—Let us now seek to determine the allowable stress σ per unit of area, from the given working strength.

According to Wöhler's conclusions, the number of repetitions may be greater the less the loading, so that when the loading sinks to a certain amount, rupture will take place only after an infinite number of repetitions. If then, we denote the stress per unit of area, for which, after removal, the member would always return to its originally unstrained condition, by p, then p will correspond to the unit load for an infinite number of repetitions. If, however, the unit stress is greater than p, then, for an infinite number of repetitions, the member will not continue to return to the unstrained condition, but will have eventually a certain set or residual strain. Such a stress, greater than p, which would therefore eventually cause rupture, if applied a sufficient number of times, we call the "crippling stress," and denote it by c, while the stress p we call the "primitive safe stress," "safe," because it admits of an infinite number of repetitions, and "primitive," because at each repetition the load is wholly removed and the piece returns to its primitive unstrained condition.

Now let the "crippling stress" c, as above defined, consist of two parts, a portion ρ which always acts, and which we may call the "residual stress," and a portion s which

may be repeated an infinite number of times without rupture, the piece after each repetition returning to the residual stress. We may then call s the "safe stress" simply, while p is the "primitive safe stress." Then we have the relation $s = c - \rho$, and hence,

$$c = s + \rho. \quad \ldots \ldots \quad (1)$$

We see then that the crippling stress c is some function of s, or in general,

$$c = ks, \quad \ldots \ldots \quad (2)$$

where k denotes some unknown coefficient.

In order to determine k, we have for the limiting values of c, when the residual stress $\rho = 0$, $c = p = s$; when the difference $s = 0$, $c = \rho = \mu =$ ultimate strength for quiescent load.

Thus ultimate strength and primitive safe strength are special cases of working strength.

Since now, for $s = 0$, $c = \mu$, we see from (2) that for this limit, $k = \infty$. Since also for $s = p$, we have $c = s$, we see from (2) that for this limit $k = 1$.

These conditions satisfy perfectly the expression which Launhardt gives, viz.,

$$k = \frac{\mu - p}{\mu - c}.$$

This expression we have still to test by experiment for intermediate values, of course, before we can accept it finally as correct. Assuming its correctness at present, we have from (2):

$$c = \frac{\mu - p}{\mu - c} s.$$

Or putting for s its value from (1):

$$c = \frac{\mu - p}{\mu - c}(c - \rho).$$

Reducing:

$$c = p\left(1 + \frac{\mu - p}{p}\frac{\rho}{s}\right). \quad \ldots \ldots \quad (3)$$

If we denote by const. S the constant and by total S the total stress on the member, then we have evidently

$$\frac{\rho}{c} = \frac{\text{const. } S}{\text{total } S};$$

and hence the crippling stress

$$c = p\left[1 + \frac{\mu - p}{p}\frac{\text{const. } S}{\text{total } S}\right]. \quad \ldots \ldots \quad (4)$$

This is Launhardt's formula. We see from equation (1) that it manifestly holds good only for the case where min. S and max. S have the same sign, that is, only for repeated tension or repeated compression. Also in the latter case it is understood that there is no tendency to flexure.

The value of p for compression has not yet been satisfactorily determined. We therefore take the values of μ and p the same for compression as for tension, a practice which seems justified by certain observations, and which, as regards μ, has always been the custom heretofore.

We have yet to show that Launhardt's expression for the coefficient k holds good for intermediate values of p and μ. For this purpose, we solve (3) with reference to c, and obtain

$$c = \frac{p}{2} + \sqrt{\left(\frac{p}{2}\right)^2 + \rho(\mu - p)},$$

where we must have + before the radical, because c must be positive and greater than p. According to the method of loading and the kind of material, μ and p vary, as also c, for any given ρ. Hence, in order that an experiment may possess any value, the results must all be obtained in the same manner and with the same material. The results best suited for comparison are beyond doubt those obtained by Wöhler with Krupp cast steel, and it may be said for Launhardt's formula that it agrees excellently well with them. Thus Wöhler found $\mu = 1,100$ centners, $p = 500$ centners, hence

$$c = 250 + \sqrt{62,500 + 600\,\rho}.$$

Below we give the comparison of the formula with the experimental results of Wöhler:

For $\rho =$	0	250	400	600	1,100
c by experiment $=$	500	700	800	900	1,100
c by formula $=$	500	711	800	900	1,100

According to previous views, the single quiescent stress of 1,100 is that necessary for rupture, but we see from the above that *all stresses down to 500 may cause rupture, if repeated often enough.*

WEYRAUCH'S FORMULA.—It frequently happens that a member may be subjected to alternate compression and tension. Since the formula of Launhardt no longer holds good in such case, we must deduce one which does. Such a formula is Weyrauch's. Wöhler has shown that the crippling strength is much less than when the repeated stress is always of one kind. He has also investigated the case in which the opposite stresses are equal. The strength in this case we call the "*vibration safe strength*," and denote it by p'. Thus if the stress in one direction is zero, p' becomes p, the primitive safe strength. Here then are two limits given.

Let now, a member of one square unit cross-section be subjected to alternate compression and tension. Then for any value c for the greater of these two stresses, there will be a corresponding value c' for the less, so that for the greatest number of alternations which can ever occur between $\pm c$ and $\mp c'$, the material remains uninjured. The difference of the stresses is then

$$\left.\begin{aligned} s &= c + c', \\ \text{or} \quad c &= s - c', \end{aligned}\right\}, \qquad (5)$$

where simply numerical values are inserted without regard to sign or character of stress.

Now, according to Wöhler's law, c decreases as s increases; and, in general, c is a function of s. We can, therefore, put

$$c = ks. \qquad (6)$$

But from (5) we have

$$\text{when } c' = 0, \qquad c = p = s,$$

$$\text{when } c = c', \qquad c = p' = \tfrac{1}{2}s.$$

We have also, from (6),

$$\text{when } c = p, \qquad k = 1,$$
$$\text{when } c = p', \qquad k = \tfrac{1}{2}.$$

These conditions are satisfied by

$$k = \frac{p - p'}{2p - p' - c'},$$

hence

$$c = \frac{p - p'}{2p - p' - c} s,$$

or since

$$c = \frac{p - p'}{2p - p' - c}(c + c'),$$

we have

$$c = p\left[1 - \frac{p - p'}{p}\frac{c'}{c}\right].$$

If now, for any member, max. S is the greatest stress whether of tension or compression, and max. S' the greatest stress of the opposite kind (less than max. S), we have

$$\frac{c'}{c} = \frac{\text{max. } S'}{\text{max. } S},$$

and hence

$$c = p\left[1 - \frac{p - p'}{p}\frac{\text{max. } S'}{\text{max. } S}\right]. \quad \dots\dots\dots\dots \quad (7)$$

This is Weyrauch's formula. All quantities are simply to be inserted numerically without reference to their signs before insertion.

The primitive safe strength is p, the vibration safe strength p', and c the crippling strength in the direction of the greatest of the two stresses, max. S. Since p is not yet known for compression, we may, as in Launhardt's formula, for the present, use its value for tension, which is rather too small if any thing.

In many constructions, the alternations occur between the limits c and c' for primitive stress of zero. In others, we have a previous stress of ρ, in most cases due to the dead weight. However we may conceive it to be, the action of each complete alternation must be similar, nor can it be changed by the long continued action of ρ, which lies far within the elastic limits.

If then generally, we denote by ϕ the ratio of the two limiting stresses of a member, the less to the greater, without reference to sign, our formulas become:

For repeated stress of one kind only

$$c = p\left[1 + \frac{\mu - p}{p}\phi\right].$$

For repeated stresses of alternate kinds

$$c = p\left[1 - \frac{p - p'}{p}\phi\right].$$

LIST OF LITERATURE.

We give for the benefit of the student a list of some of the more important works treating of the subject of the preceding articles:

STONEY, BINDON B.—"Theory of Strains in Girders and Similar Structures." London: Longmans, Green & Co., 1869.

WOOD, DE VOLSON.—"Treatise on the Resistance of Materials." New York: John Wiley & Sons, 1871.

WEYRAUCH.—"Strength and Determination of the Dimensions of Structures of Iron and Steel." Translated by A. J. Du Bois. New York: John Wiley & Sons, 1877.

OTT, KARL VON.—"Vorträge über Baumechanik." Prag: 1880.

SPANGENBERG.—"Ueber das Verhalten der metalle bei wiederholten Anstrengungen." Erbkamms Ztschr. für Bauwesen, 1874 and 1875. Also separate reprint by *Ernst* and *Korn*. Berlin, 1875.

WÖHLER.—Ztschr. für Bauwesen, 1860, 1863, 1866, 1870. Also reprint by *Ernst* and *Korn*, under the title, "Die Festigkeits Versuche mit Eisen und Stahl." Berlin, 1870.

LAUNHARDT.—"Die Inanspruchnahme des Eisens." Ztschr. d. Hannöv, Arch. u. Ing. Vereins, 1873.

WINKLER.—"Wahl der Zulässigen Inanspruchnahme der Eisen Constructionen." Wien, 1877.

NEW METHOD FOR THE DETERMINATION OF THE ALLOWABLE UNIT STRESS.—As soon as we have determined the maximum stresses in any member by statical calculation, as detailed in Part I., we can find from the preceding equations, as soon as the proper values of μ, p, and p' are known, that stress c per unit of area, which will cause rupture only after an infinite number of repetitions. These values of μ, p, and p' for various materials, will presently be given in the Recapitulation which follows.

It must, of course, be understood that thus far flexure has not been considered, that is, all struts are supposed very short, and the equations above apply therefore to pure compression or tension. No account has also been taken of those prejudicial influences which do not admit of precise estimation, such as sudden shocks, impact of moving loads, lack of homogeneity of materials, action of the atmosphere, rust, changes of temperature, etc. Of these, impact may be included by properly modifying the values of $\frac{\mu - p}{p}$ and $\frac{p - p'}{p}$ in the above formulas, and the others may be allowed for by means of a factor of safety.

If, then, const. S is the constant steady tension or compression, and total S the greatest total stress, and n the factor of safety, we have for the allowable stress σ, per unit of cross-section, for *repeated stress of one kind only*,

$$\sigma = \frac{p}{n}\left[1 + \frac{\mu - p}{p}\frac{\text{const. } S}{\text{total } S}\right], \quad \text{(I.)}$$

and for *alternating stress of opposite kinds*,

$$\sigma = \frac{p}{n}\left[1 - \frac{p - p'}{p}\frac{\text{max. } S'}{\text{max. } S}\right], \quad \text{(II.)}$$

where max. S is the greatest stress, whether of tension or compression, and max. S' the greatest stress of opposite kind, *less* than max. S. That is, the greatest of the two maximum stresses is always to be put *in the denominator*.

The difference, then, between the old and new methods, is, that while in the former a portion of the ultimate strength is taken, in the latter, a portion of the "crippling stress" is taken as the allowable stress. This portion is constant for the new method,

and the allowable stress varies according to the action of the repeated loading, while to accommodate the old method to such action, the factor of safety, or the allowable unit stress, is rather arbitrarily chosen, and varies greatly in individual practice.

NEW METHOD—APPLICATION TO LONG STRUTS.—The method just given applies to pieces in pure compression or tension, but does not take into account the extra material required for stiffening, in the case of long struts. This may easily be done, in a method similar to that adopted in the old method. Thus we have for

repeated compression, taking flexure into account,

$$\text{allowable stress} = \frac{\sigma}{1 + c\frac{l^2}{r^2}} = \frac{p}{n\left(1 + c\frac{l^2}{r^2}\right)}\left[1 + \frac{\mu - p}{p} \cdot \frac{\text{const. } S}{\text{total } S}\right], \quad . \quad . \quad (\text{III.})$$

and for

alternating stress, taking flexure into account,

$$\text{allowable stress} = \frac{\sigma}{1 + c\frac{l^2}{r^2}} = \frac{p}{n\left(1 + c\frac{l^2}{r^2}\right)}\left[1 - \frac{p - p'}{p}\frac{\text{max. } S'}{\text{max. } S}\right], \quad . \quad . \quad . \quad (\text{IV.})$$

where max. S is the *greatest* of the two stresses.

In these equations, c has the same value as in the old method, l is the length in inches, and r the least radius of gyration of cross-section in inches.

RECAPITULATION—OLD AND NEW METHODS OF DIMENSIONING.—VALUES OF $\frac{p}{n}$, $\frac{\mu - p}{p}$, AND $\frac{p-p'}{p}$.

OLD METHOD.

Let A be the cross-section of the member, max. S the greatest stress which can ever come upon it, and σ the allowable stress per square inch. Then for simple tension or compression, when flexure is not to be apprehended,

$$A = \frac{\text{max. } S}{\sigma}.$$

The customary values of σ for the various members we have to deal with, for simple tension and compression (without flexure), are given on page 369. These values are different for different members, in order to allow for the effect of repetition, shock, etc.

If the member is subjected to *alternating stress, i. e.*, sometimes tension and sometimes compression, then if flexure is not to be guarded against, we have

$$A = \frac{\text{max. tension} + \text{max. compression}}{\sigma}.$$

The values of σ being taken from page 369.

If the member is so long that flexure has to be guarded against, that is, in general when $\frac{l}{r}$ is greater than 30, or $\frac{l}{d}$ is greater than 10, we have

$$A = \frac{\text{max. compression}}{\sigma_1} \quad \text{or} \quad \frac{\text{max. compression}}{\sigma_1} + \frac{\text{max. tension}}{\sigma},$$

where σ is as before, given on page 369, and σ_1 is given by Gordon's formula,

$$\sigma_1 = \frac{1}{4 + \frac{1}{20}\frac{l}{d}}\left[\frac{\mu}{1 + c\frac{l^2}{r^2}}\right],$$

l being the length, d the least dimension, and r the least radius of gyration of cross-section in inches, and μ being taken as given on page 369.

For a member in longitudinal *tension* and at the same time acting like a beam to support a transverse load, we have (Appendix, Part I, page 313),

$$A = \frac{Mv}{\sigma r^2} + \frac{S}{\sigma},$$

where σ is given on page 369, S is the longitudinal tension in lbs., and M the greatest moment in inch lbs. due to the transverse load, v is the distance from the neutral axis to the outer fibre, and r the radius of gyration with reference to the neutral axis, in inches.

For a piece in longitudinal *compression* and at the same time acting like a beam to support a transverse load,

$$A = \frac{Mv}{\sigma_1 r^2} + \frac{S}{\sigma_1},$$

where S is the longitudinal compression, and σ_1 is given by Gordon's Formula.

NEW METHOD.

By the *new method*, we have in *all cases*,

$$A = \frac{\text{max. } S}{\sigma} \quad \text{or} \quad \frac{\text{max. } S}{\sigma_1},$$

but instead of the values of σ and σ_1 used in the old method, we have,

For repeated stress of one kind, without flexure,

$$\sigma = \frac{p}{n}\left[1 + \frac{\mu - p}{p}\,\frac{\text{const. } S}{\text{total } S}\right].$$

For repeated stress of one kind, with flexure,

$$\sigma_1 = \frac{p}{n\left(1 + c\frac{l^2}{r^2}\right)}\left[1 + \frac{\mu - p}{p}\,\frac{\text{const. } S}{\text{total } S}\right].$$

The values of $\frac{p}{n}$ and $\frac{\mu - p}{p}$ will be given presently for different materials. Const. S is the steady stress, if any, acting all the time upon the piece; total S, the greatest total stress (*including* const. S and also any repeated stress), which acts upon the member.

For alternating stress, without flexure,

$$\sigma = \frac{p}{n}\left[1 - \frac{p - p'}{p}\,\frac{\text{max. } S'}{\text{max. } S}\right].$$

For alternating stress, with flexure,

$$\sigma_1 = \frac{p}{n\left(1 + c\dfrac{l^2}{r^2}\right)}\left[1 - \frac{p - p'}{p}\,\frac{\text{max. } S'}{\text{max. } S}\right],$$

where max. S is always the *greatest* of the two opposite maximum stresses.

For a piece subjected to longitudinal tension and at the same time acting as a beam to sustain a load, we have as before,

$$A = \frac{Mv}{\sigma r^2} + \frac{S}{\sigma},$$

or if subjected to longitudinal compression,

$$A = \frac{Mv}{\sigma_1 r^2} + \frac{S}{\sigma_1},$$

where σ and σ_1 are as just given above.

The values of c, μ, and $\dfrac{1}{1 + c\dfrac{l^2}{r^2}}$ in all these formulas have been given on page 369.

It is unnecessary to repeat them here. Finally, for the values of $\frac{p}{n}$, $\frac{\mu - p}{p}$, and $\frac{p - p'}{p}$ to be used in the *new* method, we have,

	$\frac{p}{n}$	$\frac{\mu - p}{p}$	$\frac{p - p'}{p}$
Wood	400	2	$\frac{1}{2}$
* Wrought iron, double rolled (links or rods), in tension.	7500.	1	$\frac{1}{2}$
Wrought iron plates in tension	7000	1	$\frac{1}{2}$
Wrought iron in compression	6500	1	$\frac{1}{2}$
Cast iron	10000	$\frac{4}{3}$	$\frac{2}{5}$
Ordinary steel (soft)	9530	$\frac{3}{2}$	$\frac{2}{5}$
Soft cast steel	17870	1	$\frac{7}{15}$
Iron wire rope	11400	$\frac{3}{2}$	
Steel wire rope	26700	1	

For *shear*, for iron and steel, we may take $\frac{4}{5}\sigma$ as the allowable stress, where σ is to be found as above.

* The values for wrought iron are those adopted by Joseph M. Wilson, C. E., in his specifications. "Specifications for Strength of Iron Bridges," *Trans. Am. Soc. Civil Engineers*, June, 1886, also page 455.

Prof. Merriman has deduced ("*Mechanics of Materials*," Wiley & Sons, 1885) the single formula, both for repeated stress of one kind and for alternating stress also,

$$\sigma = \frac{p}{n}\left[1 + \frac{\mu - p'}{2p}R + \frac{\mu + p' - 2p}{2p}R^2\right],$$

where R stands for the ratio of the least limiting stress to the greatest limiting stress, or what we have called $\frac{\text{const. } S}{\text{total } S}$ for repeated stress of one kind, and $\frac{\text{max. } S'}{\text{max. } S}$ for alternating stress, only *regard is paid to the character or sign* of the stress. Thus, if both limiting stresses are tension or both compression, R is *positive*; if one is tension and the other compression, R is *negative*. With this understanding, the single formula of Prof. Merriman replaces Launhardt's and Weyrauch's.

To apply it to long struts we have simply to put $\dfrac{p}{n\left(1 + c\frac{l^2}{r^2}\right)}$ in place of $\frac{p}{n}$.

The values of the coefficients are as follows:

	$\frac{p}{n}$	$\frac{\mu - p}{2p}$	$\frac{\mu + p' - 2p}{2}$
Wood	400	$\frac{5}{4}$	$\frac{3}{8}$
Wrought iron, double rolled (links or rods, in tension)	7500	$\frac{3}{4}$	$\frac{1}{4}$
Wrought iron plates in tension	7000	$\frac{3}{4}$	$\frac{1}{4}$
Wrought iron in compression	6500	$\frac{3}{4}$	$\frac{1}{4}$
Cast iron	10000	$\frac{13}{15}$	$\frac{7}{15}$
Ordinary steel (soft)	9530	$\frac{19}{20}$	$\frac{11}{20}$
Soft cast steel	17870	$\frac{11}{15}$	$\frac{4}{15}$
Iron wire rope	11400	$\frac{3}{4}$	$\frac{3}{4}$
Steel wire rope	26700	$\frac{1}{2}$	$\frac{1}{2}$

THE STRAIGHT-LINE FORMULA.—Instead of the Rankine formula the straight-line formula (given by Thomas H. Johnson, C. E., *Trans. Am. Soc. C. E.*, July, 1886) is often used. This formula has been given in Part I., page 335.

It is as follows:

$$\text{For } \frac{l}{r} < n\sqrt{\frac{3E}{S_e}}, \qquad \frac{P}{A} = S_e\left[1 - \frac{2\sqrt{S_e}}{3n\sqrt{3E}} \cdot \frac{l}{r}\right],$$

where P is the crippling load, A the area of cross-section, so that $\frac{P}{A}$ is the crippling unit stress, S_e is the elastic limit unit stress, E the coefficient of elasticity, l the length and r the least radius of gyration. The same factors of safety are used as for Rankine's formula.

The values of n as given Part I., page 334, are as follows:

	Two Pin Ends	One Pin, One Flat End.	Two Flat Ends.
$n =$	$\pi\sqrt{\frac{5}{3}}$	$\frac{5\pi}{2\sqrt{3}}$	$\pi\sqrt{\frac{5}{2}}$.

Beyond the value of $\frac{l}{r}$ given above we have Euler's formula (Part I., page 333).

Hence

$$\text{For } \frac{l}{r} > n\sqrt{\frac{3E}{S_e}}, \qquad \frac{P}{A} = \frac{n^2 E r^2}{l^2}.$$

Mr. Johnson has given the following values for different materials.

JOHNSON'S STRAIGHT-LINE FORMULA FOR VARIOUS MATERIALS AND END BEARINGS.

$$\text{Factor of safety, } \begin{cases} \text{quiescent loading} & \begin{cases} \text{wrought iron} = 4 \\ \text{cast iron} = 6 \end{cases} \\ \text{variable loading } 4 + \dfrac{l}{20d} \end{cases}$$

$l =$ length in inches, $d =$ least dimension in inches, $A =$ area of cross-section in square inches, $P =$ crippling load in lbs.

$S_e =$ elastic limit unit stress.

$r =$ least radius of gyration of cross-section in inches.

For round ends $n = \pi$; for hinged ends $n = \pi\sqrt{\frac{5}{3}}$; for one pin one flat end $n = \frac{5\pi}{2\sqrt{3}}$; for flat ends $n = \pi\sqrt{\frac{5}{2}}$.

MATERIAL.	E in lbs.	S_e in lbs.	END BEARING.	$\frac{P}{A} = S_e\left[1 - \frac{2\sqrt{S_e}}{3n\sqrt{3E}} \cdot \frac{l}{r}\right]$ when $\frac{l}{r} < n\sqrt{\frac{3E}{S_e}}$	$\frac{l}{r} = n\sqrt{\frac{3E}{S_e}}$	$\frac{P}{A} = \frac{n^2Er^2}{l^2}$ when $\frac{l}{r} > n\sqrt{\frac{3E}{S_e}}$
Wrought Iron.	27000000	42000	Flat,	$\frac{P}{A} = 42000 - 128\frac{l}{r}$	218.1	$\frac{P}{A} = 666090000\frac{r^2}{l^2}$
			Hinged,	$\frac{P}{A} = 42000 - 157\frac{l}{r}$	178.1	$\frac{P}{A} = 444150000\frac{r^2}{l^2}$
			Round,	$\frac{P}{A} = 42000 - 203\frac{l}{r}$	138.1	$\frac{P}{A} = 266490000\frac{r^2}{l^2}$
Mild Steel (Carbon = 0.12).	27000000	52500	Flat,	$\frac{P}{A} = 52500 - 179\frac{l}{r}$	195.1	$\frac{P}{A} = 666090000\frac{r^2}{l^2}$
			Hinged,	$\frac{P}{A} = 52500 - 220\frac{l}{r}$	159.3	$\frac{P}{A} = 444150000\frac{r^2}{l^2}$
			Round,	$\frac{P}{A} = 52500 - 284\frac{l}{r}$	123.3	$\frac{P}{A} = 266490000\frac{r^2}{l^2}$
Hard Steel (Carbon 0.36).	27000000	80000	Flat,	$\frac{P}{A} = 80000 - 337\frac{l}{r}$	158	$\frac{P}{A} = 666090000\frac{r^2}{l^2}$
			Hinged,	$\frac{P}{A} = 80000 - 414\frac{l}{r}$	129	$\frac{P}{A} = 444150000\frac{r^2}{l^2}$
			Round,	$\frac{P}{A} = 80000 - 534\frac{l}{r}$	99.9	$\frac{P}{A} = 266490000\frac{r^2}{l^2}$
Cast Iron.	16000000	80000	Flat,	$\frac{P}{A} = 80000 - 438\frac{l}{r}$	121.6	$\frac{P}{A} = 394720000\frac{r^2}{l^2}$
			Hinged,	$\frac{P}{A} = 80000 - 537\frac{l}{r}$	99.3	$\frac{P}{A} = 263200000\frac{r^2}{l^2}$
			Round,	$\frac{P}{A} = 80000 - 693\frac{l}{r}$	77	$\frac{P}{A} = 157920000\frac{r^2}{l^2}$
Oak.	1200000	5400	Flat,	$\frac{P}{A} = 5400 - 28\frac{l}{r}$	128.1	$\frac{P}{A} = 29604000\frac{r^2}{l^2}$

Theodore Cooper, C. E., has adopted the straight-line formula in his specifications,* but varies somewhat the constants employed.

He makes the limit of length of any compression member 45 times its least width. Within this limit, for *wrought iron* he gives the following formulas, for *allowable* compression per square inch of cross-section.

For *chords*,

$$\sigma = 8000 - 30\frac{l}{r} \text{ for live load stresses.}$$

$$\sigma = 16000 - 60\frac{l}{r} \text{ for dead load stresses.}$$

For all posts,

$$\sigma = 7000 - 40\frac{l}{r} \text{ for live load stresses,}$$

$$\sigma = 14000 - 80\frac{l}{r} \text{ for dead load stresses.}$$

$$\sigma = 10500 - 60\frac{l}{r} \text{ for wind stresses.}$$

For lateral struts

$$\sigma = 9000 - 50\frac{l}{r} \text{ for assumed initial stress.}$$

PARABOLA FORMULA.—This formula (given by Prof. J. B. Johnson, Theory and Practice of Modern Framed Structures, Wiley & Sons) has been given, Part I., page 336.

It is as follows:

$$\text{For } \frac{l}{r} < n\sqrt{\frac{2E}{S_e}}, \qquad \frac{P}{A} = S_e\left[1 - \frac{S_e}{4n^2E}\cdot\frac{l^2}{r^2}\right],$$

where, as before, P is the crippling load, A the area of cross-section, so that $\frac{P}{A}$ is the crippling unit stress; S_e is the elastic limit unit stress, E the coefficient of elasticity, l the length, and r the least radius of gyration of the cross-section. The same factors of safety are used as for Rankine's formula. The values of n are the same as already given for the straight-line formula. Beyond the value of $\frac{l}{r}$ given above we have Euler's formula. Hence

$$\text{For } \frac{l}{r} > n\sqrt{\frac{2E}{S_e}}, \qquad \frac{P}{A} = \frac{n^2Er^2}{l^2}.$$

Prof. Johnson has given the following values for different materials:

$$\textit{For Wrought-iron Columns, Pin Ends,} \begin{cases} \frac{l}{r} \overline{<} 170 & \frac{P}{A} = 34000 - 0.67\frac{l^2}{r^2} \\ \frac{l}{r} \overline{>} 170 & \frac{P}{A} = \frac{432000000r^2}{l^2} \end{cases}$$

$$\textit{For Wrought-iron Columns, Flat Ends,} \begin{cases} l \overline{<} 210 & \frac{P}{A} = 34000 - 0.43\frac{l^2}{r^2} \\ l \overline{>} 210 & \frac{P}{A} = \frac{675000000r^2}{l^2} \end{cases}$$

* *General Specifications for Iron and Steel Railroad Bridges and Viaducts.* 1888, Engineering News Publishing Company, end of this work.

For Mild Steel Columns, Pin Ends,
$$\begin{cases} \dfrac{l}{r} \leqq 150 & \dfrac{P}{A} = 42000 - 0.97\dfrac{l^2}{r^2} \\ \dfrac{l}{r} \geqq 150 & \dfrac{P}{A} = \dfrac{456000000r^2}{l^2} \end{cases}$$

For Mild Steel Columns, Flat Ends,
$$\begin{cases} \dfrac{l}{r} \leqq 190 & \dfrac{P}{A} = 42000 - 0.62\dfrac{l^2}{r^2} \\ \dfrac{l}{r} \geqq 190 & \dfrac{P}{A} = \dfrac{712000000r^2}{l^2} \end{cases}$$

For Cast Iron, Round Ends,
$$\begin{cases} \dfrac{l}{r} \leqq 70 & \dfrac{P}{A} = 60000 - \dfrac{25l^2}{4r^2} \\ \dfrac{l}{r} \geqq 70 & \dfrac{P}{A} = \dfrac{144000000r^2}{l^2} \end{cases}$$

For Cast Iron, Flat Ends,
$$\begin{cases} \dfrac{l}{r} \leqq 120 & \dfrac{P}{A} = 60000 - \dfrac{9l^2}{4r^2} \\ \dfrac{l}{r} \geqq 120 & \dfrac{P}{A} = \dfrac{400000000r^2}{l^2} \end{cases}$$

For White Pine, Flat Ends, $\dfrac{l}{d} \leqq 60 \quad \dfrac{P}{A} = 2500 - \dfrac{0.6l^2}{d^2}$

For Short-leaf Yellow Pine, Flat Ends, $\dfrac{l}{d} \leqq 60 \quad \dfrac{P}{A} = 3300 - \dfrac{0.7l^2}{d^2}$

For Long-leaf Yellow Pine, Flat Ends, $\dfrac{l}{d} \leqq 60 \quad \dfrac{P}{A} = 4000 - \dfrac{0.8l^2}{d^2}$

For White Oak, Flat Ends, $\dfrac{l}{d} \leqq 60 \quad \dfrac{P}{A} = 3500 - \dfrac{0.8l^2}{d^2}$

MERRIMAN'S FORMULA.—This formula (given by Prof. Merriman, *Engineering News*, July 19, 1894, has been given, Part I., page 338.

It is as follows:

$$\frac{P}{A} = \frac{S_e}{1 + \dfrac{S_e l^2}{n^2 E r^2}},$$

where, as before, P is the crippling load, A the area of cross-section, so that $\dfrac{P}{A}$ is the crippling unit stress; S_e is the elastic limit unit stress, E the coefficient of elasticity, l the length, and r the least radius of gyration of the cross-section. The same factors of safety are used as for Rankine's formula. The values of n are the same as already given for the straight-line formula.

We have then for different materials the following formulas:

For Wrought-iron Columns, Pin Ends, $\dfrac{P}{A} = \dfrac{34000}{1 + \dfrac{l^2}{12700r^2}}$

For Wrought-iron Columns, Flat Ends, $\dfrac{P}{A} = \dfrac{34000}{1 + \dfrac{l^2}{20000r^2}}$

For Mild Steel Columns, Pin Ends, $\dfrac{P}{A} = \dfrac{42000}{1 + \dfrac{l^2}{10825r^2}}$

For Mild Steel Columns, Flat Ends, $\dfrac{P}{A} = \dfrac{42000}{1 + \dfrac{l^2}{17000r^2}}$

For Cast Iron, Round Ends, $\dfrac{P}{A} = \dfrac{60000}{1 + \dfrac{l^2}{2400r^2}}$

For Cast Iron, Flat Ends, $\dfrac{P}{A} = \dfrac{60000}{1 + \dfrac{l^2}{6666r^2}}$

For White Pine, Flat Ends, $\dfrac{P}{A} = \dfrac{2500}{1 + \dfrac{l^2}{1000d^2}}$

For Short-leaf Yellow Pine, Flat Ends, $\dfrac{P}{A} = \dfrac{3300}{1 + \dfrac{l^2}{1180d^2}}$

For Long-leaf Yellow Pine, Flat Ends, $\dfrac{P}{A} = \dfrac{4000}{1 + \dfrac{l^2}{1250d^2}}$

For White Oak, Flat Ends, $\dfrac{P}{A} = \dfrac{3500}{1 + \dfrac{l^2}{1090d^2}}$

TABLES FOR LONG STRUTS.—To lessen the labor of computation by Rankine's formula we shall now give a number of tables, from which we can find for any ratio of $\dfrac{l}{r}$ or $\dfrac{l}{d}$ the crippling stress, in accordance with the formulas already given. As the factor of safety is given by itself, and the crippling strength by itself, the working stress for any desired factor of safety can be obtained if desired. The tables for "Square," "Phœnix," "American," and "Common" columns (page 388) were given by C. Shaler Smith in *Trans. Am. Soc. of Civil Engrs.*, for October, 1880. Similar tables can be made out for Merriman's formulas.

The values of $\dfrac{1}{1 + c\dfrac{l^2}{r^2}}$ or $\dfrac{1}{1 + c\dfrac{l^2}{d^2}}$, to be used in the formulas for the *new method*, may be easily taken from these tables, by dividing the crippling strength given in the table by the value of μ or ultimate strength taken in any case.

The straight-line formulas of Johnson or Cooper require no Tables, are easily applied, and are coming into general use. We have thus several methods for finding the crippling strength for long struts—the old method, by means of the following Tables, the new method, also by use of the following Tables and the formulas already given, and the "straight-line formula."

TABLE I.

STRENGTH OF WROUGHT IRON STRUTS OF ANY CROSS SECTION—EXCEPT HOLLOW ROUND. (*For Cast Iron, take twice the tabular values. For Steel, see Carnegie's Hand-Book.*)

l = length in inches.
r = least radius of gyration in inches.
d = least dimension of rectangle enclosing the given cross section, in inches.

Factor of safety $\begin{cases} \text{for wrought iron} = 4 + \dfrac{l}{20d} \\ \text{for cast iron} = 7 + \dfrac{l}{20d} \end{cases}$ Intermittent Loading; $\begin{matrix} \text{for wrought iron} = 4 \\ \text{for cast iron} = 6 \end{matrix}\Bigg\}$ Quiescent Loading.

STRUT. *Flat ends.*	STRUT. *Pin and flat end.*	STRUT. *Pin ends.*
$\dfrac{40000}{1 + \dfrac{l^2}{36000r^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{24000r^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{18000r^2}}$

$\frac{l}{r}$.	*Crippling Strength in tons (2000 lbs.) per square inch.*			$\frac{l}{r}$.	*Crippling Strength in tons (2000 lbs.) per square inch.*		
	Flat ends.	Pin and flat.	Pin ends.		Flat ends.	Pin and flat.	Pin ends.
30	19.510	19.275	19.050	90	16.325	14.955	13.800
32	19.455	19.180	18.925	92	16.195	14.785	13.605
34	19.380	19.080	18.795	94	16.060	14.620	13.415
36	19.305	18.975	18.655	96	15.925	14.450	13.230
38	19.230	18.865	18.515	98	15.790	14.285	13.040
40	19.150	18.750	18.365	100	15.650	14.120	12.855
42	19.065	18.630	18.215	102	15.515	13.950	12.675
44	18.980	18.510	18.060	104	15.380	13.790	12.495
46	18.890	18.380	17.895	106	15.245	13.625	12.315
48	18.795	18.250	17.730	108	15.105	13.460	12.135
50	18.700	18.115	17.560	110	14.970	13.300	11.960
52	18.600	17.975	17.390	112	14.835	13.135	11.785
54	18 500	17.835	17.210	114	14 695	12.975	11.615
56	18.400	17.690	17.035	116	14.560	12.815	11.445
58	18.290	17 540	16.850	118	14.425	12.655	11.275
60	18.180	17.390	16.665	120	14.285	12.500	11.110
62	18.070	17.240	16.480	122	14.150	12.355	10 950
64	17.955	17.035	16.300	124	14.015	12.190	10.785
66	17.840	16.930	16.110	126	13.880	12.035	10.625
68	17.725	16.770	15.930	128	13.745	11.885	10.470
70	17.605	16.610	15.725	130	13.610	11.735	10.315
72	17.485	16 450	15.535	132	13.475	11.590	10.165
74	17.360	16.285	15.340	134	13.345	11.440	10.010
76	17.255	16.120	15.150	136	13.210	11.295	9.865
78	17.110	15.955	14 955	138	13.080	11.150	9.720
80	16 980	15.790	14.760	140	12.950	11.010	9.575
82	16.855	15.620	14.565	142	12.820	10.870	9.435
84	16.720	15.455	14 375	144	12.690	10.730	9.295
86	16.590	15 290	14.180	146	12.560	10.590	9.155
88	16.460	15.120	13.985	148	12.435	10.455	9.020

NEW METHOD.—*For repeated compression:* For wrought iron, $\beta = \dfrac{6500}{1 + c\dfrac{l^2}{r^2}}\left[1 + \dfrac{\text{min. } B}{\text{max. } B}\right]$; for cast iron $\beta = \dfrac{10000}{1 + c\dfrac{l^2}{r^2}}\left[1 + \dfrac{4}{3}\dfrac{\text{min. } B}{\text{max. } B}\right]$. The crippling strength in pounds, divided by 40000, gives the value of $\dfrac{1}{1 + c\dfrac{l^2}{r^2}}$ for wrought iron, and divided by 80000 for cast iron.

TABLE II.

STRENGTH OF HOLLOW CYLINDRICAL CAST AND WROUGHT IRON STRUTS.

l = length in inches.
d = least diameter in inches.

$$\text{Factor of safety} \begin{cases} \text{for wrought iron} = 3 + \dfrac{l}{10d} \\ \text{for cast iron} \quad = 6 + \dfrac{l}{10d} \end{cases} \begin{matrix} \text{Intermittent} \\ \text{Loading;} \end{matrix} \qquad \left. \begin{matrix} \text{for wrought iron} = 4 \\ \text{for cast iron} \quad = 6 \end{matrix} \right\} \begin{matrix} \text{Quiescent} \\ \text{Loading.} \end{matrix}$$

CAST IRON.			WROUGHT IRON.		
Flat ends.	*Pin and flat.*	*Pin ends.*	*Flat ends.*	*Pin and flat.*	*Pin ends.*
$\dfrac{80000}{1 + \dfrac{l^2}{800d^2}}$	$\dfrac{80000}{1 + \dfrac{3l^2}{1600d^2}}$	$\dfrac{80000}{1 + \dfrac{l^2}{400d^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{4500d^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{3000d^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{2250d^2}}$

$\frac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per sq. in.*			$6 + \frac{l}{10d}$	$\frac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per sq. in.*			$3 + \frac{l}{10d}$
	Flat ends.	Pin and flat.	Pin ends.			Flat ends.	Pin and flat.	Pin ends.	
10	35.555	33.685	32.000	7.	10	19.565	19.354	19.150	4
11	34.745	32.603	30.710	7.1	11	19.476	19.225	18.980	4.1
12	33.895	31.496	29.412	7.2	12	19.380	19.084	18.797	4.2
13	33.024	30.375	28.120	7.3	13	19.276	18.933	18.603	4.3
14	32.128	29.250	26.846	7.4	14	19.165	18.774	18.397	4.4
15	31.220	28.132	25.600	7.5	15	19.048	18.605	18.182	4.5
16	30.303	27.027	24.390	7.6	16	18.924	18.427	17.957	4.6
17	29.385	25.943	23.223	7.7	17	18.793	18.270	17.724	4.7
18	28.470	24.883	22.099	7.8	18	18.657	18.051	17.483	4.8
19	27.563	23.854	21.025	7.9	19	18.514	17.852	17.235	4.9
20	26.667	22.858	20.000	8.0	20	18.367	17.647	16.981	5.0
21	25.786	21.895	19.025	8.1	21	18.215	17.436	16.723	5.1
22	24.922	20.970	18.100	8.2	22	18.058	17.222	16.459	5.2
23	24.078	20.082	17.223	8.3	23	17.896	17.000	16.193	5.3
24	23.256	19.230	16.393	8.4	24	17.731	16.778	15.924	5.4
25	22.456	18.418	15.625	8.5	25	17.561	16.552	15.652	5.5
26	21.680	17.640	14.870	8.6	26	17.388	16.322	15.379	5.6
27	20.928	16.900	14.171	8.7	27	17.212	16.090	15.106	5.7
28	20.202	16.195	13.513	8.8	28	17.032	15.856	14.832	5.8
29	19.500	15.523	12.893	8.9	29	16.851	15.621	14.558	5.9
30	18.823	14.884	12.306	9.0	30	16.666	15.385	14.286	6.0
31	18.172	14.276	11.756	9.1	31	16.481	15.148	14.014	6.1
32	17.544	13.698	11.236	9.2	32	16.292	14.911	13.745	6.2
33	16.940	13.149	10.745	9.3	33	16.103	14.674	13.477	6.3
34	16.360	12.628	10.283	9.4	34	15.912	14.437	13.212	6.4
35	15.803	12.133	9.846	9.5	35	15.720	14.202	12.950	6.5
36	15.267	11.662	9.434	9.6	36	15.528	13.966	12.690	6.6
37	14.754	11.215	9.044	9.7	37	15.335	13.733	12.435	6.7
38	14.260	10.789	8.677	9.8	38	15.141	13.501	12.182	6.8
39	13.787	10.385	8.329	9.9	39	14.947	13.271	11.933	6.9
40	13.333	10.000	8.000	10.0	40	14.754	13.043	11.688	7.0

NEW METHOD.—*For repeated compression:* For wrought iron $\beta = \dfrac{6500}{1 + c\dfrac{l^2}{d^2}} \left[1 + \dfrac{\text{min. } B}{\text{max. } B}\right]$; for cast iron $\beta = \dfrac{10000}{1 + c\dfrac{l^2}{d^2}} \left[1 + \dfrac{4}{3}\dfrac{\text{min. } B}{\text{max. } B}\right]$. The crippling strength in pounds, divided by 40000 or 80000 gives the value of $\dfrac{1}{1 + c\dfrac{l^2}{d^2}}$ for wrought or cast iron.

TABLE III.

STRENGTH OF RECTANGULAR TIMBER STRUTS.

l = length in inches.
d = least side in inches.

Flat ends.	*Pin and flat.*	*Pin ends.*
$\dfrac{5600}{1+\dfrac{l^2}{550d^2}}$	$\dfrac{5600}{1+\dfrac{1.5\,l^2}{550d^2}}$	$\dfrac{5600}{1+\dfrac{l^2}{275d^2}}$

Factor of safety $6+\dfrac{l}{10d}$ for intermittent loading and 6 for quiescent loading.

$\dfrac{l}{d}$	*Crippling Strength in lbs. per square inch.*			$6+\dfrac{l}{10d}$	$\dfrac{l}{d}$	*Crippling Strength in lbs. per square inch.*			$6+\dfrac{l}{10d}$
	Flat.	Pin and flat.	Pin.			Flat.	Pin and flat.	Pin.	
12	4440	4020	3680	7.2	30	2120	1620	1310	9
13.2	4250	3800	3430	7.32	31.2	2020	1530	1230	9.12
14.4	4070	3580	3190	7.44	32.4	1930	1450	1160	9.24
15.6	3880	3370	2970	7.56	33.6	1830	1370	1100	9.36
16.8	3700	3160	2760	7.68	34.8	1750	1300	1040	9.48
18	3520	2970	2570	7.8	36	1670	1230	980	9.6
19.2	3350	2790	2390	7.92	37.2	1590	1170	930	9.72
20.4	3190	2620	2230	8.04	38.4	1520	1120	880	9.84
21.6	3040	2470	2080	8.16	39.6	1450	1060	840	9.96
22.8	2890	2320	1940	8.28	40.8	1390	1010	790	10.08
24	2740	2180	1810	8.4	42	1330	960	760	10.2
25.2	2600	2050	1690	8.52	43.2	1270	920	720	10.32
26.4	2470	1930	1580	8.64	44.4	1220	880	690	10.44
27.6	2350	1820	1490	8.76	45.6	1170	840	650	10.56
28.8	2230	1720	1400	8.88	46.8	1120	800	620	10.68

NEW METHOD.—*For repeated compression:* $\beta = \dfrac{400}{1+c\dfrac{l^2}{d^2}}\left[1+2\dfrac{\text{min. }B}{\text{max. }B}\right]$. Crippling strength in pounds divided by 5600 gives $\dfrac{1}{1+c\dfrac{l^2}{d^2}}$.

TABLE IV.

Strength of Wrought Iron Struts.

Square Column. Fig. 201, page 370.			Phœnix Column. Fig. 201, page, 370.		
Flat ends.	*Pin and flat.*	*Pin ends.*	*Flat ends.*	*Pin and flat.*	*Pin ends.*
$\dfrac{38500}{1+\dfrac{l^2}{5820d^2}}$	$\dfrac{38500}{1+\dfrac{l^2}{3000d^2}}$	$\dfrac{37500}{1+\dfrac{l^2}{1900d^2}}$	$\dfrac{42500}{1+\dfrac{l^2}{4500d^2}}$	$\dfrac{40000}{1+\dfrac{l^2}{2250d^2}}$	$\dfrac{36600}{1+\dfrac{l^2}{1500d^2}}$

l = length in inches.
d = least dimension in inches.

Factor $4+\dfrac{l}{20d^2}$

$\dfrac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per square inch.*			$4+\dfrac{l}{20d}$	$\dfrac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per square inch.*			$4+\dfrac{l}{20d}$
	Flat ends.	Pin and flat.	Pin ends.			Flat ends.	Pin and flat.	Pin ends.	
15	18.533	17.907	16.899	4.75	15	20.238	18.182	15.913	4.75
16	18.438	17.730	16.641	4.80	16	20.106	17.948	15.619	4.80
17	18.340	17.552	16.384	4.85	17	19.967	17.713	15.327	4.85
18	18.235	17.374	16.127	4.90	18	19.823	17.479	15.034	4.90
19	18.126	17.180	15.869	4.95	19	19.672	17.230	14.741	4.95
20	18.012	16.986	15.613	5.00	20	19.515	16.981	14.447	5.00
21	17.893	16.783	15.339	5.05	31	19.353	16.723	14.142	5.05
22	17.772	16.576	15.063	5.10	22	19.187	16.459	13.835	5.10
23	17.646	16.365	14.784	5.15	23	19.014	16.193	13.529	5.15
24	17.517	16.150	14.504	5.20	24	18.838	15.924	13.222	5.20
25	17.384	15.931	14.222	5.25	25	18.658	15.652	12.917	5.25
26	17.246	15.710	13.940	5.30	26	18.474	15.379	12.615	5.30
27	17.106	15.487	13.659	5.35	27	18.287	15.106	12.315	5.35
28	16.965	15.262	13.379	5.40	28	18.096	14.832	12.018	5.40
29	16.820	15.035	13.101	5.45	29	17.904	14.554	11.726	5.45
30	16.672	14.808	12.825	5.50	30	17.712	14.286	11.437	5.50
31	16.522	14.577	12.552	5.55	31	17.511	14.014	11.154	5.55
32	16.370	14.352	12.281	5.60	32	17.311	13.745	10.875	5.60
33	16.216	14.123	12.014	5.65	33	17.105	13.474	10.602	5.65
34	16.060	13.897	11.751	5.70	34	16.907	13.212	10.335	5.70
35	15.903	13.668	11.491	5.75	35	16.703	12.949	10.073	5.75
36	15.743	13.443	11.236	5.80	36	16.498	12.691	9.818	5.80
37	15.582	13.212	10.985	5.85	37	16.292	12.435	9.568	5.85
38	15.422	12.995	10.738	5.90	38	16.072	12.182	9.324	5.90
39	15.261	12.779	10.497	5.95	39	15.866	11.933	9.086	5.95
40	15.099	12.555	10.260	6.00	40	15.676	11.689	8.854	6.00
45	14.281	11.493	9.149	6.25	45	14.655	10.527	7.787	6.25
50	13.466	10.503	8.163	6.50	50	13.661	9.474	6.862	6.50
55	12.666	9.585	7.292	6.75	55	12.708	8.531	6.066	6.75
60	11.894	8.750	6.529	7.00	60	11.806	7.675	5.383	7.00

New Method.—*For repeated compression*: $\beta=\dfrac{6500}{1+c\dfrac{l^2}{d^2}}\left[1+\dfrac{\text{min. }B}{\text{max. }B}\right]$. Crippling strength in pounds divided by the numerators of the respective formulas, as given at the top of the Table, gives the value of $\dfrac{1}{1+c\dfrac{l^2}{d^2}}$.

TABLE V.

STRENGTH OF WROUGHT IRON STRUTS.

AMERICAN COLUMN.
Fig. 201, page 370.

Flat ends.	Pin and flat.	Pin ends.
$\dfrac{36500}{1+\dfrac{l^2}{3750d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{3000d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{1900d^2}}$

COMMON COLUMN.
Fig. 201, page 370.

Flat ends.	Pin and flat.	Pin ends.
$\dfrac{36500}{1+\dfrac{l^2}{2700d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{1500d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{1200d^2}}$

l = length in inches.
d = least dimension in inches.
Factor $4+\dfrac{l}{20d}$.

$\frac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per square inch.*			$4+\frac{l}{20d}$	$\frac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per square inch.*			$4+\frac{l}{20d}$
	Flat ends.	Pin and flat.	Pin ends.			Flat ends.	Pin and flat.	Pin ends.	
15	17.217	16.591	16.171	4.75	15	16.847	15.869	15.333	4.75
16	17.084	16.377	15.908	4.80	16	16.669	15.582	15.004	4.80
17	16.944	16.163	15.645	4.85	17	16.486	15.295	14.675	4.85
18	16.799	15.949	15.382	4.90	18	16.295	15.008	14.346	4.90
19	16.647	15.723	15.119	4.95	19	16.098	14.707	14.017	4.95
20	16.491	15.496	14.854	5.00	20	15.895	14.407	13.688	5.00
21	16.329	15.259	14.577	5.05	21	15.688	14.104	13.317	5.05
22	16.164	15.019	14.296	5.10	22	15.476	13.798	13.005	5.10
23	15.994	14.776	14.014	5.15	23	15.260	13.492	12.666	5.15
24	15.820	14.531	13.731	5.20	24	15.041	13.187	12.331	5.20
25	15.643	14.283	13.448	5.25	25	14.819	12.883	12.000	5.25
26	15.413	14.034	13.165	5.30	26	14.596	12.581	11.674	5.30
27	15.279	13.784	12.883	5.35	27	14.370	12.282	11.353	5.35
28	15.094	13.534	12.605	5.40	28	14.143	11.986	11.039	5.40
29	14.907	13.285	12.327	5.45	29	13.916	11.694	10.730	5.45
30	14.718	13.036	12.052	5.50	30	13.688	11.406	10.428	5.50
31	14.527	12.788	11.781	5.55	31	13.459	11.124	10.134	5.55
32	14.336	12.542	11.513	5.60	32	13.232	10.846	9.847	5.60
33	14.143	12.295	11.249	5.65	33	13.005	10.574	9.568	5.65
34	13.949	12.056	10.990	5.70	34	12.779	10.307	9.296	5.70
35	13.756	11.818	10.736	5.75	35	12.554	10.046	9.031	5.75
36	13.563	11.580	10.485	5.80	36	12.435	9.791	8.774	5.80
37	13.370	11.347	10.240	5.85	37	12.316	9.542	8.525	5.85
38	13.177	11.116	9.999	5.90	38	12.197	9.298	8.283	5.90
39	12.984	10.889	9.764	5.95	39	12.078	9.062	8.049	5.95
40	12.792	10.666	9.534	6.00	40	11.959	8.831	7.822	6.00
45	11.851	9.605	8.460	6.25	45	10.429	7.766	6.791	6.25
50	10.950	8.645	7.515	6.50	50	9.476	6.844	5.919	6.50
55	10.102	7.785	6.689	6.75	55	8.607	6.050	5.184	6.75
60	9.311	7.019	5.969	7.00	60	7.822	5.369	4.563	7.00

NEW METHOD.—*For repeated compression:* $\beta = \dfrac{6500}{1+c\dfrac{l^2}{d^2}}\left[1+\dfrac{\text{min. } B}{\text{max. } B}\right]$. Crippling strength in pounds divided by 36,500 gives the value of $\dfrac{1}{1+c\dfrac{l^2}{d^2}}$.

CHAPTER II.

CROSS-SECTIONING, DETERMINATION OF DIMENSIONS.

A. TENSION MEMBERS.

WE shall make constant use of the Tables and methods of Chapter I., in finding the proper cross section and size of the various members which we have to design. The student, before proceeding further, should read over carefully the general specifications at the end, look over all the plates and illustrations of various members given in the following pages, and familiarize himself at all times and at every opportunity with the way in which various members are made up and put together, by careful examination of actual structures.

CARNEGIE'S POCKET BOOK OF SHAPES.—The various members we have to design are made up, broadly speaking, of I bars and channel bars, combined with plates and rectangular bars by means of pins and rivets. When the requisite area of cross section has been found, according to the principles of the preceding chapter, it might seem that any dimensions which would give the required area would be equally good. But such is not the case. Eye bars, channels, plates, angle irons, etc., are produced by the various mills and rolling companies of *certain sizes*. These sizes are sufficiently numerous and cover a range sufficiently great to answer all practical requirements. But if in our design we specify sizes which are not rolled, such requirements evidently cannot be filled without the expense of making new rolls for the special purpose. We are limited, therefore, in our choice to the sizes actually produced by the various mills and rolling companies, and must choose such sizes as can be readily ordered and bought in the market.

For the purpose of facilitating such choice, the various mills issue for the use of their patrons, " Pocket Books," which, besides much valuable miscellaneous information, contain detailed lists of all the various sizes of iron which they roll, giving for each size and shape the weight per foot, area of section, dimensions, moment of inertia of cross section, radius of gyration of cross section, etc., etc. In short, all the information that can be desired in order to facilitate designing is given. Among the best of such pocket books are those of the PENCOYD IRON WORKS (John Wiley & Sons), and the CARNEGIE STEEL COMPANY, Phipps & Co.,* Pittsburg, Pa. These works are readily procured, and all our future calculations will be based upon the Tables given by the latter. The student who would intelligently read what follows, should always have " Carnegie's Pocket Book " within reach. All our references to it refer to the edition of 1893. We shall, whenever necessary, simply refer to this edition by page, and thus avoid the incorporation in this work of extensive Tables. The principles of designing once understood, the reader can patronize any company whose Tables furnish him with the necessary information.

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* *Pocket Companion of Useful Information and Tables Appertaining to the Use of Steel as manufactured by the Carnegie Steel Company.* Edited by F. H. Kindl, C. E., Pittsburg, Pa., 1893. This book is indispensable to students wishing to read this section. Designers will also find a very serviceable help in *Tables of Moments of Inertia* by Frank C. Osborne, C. E. Eng. News Pub. Co., New York, 1889.

We shall now illustrate the methods of designing, by means of a series of selected examples, and shall take up first the subject of *Tension Members.*

A. TENSION MEMBERS.

The lower chords of ordinary bridges, the main and counter diagonals of trusses, and the diagonals in the upper and lower horizontal wind bracing, and the vertical sway bracing, are all ordinarily in tension, and never take compression.

Sometimes one or more cross ties rest directly upon the lower chord, between the panel points, in which case the panel in question acts as a beam, and at the same time has a tensile stress. This case is rare, as ordinarily the cross ties rest upon stringers, which in turn rest upon floor beams at every panel point. There is thus usually no transverse stress upon a lower panel, but a stress of tension only. Let us now apply both the "old" and "new methods" to the designing of tension members. In every case the stresses in the member considered are supposed to have been found by statical calculation, according to the principles of Part I. of this work.

VALUES OF σ FOR TENSION MEMBERS.—For wrought iron, the value of the allowable working stress σ for tension is for the "*old method*:"

	Lbs. per sq. in.
On lateral bracing, ...	$\sigma = 15{,}000$
On bottom chords and main diagonals, ...	10,000
On counters and long vertical rods, ...	8,000 to 9,000

For the "*new method*," we have for wrought iron, for links or rods of double rolled iron, page 381,

$$\sigma = 7500\left(1 + \frac{\text{const. } S}{\text{total } S}\right),$$

where const. S is the dead load or constant tension and total S is the greatest tension which ever acts upon the piece.

EXAMPLE I. *One of the lower panels of a bridge truss is subjected to a tension of* 132,200 *lbs. due to the dead load, which is, therefore, the least tension which ever acts upon the panel in question. The live load stress is* 115,600 *lbs. The total tension is, therefore,* 247,800 *lbs. What should be the area of cross-section?*

By the "old method," we have for the area required

$$A = \frac{\max.\ S}{\sigma} = \frac{274800}{10000} = 24.78 \text{ square inches.}$$

If the panel is an end panel, we should probably distribute this area among two chord bars. Each bar should then have an area of 12.39 square inches.

A BAR OF IRON ONE YARD LONG AND ONE SQUARE INCH IN CROSS-SECTION WEIGHS TEN POUNDS.

The student should make a note of this once for all, as we shall hereafter apply it without comment.

Each bar in question then will weigh $\frac{123.9}{3} = 41.3$ lbs. per foot, whatever the shape of cross-section.

If each bar is to be a Union Iron Mills' channel, we see, by reference to *Carnegie*, page 100, that only three sizes rolled will suit us, *viz.*, a 15-inch, a 13-inch or a 12-inch channel. If we decide on a 15-inch channel, then we see from the Table, its thickness of web would be 0.566 inch, and its width of flange 3.566 inches.

If we decide upon a 12-inch channel, its thickness of web would be 0.79 inch, and width of flange 3.39 inches.

The sizes can be specified and will be rolled and furnished of these dimensions.

If we decide upon a flat rectangular bar, as we probably would if the panel were not an end panel, we see from *Carnegie*, page 202, that the nearest bar rolled will be about 11 inches by $1\frac{1}{8}$ inches, and that such a bar weighs 41.54 lbs. per foot.

The reason why we should probably use a channel bar for an end panel and a flat bar for other panels, is that the stresses in the horizontal wind bracing, when the bridge is empty, might cause a slight compression in the lower

end panel. If so, a channel bar would better resist such stress, and would admit of being laced to its fellow so as to prevent lateral deflection.

By the "*new method*," our results would be somewhat different. For σ we should have

$$\sigma = 7500\left(1 + \frac{132200}{247800}\right) = 11500 \text{ lbs.},$$

and the necessary area would be

$$A = \frac{247800}{11500} = 21.55 \text{ sq. inches.}$$

Each bar should then have an area of 10.73 sq. inches, and would weigh $\frac{107.3}{3} = 35.76$ lbs. per foot. We see from *Carnegie*, page 100, that a 12-inch channel will suit our purpose. The thickness of web of such a channel is 0.657 inch, and width of flange 3.257 inches.

We see also that we obtain by the "new method" in this case a lighter bar than by the old method. This is as it should be. The "old method" only takes account of the action of a repeated stress by its specification of a different σ for different members. But we see from our formula that σ should vary with the ratio of $\frac{\text{const. } S}{\text{total } S}$. If the dead load were equal to the maximum stress, or, in other words, if there were no live load at all, we should have the case of a steady stress, and, of course, could take σ the largest allowable. This largest allowable stress is, we see from the formula, 15000 lbs., which corresponds with the largest allowable stress by the old method, for lateral bracing, which is seldom called into action. On the other hand, if there were no dead load stress at all, we should have by our formula $\sigma = 7500$ lbs., which agrees well with the value of σ according to the old method, for counters, which are strained by every passing load, but not by dead loads. Between these limits, then, of 7500 and 15000, our values of σ will range, according to the ratio of $\frac{\text{const. } S}{\text{total } S}$, and our single formula replaces all the specifications of the old method. The value of 10000 lbs. for σ, prescribed by the old method for chords, corresponds to a ratio of $\frac{\text{const. } S}{\text{total } S} = \frac{1}{3}$. For bridge construction, this is a good average value, but as this value is really not a constant the new method gives the most rational means of taking it into account.

COMBINED TENSION AND FLEXURE.—The lower panel may have a weight resting upon it, due to a cross tie between the panel points. In this case it acts as a beam, and at the same time is in tension. This case has already been discussed, Part I., page 313.

EXAMPLE 2.—*If the panel in the preceding example is 20 feet long, and, besides the longitudinal tension already given, has a weight of 2 tons at the centre, what should be the area?*

For this case we have, page 379,

$$A = \frac{Mv}{\sigma r^2} + \frac{S}{\sigma}.$$

By the old method, $\sigma = 10000$ lbs.; by the new method we have just found for this case $\sigma = 11500$ lbs.

We cannot tell what value to take for r, however, unless we assume the depth of the bar required. Guided by the preceding example, we should choose a 15-inch channel, because a 15-inch is the largest channel we can have by Carnegie's Table, and the area in the present case must be greater than in the previous case. If, then, we can have two bars at all, we shall need 15-inch channels. We may have to use more than two bars. From *Carnegie*, page 100, we see that the value for r varies for 15-inch channels between 5.60 and 5.13 inches. Let us assume 5.3 for r therefore. We have, then,

$$M = 2000 \times 10 \times 12 = 240000 \text{ inch lbs.}, \quad v = 7.5 \text{ inches}, \quad S = 247800 \text{ lbs.}$$

Hence, by the old method,

$$A = \frac{240000 \times 7.5}{10000 \times 28.09} + \frac{247800}{10000} = 6.40 + 24.78 = 31.18 \text{ sq. inches.}$$

By the new method,

$$A = \frac{240000 \times 7.5}{11500 \times 28.09} + \frac{247800}{11500} = 5.57 + 21.55 = 27.12 \text{ sq. inches.}$$

In the first case, if we have two bars, the area of each bar will be 15.59 sq. inches, and its weight $\frac{155.9}{3} = 51.96$ lbs. per ft. From *Carnegie*, page 100, this calls for thickness of web 0.78 inch, and width of flange 3.78 inches. This corresponds to a value of r of 5.2 inches, which is sufficiently close to our assumed value of 5.3 inches, not to necessitate another calculation.

In the second case, we have for the area of each bar 13.51 square inches, and $\frac{135.1}{3} = 45.03$ lbs. per ft. From *Carnegie*, page 142, this calls for thickness of web of 0.64 inch, and width of flange of 3.64 inches. The corresponding value of r is 5.34, which agrees sufficiently well with our assumed value of 5.3 inches.

If we wish a flat bar instead of a channel, we may assume the depth of bar at, say, 12 inches. Then,

$$r^2 = \frac{I}{A} = \frac{bd^3}{12bd} = \frac{d^2}{12} = 12.$$

Hence, by the old method,

$$A = \frac{240000 \times 6}{10000 \times 12} + \frac{247800}{10000} = 12 + 24.78 = 36.78,$$

and by the new method,

$$A = \frac{240000 \times 6}{11500 \times 12} + \frac{247800}{11500} = 10.40 + 21.55 = 31.95.$$

In the first case, if we have two bars, each will have an area of 18.39 sq. inches, and will weigh $\frac{183.9}{3} = 61.3$ lbs. per ft. From *Carnegie*, page 202, the nearest size is $11\frac{1}{4}$ inches by $1\frac{5}{16}$ inches.

In the second case, each bar has an area of 15.93 sq. inches, and will weigh $\frac{159.3}{3} = 53.1$ lbs. per ft. This calls for a bar $11\frac{1}{4}$ inches by $1\frac{3}{8}$ inches.

An increase in the assumed depth would, of course, diminish the material required, but as $12\frac{3}{4}$ inches is the limit in depth of the table, we have taken nearly the greatest depth procurable.

SECONDARY STRESSES.—The members at an apex should be loaded in their axes, and these axes should meet in a point. If these conditions are not complied with, we have secondary stresses due to bending, and the unit stress must be determined as directed, page 313.

INITIAL TENSION.—Many of the tension members are made adjustable by means of turn-buckles or sleeve nuts. The screwing up of these may bring a strain upon the member independently of the stress which comes upon it from the loading. To allow for this we may add 1 ton for a rod 1″ in diameter, and $\frac{1}{4}$ of a ton for each increase of $\frac{1}{8}$″ in the diameter. That is *for round bars,*

$$\textit{Initial tension in tons} = 2d - 1,$$

where d is the diameter in inches. Flat bars are to have the same allowance as round rods of equal sectional area; or *for flat bars,*

$$\textit{Initial tension in tons} = 2.25\sqrt{A} - 1$$

where A is the area of cross-section in sq. inches.

EXAMPLE.—*The maximum tesnion in a flat bar is* 90000 *lbs., and the working stress is found to be* 10000 *lbs. per sq. inch. If the bar is adjustable, what should be the area?*

The area, without allowance for initial tension, is $\frac{90000}{10000} = 9$ sq. inches. This area would give us 5.75 tons initial tension. Let us take 6.25 tons, or 12500 lbs., for the initial tension. Then the maximum tension would be 102500 lbs., and area required would be $\frac{102500}{10000} = 10.25$ sq. inches. This area substituted, gives us 6.2 tons initial tension, which is near enough to the initial tension assumed. The area required is then 10.25 square inches, instead of 9 square inches, called for by the loading alone.

COMPRESSION IN END LOWER PANELS.—The lower panels are all in tension by reason of the live and dead load. But the wind blowing upon one side bends the truss laterally, and acts as a horizontal load. The stresses due to wind must then be resisted by the horizontal bracing, and it may happen that in one or more of the end panels there will be a compression due to wind, greater than the tension due to dead load, or even to dead and live loads combined. In such case the difference between wind compression and dead load tension, or dead and live load tension, will come as a stress of compression upon the lower end panels. The end panels should be able to take such excess, and must be treated for it as long struts in compression. It is for this reason that in the preceding examples we have taken channel bars in pairs when the panel was supposed to be an end one. Such bars can be joined to one another by lattice bars riveted to the flanges, and thus made to act together as a strut with a much greater least radius of gyration than either would have acting separately. Thus made, they will require no extra material to resist the slight compression which they may be called upon to sustain, as the area called for by the tensile stress will be ample, provided the radius of gyration is thus secured sufficiently large. As in general the chord bars go in pairs at the end, they may be spaced apart a distance always greater than their depth, and therefore, when latticed to each other, their least radius of gyration will be when the neutral axis is perpendicular to the web at centre. If not so latticed, we should have to take the radius of gyration for the axis coincident with centre line of web, which would of course be very small, and extra material might be required. As flat bars cannot easily be thus latticed, we see the propriety of making the end panels of channel bars, when compression due to the wind is to be feared.

CHOICE OF DEPTH OF LOWER CHORD BARS.—The maximum stress at any panel will determine the area required. According to the depth assumed for chord bars, the number required in each panel will vary. As the bars should go in pairs, and increase in number towards the centre, without increasing much in depth, a little preliminary figuring and judgment is required in order to assume, in any given case, such a depth as will allow of the requisite number of bars at each panel, without causing the depth to vary too greatly, or necessitating undue thickness. For this purpose, we may first find the area required in the end panel and centre panel. Then we may choose such a depth for the end panel bars as shall give the required area for a medium thickness, and at the same time will give, for about the same depth and thickness, the required number of bars at the centre. The bars in a panel need not have the same thickness necessarily.

EXAMPLE.—*The lower centre panel in a bridge truss has a tension of 526000 lbs. due to dead load and 427000 lbs. due to live load. In the first end panel, in which flat bars are used, we have 191600 lbs. due to dead load and 166400 due to live load. To choose a good depth for chord bars.*

We shall use the "new method" in our calculation. To apply the old method, we simply take $\sigma = 10000$ lbs.

By the new method, then, we have for centre panel,

$$\sigma = 7500\left(1 + \frac{526000}{953000}\right) = 11638 \text{ lbs.},$$

and for the end panel

$$\sigma = 7500\left(1 + \frac{191600}{358000}\right) = 11514 \text{ lbs.}$$

The area required in the centre panel is then $\frac{953300}{11638}$ = about 82 square inches, and in the end panel $\frac{358000}{11514}$ = about 31 square inches.

If we are to have four bars at the end, each bar will have an area of 7.75 square inches. A number of sizes may be chosen which will give this area. For such a heavy bar, we should not have less than 1 inch thickness. From *Carnegie*, page 194, we see that $7\frac{1}{2}$ inches by $1\frac{1}{16}$ inches will be ample for the end panel. If we take the same depth for the

centre panel and $1\frac{1}{4}$ inches thickness, we should have to have 4 bars $7\frac{1}{4}''$ by $1\frac{7}{16}''$ and 4 more $7\frac{1}{4}''$ by $1\frac{3}{8}''$, or 8 bars altogether in the centre panel. If this is not judged to be too many, we may then take $7\frac{1}{4}''$ for the depth. For a long truss, such as we have supposed, it would not be too many. We could not take the depth much greater than $7\frac{1}{4}''$ without getting too small a thickness for the end bars, nor much less than $7\frac{1}{4}''$ without getting too many bars in the centre panel. For constructive reasons, it is preferable to have all the eye-bars of the same depth and as near as may be of the same thickness, and to increase the number as required. A depth of $7\frac{1}{4}''$ will then be satisfactory.

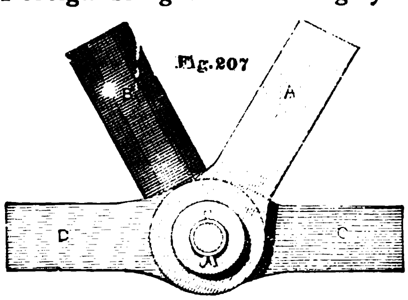
COUNTERS.—The main diagonals, lower chord, and vertical suspenders are generally made of forged eye-bars. All that is necessary for these is that the design of the head shall be such that upon being tested to destruction, the break shall occur in the bar, not in the head.

For the counter rods, square bars are preferably used. These have square loop eyes around the pin, and turn buckles for adjusting. If round bars are used with square loop eyes and turn buckles, they are more expensive.

Round bars are sometimes used for counters without turn buckles, but with loop swivels on the ends.

DETAILS OF LOWER CHORD.—The cross section is generally increased, as shown in Fig. 205, Plate 8, by increasing the number of eye-bars, and it is rarely that the dimensions are increased without increasing the number, or still more rarely that number and dimensions are both increased. A uniform size, as near as may be, at least in depth, is less expensive. This principle holds for all duplicated parts generally. In Fig. 205, Plate 8, we have shown the arrangement of eye-bars and ties at two panel points. The ties are distinguished by having their partly visible upturned ends shaded.

In Fig. 206, Plate 8, we have given an isometric drawing showing the details of bottom chords, etc. It will be seen in Fig. 206 that provision is made for an auxiliary timber stringer to support ends of ties in case of derailment. It is generally customary to use a light iron stringer for this purpose, or else to space the main stringers in such a manner as to accomplish the same purpose. There are many other points about Fig. 206 which will repay study, such as the connection of lower wind braces, the construction of posts and details at bottom of posts, etc. *No cast iron is allowed* in a bridge at the present time for ***any purpose*** except for bed plates and for the machinery of draw spans. Figs. 205, 206, 220, and 221 represent modern American practice. The other figures are specimens of riveted work. Foreign bridges consist largely of riveted work. Though eye-bars and pins are sometimes used, it is rare, comparatively. This, and the necessary details, constitute the chief distinction between foreign and American practice. American engineers use eye-bars and pins almost exclusively for the bottom chords of bridges of any ordinary span.



The figures thus far given require but little explanation. The student can acquire, by a careful study of them, a good knowledge of the system of forming lower chord and connections. For other illustrations he can consult the illustrated albums of our various bridge companies, and better still, should seize every opportunity to study existing structures on the spot.

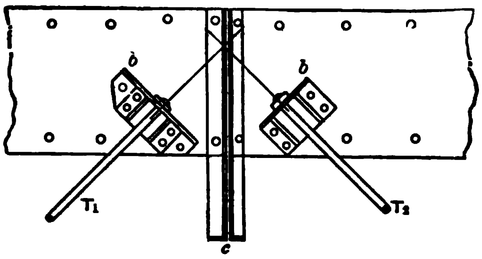
In Figs. 210 to 219, Plates 9 and 10, we have given illustrations of bottom chord riveted work, mostly foreign examples. We shall treat of riveted work hereafter in detail, and much information will be found in the specifications at the end of this work. Although riveting is but little used in the main trusses in American bridges, still it is of great importance, and a study of its application, as set forth in our illustrations, will be profitable.

In Fig. 210 we see how the area of the bottom chord may be increased by adding

plates one over the other, as also the connection of the braces. Fig. 211 show show the depth of bottom chord may be increased, as well as the use of an auxiliary plate to give greater area for riveting. Fig. 212 shows the introduction of a post, and Fig. 213 the same with auxiliary plate, when the depth of chord is not sufficient, to attach the braces directly to it. Figs. 214, 215, 216, 217, 218, 219 give different styles of end connections.

Fig. 220 shows the details for inclined end posts or "batter braces."

Plate 11*a*, page 401, gives an isometric view of a double intersection Pratt Truss R. R. bridge, taken by permission from "A System of Railroad Bridges for Japan," by Prof. J. A. L. Waddell (Memoirs of the Tokiô Daigaku, No. 11). The names of the various members are written upon the drawing, and by inspection of the drawing and of actual bridges the student should familiarize himself with the name, duty, and connection of every member.



An excellent detail for the attachment of the upper lateral rods to the top chord is given by Professor Burr, *Stresses in Bridge and Roof Trusses*, Wiley & Sons, New York, 1886, and shown in the accompanying figure. At *b* a piece of angle iron 6″ by 4″, with the 6″ leg lying on the chord, carries two pieces of angle iron 3″ by 3″, with their edges parallel to the axis of T_1. One end of each of the latter angles rests squarely against the vertical 4″ leg of the 6″ by 4″ angle. The tie T_1 passes through the 4″ leg of the heavy angle, between the 3″ angles, and is adjusted by nut at the end.

The arrangement is effective, cheap, and the axes of the ties can be made to meet at the neutral axis of the chord. The student can compare this detail with that on Fig. 221.

PLATE 8.

Fig. 205

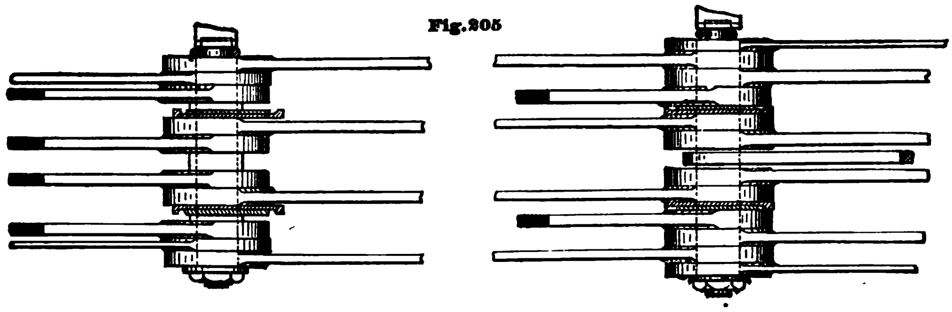


Fig. 206

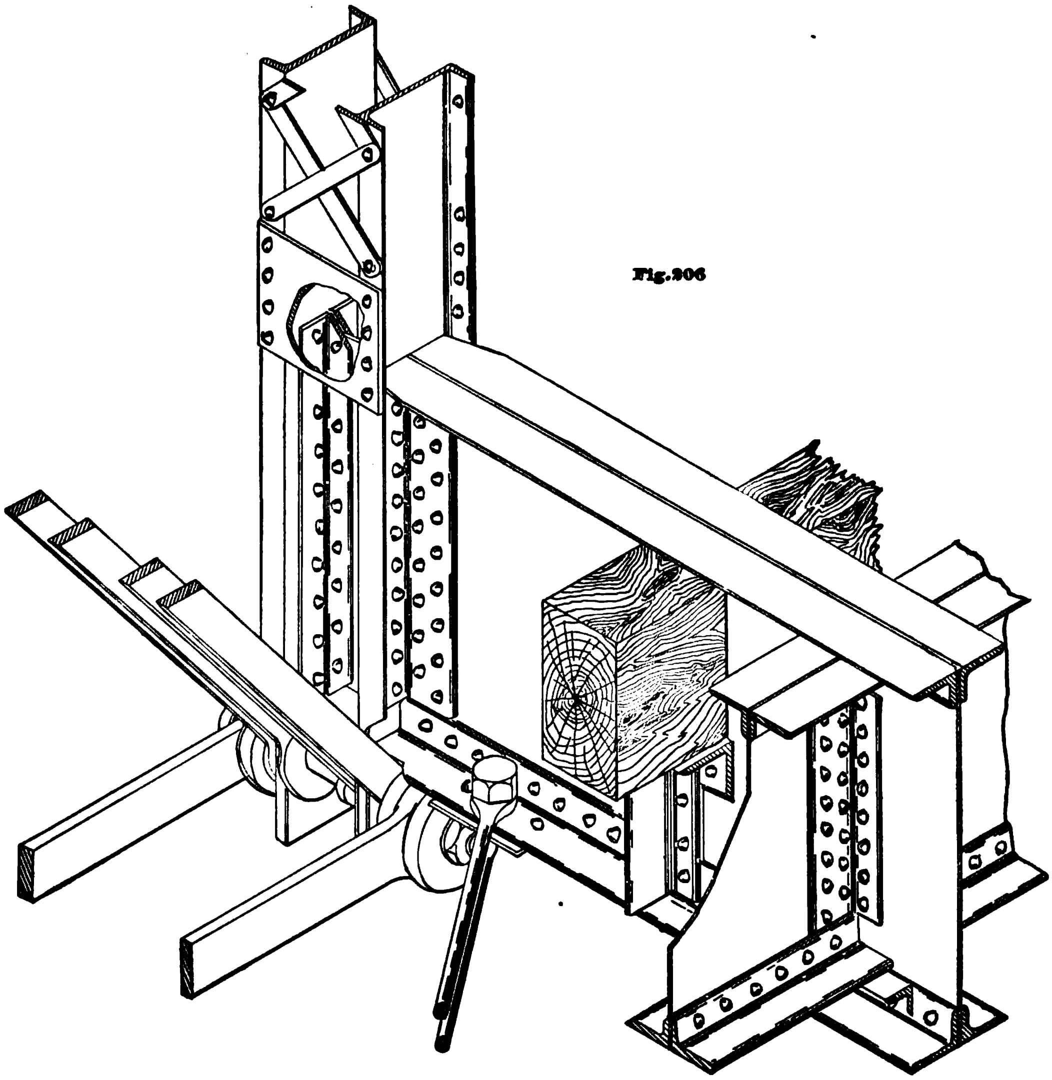


PLATE 9.

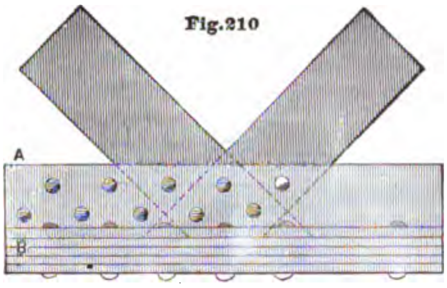


Fig. 210

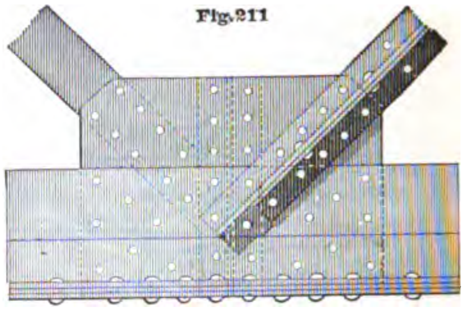
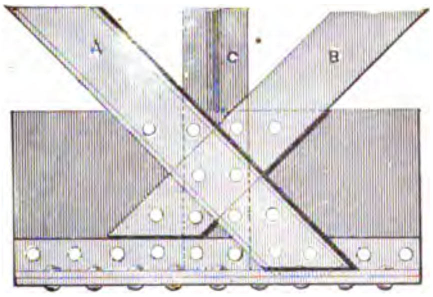


Fig. 211

Fig. 212



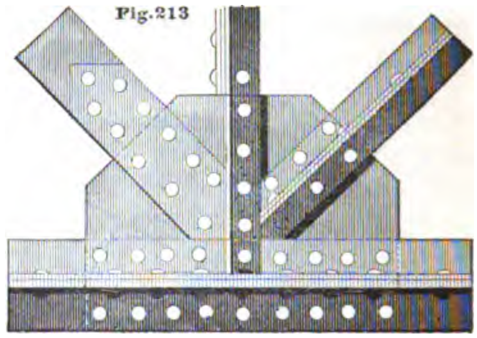


Fig. 213

Fig. 214

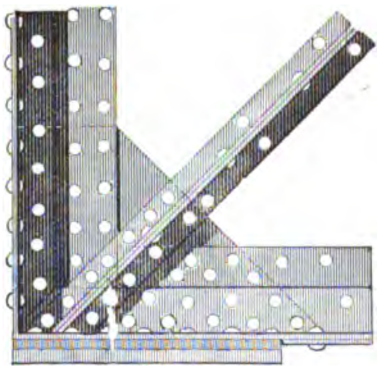
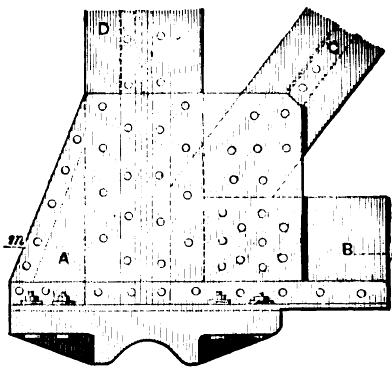


Fig. 215



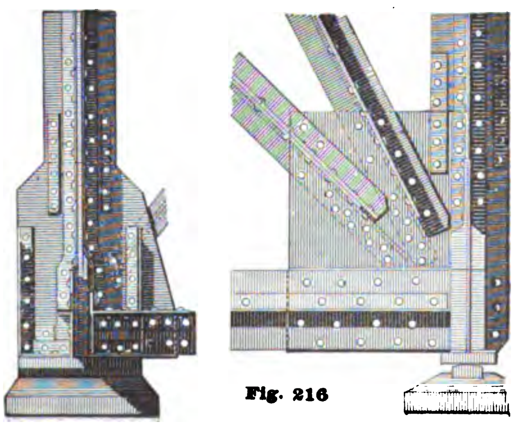


Fig. 216

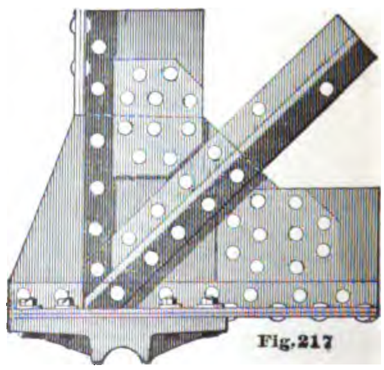


Fig. 217

PLATE 10.

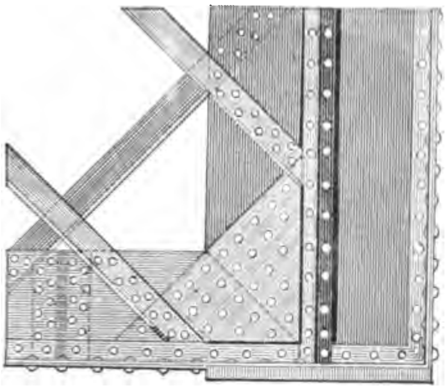


Fig. 218

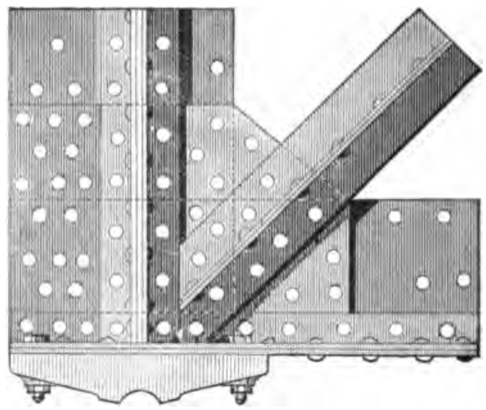


Fig. 219

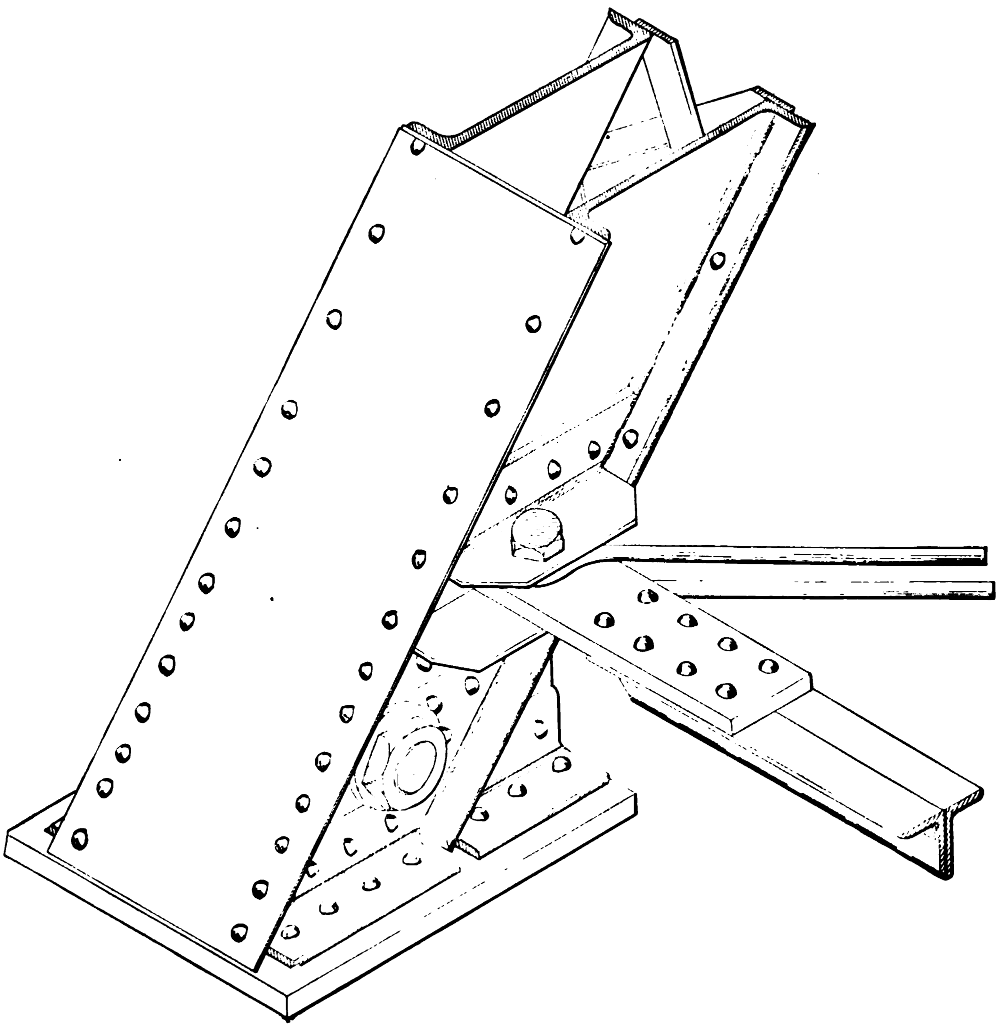
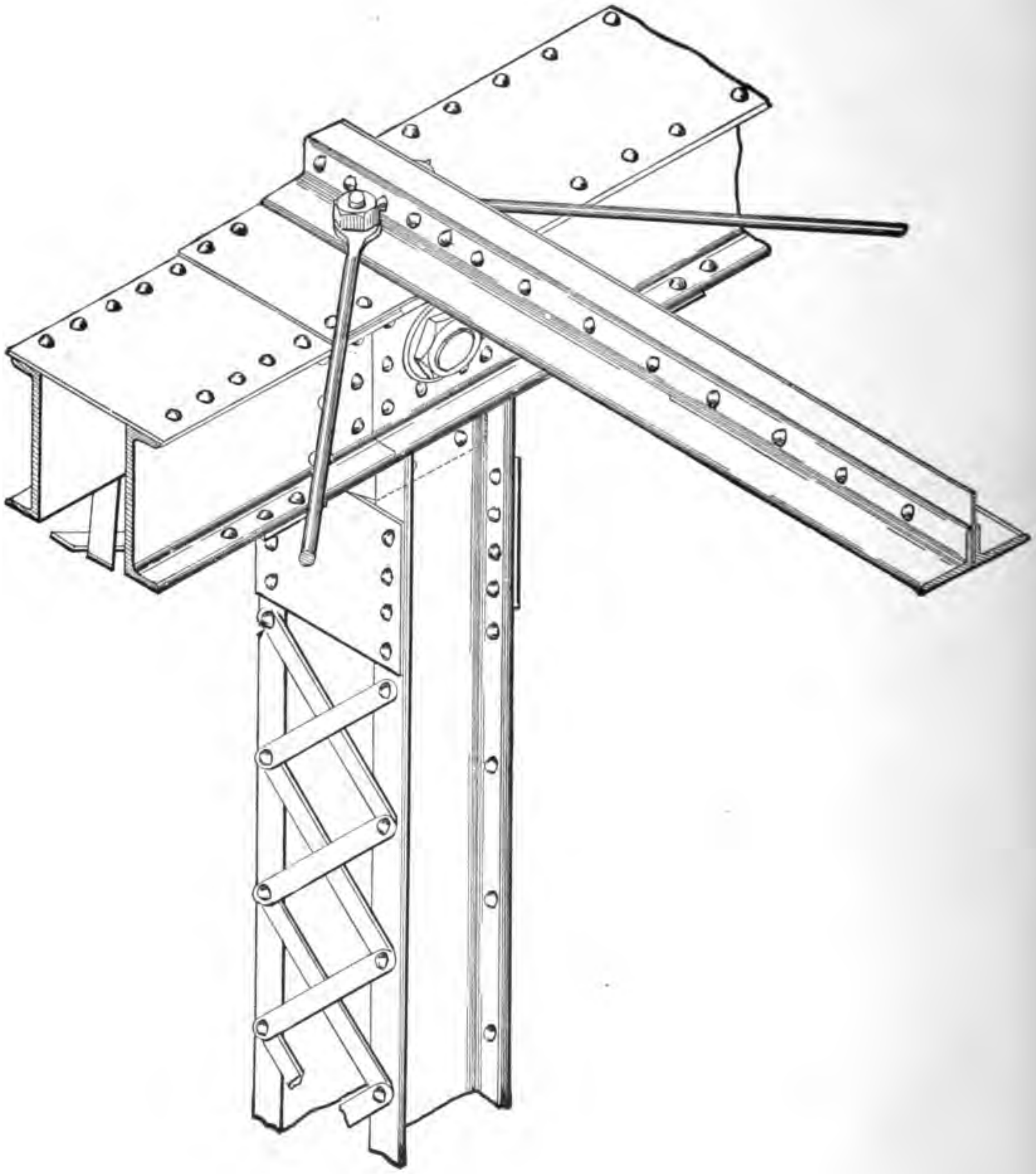
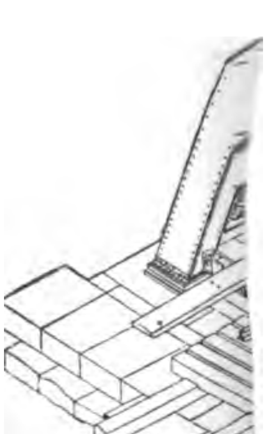


Fig. 220

PLATE II.

Fig. 221





CHAPTER III.

CROSS-SECTIONING, DETERMINATION OF DIMENSIONS.

B. COMPRESSION MEMBERS.

WE have represented in Fig. 221, Plate 11, the ordinary method of forming compression members. It will be seen that both post and chord are composed of channels. The post is composed of two channels, united by lattice or lacing bars. When there is a single system of bars, the channels are said to be "*laced.*" When there is a double system, as in Fig. 221, they are said to be "*latticed.*" The upper chord is also composed of two channels, latticed or laced on the under side, and with a top plate. This constitutes the "common chord section." Sometimes, for short compression members, instead of lattice or lacing bars, we may have a rectangular strip or plate, riveted on at intervals, these plates or strips answering the same purpose as the lattice or lacing bars, viz., to unite the channels, and make them act together to resist lateral flexure. Without such lateral connections each channel would evidently bend more easily, and the resisting power of the combination is much greater than the sum of each channel acting separately. The strut for the over-head horizontal wind bracing is also shown in Fig. 221, composed of two angle irons riveted to a central plate. We also see the method of connection of the various pieces at the panel point or pin. The post channels are shaved off at the end, and the web strengthened at top and bottom by plates called "*reinforcing plates.*" The pin goes through these plates and the web of the post channels, as well as through the web of the chord channels. The object of the reinforcing plates is not only to strengthen the ends of the posts, but also to give a sufficient bearing upon the pin. Each post has also, upon each side, at top and bottom, plates just between where the lacing or latticing ends and the pin, called "*stay plates.*"

The channels composing the struts, whether posts or chords, are generally spaced farther apart than the depth of channel, so that the least radius of gyration, when the channels are laced or latticed, is with reference to the axis perpendicular to the web, and *not* coincident with the web of the channels.

RADIUS OF GYRATION.—We see from Chapter I., page 371, that in order to find the strength of a long strut we need to know r^2, or the square of the radius of gyration. The radius of gyration of two post channels is, in general, the same as the radius of gyration of a single channel, when the axis is at right angles to the web.

We have in general,

$$r^2 = \frac{I}{A},$$

where r is the radius of gyration, I is the moment of inertia of the cross section with reference to the required axis, and A is the area of the cross section.

For two post channels, the area is twice that for one, and the moment of inertia is also twice that for one, for axis at right angles to the web. So that the radius for the two, if they are connected by lattice or lacing bars, and spaced farther apart than their depth, is the same as for one.

But for the chord cross section, composed of two channels and a top plate, we must take into account the moment of inertia of this plate, with reference to the axis perpendicular to the web of the channels.

The moment of inertia of a rectangular cross section *with reference to the axis through its own centre of gravity* parallel to its breadth is, $\frac{1}{12}bd^3$, where b is the breadth and d is its depth.

The moment of inertia of any cross section with reference to an eccentric axis outside of it, is equal to the moment of inertia with reference to a parallel axis passing through the centre of gravity, *plus* the area into the square of the distance between the two axes. Or,

$$I' = I + AD^2,$$

where I' is the moment of inertia with reference to the eccentric axis, I is the moment of inertia with reference to the parallel axis through the centre of gravity, A is the area of cross section, and D is the distance between the two axes. Carnegie's Tables give us the moment of inertia of channel cross sections with reference to axes through the centre of gravity, perpendicular to the web, and an application of the preceding principle will enable us to find the moment of inertia and the radius of gyration of any compound cross section with reference to any given axis.

EXAMPLE.—*Suppose a top chord is composed of two 6 inch 10 lb. channels, spaced say 7 inches apart, back to back, with a top plate $\frac{1}{4}$ inch thick.*

Since the channels are spaced farther apart than their depth, flexure, if any will be in the direction of their depth, and we must find the moment of inertia and radius of gyration for an axis perpendicular to the web, passing through the centre of the channel cross section.

From *Carnegie*, page 100, we see that the width of flange is 2 inches. Hence the breadth of plate is $7 + 4 = 11$ inches.

The moment of inertia of the plate with reference to the required axis is then

$$I' = I + AD^2 = \frac{bd^3}{12} + bd \times (3\tfrac{1}{8})^2 = \frac{11.08 \times (\frac{1}{4})^3}{12} + 11.08 \times \tfrac{1}{4} \times (3\tfrac{1}{8})^2,$$

or, $\qquad I' = 27.45.$

The moment of inertia of the two channels is, from *Carnegie*, page 100, $15 \times 2 = 30$. Hence total moment of inertia is 57.45, and since the total area is $6 + 2.77 = 8.77$, we have $r^2 = \frac{57.45}{8.74} = 6.57$ inches, or $r = 2.58$ inches.

In this way we find r^2, or the square of the radius of gyration, for any cross-section. Then from our formulas, Chapter I., page 369, or from the Table, page 385, we can find the load which the strut will bear. Many compound sections will be found thus worked out in *Tables of Moment of Inertia* by Frank C. Osborne, C.E., Eng. News Pub. Co., New York, 1889.

VALUE OF σ FOR COMPRESSION MEMBERS.—For wrought iron, the value of the allowable working stress σ, for compression, is, for the "*old method*," given by the formulas on page 369 or at the top of the Tables at the end of Chap. I., when we take the proper factor of safety, as given at the head of every Table. The use of these Tables will greatly abridge the labor of calculation. Table I. applies generally to any form of cross-secction except hollow round, but, as we have just seen, it requires some little calculation to find r for compound cross-sections, it will ordinarily be more convenient to make use of the other tables, which only require d, or the least depth, to be known. We shall, therefore, in general, only apply Table I. in those cases where r can be taken at once from *Carnegie's* Tables, and in other cases may make use of one of the other tables of Chap. I.

The "straight-line" formulas, page 380, can be at once applied without Table.

By the "*new method*,"

$$\sigma = \frac{6500}{1 + c\frac{l^2}{r^2}}\left[1 + \frac{\text{const. } S}{\text{total } S}\right],$$

where the value of $\dfrac{1}{1+c\dfrac{l^2}{r^2}}$ in any case may be found from Table I., by dividing the crippling strength *in pounds*, as found from the Table, by 40000. When we use one of the other Tables, we have

$$\sigma = \frac{6500}{1+c\dfrac{l^2}{d^2}}\left[1+\frac{\text{const. } S}{\text{total } S}\right],$$

where the value of $\dfrac{1}{1+c\dfrac{l^2}{d^2}}$ may be found by dividing the crippling strength *in pounds*, as found from the Table, by the ultimate strength taken for the case, as indicated by the formulas given at the head of each Table.

EXAMPLE.—*A post in a bridge truss is subjected to a compression of* 46900 *lbs. due to the dead load, and* 64100 *lbs. due to the live load. The post is* 30 *feet long. What should be its area of cross-section?*

Let us suppose that the post is composed of two channels latticed or laced. Then we should use Table I., Chapter I., page 385. We cannot enter the Table until we first know r, and therefore the value of $\frac{l}{r}$, and we cannot tell r until we first assume some size for the channels. Here judgment and experience will aid in making a suitable choice. Whatever choice we make we can soon test, however, and make another if not suitable.

Let us take two 10-inch channels between 20 and 35 lbs. per foot, and space these channels at least 10 inches apart, so that r must be taken for an axis at right angles to the web of the channels.

From *Carnegie*, page 100, we see that r varies between 3.85 and 3.40. Let us assume r then at 3.6. Then

$$\frac{l}{r} = \frac{360}{3.6} = 100.$$

Suppose the post to be pinned at both ends.

Then, by the "*old method*," we have from Table I., the factor of safety, $4 + \frac{l}{20d} = 4 + \frac{360}{200} = 5.8$, and the crippling strength = 12.855 tons, or 25710 lbs. per sq. inch. The allowable working stress is then $\frac{25710}{5.8} = 4433$ lbs. per sq. in. $= \sigma$. The area required is then $\frac{111000}{4433}$ = about 25 sq. inches. Each channel will weigh therefore $\frac{250}{3 \times 2} = 41.66$ lbs. per foot. We see from *Carnegie* that there is no 10 inch channel rolled as heavy as this, but that the size required will evidently come between 12 inch 20 lb. and 12 inch 44 lb. Taking for this size $r = 4.2$ inches, we have $\frac{l}{r} = \frac{360}{4.2} = 85.7$, and factor of safety $= 4 + \frac{360}{20 \times 12} = 5.5$. From Table I., therefore, we have the crippling strength = 14.180 tons = 28360 lbs. per sq. inch, and the allowable working stress is $\frac{28360}{5.5} = 5156$ lbs. per sq. in. $= \sigma$. The area required is then $\frac{111000}{5156} = 21.52$ sq. inches.

This will give for each channel a weight of $\frac{215.2}{3 \times 2} = 35.86$ lbs. per foot. This comes well within the limits for 12 inch channels. The 12 inch channels required will then weigh 35.86 lbs. per ft. each, the thickness of web will be 0.66 inch, and width of flange 3.26 inches. The corresponding value of r is 4.14 inches, which is near enough to our assumed value.

By the "*new method*," we have from Table I., for $\dfrac{1}{1+c\dfrac{l^2}{r^2}}$, for the 12 inch channels, $\frac{28360}{40000} = 0.709$; hence

$$\sigma = 0.709 \times 6500\left[1+\frac{46900}{111000}\right] = 6545 \text{ lbs. per sq. inch.}$$

The area required is therefore $\frac{111000}{6545}$ = about 17 sq. inches. This gives for each channel $\frac{170}{3 \times 2} = 28.33$ lbs. per ft. We see, from *Carnegie*, that this calls for 12-inch channels, 28.3 lbs. per ft., 0.47 inch thickness of web, and 3.07 inches width of flange. The corresponding value of r is 4.43 inches; our assumed value of 4.2 inches is near enough not to require recalculation, and is on the side of safety.

If we use the "*straight-line*" formula, with Cooper's values, page 382, we have, taking $r = 4.3$, for the live load, $\sigma = 7000 - 40\frac{360}{4.3} = 3651$, and for the dead load $\sigma = 14000 - 80\frac{360}{4.3} = 7302$ lbs. The area required is therefore $\frac{46900}{7302} + \frac{64100}{3651} = 23.97$ sq. inches. This will give for each channel $\frac{239.7}{3 \times 2} = 39.95$ lbs. per ft.

SPACING OF THE LATTICE OR LACING BARS.—The object of the lacing or lattice bars is to join the two channels composing the post or chord, and thus cause them to act together. Evidently, the principle which applies here is that the bars should be attached at intervals so close that there shall be no danger of failure of the channels between the points of attachment. In other words, the length of a single channel between the points of attachment of the bars, shall be as strong at least, considered as a short post, as the whole post or chord itself.

If then l is the distance in inches between the points of attachment of the bars, and r is the least radius of gyration of the channel cross section in inches, and L is the length of the whole post or chord in inches, and R its least radius of gyration in inches, we have

$$\frac{l}{r} = \frac{L}{R}, \text{ or } l = \frac{Lr}{R}.$$

The distance between the ends of bars cannot then be greater than the value of l thus determined.

Practice has made this distance much less, viz., never more than $0.6l$. Also, in order to avoid having the bars make too small an angle with the flanges, which would impair their action, lacing bars are not allowed to make an angle of more than 60° with each other, or less than 60° with the flanges. If then, the value of $0.6l$ comes out less than d or equal to d, where d is the distance between the channels in inches, we can use lacing bars with a distance of d between the points of attachment. If $0.6l$ is greater than d, we must use lattice bars. In case lattice bars are used, the ratio $\frac{l}{d}$ must not exceed $\frac{4}{3}$. The value of the $0.6l$ simply determines then whether lacing or lattice bars shall be used. This point settled, we take d for the distance between points of attachment for lacing and $\frac{4}{3}d$ for lattice bars.

LEAST RADIUS OF GYRATION FOR SINGLE CHANNELS.—The application of the preceding requires us to know the radius of gyration r for channels for axis parallel to the web, through the centre of gravity. This value of r is not given in Carnegie for the different sizes on page 109. We therefore give here these values of r, for the sizes in Carnegie's Pocket Book, with sufficient accuracy for use in the preceding formula:

No. of shape	25		26	27		28		29	30	
Designation	15" Light.	15" Heavy.	12"	12" Light.	12" Heavy.	12" Light.	12" Heavy.	10"	10" Light.	10" Heavy.
Radius of gyration, axis parallel to web. r in inches	0.93	0.90	0.85	0.84	0.82	0.74	0.75	0.67	0.68	0.66

No. of shape	31		32	33		34		35		36		37	
Designation	10" Light.	10" Heavy.	9"	9" Light.	9" Heavy.	8" Light.	8" Heavy.	8" Light.	8" Heavy.	7" Light.	7" Heavy.	7" Light.	7" Heavy.
Radius of gyration. axis parallel to web. r in inches	0.72	0.71	0.69	0.68	0.68	0.56	0.55	0.65	0.66	0.56	0.55	0.64	0.65

No. of shape...	38		39		40		41		42		43		44	
Designation....	6″ Light.	6″ Heavy.	6″ Light.	6″ Heavy.	5″ Light	5″ Heavy.	5″ Light.	5″ Heavy.	4″ Light.	4″ Heavy.	4″ Light.	4″ Heavy.	3″ Light.	3″ Heavy.
Radiusofgyration, axis parallel to web. r in inches..	0.51	0.50	0.58	0.58	0.47	0.46	0.55	0.52	0.46	0.46	0.50	0.51	0.45	0.46

EXAMPLE.—*Suppose the post channels, as determined in the last example, are 12 inch 28.3 lb. channels, spaced 15 inches apart, back to back, what should be the distance between the ends of bars, and shall we use lacing or lattice bars?*

Here we have from Carnegie, page 100, $R = 4.47$, and since $L = 360$ inches and $r = 0.84$ from our Table, we have

$$l = \frac{360 \times 0.84}{4.37} = 68 \text{ inches.}$$

Hence $0.6l = 41$ inches. This is greater than $d = 15$ inches, so we should use lattice bars, and space $\frac{4}{3} \times 15 = 20$ inches apart.

If, however, the post were only 10 feet long, instead of 30 feet, we should have

$$l = \frac{120 \times 0.84}{4.47} = 23 \text{ inches,}$$

and $0.6l = 13.8$ inches. As this is less than $d = 15$ inches, we could use lacing bars, and space 15 inches apart.

SIZE OF STAY PLATES.—Every compression member, composed of channels united by lacing or lattice bars, should have "stay plates" at the ends, as shown in Fig. 221, Plate 11, page 400. Lacing or lattice bars should never be used without such plates at the ends. No general principles can be laid down for determining the size of such plates.

In accordance with practice, we may be guided by the following rules:

Thickness of Stay Plates.—

For all depths of channel less than 8″................ $t = \frac{1}{4}$ inch.
From 8 to 10″ inclusive........................... $t = \frac{5}{16}$ "
Above 10″ .... $t = \frac{3}{8}$ "

Length of Stay Plates.—Let D = depth of channel in inches, d = distance between inner faces of the channels in inches, l = length of stay plate in inches. Then, for latticing or double riveted lacing,

$$l = 0.5D + \frac{d}{D} + 1.5;$$

for single riveted lacing,

$$l = D + \frac{2d}{D} + 2.$$

EXAMPLE.—*Thus in the preceding example, for two 12 inch channels, 15 inch spacing, what should be the size of stay plates for lattice bracing?*

We have, according to the above rules, the thickness of stay plate, $t = \frac{3}{8}$ inch, and for the length of plate,

$$l = 6 + \frac{15}{12} + 1.5 = 8.75 \text{ inches.}$$

SIZE OF LACING OR LATTICE BARS.—We can give no general principles for determining the sizes of the lacing or lattice bars, but the following rules are in accord with established practice:

Thickness of Lattice or Lacing Bars.—The same rule as for the thickness of stay plate holds good, viz.: For all depths of channel less than 8 inches, $t = \frac{1}{4}$ inch. From 8 inches to 10 inches inclusive, $t = \frac{5}{16}$ inch. Above 10 inches, $t = \frac{3}{8}$ inch.

Width of Lattice or Lacing Bars.—Let D = the depth of channel, d = the distance between the inner faces of the channels, and w = the width of the bar, all in inches. Then, *for lattice bars,*

$$w = \frac{9}{88}D + \frac{d}{4D} + \frac{37}{44};$$

for lacing bars,

$$w = \frac{17}{88}D + \frac{d}{2D} + \frac{31}{88}.$$

The ends of lattice and lacing bars are made semicircular, the centre being taken a little outside of the outer edge of the rivet hole.

EXAMPLE.—*For two 12 inch channels, 15 inch spacing, what should be the size of lattice bars adopted?*

According to our rules, we have for the thickness of bars, $t = \frac{3}{8}$ inch, the same as for the stay plates.

For the width of bars we have:

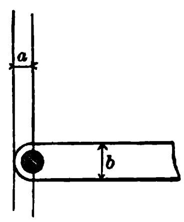
$$w = \frac{9}{88} \times 12 + \frac{15}{4 \times 12} + \frac{37}{44} = \text{about } 2\tfrac{3}{8} \text{ inches.}$$

The dimensions and weight of lattice bars may be figured from the following table adopted by the Phœnix Bridge Company:

LATTICE BARS FOR POSTS AND CHORDS.

THE DIMENSIONS OF SINGLE LATTICE BARS SHALL GENERALLY BE AS FOLLOWS:

		wt. ℔ ft.
	"	
For 6" Rolled or Built Channels	$1\frac{3}{4} \times \frac{5}{16}$	1.82
" 7 " " "	$1\frac{3}{4} \times \frac{5}{16}$	1.82
" 8 " " "	$1\frac{3}{4} \times \frac{5}{16}$	1.82
" 9 " " "	$2 \times \frac{3}{8}$	2.50
" 10 " " "	$2 \times \frac{3}{8}$	2.50
" 11 " " "	$2 \times \frac{3}{8}$	2.50
" 12 " " "	$2\frac{1}{4} \times \frac{3}{8}$	2.81
" 13 " " "	$2\frac{1}{2} \times \frac{3}{8}$	3.13
" 14 " " "	$2\frac{1}{2} \times \frac{3}{8}$	3.13
" 15 " " "	$3 \times \frac{3}{8}$	3.75
" 16 " " "	$3 \times \frac{3}{8}$	3.75
" 18 " " "	$3 \times \frac{3}{8}$	3.75
" 21 " " "	$3 \times \frac{7}{16}$	4.38
" 24 " " "	$3 \times \frac{7}{16}$	4.38
" 27 " " "	$3 \times \frac{1}{2}$	5.00
" 30 " " "	$4 \times \frac{7}{16}$	5.83

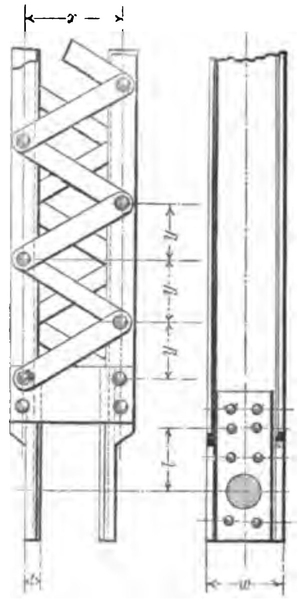


$$a = \frac{b}{2} + \frac{1''}{4}$$

	b	a
for	$1\frac{1}{2}''$	$1''$
	$1\frac{3}{4}$	$1\frac{1}{8}$
	2	$1\frac{1}{4}$
	$2\frac{1}{2}$	$1\frac{1}{2}$

LENGTHS OF LATTICE BARS FOR ORDINARY CHORDS AND POSTS.

DISTANCE "x"	DISTANCE "y"	DISTANCE c to c	DISTANCE "x"	DISTANCE "y"	DISTANCE c to c
"	"	"	"	"	"
$4\frac{1}{2}$	6	$7\frac{1}{2}$	$14\frac{1}{2}$	$8\frac{1}{4}$	$16\frac{11}{16}$
5	6	$7\frac{13}{16}$	15	$8\frac{1}{2}$	$17\frac{1}{4}$
$5\frac{1}{2}$	$6\frac{1}{2}$	$8\frac{1}{2}$	$15\frac{1}{2}$	9	$17\frac{15}{16}$
6	$6\frac{1}{2}$	$8\frac{27}{32}$	16	$9\frac{1}{4}$	$18\frac{1}{2}$
$6\frac{1}{2}$	$6\frac{1}{2}$	$9\frac{3}{16}$	$16\frac{1}{2}$	$9\frac{1}{2}$	$19\frac{1}{32}$
7	$6\frac{1}{2}$	$9\frac{9}{16}$	17	$9\frac{3}{4}$	$19\frac{19}{32}$
$7\frac{1}{2}$	$6\frac{1}{2}$	$9\frac{15}{16}$	$17\frac{1}{2}$	10	$20\frac{5}{32}$
8	7	$10\frac{5}{8}$	18	$10\frac{1}{4}$	$20\frac{11}{16}$
$8\frac{1}{2}$	7	11	$18\frac{1}{2}$	$10\frac{1}{2}$	$21\frac{9}{32}$
9	7	$11\frac{13}{32}$	19	11	$21\frac{15}{16}$
$9\frac{1}{2}$	7	$11\frac{13}{16}$	$19\frac{1}{2}$	$11\frac{1}{4}$	$22\frac{1}{2}$
10	7	$12\frac{7}{32}$	20	$11\frac{1}{2}$	$23\frac{1}{16}$
$10\frac{1}{2}$	7	$12\frac{5}{8}$	$20\frac{1}{2}$	$11\frac{3}{4}$	$23\frac{5}{8}$
11	$7\frac{1}{2}$	$13\frac{5}{16}$	21	12	$24\frac{3}{16}$
$11\frac{1}{2}$	$7\frac{1}{2}$	$13\frac{23}{32}$	$21\frac{1}{2}$	$12\frac{1}{2}$	$24\frac{7}{8}$
12	$7\frac{1}{2}$	$14\frac{5}{32}$	22	$12\frac{3}{4}$	$25\frac{7}{16}$
$12\frac{1}{2}$	$7\frac{1}{2}$	$14\frac{9}{16}$	$22\frac{1}{2}$	13	26
13	$7\frac{1}{2}$	15	23	$13\frac{1}{4}$	$26\frac{9}{16}$
$13\frac{1}{2}$	$7\frac{1}{2}$	$15\frac{7}{16}$	$23\frac{1}{2}$	$13\frac{1}{2}$	$27\frac{1}{8}$
14	8	$16\frac{1}{8}$	24	$13\frac{3}{4}$	$27\frac{21}{32}$

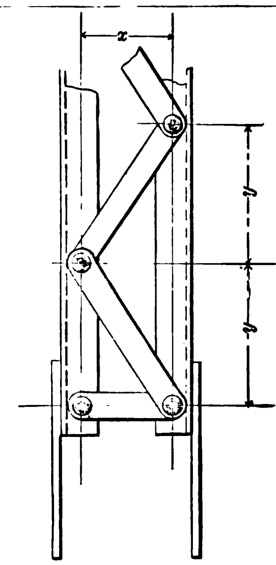


$$t = \frac{P}{7000w} + \frac{l}{27}$$

P = total compression carried by 1 Jaw.

LENGTHS OF LATTICE BARS FOR SMALL POSTS.

DISTANCE "x"	DISTANCE "y"	DISTANCE c to c	DISTANCE "x"	DISTANCE "y"	DISTANCE c to c
"	"	"	"	"	"
$4\frac{1}{2}$	$7\frac{3}{4}$	$8\frac{31}{32}$	8	$13\frac{1}{8}$	$15\frac{11}{16}$
5	$8\frac{1}{2}$	$9\frac{7}{8}$	$8\frac{1}{2}$	$14\frac{3}{4}$	$17\frac{1}{32}$
$5\frac{1}{2}$	$9\frac{1}{2}$	$10\frac{31}{32}$	9	$15\frac{3}{4}$	$18\frac{1}{8}$
6	$10\frac{1}{2}$	$12\frac{3}{32}$	$9\frac{1}{2}$	$16\frac{1}{4}$	$18\frac{27}{32}$
$6\frac{1}{2}$	$11\frac{1}{4}$	13	10	$17\frac{1}{4}$	$19\frac{15}{16}$
7	12	$13\frac{7}{8}$	$10\frac{1}{2}$	$18\frac{1}{4}$	$21\frac{1}{32}$
$7\frac{1}{2}$	13	15	11	19	$21\frac{15}{16}$



UPPER CHORDS.—The common chord section consists of two channels, latticed or laced on the under side, with a top or "cover plate." The same principles apply to this case as those already applied to posts. The size and spacing of lattice or lacing bars is the same, and the same rules hold good for stay plates.

For the common chord section, we may make use of Table IV., Chapter I., which gives the strength when the depth is known. We are thus saved the necessity of finding the radius of gyration, as described on page 403.

EXAMPLE.—*The upper chord of a bridge truss is* 25 *feet long, and is subjected to a stress of* 292700 *lbs. due to the dead load, and* 253100 *lbs. due to the live load. What should be the area of cross-section?*

We suppose each chord member to be in the condition of a strut *fixed at both ends*, owing to the action of the splicing plates, etc. We must also assume the depth of chord. Here judgment and experience must aid us to make a good choice the first time, though any choice can be tested and its suitability determined.

Suppose we take the depth in this case at 15 inches, that being the greatest depth of channel rolled.

Then we have $\frac{l}{d} = \frac{300}{15} = 20$, and from Table IV., Chapter I., page 389, we have for flat ends, the crippling strength $= 36024$ lbs. The factor of safety is $4 + \frac{l}{20d} = 4 + \frac{300}{300} = 5$. Hence, by the "old method," the safe working stress is $\sigma = \frac{36024}{5} = 7205$ lbs. per square inch. The total area required is then $\frac{545800}{7205} = 75.75$ square inches.

If the channels are spaced 20 inches apart, back to back, and the cover plate is ⅜ inch thick, then, since by reference to *Carnegie*, page 100, we see that the width of flange will not be far from 3.6 inches for a 15-inch channel, the width of cover plate will not be far from $20 + 7.2 = 27.2$ inches, and its area will be about $27.2 \times \frac{3}{8} = 10.2$ square inches.

This will leave for the required area of the two channels $75.75 - 10.2 = 65.55$ square inches, or for each channel 32.77 square inches.

From *Carnegie* we see that this is far heavier than the heaviest single channel rolled. We must therefore build up our chord section in this case, by means of plates and angle irons.* In this we may be guided by the principle of not having any plate less than ¼ inch or greater than ½ inch, or at most ⅝ inch in thickness.

We shall also find it advantageous to have a greater depth, and thus save material.

Let us take, therefore, the depth at 20 inches, then $\frac{l}{d} = \frac{300}{20} = 15$, and from Table IV. we have the crippling strength 37066 lbs. The factor of safety is 4.75, and hence $\sigma = \frac{37066}{4.75} = 7803$ lbs. per square inch. The area required now is therefore $\frac{545800}{7803} =$ about 70 square inches.

If the spacing is as before, 20 inches, and our flanges 4 inches, we have width of top plate 28 inches, and area $= 28 \times \frac{3}{8} = 10.6$ square inches. This leaves for the built channels $70 - 10.5 = 59.5$ square inches, or for each channel 29.75 square inches.

Let us take for the flanges equal-leg angle irons, 4″ by 4″ by ⅝ inch. The area of each is from *Carnegie*, page 106, 4.61 square inches. For two, we have 9.22 square inches. This leaves for the web about 20 square inches. For a depth of 20 inches and thickness of ⅝″, the web would be 12.5 square inches. We have then 7.5 square inches, remaining. We may rivet a flat plate to the bottom angle and make it 4″ by ½″. This would give 2 square inches more area, and leave 5.5 remaining. If now we add a side plate 12 inches by $\frac{7}{16}$″, it will make up the area remaining and just fit in between the legs of the angles.

The chord then may be built up as follows: 1 top plate 28″ × ⅜″, 2 web plates 20″ × ⅝″, 2 side plates 12″ × $\frac{7}{16}$″, 4 angles 4″ × 4″ × ⅝″, 2 flats 4″ × ½″.

By the "new method," we have from Table IV., Chapter I., page 389, for $\frac{l}{d} = 15$, and flat ends, $\frac{1}{1 + c\frac{l^2}{d^2}} = \frac{37066}{38500} = 0.9627$, and hence $\sigma = \frac{6500}{1 + c\frac{l^2}{d^2}}\left[1 + \frac{\text{const. } S}{\text{total } S}\right] = 0.9627 \times 6500 \left(1 + \frac{292700}{545800}\right) = 9613$ lbs. per square inch.

The area required by the new method is therefore $\frac{545800}{9613} = 56.77$ sq. inches, instead of 70 sq. inches by the old method.

Using the same top plate, we have $56.77 - 10.5 = 46.27$, or 23.19 square inches for each channel. Taking angles 4″ × 4″ × ⅝″ for the flanges, we have $23.14 - 9.22 = 13.92$ square inches remaining. A web plate 20″ × ⅝″ will about cover this.

The chord then will consist of 1 top plate 28″ × ⅜″, 2 web plates 20″ × ⅝″, 4 angles 4″ × 4″ × ⅝″.

This section may now be tested by calculating the radius of gyration according to the principles of page 403, and using Table I., page 386.

The lattice work or lacing bars on the bottom are to be then spaced and dimensioned according to the rules on page 407.

Our example is for a very long bridge, and therefore very heavy chords. Ordinarily the size of channels required will fall within the limits of Carnegie's Table. At present prices of labor and material it is, however, cheaper to build up the chords by plates and angles than to roll heavy channels, and top chords are therefore usually built up.

If we use the straight-line formula, with Cooper's values, page 382, we have, taking $r = 7.5$, for the live load,

* From Osborne's Tables we can choose a built-up section without calculation.

$\sigma = 8000 - 30\frac{l}{r} = 6800$, and for the dead load, $\sigma = 16000 - 60\frac{l}{r} = 13600$ lbs. The area required is therefore $\frac{292700}{13600} + \frac{253100}{6800} = 58.7$ sq. inches.

In using built-up chords the designer will find it indispensable to have on hand *Tables of Moments of Inertia*, by Frank Osborne, C. E. Eng. News Pub. Co., New York.

WIDTH OF UPPER CHORD AND TOP PLATE, AND THICKNESS OF TOP PLATE.—The width of top plate is determined by the conditions of the case. It must be wide enough to admit the posts and the main and counter ties.

The *least allowable width*, independently of these considerations, must be at least greater than the depth. The least allowable width of the top plate must be then equal to the distance between the channels, *plus* twice the width of the flange.

This least allowable width of the top plate may be taken at

$$w = \frac{7}{6}D + 1,$$

where D is the depth of channel, and w the width of plate in inches.

The least allowable *thickness* of top plate may be taken at $\frac{1}{4}''$ for depths of channel less than 8″. From 9 to 10 inches inclusive, $\frac{5}{16}''$. From 12 to 18 inches inclusive, $\frac{3}{8}''$. Above 20 inches, $\frac{7}{16}$ to $\frac{1}{2}''$. These thicknesses correspond to the least allowable width, as already given. Should the actual width exceed the least allowable by 50 per cent., we may add $\frac{1}{16}''$ to the thickness. If it exceeds by 75 per cent., we may add $\frac{1}{8}''$ to the thickness, as determined by the above rules.

DEPTH OF CHORD.—A little preliminary calculation will usually be necessary to fix upon a suitable depth for the top chord. As the depth ought to be constant from end to end of the bridge, and as the stress is much greater in the middle than at the ends of the truss, we must choose such a depth of channel as will allow of the necessary variation in thickness to meet the strain in centre and end panels.

If we find the area required in the end panel, then the depth which will give the least average area and allow for the area of centre panel and of end panel will be the best depth to use.

EXAMPLE.—*Suppose the end upper panel is subjected to a stress of* 47000 *lbs. due to the dead load, and* 46000 *lbs. due to the live load, and the centre panel to a stress of* 70000 *lbs. due to dead load, and* 54000 *lbs. due to live load, what should be the depth of upper chord, if the panel length is* 20 *feet?*

Let us try 9-inch channels. The ratio $\frac{l}{d} = \frac{240}{9} = 26\frac{2}{3}$, and from Table IV., Chapter I., page 389, we have for flat ends, crippling strength = 17.153 tons = 34306 lbs. The factor of safety is 5.33, hence the safe working stress is $\sigma = \frac{34306}{5.33} = 6470$ lbs. per sq. inch. The area required in the end panel, by the "*old method*," is then $\frac{93000}{6470}$ = about 14 square inches.

The minimum width of top plate, according to the rule just given, is, for 9″ channel, 11 inches, and its thickness $\frac{5}{16}''$. Its area is then $11 \times \frac{5}{16} = 3.44$ square inches. This leaves $14 - 3.44 = 10.56$ for the channels, or 5.28 sq. inches for each channel. From *Carnegie*, page 100, we see that 9-inch channels will answer.

Let us see whether 9″ channels will give us enough area at centre. Here we have $\frac{124000}{6470} = 19.16$ sq. inches. Deducting 3.44 for the top plate, we have 7.86 sq. inches for each channel. This falls beyond the limit of weight for 9-inch channels, and such a depth then will not answer.

Let us try 10-inch channels. We have then $\frac{l}{d} = \frac{240}{10} = 24$, and from Table IV., $\sigma = \frac{35034}{5.2} = 6737$ lbs. per sq. inch. This calls for an area in the end panel of $\frac{93000}{6737} = 13.8$ sq. in. The area of top plate is $13 \times \frac{5}{16} = 4$ sq. inches. Deducting this, we have 4.9 for area of each channel. The lightest 10-inch channel is just 4.9 sq. inches.

The *average area* for the 10-inch channels is $\frac{5.25 + 7.2}{2} = 6.27$ sq. inches.

Twelve inch channels will be found in like manner to call for 3.64 sq. inches at end, and 5.79 at the centre. No 12" channels are rolled as light as this. The lightest 12 inch channel, *of one weight only*, has about 6 sq. inches cross-section. We might therefore use this throughout the upper chord. It would give too great area throughout, but the average area would be only 6 square inches, a little less than for 10" channels. There is also practical advantage in having all the chords of a size, as it makes all the splice plates and top cover plates of a size also, and secures economy in price, ease of erection, and uniformity of details.

By the "*new method*," we should proceed precisely as above, only the value of σ would be determined from

$$\sigma = \frac{6500}{1 + c\frac{l^2}{d^2}}\left(1 + \frac{\text{const. } S}{\text{total } S}\right),$$

where $\frac{1}{1 + c\frac{l^2}{d^2}}$ can be found from Table IV., by dividing the crippling strength in lbs., as given by the Table, by 38500 for flat ends. By the straight-line formula we should also proceed precisely as above, only the value of σ would be $\sigma = 8000 - 30\frac{l}{r}$ for live load, and $\sigma = 16000 - 60\frac{l}{r}$ for dead load, page 382.

COMPRESSION AND FLEXURE COMBINED.—The top chord of a deck bridge may have a load upon it, due to a cross tie, between the panel points. It then acts as a beam as well as a strut.

For this case we have, page 378,

$$A = \frac{Mv}{\sigma r^2} + \frac{S}{\sigma},$$

where σ is taken according to the "old" or "new" method, or straight-line formula for struts.

EXAMPLE.—*Suppose an upper panel to be subjected to compression of* 30000 *lbs. due to dead load, and* 60000 *lbs. due to live load, and to have a weight of* 1 *ton acting at the middle. If the panel is* 15 *feet long, what should be the area?*

Let us try 10-inch channels. The ratio $\frac{l}{d} = \frac{180}{10} = 18$. For common chord section, flat ends, we have from Table IV.. the crippling strength = 36470 lbs., and factor of safety = 4.9. By the "old method," $\sigma = \frac{36470}{4.9} =$ 7443 lbs. per sq. inch.

By the "new method" we have $\frac{1}{1 + c\frac{l^2}{d^2}} = \frac{36470}{38500} = 0.947$, and hence $\sigma = 0.947 \times 6500\left[1 + \frac{30000}{90000}\right] =$ 8207 lbs. per sq. inch.

In the present case $M = 1000 \times 7.5 \times 12 = 90000$ inch lbs., $v = 4$ inches, $r =$ not far from 3 inches, according to *Carnegie*, page 100.

Hence by "old method,"

$$A = \frac{90000 \times 4}{6943 \times 9} + \frac{90000}{6943} = 5.36 + 13 = 18.7 \text{ sq. in.}$$

By the "new method,"

$$A = \frac{90000 \times 4}{8207 \times 9} + \frac{90000}{8207} = 4.87 + 10.96 = 15.83 \text{ sq. ins.}$$

The least allowable width of top plate is 8 inches, and thickness $\frac{1}{4}$". The area of top plate is then 2 sq. inches.

This leaves 16.7 sq. inches, or 8.35 sq. inches for each channel by the old method, and 13.83 sq. inches or 6.9 sq. inches for each channel by the new method. From *Carnegie*, page 100, we see that these channels can be rolled

In the first case, then, we have two 10" channels, 27.32 lbs. per foot, 0.615 inches thickness of web, and 3.015 inches wide of flange.

In the second case, we have two 10" channels, 23.3 lbs. per foot, 0.47 in. thickness of web, and 2.87 in. width of flange.

If we use the straight-line formula, with Cooper's values, page 382, we have $\sigma = 6200$ lbs. for live load, and $\sigma = 12400$ lbs. for dead load. Hence $A = \frac{30000 \times 4}{12400 \times 9} + \frac{30000}{12400} + \frac{60000 \times 4}{6200 \times 9} + \frac{6.000}{6200} = 17.47$ sq. inches.

SECONDARY STRESSES.—The members at an apex should be loaded in their axes, and these axes should meet in a point.

If these conditions are not complied with, we have secondary stresses due to bending, and the unit stress must be determined as directed, page 313.

JAW PLATES.—When the flanges at the pin ends of compression members are cut away for the purpose of close packing, the webs of the channels remaining must be strengthened by "pin plates" or "jaw plates." These must give sufficient bearing on the pin. They must also have sufficient area as posts.

Their thickness as posts is determined by the formula

$$t = \frac{P}{7000w} + \frac{b}{27},$$

where P is the compression carried by one jaw in lbs., w = width of the jaw, b = length in inches from the centre of pin hole to the first rivet beyond the point at which the full section of the post begins, t = thickness in inches.

We give, in Figs. 221 and 222, Plates 11 and 12, details of upper chords and connections. The drawings explain themselves. These represent modern American practice.

In Fig. 221, Plate 11, we have the ordinary style of posts, formed of channels latticed or laced. Fig. 222, Plate 12, shows also the inclined-end posts or "batter braces," formed, like the chords, of latticed or laced channels, with top plate. It is designed precisely like the top chord. Figs. 233 and 234, Plate 14, show methods of riveting.

In Figs. 235 and 236 we have given details for light highway bridges. Such details are only allowable in light structures, and good modern practice would avoid bending the ties, as shown in Fig. 235. Fig. 237 shows the details for the ordinary Howe Truss.

No castings are permitted in modern bridges, for any purpose, except for bed plates and for the machinery of draw spans. For best modern construction, the student should observe carefully, well-executed examples in the field, and sketch details. The recent editions of the illustrated albums of our best bridge companies will give much information. Also the reports of Geo. S. Morison, C. E., upon the Bismarck Bridge, the Plattsmouth Bridge, and the Omaha Bridge. The illustrated albums of the various bridge companies are easily obtained upon application, for a small price, and many of them are excellently illustrated.

To attempt to give such details in a work of this character is to run the risk of becoming antiquated in a few years.

A comparison of differences in practice and a consideration of the reasons for such differences is a most instructive exercise. The improvement of details is the constant aim of the designer, and the student should render himself familiar with the best practice attainable, and be on the alert to note new forms and improved methods.

PLATE 12.

Fig. 282

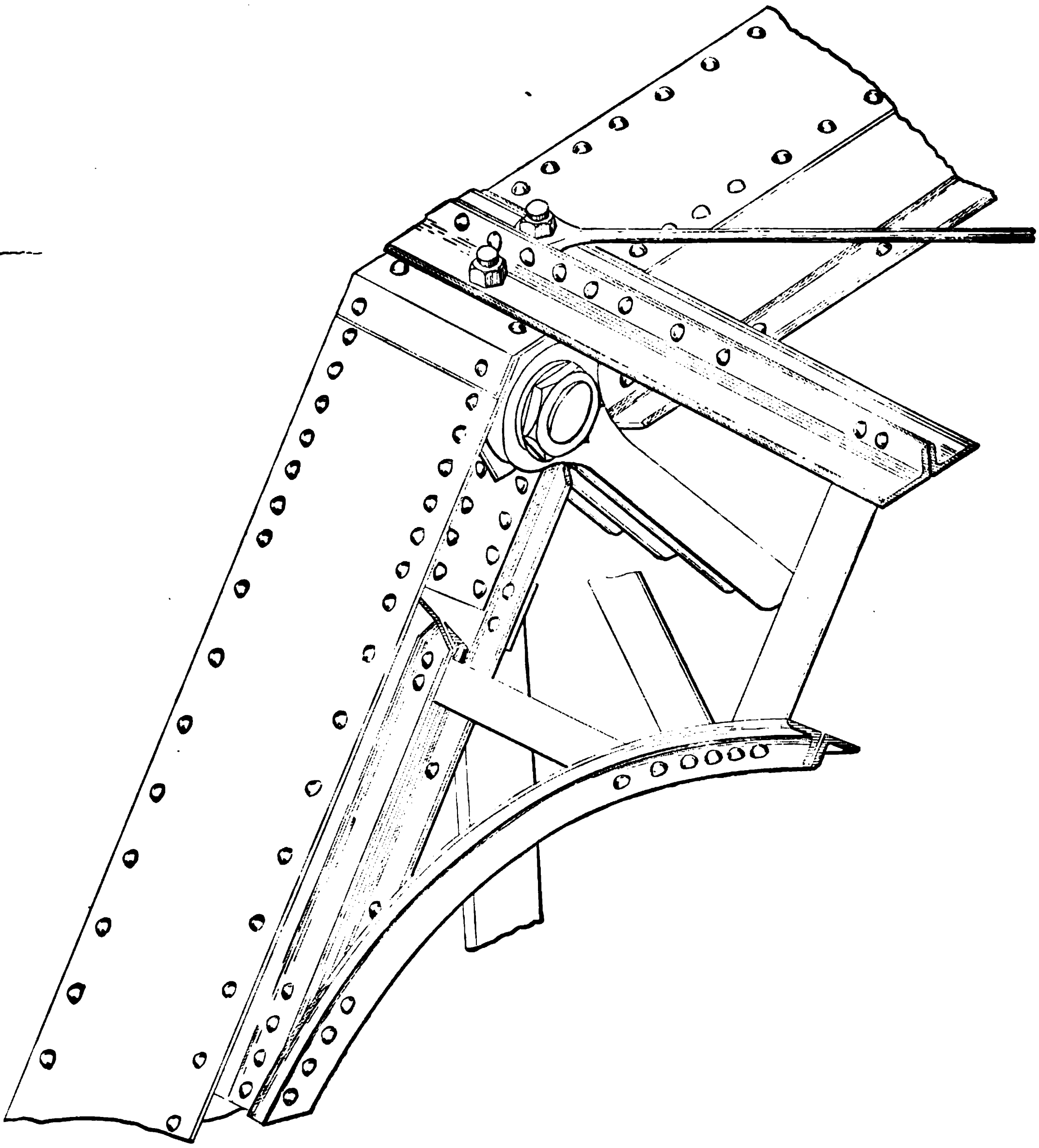


PLATE 13.

Fig. 226

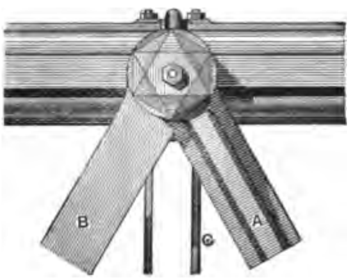


Fig. 228

Fig. 229

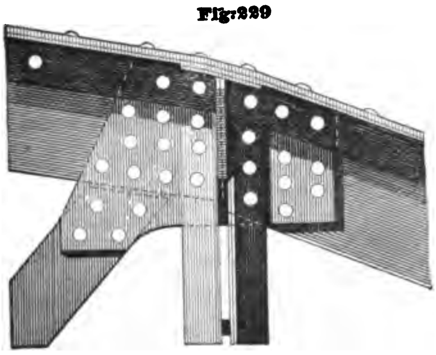


Fig. 230

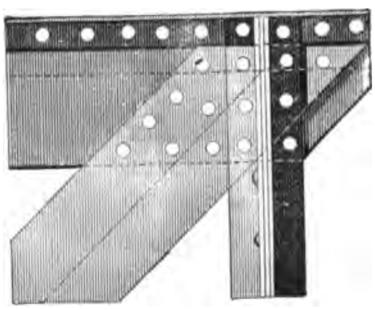


PLATE 14.

Fig. 235

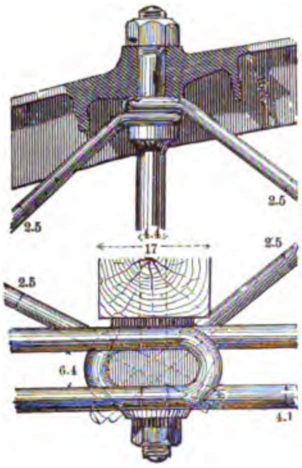


Fig. 236

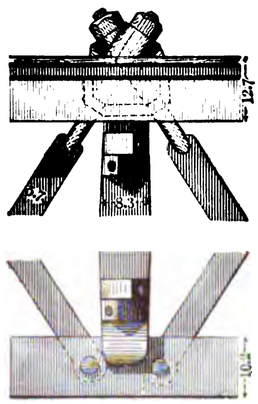


Fig. 237

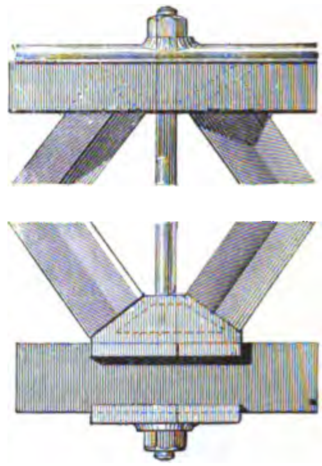


Fig. 231

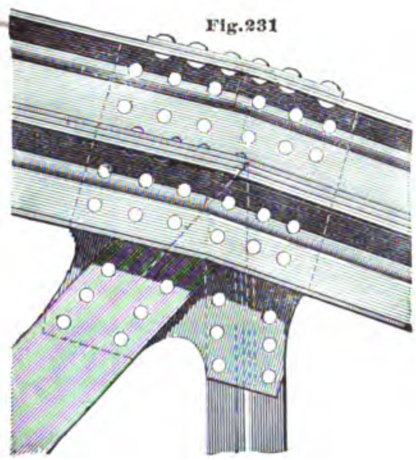


Fig. 232

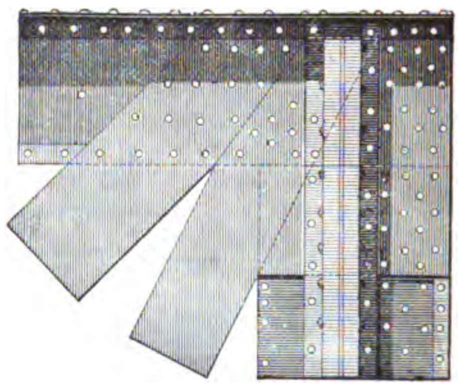


Fig. 233

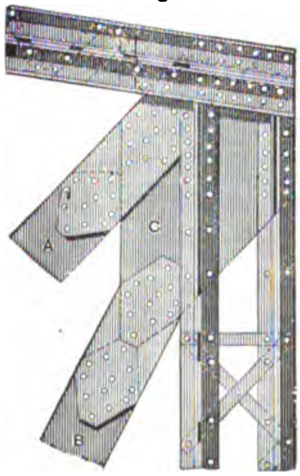


Fig. 234

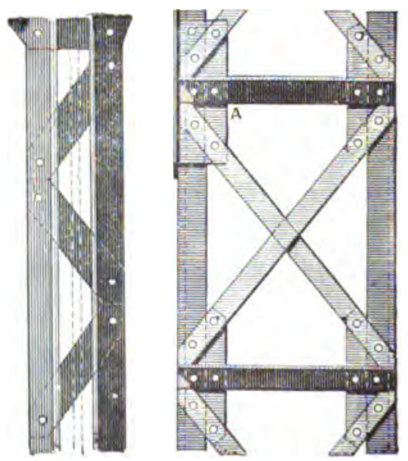


PLATE 17.

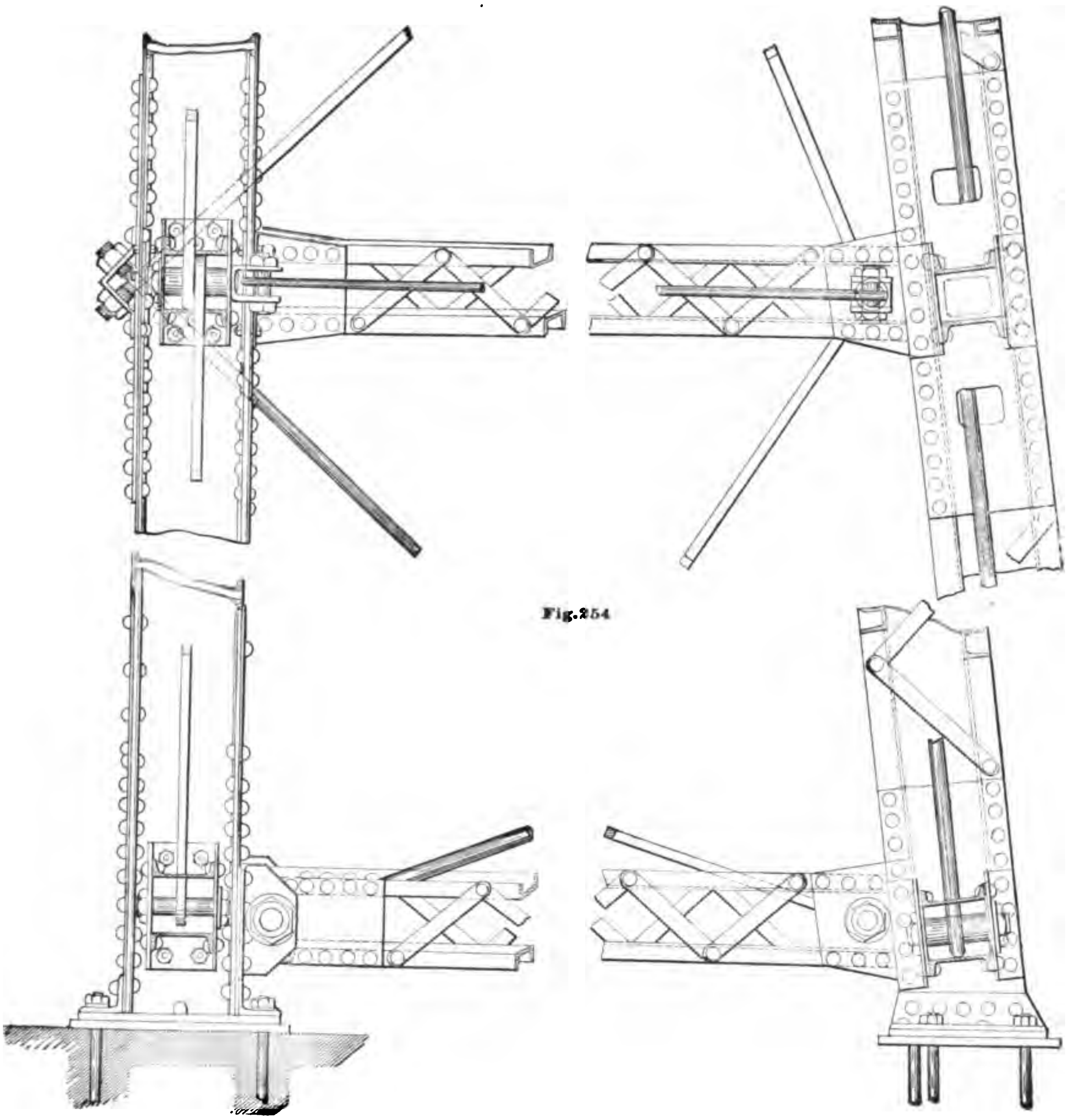


Fig.254

PLATE 18.

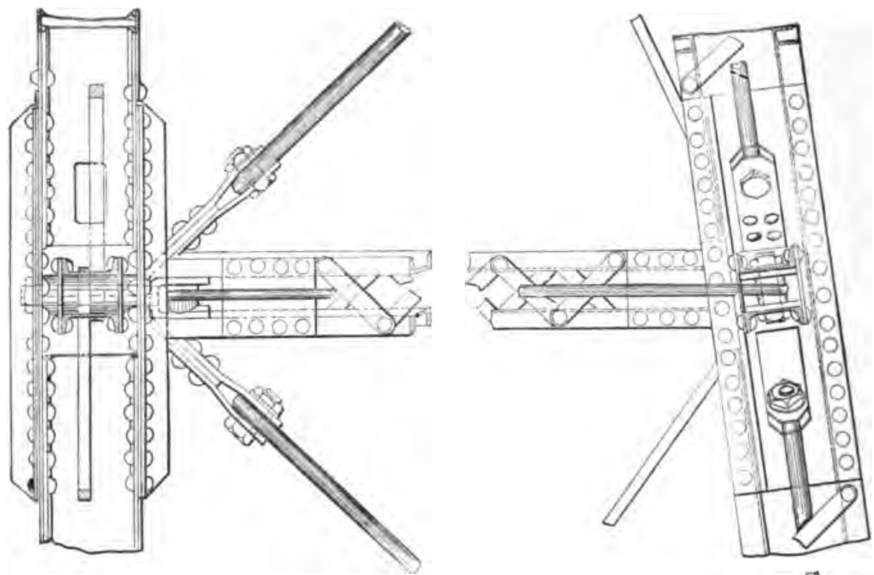
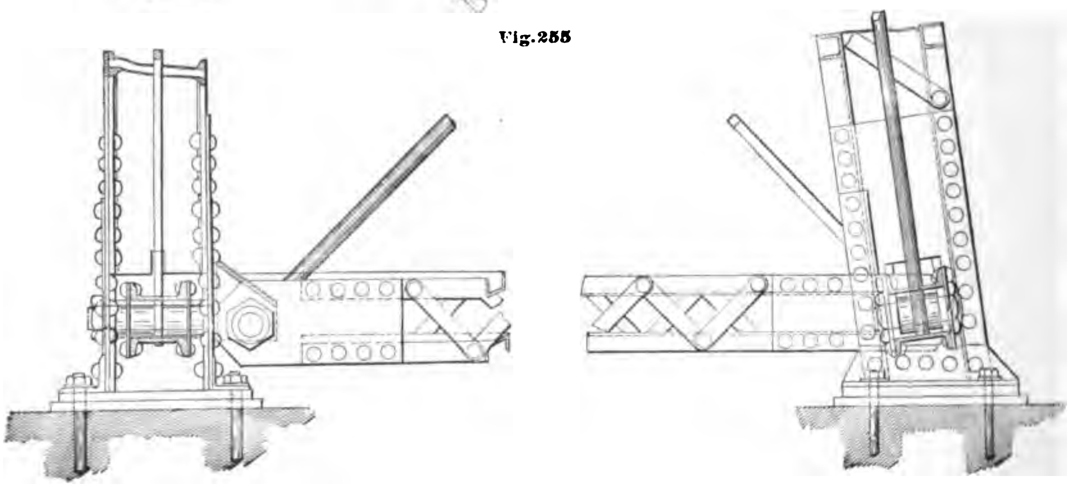


Fig. 255



CHAPTER IV.

PINS AND EYE BARS.

THE use of "pin connections" and "screw-end connections," is the characteristic of American bridge practice. Rivets are only used for such minor details as splice, cover, and re-enforcing plates, in the flanges of plate cross-girders and stringers, and their connections, and for stiffeners. The main connections of posts with chords, are by means of pins, whereas in England and on the Continent all the connections are usually riveted.

The price of labor, extent to which machine processes are employed, etc., are the main reasons which justify such diversity of practice. As regards the theoretical advantage of the two systems, the pin connection seems undoubtedly the best, and in this country has shown itself also the best practically.

Among the evident disadvantages of rivet connections, as compared with pin connections, we may mention the impossibility of getting the stresses to act along the axis of the members. An indefinite amount of twisting is thus caused at each joint, which is entirely absent in pin joints. The practice of distributing rivets assumes that each takes its equal share. But in reality the first rivets must take a greater amount, and only the elasticity of the plates brings the others into play. With many rivets it is thus questionable whether some of them act at all, and it is impossible to determine to what extent. Rivet holes, even when laid out with the greatest care, cannot be made to always coincide, and the holes are then forced to match by the use of the "drift-pin," which distorts the holes and injures and mutilates the material. Imperfect workmanship cannot be avoided, even by the most careful supervision, and a rivet hole imperfectly filled presents the same appearance as a perfect one. Rivet heads will snap off under the contraction of the rivet when cooled, and often when carelessly driven by hand will shrink away from the hole without filling it at all. To guard completely against all these sources of imperfection would seem hopeless. To guard sufficiently requires that the greatest care and every precaution be taken to eliminate them. Thus, in Europe the holes are drilled in the plates while clamped in position, and machine riveting employed.

Such precautions mean increase of expense, and in this country cannot be employed in competition with pin connections. Hence, in this country at least, such precautions are not taken, and, whatever may be the case abroad, here our riveted bridges are inferior to the pin connected.

For this reason, our American practice would seem the best adapted to the circumstances, which uses pins for all the important main connections, and only employs rivets for those details, such as splice and cover plates, etc., whose office is simply to keep the members in line, or for such connections as the flanges of plate girders, where rivets are unavoidable. In such cases, as far as practicable, the riveting should be done in shop and not in the field. Field riveting is sure to be poor, and open to all the objections named.

There are no objections to the pin joint from a theoretical standpoint, and the only practical ones urged are the difficulty of securing a tight fit, and consequent expense, and the fact that the rupture of a single joint destroys the structure. The practical objections are practically answered by machine-made bearings and connections of the nicest fit, and by existing structures both economical and safe, which have given American engineers the reputation of being among the best bridge-builders.*

THEORY OF PINS AND EYE-BARS.—THICKNESS OF RE-ENFORCING PLATES.—The bearing resistance of the pin should equal the greatest pressure upon it due to any plate through which it passes.

If d is the diameter of pin in inches, $t =$ the thickness of any plate through which it passes in inches, then dt is the bearing area in square inches. Let C be the working compressive stress per square inch, then dtC is the bearing resistance of the pin. This should equal the stress transmitted through the plate, or

$$dtC = \text{stress}.$$

We may take C at 6.25 tons. The stress transmitted is always known. If the stress is *one ton*, the requisite bearing area is

$$dt = \frac{1}{6.25}, \text{ and hence we have}$$

$$\textit{lineal bearing on pin, in inches per ton of stress} = \frac{1}{6.25d}, \quad \ldots \ldots \ldots \ldots \quad (1)$$

From equation (1), having given the diameter, we can find the corresponding lineal bearing or thickness of plate, for every ton of stress to be transmitted. We have only to multiply this by the number of tons stress in any case, to find the requisite thickness of plate in any case. This equation is therefore to be applied in finding the thickness of re-enforcing plates.

EXAMPLE.—*The stress transmitted through a 12-inch post channel is 55500 lbs. The thickness of web is $\frac{6}{10}$ths of an inch, and diameter of pin is 3 inches. What thickness of re-enforcing plate is required?*

The thickness for each ton is $\frac{1}{6.25d} = \frac{1}{6.25 \times 3} = 0.0533$ inches. For $\frac{55500}{2000} = 27.75$ tons, we should have then a thickness of $0.0533 \times 27.75 = 1.48$ inches. As the channel web is 0.6 inch, this leaves $1.48 - 0.6 = 0.88''$ for the thickness of re-enforcing plate. Two plates, $\frac{7}{16}''$ thick upon each side of channel web, will then give the required thickness.

The thickness for each ton of stress, for different diameters, has been found from the formula (1), and is given in the Table, page 387, which follows.

LEAST DIAMETER OF PIN.—If t is the thickness of eye-bar, and w its depth, then tw is the area of cross section of eye-bar. If σ is the working tensile stress for which the bar has been dimensioned, then $tw\sigma$ is the stress transmitted from the bar to the pin.

* "The typical American railroad bridge is a skeleton structure, pin-connected at all the principal articulations. Its essential characteristics, in addition to being connected by pins," are stated by Cooper as follows : "*First*—So formed as to reduce all ambiguity of strains to a minimum. *Second*—Concentration of parts. *Third*—Facility of manufacture. *Fourth*—Perfection of lengths and fitting of all the members, so as to reduce to a minimum all riveting or mechanical work in the field. *Fifth*—Readiness with which the individual members can be assembled during erection."—*Trans. Am. Soc. C. E.*, July, 1889.

Now if d is the diameter of pin, and if the thickness of head is equal to the thickness of bar, t, we have td for the bearing of pin, and tdC for its bearing resistance.

We must have then for the smallest admissible value of d,

$$tdC = tw\sigma, \qquad \text{or} \qquad d = \frac{\sigma}{C}w.$$

The ratio of the tensile working stress σ to the compressive working stress C, or $\frac{\sigma}{C}$, may be taken at $\frac{3}{4}$. We have then for the *least diameter of pin admissible,*

$$d = \tfrac{3}{4}w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The diameter of pin may need to be much greater than this, but it cannot be less, *unless the thickness of head of eye-bar is made greater than the thickness of bar itself.*

When this is the case, if t_1 is the thickness of bar and t the thickness of head, we have

$$tdC = t_1w\sigma, \qquad \text{or} \qquad d = \tfrac{3}{4}\frac{t_1}{t}w,$$

for the least diameter of pin, and $\qquad t = \dfrac{3wt_1}{4d},$

for thickness of head when diameter is given.

It is seldom desirable and often impossible to use this smallest value $d = \frac{3}{4}w$ for lower chord bars. It is well simply to note it as a limit below which we cannot go without increasing the thickness of heads. For diagonals, counters, and hip verticals, the head must sometimes be thicker than the bar.

EXAMPLE 1.—*If the depth of eye-bar is 10 inches, what is the least diameter of pin which can be used without thickening the head of eye-bar?* *Ans.* $d = 7\frac{1}{2}$ *inches.*

EXAMPLE 2.—A hip vertical bar is 8″ by $\frac{7}{8}$″. If the diameter of pin passing through it at the upper end is $4\frac{5}{8}$″, what should be the thickness of the head?

The least diameter allowable without thickening the head is $\frac{3}{4}w = \frac{3}{4}\,8 = 6''$. As the pin in this case is less than this, the head must be thicker than the bar. The thickness of head is $t = \frac{3wt_1}{4d} = \frac{3 \times 8 \times \frac{7}{8}}{4 \times 4\frac{5}{8}} = \frac{21}{18\frac{1}{2}} = 1\frac{5}{37}''$.

EXAMPLE 3.—*A main tie-bar is 5″ by $1\frac{9}{16}$″. If the diameter of pin is $3\frac{5}{8}$″, what should be the thickness of head at that end?*

Here $\frac{3}{4}w = \frac{3}{4} \times 5 = 3\frac{3}{4}$, therefore the head must be thicker than bar. We have for thickness of head $t = \frac{3wt_1}{4d} = \frac{3 \times 5 \times 1\frac{9}{16}}{4 \times 3\frac{5}{8}} = 1.616''$.

EXAMPLE 4.—A counter rod is 1″ diameter.. What should be the thickness of head, if the pin is $3\frac{3}{4}$″?

We must replace here t_1w in the formula by $\frac{\pi w^2}{4}$, where w is the diameter of the rod. We have then

$$t = \frac{3\pi w^2}{16 \times 3\frac{3}{4}} = 0.157''.$$

If the head, then, is a loop of same diameter as rod, it will afford ample bearing.

SIZE OF PIN.—The pin should be treated as a beam which fails by flexure. The size as thus determined is greater than the diameter required for safe bearing or shearing resistance.

From the theory of flexure, Part I, page 286, we have

$$M = \frac{RI}{v} = \frac{RI}{r},$$

where r is the radius of pin, and R is the working stress in the outer fibre, and $I =$ the moment of inertia $= \frac{\pi r^4}{4}$. Hence the moment of resistance of the pin is

$$M = \frac{\pi R d^3}{32}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From this formula we may calculate the bending moment or moment of resistance of the pin for different diameters. The usual value for R is 15000 lbs. for iron and 20000 lbs. for steel. We have given the value of M for these two cases in the Table, page 387.

Now to find the requisite diameter in any case, we must find the maximum resultant moment M of all the forces acting upon the pin. From the Table, we can then pick out a diameter which gives a bending moment or moment of resistance equal to this maximum resultant moment.

It remains therefore to show how to find the maximum resultant moment M of all the forces acting upon the pin.

We have represented in Figs. 202 (c) and 202 (d), a pin joint in elevation and plan. See also Plate 8, Fig. 205.

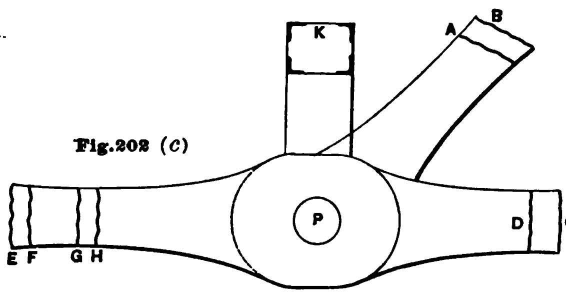


Fig. 202 (c)

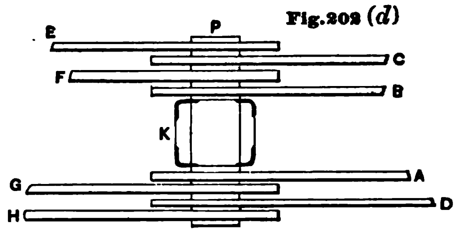


Fig. 202 (d)

K is the post, in this case composed of plate and angle irons, which takes compression only. A and B are the two main ties. The counter, if any, would be attached to centre of pin, but as the greatest stress on the pin will be for a full load, there will be no stress for this loading in the counter, and *hence it may always be omitted in finding the size of pin at any joint in either upper or lower chord.* The main ties, A and B, come next to the post upon each side. The chord bars, G, H, come next with the bar D between. The same arrangement holds on the other side of the middle.

The main ties are the only inclined members. All the others are either horizontal or vertical. The sum of the horizontal forces upon one side of the pin must be equal to the sum of all the horizontal forces upon the other side of the pin, including, of course, in this sum the horizontal component of the stress in the ties. The vertical component of the strain in the ties is equal to the post compression. These components may then be easily found. Thus the entire horizontal stress upon one side of the pin (the left in Fig. 202) is equal to the area of all the bars on that side $\times$ by the working stress β for which the bars were dimensioned. The entire stress upon the other side must be the same. The *horizontal component of the tie stress*, may then be found by subtracting the sum of the stresses in the chord bars upon the main tie side, from the sum of the stresses in the chord bars on the other side. Thus in Fig. 202 (d), we multiply the area of D by β and subtract from the area of $H \times \beta$ + the area of $G \times \beta$. The result is the horizontal component of the stress in A. The *vertical component* is equal to the half post compression *calculated for full loading.*

In general, for any pin, we must resolve the stress in every member through which that pin passes, *as found for full loading*, into its vertical and horizontal components. *The stress in each member is considered as acting along the centre line or axis*, and hence the point of application of each vertical and horizontal component is *at the centre of the bearing of the corresponding member.*

Let M_H be the maximum moment of all the horizontal stresses, and M_V the maximum moment of all the vertical stresses. Then the resultant moment is

$$M = \sqrt{M_H^2 + M_V^2},$$

and the size of pin required may be found from the Table, page 427, by taking that size whose bending moment is equal to M, or from the formula

$$d = \sqrt[3]{\frac{32M}{\pi R}}.$$

It remains to find M_H and M_V, or the maximum moment of all the horizontal forces and the maximum moment of all the vertical forces.

Let the horizontal forces or chord bar stresses acting upon the pin *on one side of the centre* be P_1, P_2, P_3, P_4, etc., the *odd* indices P_1, P_3, etc., acting in one direction, and the *even* indices P_2, P_4, etc., acting in the other direction. Let l_1 be the distance between centres of bearing of P_1 and P_2, l_2 the distance from P_2 to P_3, etc. Now the maximum moment will be at the point of application of some one of the forces. It is therefore easily found by trial.

Thus the moment at P_2 is P_1l_1. Add to this $(P_1 - P_2)l_2$, and we have the moment at P_3. Add again $(P_1 - P_2 + P_3)l_3$, and we have the moment at P_4, and so on. The greatest of all these is the moment required. Since all the forces upon one side, P_1, P_3, P_5, etc., are equal to all upon the other, P_2, P_4, P_6, etc., they will reduce to a couple at each end of the pin, and hence the moment at any point beyond the last force, that is, between the two inside horizontal forces (A and B in Fig. 202) is constant. We have only then to find the greatest moment by trial as above. To find M_V, we have simply to find the half post compression for full loading, and multiply by the distance between the centres of bearing of tie and post.

CHORD PACKING.—By means of washers, any two members may be separated and kept at any distance, so that the ties and posts may be in vertical planes. It is evident that a skilful packing of the bottom chord may diminish the value of M_H, and hence the size of pin. We should, in general, so arrange the packing that the points of application of the resultant on each side may as nearly as possible coincide. As the chord bars usually go in pairs of equal size, this is not difficult to arrange.

Thus, if we have two chord bars, P_1 and P_3, on one side, and one bar P_2 between them on the other side, with a tie P_4 beyond P_3, the distances being l_1, l_2, l_3; then the distance of one resultant from P_1 is $\frac{P_3}{R}(l_2 + l_1)$, and of the other, $\frac{P_4}{R}(l_3 + l_2) + l_1$. Equating the two and putting $R = P_1 + P_3$, we have, when the resultants coincide,

$$l_2 = \frac{P_1l_1 + P_4l_3}{P_3 - P_4}.$$

In such a case we should pack the first two bars snug, and the other bar and tie as close as possible, thus making l_1 and l_3 as small as circumstances allow. The distance l_2 can then be found.

If $P_1 = P_3 = 10$ and $P_2 = 15$, then $P_4 = 5$, and if $l_1 = 2$ and $l_3 = 3$, we have

$$l_2 = \frac{20 + 15}{10} = 3.5.$$

See Plate 24 at end of Work for this detail.

SIZE OF PIN AT CENTRE OF LOWER CHORD.—By an intelligent application of the preceding principles, we can find the size of pin at any joint. The application, however, admits of modification in special cases.

The largest pin in the lower chord will be at the centre for an even number of panels, or at the first joint right or left of the centre for an odd number of panels. The chord bars are fully strained by a full load at every panel point. But for such a loading the post compression at the centre is very small, and as the tie can always be packed quite close to the post, the moment M_V can be disregarded.

We have, therefore, for the centre joint in the bottom chord, simply

$$M = M_H.$$

For ordinary spans all the pins in both upper and lower chord, except the pin at the hip, are made of the same size. In general, then, two calculations of size for *hip* and *centre of the lower chord* are sufficient. If we wish, however, to find the size of all the pins we may calculate the size of pin at end, at first joint or hip vertical, and at second joint and centre, and interpolate between these last for intermediate joints of the lower chord. For the upper chord, we may calculate the pin at the hip, at the first joint, and at the centre, and interpolate between the last two for intermediate joints of the upper chord. This is, as we have said, unnecessary in practice. The pins being so important, an excess of strength is desirable, and hence only two sizes are usually used, one for the *hip*, and the other for *all the other* joints, top and bottom. We shall, however, in what follows, illustrate the method of calculation very fully for any pin.

PRACTICAL SIZES FOR PINS.—PRACTICAL HINTS.—As we see by *Carnegie*, page 173, pins are furnished only in sizes differing either by $\frac{1}{4}$ or $\frac{1}{8}$ inch, and there are no intermediate sizes. All sizes are therefore an even number of 16ths.

When the size of a pin is calculated, we should always order it at least $\frac{1}{16}$ inch larger, in order that it may be turned down to exactly fit the hole.

We must therefore add $\frac{1}{16}''$ to the calculated size, and if this gives an even number of 16ths, it can usually be ordered. If not, we must order it $\frac{1}{16}$ larger still.

Thus, if the size of a pin is found by calculation to be $4\frac{3}{8}''$, it should be at least $4\frac{7}{16}''$, but from *Carnegie* we see that $4\frac{7}{16}$ is not rolled. We must therefore order $4\frac{1}{2}''$, and turn it down to fit the hole.

If the calculated size is $3\frac{1}{2}''$, it should be at least $3\frac{9}{16}$. But we see from *Carnegie* that only $3\frac{1}{2}$ and $3\frac{3}{4}$ are rolled. We must therefore order $3\frac{3}{4}''$, and turn down.

In general, pins in practice are between $4''$ and $6''$, and as these sizes are furnished at intervals of $\frac{1}{8}$ inch, we have, in all practical cases, between 4 and 6 inches, simply to add $\frac{1}{16}''$ to the calculated size, and if this gives an even number of 16ths, it can be ordered; if an odd number of 16ths, increase by $\frac{1}{16}$ inch.

The following Table gives about the sizes of pins used in practice. The pins should not be smaller than given in this Table, but if necessary can be made larger.

It will be noticed that the sizes given are all odd sixteenths. This is to allow turning off of $\frac{1}{16}$ as noticed above.

As the pin bends under the action of the chord bars, the adjacent edges of each pair of opposing bars are compressed more than the outer edges. The result is to diminish the distances l_1, l_2, etc., between the bearing centres, and hence to diminish the moment. It may thus happen that a pin will safely take much more than the method of calculation given justifies. Nevertheless, such action should not be relied upon, but the full bearing distances taken, and the pin proportioned accordingly.

MINIMUM SIZES OF PINS.

Span in ft.	End Pins.	Intermediate Pins.	Span in ft.	End Pins.	Intermediate Pins.
75 to 85	$3\frac{7}{16}''$	$2\frac{15}{16}''$	140 to 155	$5\frac{1}{16}''$	$4\frac{9}{16}''$
85 " 95	$3\frac{11}{16}$	$3\frac{3}{16}$	155 " 175	$5\frac{5}{16}$	$4\frac{13}{16}$
95 " 105	$4\frac{1}{16}$	$3\frac{7}{16}$	175 " 200	$5\frac{9}{16}$	$4\frac{13}{16}$
105 " 115	$4\frac{5}{16}$	$3\frac{11}{16}$	200 " 225	$5\frac{13}{16}$	$4\frac{13}{16}$
115 " 125	$4\frac{9}{16}$	$4\frac{1}{16}$	225 " 250	$6\frac{1}{16}$	$5\frac{1}{16}$
125 " 140	$4\frac{13}{16}$	$4\frac{5}{16}$	250 " 300	$6\frac{9}{16}$	$5\frac{5}{16}$

DOUBLE TRACK.

Span in ft.	End Pins.	Intermediate Pins.
150′	$6\frac{9}{16}''$	$5\frac{5}{16}''$
140	$6\frac{1}{16}$	$5\frac{1}{16}$
130	$5\frac{13}{16}$	$4\frac{13}{16}$

Pin holes are bored about $\frac{1}{40}$ of an inch (0.025) larger than the pin. The size of pin taken from this Table must be tested at every joint, as illustrated by the following examples. We should not use less size than given by the Table, but may use larger. If, however, we find a larger pin required, we may very often reduce the required size, by re-arranging the chord bars, or by increasing their number. Only two sizes are used for end and intermediate pins.

Small pins should be tested for shear as well as bending, but in general, if the pin is safe against bending and bearing, it will be safe against shearing.

In finding the maximum bending moment, the counters may always be omitted, as they are not strained by a full load. The horizontal component of the stress in the tie bars at any point is the difference in the stresses of the chord bars on each side, and the vertical component is the post stress.

Chord bars are frequently not allowed to be placed next to each other in the same direction, owing to the difficulty of painting between them. They are, therefore, in couples, one in one direction and the next in the other direction. The lighter bar is always placed outside, so as to make the first moment, which is often the greatest, as small as possible.

All vacant spaces between pins should be filled by cast or wrought iron fillers.

In packing chords, and figuring the length of pin, it is customary to allow $\frac{1}{16}$ of an inch clearance for every thickness of metal on pin.

A cast-iron filler is never used for a space less than $\frac{15}{16}$ of an inch, after all clearances are allowed for. Over $\frac{15}{16}$ of an inch, we may use cast-iron fillers, and allow $\frac{3}{16}''$ for clearance, on account of the roughness of the casting.

If we use wrought-iron fillers, it is usual to allow $\frac{1}{16}''$ for clearance.

If pilot-points are used the ends of the pin are smaller in diameter than the pin, have a thread cut on them, and a hexagonal nut is screwed on, as shown in Fig. 268, Plate 20. The nut overlaps the shoulder of the pin $\frac{1}{4}''$ on each end, making $\frac{1}{2}''$ to be added to the figured length of pin, including all clearances, in order to find the length from shoulder to shoulder.

The following examples show how to test for size of pin at various points.

EXAMPLE 1.—*In the centre panel of a bridge truss, we have 4 chord bars 7" by $1\frac{1}{2}''$, two at one end of pin and two at the other, with post between. In the next panel we have also 4 bars, 7" by 1", two on one side and two on the other side of post. We have also, on each side of centre of pin, a tie 1" thick. The tie is packed close to the inner re-enforcing plate of post, making the clearance between it and the next chord bar $1\frac{1}{2}''$. The chords are all packed snug, the lightest one on the outside, then a heavy and light one alternately. If the working stress $\sigma = 10000$ lbs. per square inch, what size pin is required?*

The area of each chord bar on one side is $7 \times 1\frac{1}{2} = 10.5$ sq. inches, and on the other side $7 \times 1 = 7$ sq. inches. We have, then, $P_1 = P_3 = 7 \times 10000 = 70000$ lbs., and $P_2 = P_4 = 10.5 \times 10000 = 105000$ lbs. The horizontal component of the tie stress is $2 \times 105000 - 2 \times 70000 = 70000 = P_5$. The distances apart are $l_1 = l_2 = l_3 = \frac{1}{2}(1\frac{1}{2} + 1) = 1\frac{1}{4}''$ and $l_4 = \frac{1}{2}(1\frac{1}{2} + 1) + 1\frac{1}{2} = 2\frac{3}{4}''$.

Then the moment at P_2 is $P_1 l_1 = 70000 \times 1\frac{1}{4} = 87500$ inch lbs.

at P_3 we have $87500 + (P_1 - P_2(l_2 = 87500 - 43750 = 43750$ inch lbs.,

at P_4 we have $43750 + (P_1 - P_2 + P_3)l_3 = 43750 + 43750 = 87500$ inch lbs.,

at P_5 we have $87500 + (P_1 - P_2 + P_3 - P_4)l_4 = 105000$ inch lbs.

The maximum moment then is at P_5 and equal to

$$M = M_H = 105000 \text{ inch lbs.}$$

From the Table, page 427, we see that this will require an iron pin of about $4\frac{1}{4}''$ diameter.

But for a bar 7" deep, we have already seen that if the diameter of pin is less than $\frac{3}{4}w = 5\frac{1}{4}''$ in this case, the head must be thickened for safe bearing. If, then, we use this diameter of $4\frac{1}{4}''$, we have (page 429) for the thickness of head,

$$t = \frac{3wt_1}{4d} = \frac{3 \times 7 \times 1}{17} = 1\frac{4}{17}'', \text{ and}$$

$$t = \frac{3 \times 7 \times 1\frac{1}{2}}{17} = 1\frac{14}{17}'',$$

for the thickness of heads of eye-bars. These thicknesses would increase the moment and make a new determination of the size of pin necessary.

If we take the diameter at $5\frac{1}{4}''$, the heads need not be thicker than the bars. *We should always make this test for bearing.* The pin can be ordered $5\frac{3}{8}''$ commercial size.

EXAMPLE 2.—*Suppose the same arrangement as in the preceding example, but the bars to be 5" by $1\frac{3}{8}''$ and 5" by $1\frac{1}{4}''$. The tie is $1\frac{5}{16}''$ thick, and centre distance of its bearing from bearing of adjacent chord $1\frac{1}{2}''$. The outside bar is then $1\frac{1}{4}''$, the next $1\frac{3}{8}''$, the next $1\frac{1}{4}''$, the next $1\frac{3}{8}''$, and finally, with a clearance of $1\frac{1}{2}''$, comes the tie. The chord bars are packed snug. If the working stress $\sigma = 10000$ lbs., what size pin is required?*

The area of each bar on one side is $5 \times 1\frac{3}{8} = 6\frac{7}{8}$ sq. inches, and on the other side $5 \times 1\frac{1}{4} = 6\frac{1}{4}$ sq. inches. Putting the lightest outside and alternating, we have $P_1 = P_3 = 6\frac{1}{4} \times 10000 = 625000$, and

$$P_2 = P_4 = 6\frac{7}{8} \times 10000 = 68750 \text{ lbs.} \quad P_5 = 2 \times 68750 - 2 \times 62500 = 12500 \text{ lbs.}$$

The distances are $l_1 = l_2 = l_3 = \frac{1}{2}(1\frac{1}{4} + 1\frac{3}{8}) = 1\frac{5}{16}$ and $l_4 = \frac{1}{2}(1\frac{3}{8} + 1\frac{5}{16}) + 1\frac{1}{2} = 2\frac{11}{32}''$.

The moment at P_2 is $P_1 l_1 = 62500 \times 1\frac{5}{16} = 82031$ inch lbs.

At P_3 we have $82031 + (P_1 - P_2)l_2 = 73828$ inch lbs.

At P_4 we have $73828 + (P_1 - P_2 + P_3)l_3 = 147656$ inch lbs.

At P_5 we have $147656 + (P_1 - P_2 + P_3 - P_4)l_4 = 114453$ lbs.

The maximum moment is then at P_4, and is equal to 147656. From the Table, page 427, this calls for a pin $4\frac{3}{4}''$ diameter. The least allowable diameter is $\frac{3}{4}w = 3\frac{3}{4}''$. The heads of bars do not require, therefore, to be thickened, and $4\frac{3}{4}''$ diameter may be taken. This gives $4\frac{7}{8}''$ commercial size.

SIZE OF PIN AT SECOND LOWER JOINT FROM END.—At this joint we must take into account M_V or the moment of the vertical forces. We have then

$$M = \sqrt{M_H^2 + M_V^2}.$$

EXAMPLE.—*Suppose we have 4 chord bars 4″ by $1\frac{3}{16}$″ on one side, and on the other 2 chord bars 4″ by $1\frac{7}{16}$″. The ties are $1\frac{9}{16}$″ thick. The tie is packed close to the post channel, the thickness of which, including the re-enforcing plate, is $\frac{7}{8}$″. The bars are packed snug. The vertical compression in the half post is 40000 lbs. for full loading. What is the size of pin required, taking the working stress σ at 10000 lbs.*

We have here at each end of pin, 2 chord bars on one side, and one bar between them on the other. Then $P_1 = P_3 = 4 \times 1\frac{3}{16} \times 10000 = 47500$, and $P_2 = 4 \times 1\frac{7}{16} \times 10000 = 57500$. The horizontal component of the tie stress is $P_4 = 2 \times 47500 - 57500 = 37500$ lbs.

The distances are $l_1 = l_2 = \frac{1}{2}(1\frac{3}{16} + 1\frac{7}{16}) = 1\frac{5}{16}$″, $l_3 = \frac{1}{2}(1\frac{3}{16} + 1\frac{9}{16}) + \frac{7}{8} = 2\frac{1}{4}$.

We have, then, at P_2 the moment $P_1 l_1 = 47500 \times 1\frac{5}{16} = 62344$ inch lbs.

At P_3 we have $62344 + (P_1 - P_2)l_2 = 49219$ inch lbs.

At P_4 we have $49219 + (P_1 - P_2 + P_3)l_3 = 133594$ inch lbs.

The maximum horizontal moment then is $M_H = 133594$ inch lbs. $= 66.797$ inch tons.

The vertical compression in post is 40000 lbs. Its lever arm is $\frac{1}{2}(1\frac{9}{16} + \frac{7}{8}) = 1\frac{7}{32}$. Hence $M_V = 40000 \times 1\frac{7}{32} = 48750$ inch lbs. $= 24.375$ inch tons.

The resultant moment is

$$M = \sqrt{M_H^2 + M_V^2} \times \sqrt{(66.8)^2 + (24.4)^2} = 71.11 \text{ inch tons} = 142220 \text{ inch lbs.}$$

This calls for a pin $4\frac{5}{8}$″ diameter, or $4\frac{3}{4}$″ commercial size.

The least diameter allowable is $\frac{3}{4}w = 3$″. Hence the bearing is abundant.

SIZE OF PIN AT FIRST LOWER JOINT FROM END AND AT END.—At this joint there are no ties or counters. We have the pin passing through the chord bars and hip vertical only. The chord bars on each side are equal and equal in number. The horizontal moment, then, is simply the stress on either side of pin at one end of pin, multiplied by the thickness of a chord bar.

If the cross-girder is riveted to the hip vertical above the pin, there is no vertical moment. If it is hung on floor-beam hangers, the vertical moment is the load supported by a hanger × by the distance from centre of bearing of a hanger to centre of bearing of hip vertical.

EXAMPLE 1.—*Suppose we have 2 bars in the first two panels, 4 by $1\frac{7}{16}$″, or 4 bars in all, one pair at one end of pin and one pair at the other. If the cross-girder is riveted to the hip vertical, and the working stress $\sigma = 10000$ lbs., what size of pin is required?*

The stress on one side of pin at one end, in one direction is $P_1 = 4 \times 1\frac{7}{16} \times 10000 = 57500$ lbs., and the stress on the other side at one end, in the other direction, is $P_2 = 57500$ lbs. The distance $l_1 = 1\frac{7}{16}$″. Hence the horizontal moment is $M_H = 57500 \times 1\frac{7}{16} = 82656$ inch lbs. From the Table, page 427, this calls for a pin $3\frac{7}{8}$″. The least allowable for bearing is $\frac{3}{4}w = \frac{3}{4} \times 4 = 3$″. Hence $3\frac{7}{8}$″ can be used.

This diameter may be used also for the end pin.

EXAMPLE 2.—*Suppose at the same joint the load sustained by a beam hanger to be 32000 lbs., and the distance from centre of bearing of a beam hanger to centre of hip vertical $1\frac{1}{2}$″.*

Then

$$M_V = 32000 \times 1\frac{1}{2} = 48000 \text{ inch lbs.} = 24 \text{ inch tons.}$$

We have already found $M_H = 82656$ inch lbs. $= 41.328$ inch tons.

Hence, $$M = \sqrt{M_H^2 + M_V^2} = \sqrt{(41.33)^2 + (24)^2} = 47.79 \text{ inch tons} = 95580 \text{ inch lbs.}$$

This calls, from Table page 427, for a 4″ pin, or $4\frac{1}{8}$″ commercial size. The least allowable pin is $\frac{3}{4}w = 3$″, hence we need not increase thickness of head.

For the end pin, we have the diameter given in the preceding example.

SIZE OF PIN AT ANY INTERMEDIATE TOP CHORD JOINT.—Any pin in the top chord is acted upon simply by the full stress of the main ties through which it passes. The horizontal component of the tie stress gives the chord stress, and the vertical component

the post stress. The main ties are packed close to the post end on the inside of post, and the counters, if any, between the main ties.

The horizontal moment, therefore, is the horizontal component of the stress in one main tie, multiplied by the distance between the tie and *chord bearings.*

The vertical moment is the vertical component of the stress in one main tie, multiplied by the distance between the tie and *post bearings.*

EXAMPLE.—*Suppose we have two main ties 5" by $1\frac{1}{8}$", the distance from centre of tie bearing to centre of chord bearing being $2\frac{5}{16}$", and the distance from centre of tie bearing to centre of post bearing being $1\frac{1}{4}$". If the working stress $\sigma = 10000$ lbs., and the angle of tie with vertical 33° 11', what size of pin is required?*

We have $\sin 33° 11' = 0.547$, and $\cos 33° 11' = 0.837$. The horizontal component of the tie stress is then $5 \times 1\frac{1}{8} \times 10000 \times 0.547 = 30769$ lbs., and the vertical component is $5 \times 1\frac{1}{8} \times 10000 \times 0.837 = 47081$ lbs.

Hence the horizontal moment is $M_H = 30769 \times 2\frac{5}{16} = 71153$ inch lbs. $= 35.576$ inch tons, and the vertical moment is $M_V = 47081 \times 1\frac{1}{4} = 58851$ inch lbs. $= 29.425$ inch tons.

The maximum moment then is

$$M = \sqrt{M_H^2 + M_V^2} = \sqrt{(35.6)^2 + (29.4)^2} = 46.17 \text{ inch tons} = 92340 \text{ inch lbs.}$$

From the Table, page 427, this calls for a pin 4" diameter, or $4\frac{1}{8}$" commercial size.

SIZE OF PIN AT HIP JOINT.—At the hip joint we have no post, but simply one, or at most two, hip verticals. The hip vertical is at the centre of pin, if there is but one, or packed as close to tie as possible on each side, if there are two. In either case, the pressure upon the chord bearing, due to the stress in the hip verticals, is *one half* of the full panel load for the truss. We have also the vertical component of the stress in a main tie, or if two ties meet at the hip on each end of pin, as is the case for a *double system,* then the sum of the vertical components of each. The vertical moment is then found as for a beam supported at the ends, with given vertical forces at given points. The horizontal moment is as before the horizontal component of the tie stress multiplied by the distance between the chord and tie bearing.

EXAMPLE.—*Suppose at the hip we have two main ties, 5" by $1\frac{9}{16}$", and one hip vertical at the centre. Let the load supported by the hip vertical be 60000 lbs., the distance between chord and tie bearing be $1\frac{1}{2}$", and between tie and hip vertical bearing 3". If the working stress σ is 10000 lbs. and the angle of ties with vertical 33° 11', what size of pin is required?*

One half of the hip vertical stress acts upon the chord bearing, or 30000 lbs. We have $\sin 33° 11' = 0.547$ and $\cos 33° 11' = 0.837$. The vertical component of the tie stress is $5 \times 1\frac{9}{16} \times 10000 \times 0.837 = 44204$ lbs. The total pressure on chord bearing is $44204 + 30000 = 74204$ lbs. The moment at centre of hip vertical is

$$M_V = 74204 \times 4\frac{1}{2} - 44204 \times 3 = 20136 \text{ inch lbs.} = 100.653 \text{ inch tons.}$$

The horizontal component of the tie stress is

$$5 \times 1\frac{9}{16} \times 10000 \times 0.547 = 42734 \text{ lbs.}$$

The horizontal moment is $M_H = 42734 \times 1\frac{1}{2} = 64101$ inch lbs. $= 32$ inch tons.

The maximum moment is

$$M = \sqrt{M_H^2 + M_V^2} = \sqrt{100^2 + 32^2} = 105 \text{ inch tons} = 210000 \text{ inch lbs.}$$

From the Table, this calls for a pin $5\frac{1}{2}$" diameter, or $5\frac{5}{8}$" commercial size.

TABLE FOR PINS.—We give below the Table referred to repeatedly in the preceding examples. The first column gives size of pin. The second the proper bearing for each size for one ton stress. This will enable us to find thickness of re-enforcing plates. The third and fourth give the bending moment for iron and steel pins.

PIN TABLE I.

LINEAL BEARING PER TON AND MAXIMUM BENDING MOMENT FOR PINS, FOR FIBRE STRESS OF 15000 LBS. IRON. AND 20000 LBS. STEEL.

Lineal bearing in inches per ton $= \dfrac{1}{6.25d}$. Max. bending moment $= \dfrac{\pi R d^3}{32}$.

Least allowable diameter without thickening the head $= \frac{3}{4}w$.

Diameter of pin in inches.	Lineal bearing on pin in inches per ton.	Moment for $R = 15000$ for iron.	Moment for $R = 20000$ for steel.	Diameter of pin in inches.	Lineal bearing on pin in inches per ton.	Moment for $R = 15000$ for iron.	Moment for $R = 20000$ for steel.
1	0.16	1470	1960	4	0.04	94200	125700
$1\frac{1}{8}$	0.142	2100	2800	$4\frac{1}{8}$	0.038	103400	137800
$1\frac{1}{4}$	0.128	2880	3830	$4\frac{1}{4}$	0.038	113000	150700
$1\frac{3}{8}$	0.116	3830	5100	$4\frac{3}{8}$	0.037	123000	164400
$1\frac{1}{2}$	0.106	4970	6630	$4\frac{1}{2}$	0.035	134200	178900
$1\frac{5}{8}$	0.098	6320	8430	$4\frac{5}{8}$	0.034	145700	194300
$1\frac{3}{4}$	0.091	7890	10500	$4\frac{3}{4}$	0.034	157800	210400
$1\frac{7}{8}$	0.085	9710	12900	$4\frac{7}{8}$	0.033	170600	227500
2	0.08	11800	15700	5	0.032	184100	245400
$2\frac{1}{8}$	0.075	14100	18800	$5\frac{1}{8}$	0.031	198200	264300
$2\frac{1}{4}$	0.071	16800	22400	$5\frac{1}{4}$	0 03	213100	284100
$2\frac{3}{8}$	0.067	19700	26300	$5\frac{3}{8}$	0.03	228700	304900
$2\frac{1}{2}$	0.064	23000	30700	$5\frac{1}{2}$	0.029	245000	326700
$2\frac{5}{8}$	0.061	26600	35500	$5\frac{5}{8}$	0.028	262100	349500
$2\frac{3}{4}$	0.058	30600	40800	$5\frac{3}{4}$	0.028	280000	373300
$2\frac{7}{8}$	0.056	35000	46700	$5\frac{7}{8}$	0.027	298600	398200
3	0.053	39800	53000	6	0.026	318100	424100
$3\frac{1}{8}$	0.051	44900	59900	$6\frac{1}{8}$	0.026	338400	451200
$3\frac{1}{4}$	0.049	50600	67400	$6\frac{1}{4}$	0.025	359500	479400
$3\frac{3}{8}$	0.047	56600	75500	$6\frac{3}{8}$	0 025	381500	508700
$3\frac{1}{2}$	0.046	63100	84200	$6\frac{1}{2}$	0.025	404400	539200
$3\frac{5}{8}$	0.044	70100	93500	$6\frac{5}{8}$	0.024	428200	570900
$3\frac{3}{4}$	0.042	77700	103500	$6\frac{3}{4}$	0.023	452900	603900
$3\frac{7}{8}$	0.041	85700	114200	$6\frac{7}{8}$	0.023	478500	638000

EYE-BAR HEADS.—An eye-bar should be so proportioned that it will break first in the body rather than in the eye or head. Many experiments have been made to determine the proper relative dimensions of head and bar.

The following simple formulas agree well with these experiments: Let D be the diameter of the head, d the diameter of pin, and w the depth of bar, then for *thickened heads*, or for

$$d < \tfrac{3}{4}w,$$

$D = d + 1.5w$, and *thickness of head*, $t = \dfrac{3wt_1}{4d}$, where t_1 is the thickness of bar.

For heads the same thickness as bar, or for

$$d > \tfrac{3}{4}w,$$

$$D = 1.25d + 1.29w, \text{ and } \textit{thickness of head},\ t = t_1.$$

As to the shape of head, it is nearly always made circular. Prof. Burr, in his *Stresses in Bridge and Roof Trusses*, gives a method of laying down an eye-bar head, as determined

by an extensive series of experiments, which is stated to have stood the test of long American experience, but is not now in general use.

As modified by the formulas just given, it may be given as follows: Make DK equal to the value of D as found from the formulas just given, and draw the semi-circle $DRBK$.

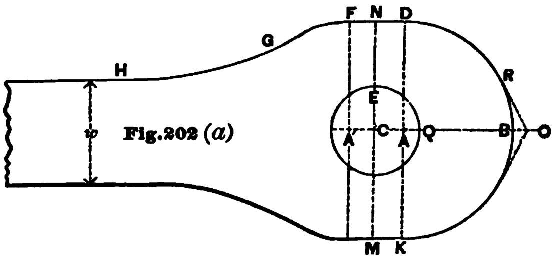


Fig. 202 (a)

Take the distance $QB = 0.87w$, that is, lay off $BC = 087w + \frac{d}{2}$ and draw the pin. Make $CA' = CA$. The curve GF is drawn with the centre A' and radius AD. GH is any curve with long radius, joining GF gradually with the body of the bar. HG should be gradual in order that there may be a large amount of metal in the vicinity of CG, for there the metal is subjected to flexure as well as direct tension. FD is a straight line parallel to the axis of the bar.

We give in the following Table the diameter of the eye D, for different values of depth of bar and size of pin, according to the preceding formulas; also the length of bar necessary to make an eye. This will be found useful in estimating the weight of iron in an eye-bar. These values we have taken approximately from Tables kindly furnished us by Jos. M. Wilson, C. E. In the column for diameter of eye, for each value of depth of bar, the value of D enclosed by lines is that for which the thickness of head is just equal to the thickness of bar, or $d = \frac{3}{4}w$. For all diameters less than this, the head must be thicker than the bar. Thus for depth of bar $w = 5''$, for all diameters of pin less than $3\frac{3}{4}$, the head must be thicker than the bar. For greater diameters than $3\frac{3}{4}$, the head and bar have the same thickness.

Different companies have different dies for heads, and it is only necessary that the designer shall know the form and size of head he has to expect. These are given from Pin Table II., or some similar Table furnished by the company. Specifications require only that upon being tested to destruction, the bar shall break in its body rather than in its head, and leave the form and size unspecified.

PIN TABLE II.

When $d < \frac{3}{4}w$, $D = d + 1.5w$. *When* $d > \frac{3}{4}w$, $D = 1.25a + 1.29w$.

$t = \frac{3wt_1}{4d}$. d = *diameter of pin in inches.* D = *diameter of eye in inches.* $t = t_1$.

t = *thickness of head.* t_1 = *thickness of bar.* w = *depth of bar in inches.*

Diameter of pin d in inches.	$w = 3''$.		$w = 4''$.		$w = 5''$.		$w = 6''$.		$w = 7''$.		$w = 8''$.		$w = 9''$.		$w = 10''$.	
	D.	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.
2	6.5	1′ 4″	8.00	0′ 11″												
2¼	6.68	1′ 5″	8.25	1′ 0″												
2½	7.00	1′ 7″	8.50	1′ 1″												
2¾	7.31	1′ 8″	8.75	1′ 2″												
3	7.62	1′ 10″	9.00	1′ 3″	10.5	1′ 3″	12.00	1′ 4″								
3¼	7.93	2′	9.23	1′ 4″	10.75	1′ 4″	12.25	1′ 5″								
3½	8.24	2′ 2″	9.54	1′ 5″	11.00	1′ 5″	12.50	1′ 6″								
3¾	8.56	2′ 5″	9.86	1′ 6″	11.25	1′ 6″	12.75	1′ 7″								
4	8.87	2′ 7″	10.16	1′ 7″	11.45	1′ 7″	13.00	1′ 8″	14.50	1′ 8″	16.00	1′ 8″				
4¼	9.18	2′ 9″	10.47	1′ 8″	11 76	1′ 8″	13.25	1′ 9″	14.75	1′ 9″	16.25	1′ 9″				
4½	9.50	2′ 11″	10.78	1′ 9″	12.07	1′ 9″	13.50	1′ 9″	15.00	1′ 10″	16.50	1′ 10″				
4¾	9.80	3′ 2″	11.10	1′ 10″	12.39	1′ 10″	13.68	1′ 10″	15.25	1′ 11″	16.75	1′ 11″				
5	10.12	3′ 4″	11.41	1′ 11″	12.70	1′ 11″	14.00	1′ 11″	15.50	2′ 0″	17.00	2′ 0″	18.50	2′ 0″	20.00	2′ 0″
5¼	10.43	3′ 6″	11.72	2′ 1″	13.01	2′ 0″	14.30	2′ 0″	15.75	2′ 1″	17.25	2′ 1″	18.75	2′ 1″	20.25	2′ 1′
5½	10.74	3′ 8″	12.03	2′ 2″	13.32	2′ 3″	14.61	2′ 0″	15.90	2′ 2″	17.50	2′ 2″	19.00	2′ 2″	20.50	2′ 2′
5¾	....	....	....	....	13.64	2′ 4″	14.93	2′ 1″	16.22	2′ 3″	17.75	2′ 3″	19.25	2′ 3″	20.75	2′ 3″
6	....	....	....	......	13.95	2′ 5″	15.24	2′ 2″	16.53	2′ 4″	18.00	2′ 4″	19.50	2′ 4″	21.00	2′ 4″
6¼	....	....	....	....	14.26	2′ 6″	15.55	2′ 3″	16.84	2′ 5″	18.13	2′ 5″	19.75	2′ 5″	21.25	2′ 5″
6½	....	....	....	....	14.57	2′ 7″	15.86	2′ 4″	17.15	2′ 6″	18.44	2′ 6″	20.00	2′ 6″	21.50	2′ 6″
6¾	....	......	....	....	....	....	....	....	17.47	2′ 7″	18.76	2′ 7″	20.25	2′ 7″	21.75	2′ 7″
7	... .	....	....	... ...	....	....	....	....	17.78	2′ 8″	19.07	2′ 8″	20.36	2′ 8″	22.00	2′ 8″
7¼	....	....	....	....	....	....	....	....	18.01	2′ 9″	19.38	2′ 9″	20.67	2′ 9″	22.25	2′ 9″
7½	....	....	....	....	....	....	....	....	18.40	2′ 10″	19.70	2′ 10″	20.98	2′ 10″	22.50	2′ 10″
7¾	....	....	....	....	....	....	....	....	....	....	....	....	21.30	2′ 11″	22.58	2′ 11″
8	....	....	.. ....	....	....	....	....	....	....	......	....	....	21.61	2′ 11″	22.90	3′ 0″
8¼	....	....	... .	....	....	....	... ...	....	....	. ....	....	....	21.92	3′ 0″	23.22	3′ 1″
8½	....	......	....	....	....	....	......	....	....	. ....	....	....	22.23	3′ 1″	23.50	3′ 2″

In the following Table we have given for different values of *d*, or for different sizes of pin, the weight of pin per inch in length, the corresponding diameter of screw at end, the size of hexagonal nut, and weight of one nut.

PIN TABLE III.

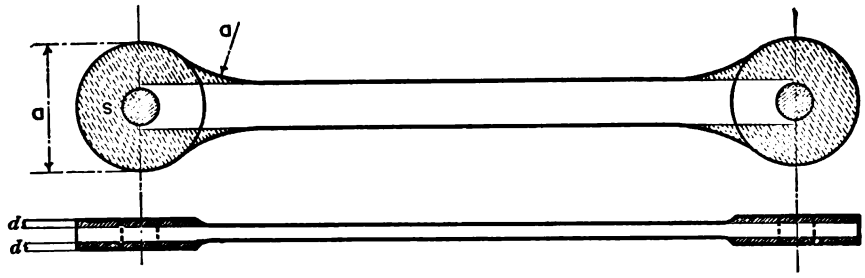
Diameter of pin.	Weight of pin per inch of length.	Diameter of screw.	Diameter of Hexagonal Nut.		Weight of one Nut.	Diameter of pin.	Weight of pin per inch of length.	Diameter of screw.	Diameter of Hexagonal Nut.		Weight of one Nut.
			Least.	Greatest.					Least.	Greatest.	
2	0.87	1½	2⅜	2.74	1.20	5½	6.60	5	7⅝	8.81	12.32
2¼	1.10	1½	2⅜	2.74	1.20	5¾	7.21	5	7⅝	8.81	12.32
2½	1.36	2	3⅛	3.61	2.07	6	7.85	5½	8⅜	9.67	14.87
2¾	1.65	2	3⅛	3.61	2.07	6¼	8.52	5½	8⅜	9.67	14.87
3	1.96	2½	3⅞	4.47	3.18	6½	9.22	6	9⅛	10.54	17.65
3¼	2.30	2½	3⅞	4.47	3.18	6¾	9.94	6	9⅛	10.54	17.65
3½	2.67	3	4⅝	5.34	4.53	7	10.69	6½	9⅞	11.40	20.67
3¾	3.07	3	4⅝	5 34	4.53	7¼	11.47	6½	9⅞	11.40	20.67
4	3.49	3½	5⅜	6.21	6.13	7½	12.27	7	10⅝	12.27	23.96
4¼	3.94	3½	5⅜	6.21	6.13	7¾	13.10	7	10⅝	12.27	23.96
4½	4.42	4	6⅛	7.07	7.95	8	13.96	7½	11⅜	13.14	27.45
4¾	4.92	4	6⅛	7.07	7.95	8¼	14.85	7½	11⅜	13.14	27.45
5	5.45	4½	6⅛	7.94	10.02	8½	15.76	8	12⅛	14.0	31.19
5¼	6.01	4½	6⅞	7.94	10.02						

From these Tables we can find the weight of chord bars including heads, and also the weight of pins and nuts, and the proper sizes for every size of pin.

The rule upon which Table 3 is based is that the *least diameter of hexagonal nut or side of square nut in rough* = $1\frac{1}{2}$ *diameter of screw* + $\frac{1}{8}''$. *The greatest diameter of hexagonal nut in rough* = $1\frac{5}{32}$ *times the least diameter. The greatest diameter of square nut in rough* = 1.414 *times the side. Height of nut* = *diameter of screw. For finished sizes subtract* $\frac{1}{16}''$. These rules are the standards of the *Franklin Institute*, recommended Dec., 1864. Tables for size and weight of nuts will be found in *Carnegie's Pocket Book.*

We give here a table for figuring the weight of eye-bars.

TABLE FOR FIGURING WEIGHT OF EYE-BARS.



SIZE OF BAR.	DIA. OF PIN.	SIZE OF HEAD.	HEAD THICKER THAN BAR.	NO. OF DIE.	CUBIC INCHES FOR 1/16" THICKNESS OF SURFACE S.	CUBIC INCHES OF THICKENED EYE 2p.	WEIGHT OF PIN 1/16" LONG.	SIZE OF BAR.	DIA. OF PIN.	SIZE OF HEAD.	HEAD THICKER THAN BAR.	NO. OF DIE.	CUBIC INCHES FOR 1/16" THICKNESS OF SURFACE S.	CUBIC INCHES OF THICKENED EYE 2p.	WEIGHT OF PIN 1/16" LONG.
"	"	"	"		cu. ins.	cu. ins.	lbs.	"	"	"	"		cu. ins.	cu. ins.	lbs.
2 × 5/8	2 1/16	4 × 7/8	1/4	206	0.581	3.143	0.0580	4 × 1 1/4	3 15/16	8 3/4 × 1 5/8	3/8	171	2.860	22.549	0.2114
2 × 3/4	2 3/16	4 1/2 × 1	1/4	207	0.768	4.000	0.0653	4 × 1	4 3/16	8 3/4 × 1 3/8	3/8	167	2.860	22.549	0.2391
2 × 7/8	2 7/16	5 × 1 1/8	1/4	204	0.985	5.000	0.0810	4 × 1	4 7/16	9 1/4 × 1 3/8	3/8	158	3.273	25.200	9.2685
2 × 1	2 9/16	5 1/2 × 1 1/4	1/4	205	1.256	6.000	0.0895	4 × 1	4 15/16	9 1/2 × 1 3/8	3/8	168	3.479	26.580	0.3324
2 1/2 × 3/4	2 1/16	4 1/2 × 1 1/16	5/16	203	0.701	4.970	0.0580	4 × 1 1/16	5 3/16	10 × 1 7/16	3/8	97	3.940	29.451	0.3669
2 1/2 × 3/4	2 11/16	5 1/2 × 1 1/8	3/8	156	1.137	8.610	0.0985	4 × 1 1/16	5 9/16	10 3/4 × 1 7/16	3/8	2	4.642	34.035	0.4219
2 1/2 × 5/8	2 15/16	6 × 7/8	1/4	77	1.421	7.070	0.1177	4 × 1	6 7/8	12 1/2 × 1 3/8	3/8	7	6.610	46.017	0.6443
2 1/2 × 3/4	3 7/16	6 1/4 × 1	1/4	160	1.552	7.670	0.1611	4 1/2 × 1 1/2	3 7/16	9 × 1 7/8	3/8	149	2.946	23.856	0.1611
3 × 3/4	2 11/16	6 × 1	1/4	172	1.286	7.070	0.0985	4 1/2 × 3/4	3 15/16	9 1/2 × 1 1/8	3/8	170	3.329	26.580	0.2114
3 × 3/4	2 15/16	7 × 1	1/4	1	1.880	9.621	0.1177	4 1/2 × 1 1/4	4 11/16	10 × 1 5/8	3/8	151	3.778	29.451	0.2996
3 × 15/16	3 7/16	7 1/4 × 1 3/16	1/4	153	2.065	10.321	0.1611	4 1/2 × 1 5/16	5 5/16	11 1/2 × 1 9/16	1/4	9	4.253	25.970	0.3854
3 × 1 1/8	3 11/16	7 1/2 × 1 3/8	1/4	152	2.222	11.045	0.1942	4 1/2 × 1 1/4	5 7/16	10 1/2 × 1 5/8	3/8	62	5.261	32.472	0.4031
3 × 1	4	8 1/2 × 1 1/8	1/8	6	3.009	7.100	0.2182	5 × 2	3 11/16	9 1/2 × 2 1/2	1/2	194	3.176	35.441	0.1942
3 × 1	4 3/16	7 3/4 × 1 1/4	1/4	169	2.409	11.793	0.2391	5 × 1	4 3/16	10 × 1 1/2	1/2	162	3.588	39.270	0.2391
3 × 1 1/8	4 3/16	8 1/8 × 1 1/2	3/8	144	2.682	19.443	0.2391	5 × 2	4 3/16	10 × 2 1/2	1/2	161	3.588	39.270	0.2391
3 × 1	5 3/16	8 5/8 × 1 3/8	3/8	137	3.060	21.909	0.3669	5 × 1	4 11/16	10 1/2 × 1 1/2	1/2	164	4.068	43.259	0.2996
3 × 1	5 1/4	10 1/2 × 1 1/8	1/8	5	4.789	10.824	0.3758	5 × 2	4 11/16	10 1/2 × 2 1/2	1/2	163	4 068	43.259	0.2996
3 1/2 × 7/8	2 11/16	7 × 1 1/4	3/8	155	1.773	14.432	0.0985	5 × 1 3/8	5 3/16	11 × 1 7/8	1/2	91	4.549	47.517	0.3669
3 1/2 × 3/4	3 3/16	7 1/2 × 1 1/8	3/8	176	2.076	16.566	0.1385	5 × 1 5/8	5 11/16	11 1/2 × 2 1/8	1/2	166	5.090	51.935	0.4411
3 1/2 × 1 3/16	3 7/16	8 × 1 7/16	1/4	154	2.435	12.566	0.1611	5 × 2	5 11/16	11 1/2 × 2 1/2	1/2	165	5.090	51.935	0.4411
3 1/2 × 7/8	3 15/16	8 1/4 × 1 1/4	3/8	175	2.618	20.046	0.2114	5 × 1 3/4	6 3/16	12 × 2 1/4	1/2	93	5.606	56.548	0.5219
3 1/2 × 1	4	9 1/4 × 1 1/8	1/8	4	3.406	8.400	0.2182	5 × 1 3/4	6 11/16	12 1/2 × 2 1/4	1/2	71	6.195	61.359	0.6099
3 1/2 × 3/4	4 7/16	8 1/2 × 1 1/8	3/8	157	2.781	7.093	0.2685	6 × 1 3/4	4 9/16	11 × 2 3/8	5/8	178	4.147	59.396	0 2841
3 1/2 × 1 3/16	4 9/16	9 × 1 9/16	3/8	8	3.197	23.856	0.2841	6 × 2	4 15/16	12 × 2 5/8	5/8	173	5.240	70.686	0.3324
3 1/2 × 1	5 1/4	11 × 1 1/8	1/8	3	5.135	11.879	0.3758	6 × 2 3/8	4 15/16	12 × 3	5/8	174	5.240	70.686	0.3324
4 × 1	3	7 1/4 × 1 3/8	3/8	159	1.815	15.768	0.1227	6 × 1 3/4	6 5/16	13 × 2 3/8	5/8	68	6.304	82.958	0.5432
4 × 1	3 1/16	7 1/4 × 1 3/8	3/8	177	2.089	17.691	0.1279	6 × 1 5/8	6 15/16	14 × 2 1/4	5/8	179	7.510	96 211	0.6563
4 × 1 1/4	3 7/16	8 1/2 × 1 5/8	3/8	150	2.629	21.279	0.1611	6 × 2 1/2	6 13/16	15 × 2 15/16	7/16	10	8.888	77.313	0.6328

EXAMPLE.—4" × 1" bar — 3" pin — 7 1/4" × 1 3/8" head — 20′—0" *c.* of pins.

Thickness of bar = 1" = 16/16". In table find $S = 1.815$; 1.815 × 16 = 29.040 cu. ins. for 1 eye of same thickness as bar.
in table find, $2p$ = 15.768 for additional thickness of 1 eye.
44.808 cub. ins.

Section of bar = 4" × 1" = 4 sq. ins.; $\frac{44.808}{4}$ = 11".2 to be added to dist. *c.* of pins to make 1 eye.

Total length of 4" × 1" bar needed = 20′ + (2 × 11".2) = say 21′— 10 1/2" = 21′.875.
Weight of 4" × 1" bar per foot (*Carnegie Handbook*) = 13.33 lbs.; 21.875 × 13.33 = 291.6 lbs.

Weight of 3" pin 1/16" long given in table = 0.1227; 2 × 1 3/8" = 2 3/4" = $\frac{44''}{16}$.

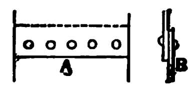
0.1227 × 44 = 5.4 = weight of cylinder bored out for both pins.
Weight of finished bar = 291.6 — 5.4 = 286.2 lbs.

CHAPTER V.

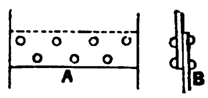
RIVETING.

In transmitting stress by rivets, it is customary to disregard the friction between the parts joined, as too uncertain an element to be relied upon to any extent. The rivets, then, must be proportioned for the entire stress transmitted.

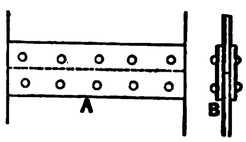
Kinds of Riveted Joints.—We may distinguish the following joints: 1st. *Simple "lap" joint, single riveted.* The Figure shows this joint, front and side view. The two plates to be joined are simply overlapped, by an amount equal to the "*lap*," and united by a single line of rivets. The distance from centre to centre of rivet, parallel to the joint, is called the "*pitch.*"



2d. "*Lap*" *joint, double riveted.*—This joint is similar to the preceding, except two lines of rivets are used. In both cases, the rivets are in *single shear.*

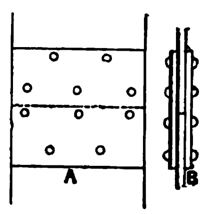


3d. "*Butt*" *joint, single riveted, two cover plates.*—Here the two plates are set end to end, making a "*butt joint,*" and a pair of "*cover plates*" are placed on the back and front, and riveted through by a single line of rivets on each side of the joint. The plates in such a joint are not suffered to touch, and the entire stress, whether tensile or compressive, is transmitted through the rivets. The thickness of the cover plates should not be less than half the thickness of the plates joined, and when this rule would give a less thickness than the least allowable, *viz.*, ¼ inch, they should have this latter thickness. Owing to deterioration of the metal by the action of the weather, no plate is used less than ¼ inch in thickness, and this, therefore, makes a limit for the thickness of the cover plates.



4th. "*Butt*" *joint, one cover plate, single riveted.*—This is the same as the preceding, except that only one cover plate is used, of the same thickness as the plates themselves.

5th. *Double riveted "butt" joint, two cover plates.*—This joint is the same as case 3, except that we have two lines of rivets on each side of the joint. The thickness of the cover plates is determined by the same considerations as in case 3. In all cases where more than one row of rivets is used, the rivets are "staggered," or so spaced that those in one row come midway between those in the next, as shown in the Figure.



6th. "*Butt*" *joint, one cover plate, double riveted.*—This is the same as the preceding case, except that there is only one cover plate, the thickness of which is equal to that of the plates themselves.

7th. Chain Riveting.—When we have more than two rows of rivets on each side of the joint, the system is called "*chain*" riveting. Such a disposition becomes necessary

when the requisite number of rivets is so great that they cannot be placed in one or two rows without weakening the plates. We give in Figs. 238, 239, and 240, different forms

Fig. 238

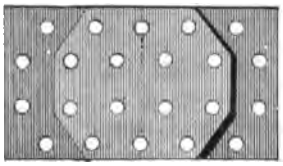


Fig. 239

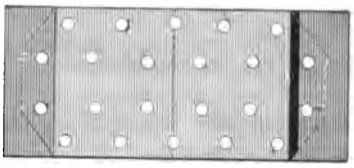
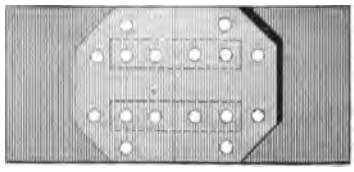


Fig. 240



of cover plate with chain riveting, and in Figs. 241, 242, 243, different methods of connections of chords by plates and angle irons.

Fig. 241

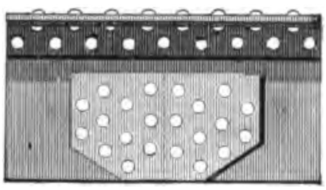


Fig. 242

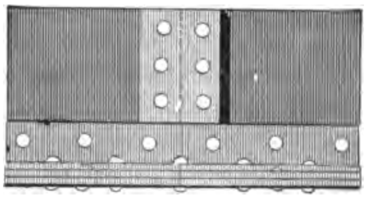
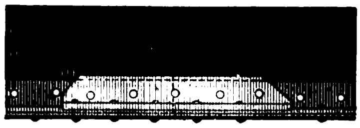


Fig. 243



THEORY OF RIVETING.—A rivet may fail by shearing across, or by being crushed. The plate may fail by rupture between the rivets, or by tearing out of the rivets at the end. The rivets should be so proportioned and spaced that the strength for any case may be equal, and the plates weakened as little as possible.

Let $b =$ the breadth of the joint in inches. This is usually a known quantity in any case. Let $t =$ the thickness of the plates to be united, in inches, also a known quantity in any given case. Let $d =$ the diameter of the rivet in inches, $m =$ the number of rivets in a row, parallel to the joint, and n the number of rivets. Then $\frac{b}{m}$ will equal the "*pitch*" c, or the distance from centre to centre of rivet parallel to the joint, the distance of the end rivets from the edge being half of the pitch.

If W is the total stress to be transmitted by the joint, and T the unit stress, or allowable stress per square inch, of the material in tension, then, since the effective area of plate in a line through a row of rivet holes parallel to the joint, is $(b - md)t$, we have

Tearing area, ... $(b - md)t = \dfrac{W}{T}$ . . . . (1)

If C is the crushing stress per square inch, then, since the bearing area of a rivet is dt, we have

Bearing area, ... $ndt = \dfrac{W}{C}$ . . . . (2)

If S is the shearing stress per square inch, then, since the shearing area of a rivet is $0.7854d^2$, we have

Shearing area:

$$\left.\begin{array}{ll}\text{Single shear, or one cover plate, ...} & 0.7854nd^2 = \dfrac{W}{S} \\ \text{Double shear, or two cover plates, ...} & 1.5708nd^2 = \dfrac{W}{S}\end{array}\right\} \quad . . . . . . (3)$$

Here we have three equations, and, in general, three quantities to be determined, *viz.*, m, n, and d.

We have, then, for the diameter in inches, for single shear,

$$d = \frac{tC}{0.7854S},$$

where t is the thickness of plate in inches, and C and S are the maximum allowable crushing and shearing stresses in lbs. per square inch.

For double shear, we substitute in place of 0.7854, 2×0.7854, for three-fold shear, 3×0.7854, and so on.

For the number of rivets we have

$$n = \frac{0.7854SW}{C^2t^2},$$

where W is the total stress transmitted by the joint, in lbs.

For the number of rivets in a row,

$$m = \frac{0.7854S}{tC}\left(b - \frac{W}{tT}\right),$$

where b = breadth of joint in inches, and T is the allowable tensile stress in lbs. per square inch.

It is customary to take $T = 10000$ lbs., $S = 7500$ lbs., $C = 12500$ lbs. Hence, for single shear,

$$d = 2.12t, \qquad n = \frac{W}{26500t^2}, \qquad m = \frac{1}{2.12t}\left(b - \frac{W}{10000t}\right).$$

PRACTICAL VALUES OF d.—SIZE OF RIVETS.—These are theoretical values, based upon the principle of equal strength, without restriction as to the diameter of the rivet. Practically, owing to risk of fracture and injury to the material, the diameter of the punch must be somewhat larger than the thickness of the plate.

Hence, we have the practical rule:

The diameter of rivet hole must not be less than the thickness of the thickest plate through which it passes.

As the least allowable thickness of plate is $\frac{1}{4}$ inch, this gives a practical lower limit of $\frac{3}{8}$ths of an inch for the rivet hole.

Rivets, however, as small as this are very rarely used. Diameters of $\frac{3}{4}$ to $\frac{7}{8}$ inch are of most frequent occurrence in girder work.

For all cross girders, stringers, and main compression members made of built sections $\frac{7}{8}$ inch rivets is the size generally used.

In other cases, we may be guided by the rule

$$d = 1\tfrac{1}{4}t + \tfrac{3}{16},$$

where the result is greater than $\frac{3}{4}''$, where d is the diameter of the rivet hole, and t the thickness of the plate in inches.

The rivet *hole* is punched the same size as the rivet and reamed out $\frac{1}{16}''$ larger to allow for the increase in size of the hot rivet. The hole must then be assumed *as* $\frac{1}{8}''$ *larger*

than the rivet, in finding net section of tension members. The diameter of the hole is to be taken, rather than that of the cold rivet, which is always smaller, but when riveted fills the hole completely. The strength is therefore governed by the size of hole, and this, therefore, is our value of *d*.

NUMBER OF RIVETS.—Guided by these considerations and rules, we may select in any case a suitable size of rivet. This done, we may easily determine the requisite number.

A rivet is considered as failing in one of two ways—either by shearing across, or by crushing. In any case, then, the diameter being fixed, we must use such a number of rivets as shall give security against these two methods of failure. In general, if we determine the number required to resist crushing, it will be found ample to resist shear. It is, however, a work of little labor to determine the number of rivets required to resist either kind of stress, and to use the greatest of these two numbers. The bearing area of a rivet is the projection of the hole upon the diameter, or is equal to the diameter of the rivet, multiplied by the thickness of the plate. If both these dimensions are taken in inches, we obtain the bearing area in square inches.

The maximum allowable *bearing pressure* per square inch varies in practice from 15000 lbs. to 12000 lbs. In girder work 12500 lbs. seems sanctioned by the best practice.

Thus, for a $\frac{1}{2}$ inch rivet and $\frac{1}{4}$ inch plate, the bearing area is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ square inch, and taking 12500 lbs. per square inch allowable pressure, we have, for the safe resistance of the rivet to crushing, 1562 lbs. If, now, the total stress to be transmitted is, say, 18750 lbs., we should require $\frac{18750}{1562} = 12$ rivets.

The allowable *shear* is taken at 7500 lbs. per square inch for single shear. Thus, in our example, the area of the rivet is $\frac{\pi d^2}{4} = \frac{3.1416 \times 1}{4 \times 4} = 0.1963$ square inches, and hence its resistance to shear would be $0.1963 \times 7500 = 1472$ lbs. If the stress transmitted is 18750 lbs., we should require, then, $\frac{18750}{1472} = 13$ rivets. In this case, then, we see at once that about 13 rivets would be required, and this number would give ample security against crushing, which only requires 12 rivets. If, however, we had two plates of $\frac{1}{4}$ inch each, on each side of a central plate of $\frac{1}{2}$ inch, the rivets would be in double shear. The stress transmitted by each outer plate would be only one half of the whole stress upon the centre plate, and we should require for shear only 7 rivets, while for bearing we should still require 12. The number in this case would then be determined by the crushing strength.

In the following Table we have given the safe shearing and bearing resistance for rivets of different sizes, and for different thicknesses of plate, calculated as in the preceding example. Having chosen, then, the size of rivet, according to the rule already given, an inspection of the Table will give at once the number required in any given case, to resist either shear or crushing. The greatest of these two is to be taken. As most practical cases are in double shear, the greatest number will usually be determined by the crushing resistance.

We must then test the rivets in at least two ways, for shear and for bearing. In some cases it may be necessary also to test for bending, as in the case of pins. This is not usually done with rivets, however, as it is assumed that the head would be sheared off before the maximum bending would occur. A case where a rivet might fail by bending is in the attachment of a stringer to a floor beam, or a floor beam to a post, when there are filling plates. The filling plate increases the leverage, and may cause bending. For this reason this construction is to be avoided if possible.

Upon *field rivets the allowable stress is usually reduced* by $\frac{1}{3}$d, or we take $\frac{1}{3}$d more than

would be given by our Table. This is to allow for the imperfection of hand work. Of course, no rivet is ever to be used in direct tension, as the heads would be torn off.

RIVET TABLE I.

SHEARING AND BEARING RESISTANCE OF RIVETS.

Diameter of Rivet in inches.		Area of Rivet in square inches.	Single Shear at 7500 lbs. per square inch.	Bearing Resistance in lbs. for different thicknesses of plate at 12500 lbs. per square inch (= diameter × thickness of plate × 12500).										
Fraction.	Decimal.			$\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{7}{16}''$	$\frac{1}{2}''$	$\frac{9}{16}''$	$\frac{5}{8}''$	$\frac{11}{16}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$\frac{7}{8}''$
$\frac{3}{8}$	0.375	0.1104	828	1170	1465	1760								
$\frac{7}{16}$	0.4375	0.1503	1130	1370	1710	2050	2390							
$\frac{1}{2}$	0.5	0.1963	1470	1560	1950	2340	2730	3125						
$\frac{9}{16}$	0.5625	0.2485	1860	1760	2200	2640	3080	3520	3955					
$\frac{5}{8}$	0.625	0.3068	2300	1950	2440	2930	3420	3900	4390	4880				
$\frac{11}{16}$	0.6875	0.3712	2780	2150	2680	3220	3760	4290	4830	5370	5908			
$\frac{3}{4}$	0.75	0.4418	3310	2340	2930	3520	4100	4690	5270	5860	6440	7030		
$\frac{13}{16}$	0.8125	0.5185	3890	2540	3170	3800	4440	5080	5710	6350	6980	7620	8250	
$\frac{7}{8}$	0.875	0.6013	4510	2730	3420	4100	4780	5470	6150	6840	7520	8200	8890	9570
$\frac{15}{16}$	0.9375	0.6903	5180	2930	3660	4390	5130	5860	6590	7320	8050	8790	9520	10250
1	1	0.7854	5890	3125	3900	4690	5470	6250	7030	7810	8590	9370	10160	10940
$1\frac{1}{16}$	1.0625	0.8866	6650	3320	4150	4980	5810	6640	7470	8300	9130	9960	10790	11620
$1\frac{1}{8}$	1.125	0.9940	7460	3520	4390	5270	6150	7030	7910	8790	9667	10550	11420	12300
$1\frac{3}{16}$	1.1875	1.1075	8310	3710	4640	5570	6490	7420	8350	9280	10200	11130	12060	12990

EXAMPLE.—*Required to unite two $\frac{1}{2}$ inch plates by a butt joint with two cover plates. The stress transmitted at the joint being 20000 lbs., what size of rivet and how many rivets are necessary?*

By our rule, we have for the diameter of rivet,

$$d = 1\tfrac{1}{4}\,t + \tfrac{3}{16} = \tfrac{5}{4} \times \tfrac{1}{2} + \tfrac{3}{16} = \tfrac{13}{16} \text{ inch.}$$

The stress in each cover plate is 10000 lbs. From our Table, we have for the resistance to shear of a $\frac{13}{16}$ inch rivet 3890 lbs. The shear will require then $\frac{10000}{3890}$ = about 3 rivets. The rivets in a butt joint with two plates are always in double shear.

From the Table we also have the bearing resistance of a $\frac{13}{16}$ inch rivet in a $\frac{1}{2}$ inch plate 5080 lbs. We shall require then for bearing $\frac{20000}{5080}$, about 4 rivets. This then is the number to be used.

RIVET SPACING, PITCH.—We thus know how to determine the size of the rivets and the required number. It remains to properly space the rivets, so that the plate shall be as strong as the rivets.

For this purpose we may take the shearing strength as equal to the tensile strength. The area then of a rivet cross section should be equal to the area of plate between the rivets. If c is the pitch or distance from centre to centre of rivets, and d the diameter of rivet, and t the thickness of plate, all in inches, and A the area of cross section of rivet in square inches, we have then

$$(c - d)\,t = A, \qquad \text{or} \qquad c = \frac{A}{t} + d,$$

for single shear. For double shear we put $2A$ in place of A, and so on.

EXAMPLE.—Thus, in the preceding example, the diameter being $\frac{13}{16}$ inch, $t = \frac{1}{2}$ inch, and the rivets in double shear, we have from our Table, $A = 0.5185$, and hence the pitch in inches is

$$C = \frac{2 \times 0.5185}{\frac{1}{2}} + \tfrac{13}{16} = 2.887 \text{ inches.}$$

This rule, however, is subject to practical restrictions. ***Rivets are not allowed to be placed nearer than* 3 *diameters, centre to centre.*** If this distance is less than 3 inches, as it usually is, *we should take* 3 *inches for the pitch.*

If the rivets were spaced nearer than 3 diameters pitch, the holes would be liable to tear out, and there is danger of injury by punching.

Rivets should not have a pitch of *more than* 6 *inches in any case*, or when the plate is in compression, 16 *times the thickness of the thinnest outside plate.*

This is to guard against buckling of the plate between rivets.

With these restrictions, we may apply the preceding formula for the pitch c. In the preceding example, therefore, we are limited practically by $3 \times \frac{13}{16} = 2.44$ inches. But if this is less than 3 inches, we should take 3 inches for the pitch, or distance from centre to centre of rivets.

If the joint is in tension, the outside limit is 6 inches. If in compression, and the cover plates are $\frac{1}{4}$ inch, the outer limit would be 4 inches. Between 3 and 6 inches, or 3 and 4 inches, then, we should space our rivets in this case.

DISTANCE FROM END AND EDGE.—The distance between the end or edge of any plate and the centre of the rivet hole, or between rows, is fixed by practice at *never less than* $1\frac{1}{4}$ *inches*, except for bars less than $2\frac{1}{2}$ inches wide, and, whenever practicable, it should be at least 2 diameters for rivets over $\frac{5}{8}''$.

Since, now, we can find the diameter of rivet, the number of rivets, the pitch, and distance from end and edge, and between rows, we can space the rivets properly in any case where the breadth of plate is known, and determine the proper size of the cover plates.

EXAMPLE.—Let us take the same example as before, viz., butt joint with two cover plates each $\frac{1}{4}$ inch and a centre plate $\frac{1}{2}$ inch. The transmitted stress 20000 lbs.

We have already found the size of rivets $\frac{13}{16}''$, the number required 4, and the spacing or pitch 3 inches. Suppose the width of plate is 8 inches.

We should have for distance from each edge at least $1\frac{1}{4}$ inches. This leaves 5.5 inches, and for 3 rivets in a row we would have a pitch of 2.75 inches. We should have to have another row of two rivets, staggered with the first row, which would give 5 rivets in all, or one more than is strictly required. Taking 3 inches for distance from end and joint and between rows, we should have 9 inches for the half length of plate, or 18 inches for whole length.

It would be better, however, to make the pitch 3 inches, and use two rows of two rivets each. This would give same length of plate, 4 rivets, and distance from each edge of 2.5 inches.

JOINTS IN COMPRESSION.—The size and number of rivets are determined for joints in compression precisely as for joints in tension, because the joints are usually not considered as in contact, and hence the rivets must transmit the stress. The thickness and length of cover plates must also be the same as in tension joints. In general, compression joints are identical in proportions with tension joints, and have the same amount of shearing and bearing area. We may, if desirable, however, space the rivets somewhat more closely at right angles to the stress or across the plate. As the metal punched out does not affect the strength of a compression joint as it does that of a tension joint, the minimum pitch is determined by the nearest distance that holes can be punched without risk of cracking or injury to the metal. The pitch, for such reasons, should never be *less than two diameters*, or one diameter from edge to edge of holes, and, in any case, never less than $1\frac{1}{4}$ inches.

COMPRESSION CHORDS.—An exception to the preceding rule, that compression joints are not to be considered in contact, is found in the case of the main compression chords of a bridge. The joints in this chord being carefully planed, are considered in close contact, and hence the faces of the abutting joints are relied upon to transmit the stress. The splice plates at the side and on top of cover plate serve, therefore, merely to resist the dis-

placing action of the live load, jolts, jars, etc., and to hold the chords in line. The rivets for such plates are not calculated.

But at the hip, although the joint there may be carefully planed, it is not relied upon, and the web is re-enforced by pin plates if greater thickness is required for pin bearing. The rivets, therefore, must be calculated for the stress transmitted from the pin through these plates to the main member itself.

SIZE OF RIVETS FOR STAY PLATES, RE-ENFORCING PLATES, LATTICE AND LACING BARS.—The preceding principles will enable us to find the size of rivets, number of rivets, pitch, distance from edge and side, distance between rows, number of rows, and length of cover or splice plate, for all tension or compression joints which occur in girder work. The same rules hold good for spacing, for the stay plates and re-enforcing plates at the ends of posts, as also for the lattice or lacing bars connecting the post channels. The *size* and corresponding number of rivets required however, for these details are best determined from the following rule, which conforms to established practice. For all post channels under 6″, the diameter of rivet employed to be not less than $\frac{1}{2}''$ or more than $\frac{5}{8}''$ or for $D < 6''$, $d \geqq \frac{1}{2}$ and $< \frac{5}{8}$.

For channels over 6″, *up to* 12″ *inclusive*, we have

$$d = \frac{1}{16} D + \frac{1}{8}.$$

For 12″ channels we have

$$d = \frac{13}{16} \text{ to } \frac{15''}{16}.$$

TOP CHORD RIVETING.—Rivets are required for the splice plates and cover plates of the top chord, and top plate of chord and batter braces; for the stay plates and re-enforcing plates of the posts, or lateral and portal struts, which like the posts are formed of channels, laced or latticed; for the top and bottom flanges of plate floor girders and stringers; for lattice or lacing bars; and sometimes for the connection of floor girders and stringers with the posts and each other. The preceding rules and principles will enable us to properly treat any given case, and we shall proceed to illustrate their application by examples such as arise in practice.

The top chord is made up, as already described, of channels with a top plate. The joint in every panel does not come at the panel point, but a little to one side, towards the nearer end of the span. By this arrangement, the pin hole goes through the solid web, and is not bored partly through each abutting end, except at the hip, where this is unavoidable.

At each joint of the top chord we have two splice plates, besides a splice plate on top, which covers the abutting ends of the two chord plates. The rivets in these are not calculated, as they simply hold the chords in place.

At the hip there are usually one or more pin plates on the inside and outside of each channel web of both the top chord and batter brace, the pin passing through all. The channels and plates of the batter brace are *not* considered as abutting against the channels and plates of the top chord. It is now customary in fact to so plane the ends that there shall be a small space between them when in position, thus giving a true hinge-joint. A small jaw plate on the outside of the top chord and the inside of the batter brace, or *vice versa*, sometimes extends beyond the pin to guard against displacement. (See Plate 12, Fig. 222, and Plate 26 at end.)

EXAMPLE.—*The upper chord at the hip is composed of two 12-inch channels, each 35 lbs. per ft., area 10.5 sq. in., and a cover plate 15″ × ¼″. The allowable stress per sq. in. which was used in dimensioning the chord is $\beta = 6978$ If the size of pin is 4⅝″, what should be the dimensions of the pin plates, and the size, number, and spacing of rivets?*

The area of top chord is $2 \times 10.5 + 3.75 = 24.75$ sq. in. $\frac{1}{2}(24.75)\beta = 86352$ lbs., the stress in one channel for which the pin bearing is calculated. The bearing value of a 4⅝″ pin @ 12500 lbs. per sq. in. in a plate 1″ thick is $12500 \times 4\frac{5}{8}'' = 57812$ lbs. From Carnegie the web of a 12″ channel @ 35 lbs. is $\frac{9}{16}''$ thick. Bearing value of pin in the web is $57812 \times \frac{9}{16} = 32519$ lbs. This leaves $86352 - 32519 = 53833$ lbs. to be carried by re-enforcing plates. $\frac{53833}{57812} = \frac{15}{16}''$ is therefore total thickness of these plates. Let us take a $\frac{9}{16}''$ plate next to the web and a ⅜″ coupler. These plates being both the same side of channel web (outside), the rivets are in single shear. The $\frac{9}{16}''$ plate takes $57812 \times \frac{9}{16} = 32519$ lbs., the remaining 21314 being taken by the ⅜″ plate. Our rule $d = \frac{1}{16}D + \frac{1}{8}$ gives in this case $\frac{12}{16} + \frac{1}{8} = \frac{7}{8}''$ for size of rivets. This is larger than would be used in the flanges, but may be used in web of channel. From Rivet Table, page 437, the bearing value of a ⅞″ rivet in a $\frac{9}{16}''$ plate = 6150 lbs., and in a ⅜″ plate = 4100 lbs. The shearing strength is 4510 lbs. Hence for the $\frac{9}{16}''$ plate we figure for shear, and for the ⅜″ plate we figure for bearing, as this gives largest number of rivets. The number required will then be $\frac{32519}{4510} = 7$ (about), and $\frac{21314}{4100} = 6$, or 13 rivets in all.

NOTE.—In the above example each of the six rivets in the ⅜″ plate can carry $4510 - 4100 = 410$ lbs. of stress, to be taken by $\frac{9}{16}''$ plate, or 2460 lbs. Hence 12 rivets would probably be sufficient, but in practice it is customary to figure as above, it being on the safe side. If, to save changing punches or drills, we used ¾″ rivets, $\frac{53833}{3310} = 17$ rivets would be required.

RIVETS IN TOP CHORD AND BATTER BRACE COVER PLATES.—The size of rivet may be chosen by our rule, $d = \frac{1}{4}t + \frac{9}{16}$, provided this gives a greater diameter than ¾″, otherwise we take $d = \frac{3}{4}''$.

It is usually customary to space the rivets 3″ pitch for a distance on each side of joint equal to about 1½ times the width of top cover plate, and 6″ pitch in the centre, unless this distance is greater than 16 times the thickness of the thinnest plate, in which case the centre rivets are spaced about 4½″ pitch.

RIVETS IN LATTICE AND LACING BARS AND RE-ENFORCING PLATES.—The rule for size of bars has been already given, page 406, and for size of rivets for bars, page 438.

The same rule holds for re-enforcing plates. The *size* of rivets is thus easily determined in any case. The rules for spacing are the same as in all the preceding cases. The *number* required may now be readily determined.

The object of the re-enforcing plates, or extension plates, at the ends of post channels, is to give sufficient bearing area upon the pin. The proper thickness, therefore, can only be determined when the size of pin is known, as well as the thickness of channel. For practical reasons, the thickness of plate cannot be less than ¼″ in any case. When this thickness is known the area can be found, because the width of plate is the same as that of the channel. The area of channel multiplied by the working stress σ, used in dimensioning the channel, will give the stress on the channel. This stress must be divided between the end channel area and plate area (or plates, if there are two to a channel) in proportion to their respective areas. We thus find the stress transmitted by a plate. Thus the stress transmitted by a plate is equal to $\frac{\text{stress on channel} \times \text{area of plate}}{\text{total area of channel and plate (or plates)}}$.

The size of rivet being then fixed as above, the number of rivets can be found by using Rivet Table I., page 436, just as in the preceding examples. The rivets may then be spaced as in the preceding examples. Making allowance for the size of pin, we may then determine the length of plate. See Plate 25, at end of work, for this detail.

EXAMPLE.—*The stress in a 9-inch 30 lbs. post channel is 6000 lbs. per sq. in. The flanges are shaved off for a certain distance from each end. The size of pin is 3⅛″. Find the sizes of re-enforcing plates needed and number of rivets.*

The area of the channel is 9 sq. in. The stress is $9 \times 6000 = 54000$ lbs. The thickness of web from Carnegie is 0.7″. The bearing value of a 3½″ pin at $12500 = 43750$. Hence web is good in bearing for $43750 \times 0.7 = 30625$ lbs. This leaves $54000 - 30625 = 23375$ lbs. for re-enforcing plates. Hence total thickness of plates must not be less than $\frac{23375}{43750} = .535$, or about $\frac{9}{16}$″.

Let us take a $\frac{5}{16}$″ plate one side, a ¼″ plate the other side. Then 6.3 sq. in. sectional area remains of the channel when flanges are shaved off. The sectional area of the two plates is $\frac{5}{16} \times 9 + \frac{1}{4} \times 8 = 4.81$ sq. in., which added to 6.3 gives 11.11 sq. in., or more than the original area of the channel. Hence cross-section is sufficient.

The stress transmitted by the plates is 23375 lbs. A $\frac{5}{16}$″ plate can transmit from a 3½″ pin 13671 lbs., and a ¼″ plate 10937 lbs., or 24608 lbs. in all. As this is slightly in excess of the stress to be transmitted (23375 lbs.), we are on the safe side. The rivets are in double shear, and as total thickness of the two pin plates is less than the thickness of channel web, we take the bearing value of the rivet in a $\frac{9}{16}$″ plate. If the web had been thinner, we would have taken bearing value of rivet in that. Our rule gives $\frac{1}{16}D + \frac{1}{8} = \frac{9}{16} + \frac{1}{8} = \frac{11}{16}$″ rivet. Bearing value of a $\frac{11}{16}$″ rivet in a $\frac{9}{16}$″ plate, from Rivet Table, is 4830 lbs. This is less than value of rivet in double shear (5560 lbs.); hence we have $\frac{24608}{4830}$ lbs. $= 5 +$ rivets, say 6 rivets. The plate can now be drawn, the pin hole located, and the rivets properly spaced.

RIVETS IN TRACK STRINGERS AND FLOOR BEAMS.—The *size* of rivets for track stringers and floor beams may be taken without discussion at ¾″ or ⅝″ for light beams. This size is to be used for flanges, for all connections of floor beams with posts, of track stringers with floor beams, and for all stiffeners, unless the rule

$$d = 1\tfrac{1}{4}t + \tfrac{3}{16}$$

gives a greater value than ¾, when this greater value may be taken. The construction of floor beams and track stringers and their connections is shown on Plate 8, Fig. 206.

The spacing of rivets need not be calculated, as that will be determined by the number required. The pitch should not exceed 6 inches, nor be less than 3 diameters. At the ends it should be least, say about 3″ for a distance of 18 or 24″, but never less than 3 inches.

We need therefore to calculate only the *number*, which will determine the spacing in accordance with these rules.

To find the number of rivets necessary to connect a track stringer with a floor girder, or a floor girder with the post, also the number of rivets to connect the flanges with the web.

Half the total load W, carried by the girder, acts at each end, or $\frac{W}{2}$. Half of this is taken by each connecting angle. $\frac{W}{4}$ is then the stress transmitted by each angle. $\frac{W}{2}$ divided by the *bearing resistance* of a rivet, taken from Rivet Table I., page 436, will give then the number of rivets to resist the *bearing pressure;* and $\frac{W}{4}$ divided by the *shearing resistance* of a rivet, will give the number required to resist shear. The greatest of these two numbers is to be taken.

For the flanges the number of rivets must be calculated for the resultant stress. If H is the horizontal stress in the flanges, and V the vertical stress at any point, the resultant stress is $\sqrt{H^2 + V^2}$. It is customary to divide the girder into a number of lengths, say 4 or 6 or 8, and find the horizontal stress at each point of division. Then the horizontal stress at the first point from the end, together with the load on the first division, gives the resultant stress at the first point of division. The *difference* between the horizontal stress at the second and first points, together with the load on the second division, gives the resultant stress at the second point from the end. The difference between the hori-

zontal stress at the third and second, together with the load on the third, gives the resultant at the third point from the end, and so on.

If W is the total load uniformly distributed, and l the length and d the effective depth of girder in feet,* the moment at any point distant x feet from the end, is $\frac{W}{2}x - \frac{W}{l}\frac{x^2}{2}$ $= \frac{Wx}{2}\left(1 - \frac{x}{l}\right)$. The horizontal flange stress at any point is then

$$\frac{Wx}{2d}\left(1 - \frac{x}{l}\right).$$

EXAMPLE.—*A railway bridge track stringer is 17 feet long and 27 inches deep. The total load is equivalent to a distributed load of 55000 lbs. The thickness of the web is $\frac{1}{4}$ inch, and of the flange angles $\frac{9}{16}$ of an inch. Find the size, number and spacing of the rivets.*

The size of rivets is $d = 1\frac{1}{4}t + \frac{3}{16} = \frac{5}{4} \times \frac{9}{16} + \frac{3}{16} = \frac{7}{8}''$. The bearing resistance for this size and $\frac{1}{4}$ inch plate is, from Table I., 2730 lbs. The horizontal flange stresses at 2.5, 5 and 8.5 feet from the end, are given by $\frac{55000x}{4.5}\left(1 - \frac{x}{17}\right)$, where for x we put 2.5, 5 and 8.5. We have then 26062 lbs., 43137 lbs., and 51944 lbs. Subtracting each from the one following, we have 26062 lbs., 17075 lbs., 8807 lbs., for the horizontal stresses to be taken by the rivets in the different lengths.

The load on the first division of 2.5 feet is 8090 lbs., on the second division of 2.5 feet is 8090 lbs., on the third division of 3.5 feet is 11320 lbs. The resultant stress then for the first division is $\sqrt{13^2 + 4^2} = 13.6$ tons $= 27200$ lbs. In the next division it is $\sqrt{(8.5)^2 + 4^2} = 9.4$ tons $= 18800$ lbs. In the next division it is $\sqrt{(4.4)^2 + (5.66)^2} =$ 7.17 tons $= 14340$ lbs. We require for bearing then, in the first 2.5 feet, $\frac{27000}{2730}$ or 10 rivets; in the next 2.5 feet, $\frac{18800}{2730}$ or 8 rivets; in the next 3.5 feet, $\frac{14340}{2730}$ or 6 rivets. We have then a pitch of about 3 inches for the first 2.5 feet, and if we take a pitch of 4 inches for the next 2.5 feet, and 5 inches for the 3.5 feet to the centre, we shall have more rivets than are needed.

Floor beams are treated in the same way. If W is the total load and a is the distance in feet from the ends to the point of attachment of the stringers, and d is the effective depth in feet,* then the flange stress at the distance a is $\frac{Wa}{2d}$. The vertical load between the end and point a, is $\frac{W}{2}$. The resultant stress is then $\sqrt{H^2 + V^2} = \sqrt{\left(\frac{Wa}{2d}\right)^2 + \left(\frac{W}{2}\right)^2}$. This stress divided by the bearing value of a rivet will give the number of rivets in the length a, or, since for two stringers *for single track*, the resultant stress beyond a is zero, the number of rivets *in the whole half length.*

See page 479 for an example.

As the load on the stringers comes through ties spaced about every 2 feet, the greatest wheel load may be taken as uniformly distributed. For the total load on stringers see page 475.

As the stringers are usually attached to the floor beams at the quarter points, the total load on the floor beam gives the same moment at centre as if it were uniformly distributed.

If the preceding remarks are carefully read and understood, and the examples worked over and checked, the student will find no difficulty in designing any riveted work.

RIVET HEADS—LENGTH FOR HEAD.—For button head rivets, if it is desired to know the size of head it may be found as follows:

Height of head $= \frac{6}{10}d$.
Radius of head $= \frac{3}{4}d + \frac{1}{16}''$.

* The distance between centres of gravity of the flange areas is the effective depth, and should be used in figuring all stresses. Usually the effective depth of an ordinary girder is about two inches less than the depth over all.

For different sizes, these rules give the following dimensions:

Diameter of rivet =	$\frac{1}{2}''$	$\frac{5}{8}''$	$\frac{3}{4}''$	$\frac{7}{8}''$	1″.
Height of head =	0.3	0.375	0.45	0.525	0.6.
Radius of head =	$\frac{7}{16}$	$\frac{17}{32}$	$\frac{5}{8}$	$\frac{23}{32}$	$\frac{13}{16}$.

For countersunk heads, the greatest diameter is the same as for button heads. The angle of countersink = 30°.

Rivets are furnished with one head. The other head is made when the rivet is put in. The length of rivet should therefore be longer than the "*grip*" or thickness of metal through which it passes, by enough to make the head. This excess of length may be determined by the rule—excess of length to make head $= d + \frac{3}{16}\left(\frac{\text{grip}}{d}\right)$. The diameter of rivet should be $\frac{1}{16}''$ less than that of the hole, so as to allow it to be inserted when hot. We give in the following Table the extra length in inches necessary to make one button head, for different diameters and length of grip.

DIAMETER OF RIVET IN INCHES.	EXTRA LENGTH IN INCHES FOR ONE BUTTON HEAD FOR DIFFERENT LENGTHS OF GRIP.			
	$\frac{3}{4}''$ and below.	$1\frac{1}{4}''$ to $\frac{3}{4}''$.	$2\frac{1}{4}''$ to $1\frac{1}{4}''$.	Above $2\frac{1}{4}''$.
	″	″	″	″
$\frac{1}{2}$	$\frac{7}{8}$	1	1	1
$\frac{5}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$
$\frac{3}{4}$	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$
	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$

To make one countersunk head, add $\frac{1}{2}''$ to the grip, or thickness of metal passed through.

Rivets have round or "button" heads, flat heads, and countersunk. If there is almost enough clearance for a button head, but not quite, a flat-head rivet, with a head $\frac{3}{8}''$ thick, may be used in preference to a countersunk rivet. Sometimes, however, either on account of clearance, or by reason of plates being in contact, it is impossible to avoid a countersunk rivet. These rivets are not nearly as capable of resisting stress as a button head, and should be counted upon very little, if at all, in figuring the number of rivets required at a joint.

Pin Plates on Compression Members.—For members in compression, there need only be an inch or two of metal between the edge of the pin hole and the end of the plate, as there is no stress, on the metal on that side of the pin.

Sometimes, indeed, the end of a compression member may simply rest on the pin. This, however, is to be avoided, as a sudden jar might cause displacement. It is for this reason that an inch or two of metal is left beyond the pin hole.

To find the number of rivets, we assume each plate to take from the web its share of the stress and figure the number as in the example, page 439.

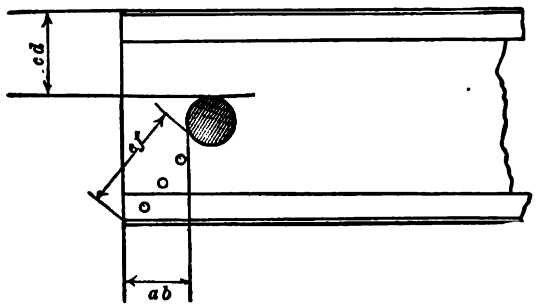
In this example, the linear bearing on pin is 0.25 + 0.7 + 0.25 = 1.2 inches. The strain is 54000 lbs., or 27 tons. If the diameter of pin were 4″, we see, from our Pin Table, page 427, that a linear bearing of 0.04 inch per ton is required. We therefore require 27 × 0.04 = 1.08 inches. As we have 1.2 inches, the bearing is sufficient. If the pin were $3\frac{1}{2}''$, the bearing would be insufficient, and the pin plates would have to be thicker.

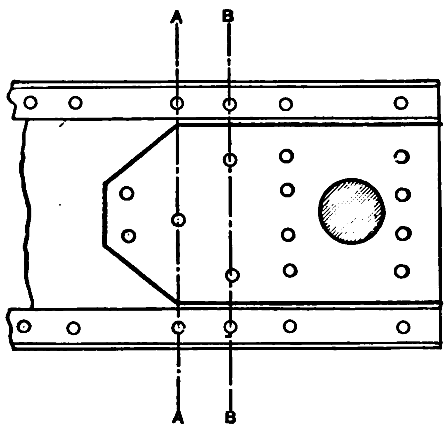
The number of rivets found for this example was 5. Knowing size of pin, we can distribute the rivets by making a sketch to scale of the end of channel, with pin hole in position.

A compression joint is easier to arrange than a tension joint, because, in the latter, rivet holes weaken the section, while, in the compression joint, the section is not impaired by the rivet holes.

PIN PLATE ON TENSION MEMBERS.—In a tension joint, stress comes on the metal behind the pin, and we should have 50 per cent. more metal in the section *ab* than along *cd*. We should also guard against reducing the strength in any direction *ef*, by putting too many rivets in a line. We should also avoid putting rivets opposite the pin hole in either direction, that is, directly above, below, or behind. They should be put to one side, above and below.

The number of rivets in the lines *AA*, *BB*, at the end of the pin plate, should be





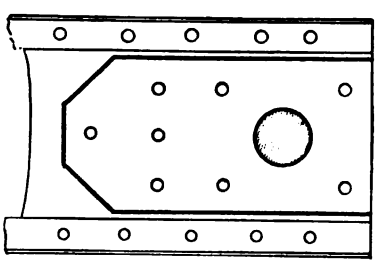
reduced gradually, in order to take out as little section as possible along *AA*, as here we do not have the full section of the pin plate.

In tension pin plates we should always put some of the rivets on the side of the pin hole next to the end.

EXAMPLE.—*Let the section of a tension member be made up of two web plates* 15" × ½", *total area* 15 *sq. in. and four angles* 3" × 3", 7 *lbs. per ft., area* 8 4 *sq. in., laced on both sides.*

The total area is 23.4 sq. in., or 11.7 sq. in. for one plate and two angles. The angles are ⅜" thick. We assume ⅞" rivets, and in calculating net section we may assume the hole 1" diameter for a ⅞" rivet. The lace bar rivets will take out one hole 1" × ⅜", or 0.38 sq. in. Only one hole is taken out, because the lace-bar rivets are of course staggered, and only one hole can come in a line at right angles to the length of the member.

The rivets attaching the angles to the web plate will take out two holes 1" × ⅞", or 1.75 sq. in. As the pin plate is 9 inches wide, we can have at most three rivet holes in a line. These take out 3 × 1 × ½ = 1.5 sq. in. The total section taken out then is 3.63 sq. in., and the net section available for tension is 11.7 − 3.63, or 8.07 sq. in. At 8000 lbs. per sq. in. this gives a stress in one jaw of 64000 lbs., or 32 tons. If pin hole is 5", we have, from Pin Table, page 427, for the necessary linear bearing 0.032 × 32 = 1.024". As the web is only ½", we must have a pin plate of at least .524", or a little over ½", thick for bearing. A ½" plate can transmit from a 5" pin 5 × ½ × 12500 = 31250 lbs. The bearing value of a ⅞" rivet in a ½" plate from our table is 5470. The shearing value is 4510. The number of rivets required then is $\frac{31250}{4510} = 7$ rivets. Two of these rivets are on the side of pin hole next to the end. Since the member is in tension, the net section of the pin plate at the pin must be sufficient to carry the stress taken by the remaining five rivets. This section is (9" − 5") × ½ = 2 sq. in., which at 8000 lbs. = 16000.



But the stress to be transmitted is 31250 − 2 × 4510 = 22230 lbs. This would require the pin plate to be 11/16" thick. In this case a ⅜" filler and a plate extending over the flanges would probably be used instead of one plate, as shown in the figure.

If one plate extends over the flanges, the width being now greater than 9", in order to obtain sufficient net section at the pin it no longer needs to be $\frac{3}{8}$" thick, a $\frac{1}{4}$" plate meeting the requirements.

This would also be true if instead of extending over the flanges the second plate is put on the back of the web, Its width being 15", the area would be still larger. In either case the rivets in the angles help secure the plate to the web; hence the number of rivets may be reduced.

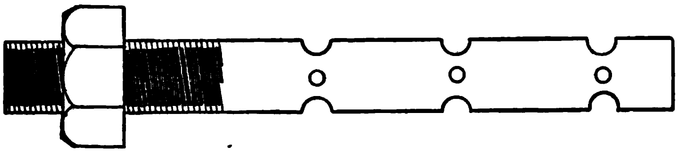
Finally, the net section of the whole member at the pin must be such that the allowable stress per sq. in. used in designing the chord shall not be exceeded. In the above example the worst possible case would be when the flanges are shaved off. We would then have about 9.8 sq. in. gross section for flanges and web, and $9 \times \frac{3}{8} + 15 \times \frac{1}{4} = 7.1$ sq. in. for pin plates, or 16.9 sq. in. total gross section. Deducting $5 \times (\frac{1}{2} + \frac{3}{8} + \frac{1}{4}) = 5\frac{5}{8}$" for pin hole, we have for net section 11.275 sq. in., which is 3.2 sq. in. more than the original section (8.07). Hence net sectional area at the pin is sufficient.

BOLTS.—Bolts should be figured for shear and bearing, and if necessary for bending, just like pins and rivets.

A "joint" bolt is simply a rough bolt. A "skinned" bolt has the roughness taken off. A "turned" bolt is turned perfectly smooth.

The diameter of the bolt and the diameter of the thread are always the same, and, by the United States Standard, the nut is of the same thickness as the bolt. In ordering the bolt, the length of thread required must be specified.

A "swedged" bolt is a bolt which has indentations on its surface, as shown in the accompanying figure. They are used for foundation bolts, and the indentations allow the melted sulphur which is poured into the hole to obtain a better grip on the bolt.



Unless ordered to the contrary, pins, bolts, and nuts always come with a right-hand thread, as shown in figure.

CHAPTER VI.

WIND BRACING—MISCELLANEOUS DETAILS.

WIND FORCE.—The wind bracing should be proportioned upon precisely the same principles as the main truss members. The only difference is in the loading assumed.

The train surface is taken at 10 square feet for every foot in length, and the wind pressure, when the train is on, at 30 lbs. per square foot, or 300 lbs. per foot of length, *plus* 30 lbs. per square foot of exposed surface of truss. The 300 lbs. per lineal foot due to the train surface is treated as a moving load, and the pressure on the exposed surface of the trusses as a fixed load. When the bridge is empty we take 50 lbs. per square foot of exposed surface as the loading, and the greatest stress by either loading is used in determining the sectional area of the bracing.

The *exposed surface of truss* is estimated by the following rule: *Add to the surface, as shown on the drawing for upper chords and posts, one and a half times the surface of the ties and twice the surface of the lower chord.**

As soon, therefore, as the design has progressed far enough for us to determine the exposed surface of truss by this rule, we multiply this exposed surface in square feet by 30 and divide by the span in feet. To the result, we add 300 lbs., and we get the load per foot for which the wind bracing is to be calculated, when the train is on the bridge.

Again, we multiply this exposed surface in square feet by 50, and divide by the span in feet, and we obtain the load per foot for bridge empty. The greatest stresses due to either loading are to be taken.

In preliminary estimates we may take the exposed surface for *both trusses* at 10 square feet per linear foot. At 30 lbs. per square foot this gives 300 lbs. per linear foot of truss, or 75 lbs. for each upper chord and 75 lbs. for each lower chord. This gives 450 lbs. per linear foot for top lateral bracing in deck bridges or bottom lateral bracing in through bridges, of which 300 lbs. is moving load. On the other chords we have 150 lbs. per lineal foot, or 75 lbs. for each chord, fixed load.

WIND BRACING.—The wind bracing consists of horizontal bracing under the floor; of horizontal bracing between the top chords in a through bridge, or the bottom chords in a deck bridge; and vertical sway bracing at every panel point of a through bridge when the truss is deep enough, and at every panel point of a deck bridge of any depth.

In through bridges the clear headway or vertical distance between the upper surface of the rails and the lowest part of the over-head bracing should be at least 12.5 feet. From 12.5 to 24 feet, we use horizontal over-head bracing only. Above 24 feet, vertical sway bracing is to be used also. Of course, *all* deck bridges have both upper and lower horizontal bracing, and vertical sway bracing also.

Of the wind load per foot found according to the preceding rule, ⅔ds may be taken as acting at the panel points at the floor, and ⅓d at the panel points of the other chord. Upon these assumptions we may find the stresses in upper and lower horizontal wind bracing. The ties and posts of the vertical sway bracing may be taken the same as those of the horizontal bracing at the centre, without special calculation.

The reasons which have led to the adoption of 30 lbs. per square foot with train, or 50 lbs. without, although wind pressures have been registered as high as 90 lbs. per square foot, are that such extreme pressures are limited to narrow belts, less than the length of ordinary spans, so that the entire span is not subjected to this pressure, and also that no

* This is because the ties are in pairs and the lower chords consist of several bars.

train would venture across a bridge during such a tornado; so that the allowance of 30 lbs. per square foot, *with train*, may give higher stresses even than 90 lbs. without, or even if not, still the stresses on bracing so proportioned, due to the extreme pressure, will be within the limits of elasticity.

The working stresses used in proportioning the wind bracing are 15,000 lbs. per square inch in the ties, and rivets, pins, and struts the same as for the main trusses. The method of proportioning ties and struts is precisely the same as for the main trusses.

DETAILS.—When the cross girders are riveted to the posts, as in Fig. 206, Plate 8, the ties for the lower horizontal bracing may be pinned to the lower flange of the cross girder, as shown in Fig. 206, and no struts are necessary, the cross girder answering the purpose of a strut.

When the cross girder is slung below the chord by beam hangers from the pin, it is often customary to pin the ties to the upper flange in the same manner. This is evidently not good construction, and it is better, though of course somewhat more expensive, to insert struts above the cross girders. These struts may be of timber, shod with iron, riveted to the posts, or may be of latticed channels, with stay plates, riveted by angle irons to the posts.

The upper horizontal bracing for medium spans may be, as in Fig. 222, Plate 12, composed of two angle irons with the ties pinned to the flanges, the pin extending through the top chord plate.

When vertical sway bracing is required, the upper struts may be made like the posts, of two channels, with webs horizontal, latticed, with stay plates. The horizontal ties may be attached to a vertical pin through the horizontal webs of the channels, and the vertical sway braces to horizontal pins passing through the extension plates of the struts and the *ends of the chord pins*. The lower or intermediate struts may also be channels, latticed, the channel webs being in a vertical plane, riveted by angles to the post, and the vertical sway braces attached by pins passing through the vertical webs of the channels.

INCREASE OF CHORD SECTION DUE TO WIND.—When the train covers the span and the wind acts, the bridge is bent sideways and the lower chord on the side away from the wind has its maximum tension increased by the tension due to the wind. The chord on each side should be able to sustain the total maximum.

Again, the compression in the windward chord at end, when the bridge is empty, due to the wind, may exceed the tension due to dead load, in which case the chord may buckle unless made to resist compression. It is well, therefore, in the last two panels, to strap the inner lower chord bars to each other, so that they may act as a strut to resist compression. (See page 392.)

UPPER AND LOWER LATERAL WIND BRACING.—The upper and lower lateral wind bracing is calculated just as for a Pratt Truss. The upper lateral system in a deck bridge and the lower lateral system in a through bridge, or pony bridge, are calculated for a dead load of 30 lbs. per square foot of exposed surface of *both trusses*, and a live load of 300 lbs. per linear foot, or a dead load of 50 lbs. per square foot of exposed surface of *both trusses*, and the greatest stresses in either case taken.

The exposed surface of truss may be found by the preceding rule, or may be assumed without calculation at 10 square feet per linear foot for *both trusses*. This is 5 square feet per linear foot for one truss, or $2\frac{1}{2}$ square feet per linear foot for each chord. At 30 lbs. per square foot, this is 75 *lbs. per linear foot for each chord*, and at 50 lbs. per square foot, it is 125 *lbs. per linear foot for each chord.*

When the panel length is known we can then easily find the panel load at each apex.

Although the train partially shelters one truss, it will be observed that this is disregarded, and each truss is considered as fully exposed, even when train is on.

For the lower lateral system in a deck bridge or the upper lateral system in a through bridge, we have simply to calculate for a dead load of 30 lbs. per square foot of exposed surface of both trusses, or 75 *lbs. per linear foot for each chord.*

The stresses in the wind braces thus obtained are to be increased for *initial tension,* page 393, since each is furnished with a turn buckle.

CENTRIFUGAL FORCE.—If the track upon the bridge is curved, the stresses in the lateral system under the train are increased by the centrifugal force.

The train load must first be reduced to an equivalent uniform load per foot; that is, a load per foot, which, spread over the whole span, will give the same moment at the centre of span as the maximum moment at centre due to the train. This is easily found.

Thus, if M is the maximum moment at the centre due to the train, and w is the equivalent load per foot, and $l =$ the span, we have

$$\frac{wl^2}{8} = M, \text{ or } w = \frac{8M}{l^2}.$$

Now, if p is the panel length, the panel load W at each panel point, is

$$W = wp = \frac{8Mp}{l^2}.$$

The centrifugal force at each panel point is then

$$C = \frac{Wv^2}{gr};$$

where v is the velocity in feet per second, $r =$ radius of curve, $g = 32\frac{1}{6}$.

We give in the following Table the values of $\frac{v^2}{gr}$ for a 1° curve, and different velocities. For any other degree multiply the tabular values by the degree of the curve.

TABLE FOR CENTRIFUGAL FORCE $C = \frac{Wv^2}{gr}$.

Values of $\frac{v^2}{gr}$ given for a 1° curve.

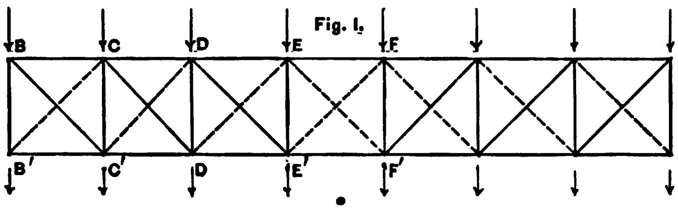
For any other degree, multiply by degree of curve.

v in miles per hour.	v in feet per sec.	$\frac{v^2}{gr}$ for 1° curve.	v in miles per hour.	v in feet per sec.	$\frac{v^2}{gr}$ for 1° curve.
10	$14\frac{2}{3}$	0.00117	40	$58\frac{2}{3}$	0.01866
15	22	0.00262	45	66	0.02361
20	$29\frac{1}{3}$	0.00467	50	$73\frac{1}{3}$	0.02915
25	$36\frac{2}{3}$	0.00729	55	$80\frac{2}{3}$	0.03527
30	44	0.01049	60	88	0.04196
35	$51\frac{1}{3}$	0.01428			

The "degree of a curve" is the angle subtended at the centre by a chord of 100 *feet.*

We can thus find the centrifugal force at each panel point for a given train, degree of curve, and assumed maximum velocity, and find the stresses in the lateral system for this loading. These stresses are to be added to the wind stresses already found, and ***initial tension*** added, page 393.

Example.—*Through bridge, span c to c 153 feet, no. of panels* = *9, panel length 17 feet, width c* to *c 16½* feet. *Find the stresses in upper and lower lateral bracing.*



We have in this case the panel load for the upper lateral system, 75 × 17 = 1275 lbs. This load acts at each apex of the windward and leeward chords. In the upper system there will be seven panels, as shown by the Fig. 1.

We have sec $\theta = 1.447$, tan $\theta = 1.046$, and hence, in the upper lateral system,

$EE' = -1275$ lbs. $\qquad DE' = +2 \times 1275 \times 1.447 = +\ 3690$ lbs.

$DD' = -3 \times 1275 = -3825$ lbs. $\qquad CD' = +4 \times 1275 \times 1.447 = +\ 7380$ "

$CC' = -5 \times 1275 = -6375$ " $\qquad BC' = +6 \times 1275 \times 1.447 = +11070$ "

$BB' = -7 \times 1275 = -8925$ " $\qquad BC = -3 \times 2550 \times 1.046 = -\ 7000$ "

$CD = -5 \times 2550 \times 1.046 = -13340$ lbs. $\qquad DE = EF = -6 \times 2550 \times 1.046 = -16000$ lbs.

The chord *BCDE* is in compression under the action of the train. The compression due to the train is increased by that due to the wind, as given above.

The stresses in the braces *BC'*, *CD'*, etc., must be increased for initial tension, page 393.

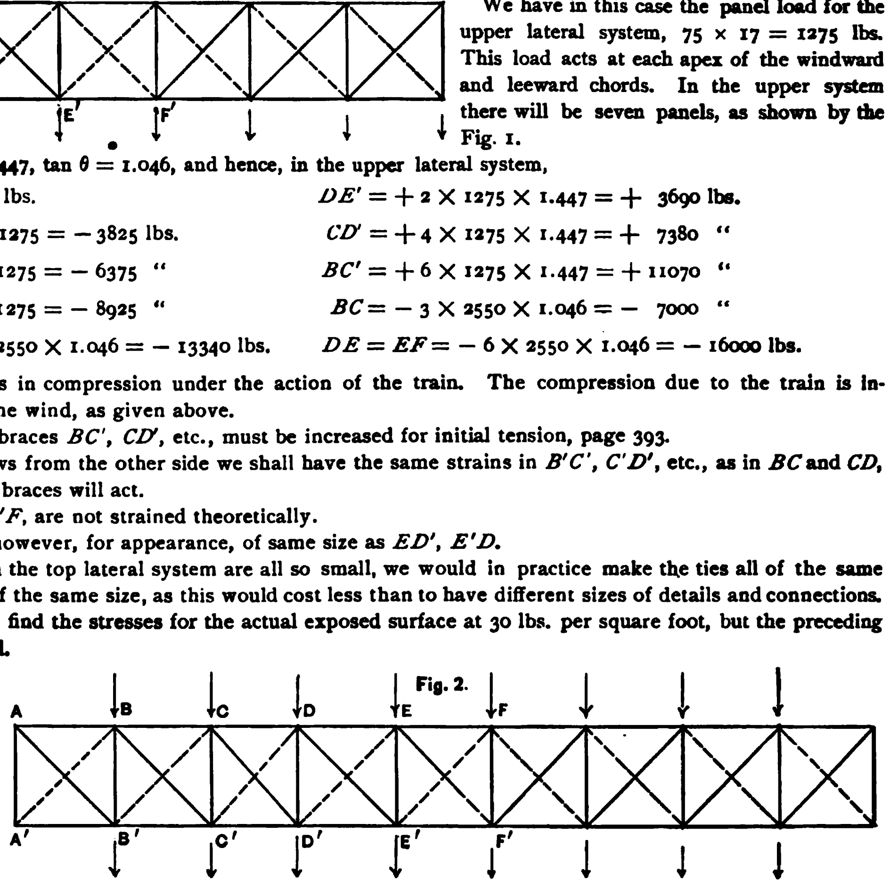
When the wind blows from the other side we shall have the same strains in *B'C'*, *C'D'*, etc., as in *BC* and *CD*, and the other system of braces will act.

The braces *EF'*, *E'F*, are not strained theoretically.

They are inserted, however, for appearance, of same size as *ED'*, *E'D*.

Since the stresses in the top lateral system are all so small, we would in practice make the ties all of the same size, and the struts all of the same size, as this would cost less than to have different sizes of details and connections.

We may, if desired, find the stresses for the actual exposed surface at 30 lbs. per square foot, but the preceding is the customary method.



For the lower lateral system, Fig. 2, we have nine panels, and the panel load for 30 lbs. per square foot will be, as before, 1275 lbs. at each apex right and left. This we treat as a dead or fixed load. The train wind load of 300 lbs. per linear foot gives a panel load of 17 × 300 = 5100 lbs. This we treat as a moving load.

We may, if desired, find the actual exposed surface, and take this at 30 lbs. per square foot, but the preceding is the customary method.

We have then

For the Chords.

$B'C' = +7650 \times 4 \times 1.046 = +32008$ lbs. $\qquad AB = -32008$ lbs.

$C'D' = +7650 \times 7 \times 1.046 = +56013$ " $\qquad BC = -56013$ "

$D'E' = +7650 \times 9 \times 1.046 = +72017$ " $\qquad CD = -72017$ "

$E'F' = +7650 \times 10 \times 1.046 = +80019$ " $\qquad DE = -80019$ "

For the Braces.

$$EF' = +\frac{10}{9} \times 5100 \times 1.447 = +8200 \text{ lbs.} \qquad DE' = +\left(2550 + \frac{15}{9} \times 5100\right)1.447 = +15989 \text{ lbs.}$$

$$CD' = \left(2 \times 2550 + \frac{21}{9} \times 5100\right)1.447 = +24599 \text{ lbs.} \qquad BC' = +\left(3 \times 2550 + \frac{28}{9} \times 5100\right)1.447 = +34028 \text{ lbs.}$$

$$AB' = +(4 \times 2550 + 4 \times 5100)1.447 = +44280 \text{ lbs.}$$

For the Struts.

$AA' = -8 \times 1275 - 4 \times 5100 = -30600$ lbs. $\quad BB' = -7 \times 1275 - \frac{28}{9} \times 5100 = -24791$ lbs.

$CC' = -5 \times 1275 - \frac{21}{9} \times 5100 = -18275$ " $\quad DD' = -3 \times 1275 - \frac{15}{9} \times 5100 = -12325$ "

$$EE' = -1275 - \frac{10}{9} \times 5100 = -6941 \text{ lbs.}$$

If we should take 50 lbs. per square foot as a fixed load, we have the panel load at each apex, right and left, 2125 lbs., and the stresses would be all less than those already found. We therefore take the latter.

When the wind blows from the other side, the other system of braces will act, and $A'F'$ will be in compression and AF in tension.

The tensile stresses in the lower chords due to the train should therefore be increased by the tension just found due to wind, and the chords designed for the combined result.

The compression in AB due to the fixed wind load of 50 lbs. per square foot, when bridge is empty, is $-4 \times 4250 \times 1.046 = -17782$ lbs.

If this compression were greater than the tension due to the dead load of the bridge itself, the chord bars in AB would have to be stiffened to take the difference, or resultant compression.

The tie rods of the lower lateral system are usually fastened to the bottom flanges of the cross-girders, thereby relieving the tensile stresses in those flanges. There need, therefore, be no bottom lateral struts at all.

If the track were on a curve, we should find the stresses due to centrifugal force as directed in the preceding article, and add them to those already found.

VERTICAL SWAY BRACING.—In deck bridges, besides the upper and lower lateral wind bracing, there is always vertical sway bracing at each panel, at right angles to the axis of the bridge. In through bridges also, if the headway allows of it, we have vertical sway bracing.

In Fig. 3, let P be the pressure concentrated at the upper panel point of a through bridge, windward and leeward. It is, according to usual assumptions, 75 lbs. per lineal ft., and hence, if p is the panel length, $P = 75p$.

If it is desired to take the actual exposed surface, P is the pressure at 30 lbs. per sq. ft. upon the surface of one panel length of top chord, one-half the surface of the diagonal braces meeting at the top chord, and one-half of the distance BC on a post. It is customary in most ordinary cases to take P as $75p$ lbs.

Let P' be the pressure concentrated at one end of the intermediate strut CC'. It is the pressure at 30 lbs. per sq. ft. upon one-half of the post. If we take the post as one foot wide, and d = depth of truss c to c,

$$P' = 30\frac{d}{2} = 15d \text{ lbs.}$$

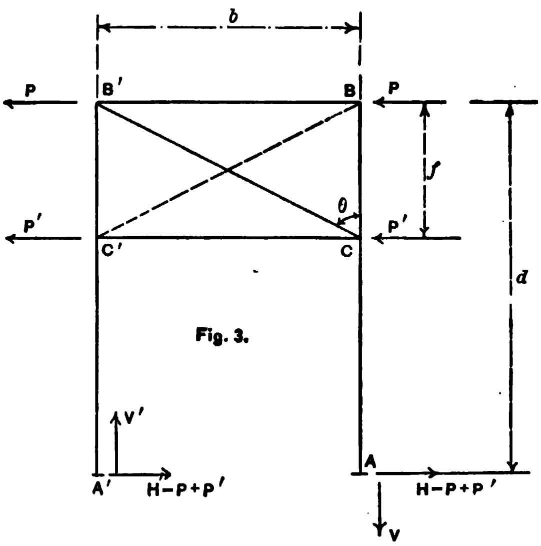


Fig. 3.

Let f = the distance BC, b = width of bridge c to c, θ = angle of vibration rods, CB' or $C'B$, with vertical.

The total pressure $2(P + P')$ is resisted by H and H', the horizontal forces at the foot of the posts, and V and V' acting vertically. V' is an increase of pressure on the lee side and acts up; V acts down on the windward side.

We have for equilibrium the three conditions,

$$2(P + P') + H + H' = 0, \qquad V + V' = 0,$$

and, taking moments about B',

$$(H + H')\,d + Vb + 2P'f = 0.$$

From the first and third of these equations, we have

$$V = \frac{2(P + P')\,d - 2P'f}{b} = -V', \quad \ldots \ldots \ldots \quad (1)$$

which is independent of the values of H and H'.

Since, then, we have three equations only, and four unknown quantities, V, V', H and H', the latter are strictly indeterminate.

It is customary to assume

$$H = H' = -(P + P'), \quad \ldots \ldots \ldots \ldots \quad (2)$$

and this assumption is probably as correct as any other that can be made.

For the stress in the vibration rod CB', we have stress in

$$CB' = +V \sec\theta = -\frac{2(P + P')d - 2P'f}{b} \sec\theta, \quad \ldots \ldots \quad (3)$$

the plus sign denoting tension.

To find the stress in the intermediate strut CC', consider it cut, and take moments about B', and we have stress in $CC' \times f + H'd + P'f = 0$; or, putting for H' its value,

$$\text{stress in } CC' = (P + P')\frac{d}{f} - P'. \quad \ldots \ldots \ldots \ldots \quad (4)$$

The maximum stress in BB' has already been found, since it is in the upper lateral system. Its stress in this case is not, therefore, needed, as it will be less than already found.

When the wind blows from the other side, we have the same compression in CC' and tension in $C'B$, instead of CB'.

The moment at C or C' on the post is, if the ends are free, $H(d - f)$, or $(P + P')(d - f)$. But it will be more correct to consider the ends as fixed, since they are rigidly attached to the cross girders. We have, therefore, the moment only one-half as much as for free ends, or

$$\text{moment at } C = \frac{1}{2}(P + P')\,(d - f).$$

The post is composed of channels latticed together. If the distance between the channels c to c is m, we have the compression on one post channel due to the bending alone, $\frac{(P + P')\,(d - f)}{2m}$. The post also has a direct compression of V, or for one channel $\frac{V}{2}$. The total compression on one post channel is, therefore,

$$\text{compression on one post channel} = \frac{V}{2} + \frac{(P + P')\,(d - f)}{2m} \quad \ldots \ldots \quad (5)$$

This is to be added to the compression due to train and weight of bridge where V is given by (1). This compression is to be added to that due to the train and the weight of the bridge itself.

Formulas (3), (4), and (5), for the vibration rod, intermediate strut and post channels,

will hold equally for the inclined portal and batter braces, if for d we put the length of batter brace $= \sqrt{d^2+p^2}$, for f the distance f_1 between upper and lower portal struts, for P' the pressure on one-half the batter brace $= P'_1$, and for P *one-fourth the sum of all the pressures concentrated at windward and leeward panel points of the upper lateral system* $= P_1$.

For the stress in the strut BB' *at the portal*, we have, then, considering this strut as part of the sway bracing only, by taking moments about C,

$$+BB' \times f_1 - P_1 f_1 - (P_1 + P'_1)(\sqrt{d^2+p^2} - f_1) = 0,$$

or

$$BB' = \frac{\sqrt{d^2+p^2}}{f_1}(P_1 + P'_1) - P'_1.$$

But this only gives the stress in BB' as part of the sway bracing. It is also part of the top lateral system, and as such has the compression $P_1 - P_e$, where P_e is the pressure concentrated at the leeward hip. Hence, *for the portal*

$$\text{stress in } BB' = \frac{\sqrt{d^2+p^2}}{f_1}(P_1 + P'_1) - P'_1 + P_1 - P_e. \quad . \quad . \quad . \quad . \quad (6)$$

where p is the panel length, d is the depth of truss, and f_1, P_1, P'_1, and P_e, have the values given above.

For the portal, (3), (4), and (5) become, therefore,

$$\text{stress in } CB' = + \frac{2(P_1 + P'_1)\sqrt{d^2+p^2} - 2P'_1 f_1}{b} \sec\theta_1; \quad . \quad . \quad . \quad . \quad (7)$$

where θ_1 is the angle made by CB' with the batter brace,

$$\text{stress in } CC' = (P_1 + P'_1)\frac{\sqrt{d^2+p^2}}{f_1} - P'_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\text{compression on one batter-brace channel} = \frac{V}{2} + \frac{(P_1 + P'_1)(\sqrt{d^2+p^2} - f_1)}{2m}; \quad . \quad (9)$$

$$\text{where } V \text{ is given by } V = \frac{2(P_1 + P'_1)\sqrt{d^2+p^2} - 2P'_1 f_1}{b}. \quad . \quad . \quad . \quad . \quad (10)$$

DECK BRIDGE.—SWAY BRACING.—For a *deck bridge*, we have simply to make $f = d$, $f_1 = \sqrt{d^2+p^2}$, and as now CC' is part of the lower lateral system, and BB' of the top, we only have to find at any intermediate panel

$$\text{stress in } CB' = + \frac{2Pd}{b}\sec\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\text{Compression on post channel} = \frac{V}{2} = \frac{2Pd}{b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

And at the end,

$$\text{stress in } BB' = 2P_1 - P_e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Knee Braces.—When, in a through bridge, there is not headway enough for sway bracing, as in Fig. 3, stiffness is obtained by the use of knee braces or brackets, as in Fig. 4.

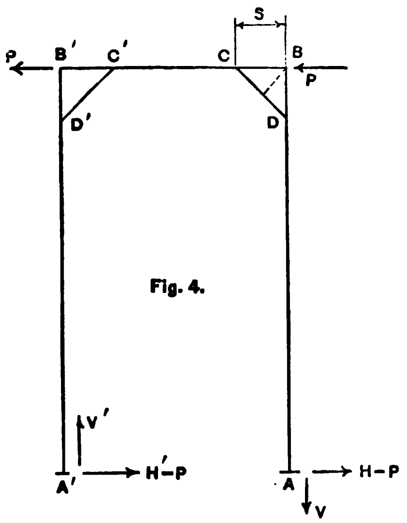


Fig. 4.

In this case, taking moments about A', we have,

$$Vb = 2Pd, \text{ or } V = \frac{2Pd}{b}, \quad . \quad . \quad . \quad . \quad (14)$$

and the compression on a post channel is $\frac{V}{2}$. This is to be added to the compression due to train and weight of bridge.

The strain in CD is found by taking moments about B.

The lever arm is $s \cos \theta$, where s is the distance CB, and θ the angle of CD with vertical.

Hence, $CD \times s \cos \theta = Pd$, or,

$$\text{stress in } CD = + \frac{Pd}{s \cos \theta}, \quad . \quad . \quad . \quad (15)$$

where the plus sign denotes tension.

There is a moment at C and C', which is equal to

$$V(b - s) - Pd = \frac{Pd}{b}(b - 2s). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

If m' is the distance c to c between the two channels of which the upper lateral strut is composed, then we have,

$$\text{compression on each channel of } BB' = \frac{Pd}{bm'}(b - 2s). \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Twice this is to be added to the compression on BB' as part of the upper lateral system.

At the portal we have to put, for d, $\sqrt{d^2 + p^2}$, and for P, P_1 as before, and θ_1, and (14), (15), and (16) become

$$V = \frac{2P_1 \sqrt{d^2 + p^2}}{b}, \qquad CD = -\frac{P_1 \sqrt{p^2 + d^2}}{s \cos \theta_1}, \qquad \frac{P_1 \sqrt{d^2 + p^2}(b - 2s)}{b}. \quad . \quad . \quad (18)$$

The compression on BB' at the portal as part of the upper lateral system is $2P_1 - P_e$, and adding to this the compression due to the moment at C, we have at the portal,

$$\text{compression in } BB' = \frac{2P_1 \sqrt{d^2 + p^2}(b - 2s)}{bm'} + 2P_1 - P_e. \quad . \quad . \quad . \quad . \quad (19)$$

If there are no knee braces, but the strut BB' is a flanged beam, rigidly fastened at the ends B and B' to the posts, the post compression is, as in the previous case, $V = \frac{2Pd}{b}$.

There is a moment at the end of the strut equal to $Vb - Pd$ or Pd. If the effective depth of the beam BB' is d', the compression in each flange is $\frac{Pd}{d'}$, or $\frac{2Pd}{d'}$ for both flanges due to this moment. This compression must be added to that in BB' as part of the upper lateral system.

DOUBLE TRACK.—The preceding holds good for wind bracing for single track. For double track we have not only the stresses, as already found, but also stresses due to transference of the load when one track only is covered by the train, the other being empty.

Thus, in Fig. 5, let W be the weight on a cross-girder, for one track loaded. Then the reaction R is given by

$$R \times 2\,(a+b) = W\,(2a+b), \text{ or}$$

$$R = W\frac{2a+b}{2\,(a+b)}.$$

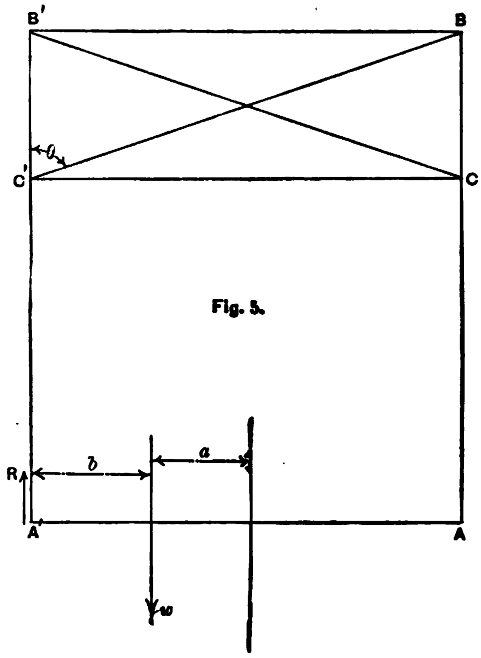


Fig. 5.

If the sway bracing were perfectly rigid, however, both trusses would have to deflect together, and each truss would carry half the load.

The weight G, transferred by the sway bracing, would then be,

$$G = R - \frac{W}{2} = \frac{Wa}{2(a+b)}.$$

The stress in the vibration rod due to this action is,

$$\text{stress in } C'B = +\frac{Wa \sec\theta}{2(a+b)}. \quad \dots\dots\dots\dots \quad (20)$$

For $C'C$, taking moments about B, we have

$$C'C \times f = G \times 2(a+b), \text{ or } C'C = \frac{Wa}{f}. \quad \dots\dots\dots \quad (21)$$

The stress in BB' is zero.
The moment at C is Wa.
If m is the distance between post channels, the compression on one channel is $\frac{Wa}{m}$.

$$\text{The compression due to bending is} = \frac{2\,Wa}{m}. \quad \dots\dots\dots \quad (22)$$

These stresses must be *added* to the wind stresses as already found.
For deck bridge, make $f = d$ in (21); (20) and (22) remain unchanged.
This action does not take place at the portal.
When knee braces are used, eq. (20) gives the stress in $D'C'$, Fig. 4.

$$\text{For the moment at } C', \text{ we have } \frac{Wa}{2(a+b)}[2(a+b)-s].$$

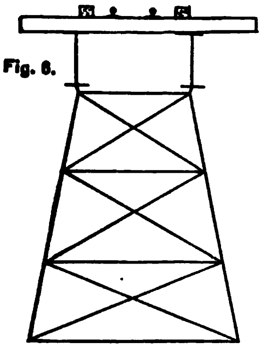
If d' is the depth of strut BB', Fig. 4, this gives a compression on the strut BB' of

$$\frac{Wa}{(a+b)d'}[2(a+b)-s].$$

These stresses must be added to wind stresses.

STRESSES IN BRACED PIERS AND TRESTLE BENTS.—A trestle "bent" is simply a pair of columns connected transversely by bracing, as shown in Fig. 6. A trestle tower consists of two bents, or four columns, united by longitudinal and transverse bracing. It is customary to unite only every other span in this manner. The usual transverse batter given to the "bent" column is 6 vertical to 1 horizontal.

Fig. 6.



In Fig. 7 we show a side view of the towers. Every other two bents are united by longitudinal bracing.

Each tower must have sufficient base, longitudinally, to be stable when standing alone without other support than its anchorage. That is, no dependence is to be placed on the girder connection between two towers at top, but the entire tower should be capable of standing alone, with the maximum wind-force on either side transverse to axis of bridge. Tower spans for high trestles are usually about 30 feet, the intermediate spans 60 feet.

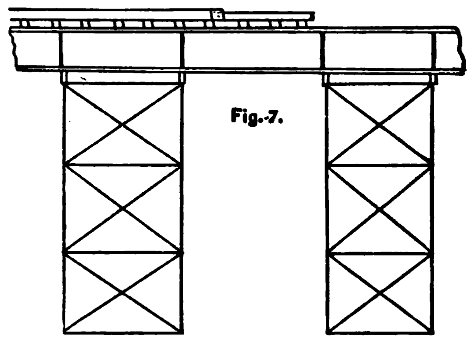


Fig. 7.

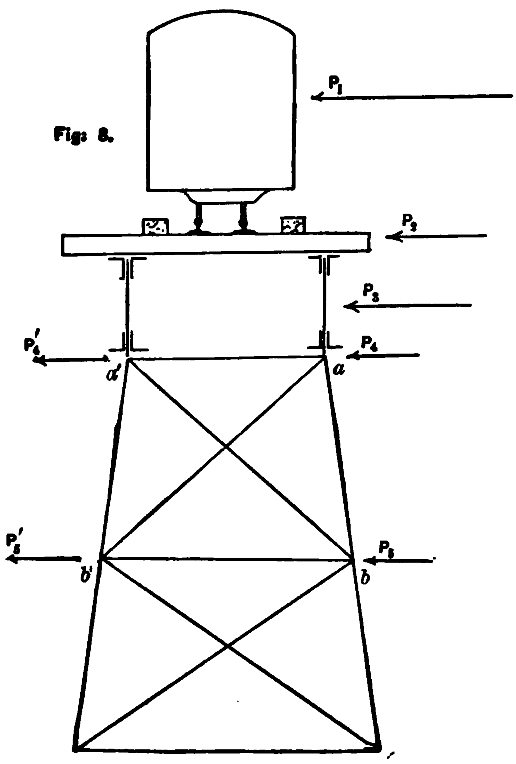
The longitudinal bracing of each tower must be capable of resisting the greatest tractive force of the engines, or any force induced by suddenly stopping, upon any part of the trestle, the assumed maximum trains.

If W is the maximum weight due to train on a bent, and ϕ is the coefficient of friction, usually taken at $\frac{1}{8}$th, then ϕW is the tractive force acting longitudinally at the top of the tower, for which the longitudinal bracing must be figured.

OUTER FORCES.

WIND STRAINS IN A BENT.—We have first the wind force on train P_1, Fig. 8. This is taken at 300 lbs. per linear foot of span, and the span *on each side* of the bent is covered by train. We may take P_1 as acting 9 feet above the base of the rail on the windward side only. P_2 is the wind on the ties and guard-rails, considered as acting at the foot of the rail, and may be taken at 30 lbs. per square foot of exposed surface. The exposed surface of ties and guard-rails may be taken at 1 square foot per linear foot; so that the wind force may be taken at 30 lbs. per linear foot on ties and guard-rails. P_2 will then be 30 × the half span on each side of the bent, and acts on the windward side only.

Fig. 8.



We have, next, the wind force P_3 on the truss. This also is taken at 30 lbs. per square foot of exposed surface. If the girder is a plate girder, it acts on the windward side only. If a framed truss, it acts upon both windward and leeward sides. In the first case P_3 is $\frac{1}{2}$ (30 × area of girder on one side of bent) + $\frac{1}{2}$

(30 × area of girder on other side of bent). In the second case P_3 is $\frac{1}{2}$ (30 × area of truss on one side of bent) + $\frac{1}{2}$ (30 × area of truss on other side of bent), and it acts at *both* windward and leeward sides. For a framed girder we may take the area of a truss of 4 square feet per linear foot, and hence we have 120 lbs. per linear foot for wind on each truss.

The wind on the towers is taken at 125 lbs. per vertical linear foot for the whole side, or one-half of this for one column; and it acts on both windward and leeward sides of bent. Hence, we have for P_4, $\frac{125}{2} \times ab$, and P'_4 the same. For P_5, $\frac{125}{2} \times bc$, and P'_5 the same.

We have also, at the top of the bent at a and a', the quarter weight of the superstructure with train for span on each side, and that part of the weight of tower itself, which is concentrated at a and a'. We denote these forces by W_4, W'_4.

Also at P_5 and P'_5, we have that part of the weight of the tower itself which is concentrated at these points, W_5, W'_5.

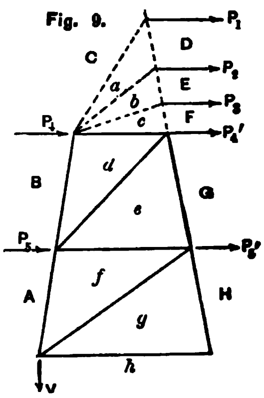
These constitute all the outer forces, and in any given case they are easily estimated.

STRESSES IN A BENT.—The simplest and easiest method of finding the stresses due to the wind is by diagram.

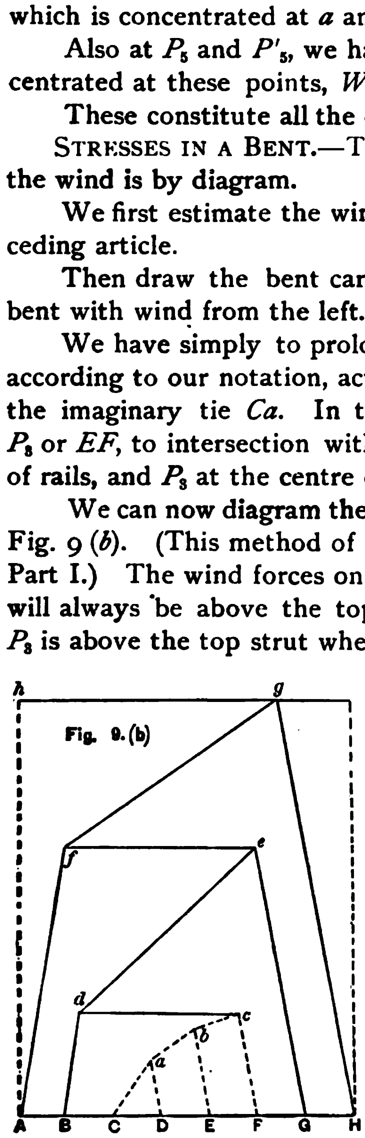
We first estimate the wind forces P_1, P_2, P_3, P_4, P'_4, P_5, P'_5, etc., as directed in the preceding article.

Then draw the bent carefully to scale, as shown in Fig. 9 (a), which represents the bent with wind from the left.

We have simply to prolong the leeward column as indicated, till it meets P_1, or CD, according to our notation, acting at 9 feet above the rails, and draw the imaginary tie Ca. In the same way, prolong P_2, or DE, and P_3 or EF, to intersection with the leeward column, P_2 acting at foot of rails, and P_3 at the centre of the girder, and draw the ties ab, bc.



We can now diagram the stresses due to these forces, as shown in Fig. 9 (b). (This method of diagram is explained in Chapter I. of Part I.) The wind forces on train P_1, and on ties and guard-rails P_2, will always be above the top strut. The wind force on the girder P_3 is above the top strut when the girder is on top of the bent. But in the case of a deck span, it may be below the top strut. In any case, prolong it to intersection with leeward column, and draw bc, either above or below.



If the girder is framed, P_3 is to be taken as the *sum* of the pressures on both windward and leeward trusses. For a plate girder, it is, of course, only the pressure upon the exposed side.

The vertical forces W_4, W'_4, include the weight of girder, weight of track, dead weight of structure, and weight of train; W_5 and W'_5 are simply the apex loads due to weight of structure.

The calculation of stresses due to the vertical forces is very simple. If θ is the angle which a column makes with the vertical, the stress in the top strut, cd, is $W_4 \tan \theta$, and in Bd or Ge, $W_4 \sec \theta$. In the next strut, ef, we have $(W_4 + W_5) \tan \theta$, and Af or Hg $= (W_4 + W_5) \sec \theta$, and so on.

All these stresses are compression. There are no stresses due to vertical loading in the inclined braces.

When the train is off, W_4 and W'_4 will be diminished by the weight of train, and the wind stresses will be diminished by the stresses due to P_1. There should be no tension in the windward columns, Af and Bd, under any circumstances. We can easily calculate the stresses in Af and Bd for train off.

One of the last members in our diagram should always be checked by calculation.

EXAMPLE.—*A bent is 20′ 4⅝″ at bottom, and 9′ 2¼″ at top, c to c. The height from base to top strut is 45 feet. From top strut to next strut, 18′ 3⅛″, and from there to bottom strut, 26′ 8⅞″. The span on each side of the bent is 30 feet. The plate girder on top is 3 feet deep, and its bottom is 6 inches above the top strut. The foot of rail is 4 feet above the top strut. Find the stresses.*

Let us first estimate the outer forces.

We have for P_1, the wind force on train, 300 lbs. per linear foot for both spans covered, or $\frac{300 \times 30}{2} + \frac{300 \times 30}{2} = 9000$ lbs.

This acts at 9 feet above foot of rail, or 13 feet above top strut.

For P_2, the wind force on ties and guard-rails, we have 30 lbs. per linear foot, or 90 lbs. acting at foot of rail, or 4 feet above the top strut.

For P_3, the wind force upon the exposed surface of the girder, we have 30 lbs. per square foot, or $\frac{90 \times 30}{2} + \frac{90 \times 30}{2} = 2700$ lbs., acting at $1.5 + 0.5 = 2$ feet above the top strut.

For P_4, the wind force on bent, we have $\frac{125}{2} \times 9 = 560$ lbs., and P'_4 the same.

For P_5, we have $\frac{125}{2}(9 + 13.5) = 1400$ lbs., and P'_5 the same.

An estimate of the weight of the structure gives $W_5 = W'_5 = 2000$ lbs. For W_4, we have for weight of structure 500 lbs. at each cap; taking the track at 400 lbs. per linear foot, we have 6000 lbs. at each cap; the weight of the girder is estimated at 4850 lbs. at each cap; the train, taking the loading of our diagram, page 243, is 68715 lbs. at each cap.

For the train on, $W_4 = W'_4 = 80000$ lbs., $W_5 = W'_5 = 2000$ lbs. We have then, for the stresses due to vertical loading, *train on*, since $\tan\theta = 0.124$, $\sec\theta = 1.007$,

$$Bd = Ge = -80000 \times 1.007 = -80560 \text{ lbs.} \qquad Af = Hg = -82000 \times 1.007 = -82574 \text{ lbs.}$$

$$cd = -80000 \times 0.124 = -9920 \text{ lbs.} \qquad ef = -82000 \times 0.124 = -10168 \text{ lbs.}$$

$$V = -82000 \text{ lbs.} \qquad de = fg = 0.$$

For the *train off*, $W_4 = W'_4 = 11285$, $W_5 = W'_5 = 2000$ lbs., and we have

$$Bd = -11285 \times 1.007 = -11364 \text{ lbs.} \qquad Af = -13285 \times 1.007 = -13378 \text{ lbs.}$$

$$V = -13285 \text{ lbs.}$$

V is the pressure on the anchorage of the windward side.

For the stresses due to P_1 *alone*, in Af and Bd, we have lever arm for $Bd = 9.2 \cos\theta = 9.13$, lever arm for $Af = 15.7 \cos\theta = 15.6$ feet, and

$$Af \times 15.6 = +9000 \times 31.3, \quad \text{or} \quad Af = +18057 \text{ lbs.}$$

$$Bd \times 9.13 = +9000 \times 13, \quad \text{or} \quad Bd = +12814 \text{ ''}$$

$$V \times 20.36 = +9000 \times 58, \quad \text{or} \quad V = +25638 \text{ ''}$$

where V is the tension on anchorage of windward side.

Making now our diagram, we have, for wind stresses, *train on*,

$$cd = -9700 \text{ lbs.} \qquad ef = -8300 \text{ lbs.} \qquad gh = -11600 \text{ lbs.} \qquad de = +16250 \text{ lbs.}$$

$$fg = +15000 \text{ lbs.} \qquad Bd = +14000 \text{ lbs.} \qquad Af = +27750 \text{ lbs.} \qquad Ge = -27850 \text{ lbs.}$$

$$Hg = -40650 \text{ lbs.} \qquad V = +40340 \text{ lbs.}$$

A convenient scale for the diagram, which has been adopted in finding these results, is ten feet to an inch for the bent, and 4000 lbs. to an inch for the forces.

We can check the value of V as follows:

$$V \times 20.36 = +9000 \times 58 + 900 \times 49 + 2700 \times 47 + 2(560 \times 45) + 2(1400 \times 26.7), \quad \text{or} \quad V = +40184 \text{ lbs.}$$

We have now for the maximum stresses,

$cd = -9700 - 9920 = -19620$ lbs. $\qquad ef = -8300 - 10168 = -18468$ lbs.

$gh = -11600$ lbs. $\qquad de = +16250$ lbs. $\qquad fg = +15000$ lbs.

$Ge = -27850 - 80560 = -108410$ lbs. $\qquad Hg = -40650 - 82574 = -123224$ lbs.

$V = -82000 + 40340 = -41660$ lbs.

These stresses are obtained by combining the stresses due to wind, *train on*, with vertical loading, *train on*. If the wind were from the other side, we would have same stresses in Bd and Af as in Ge and Hg, already found, the struts cd, ef, and gh would be the same, and the other ties would act. We do not care to put down the stresses for Bd and Af, for wind from left, because they are less than Ge and Hg, already found. But we should see if they are in tension or not. They will be

$Bd = -80560 + 14000 = -66560$ lbs. $\qquad Af = -82574 - 27750 = -54824$ lbs.

Both are therefore in compression, but maximum compression is when wind is on other side.

Also, on windward side,

$$V = -82000 + 40340 = -41660 \text{ lbs.},$$

or there is no tension on anchorage

Let us now see whether the bent with the *train off* has no tension in the columns.

Subtract from the diagram stresses the stresses for P_1 alone, which we have calculated, and we have wind stresses when *train is off*. Combine these with vertical load stresses when train is off, and see if Af and Bd are still in compression.

We have

$$Bd = -11364 + (14000 - 12814) = -10178 \text{ lbs.}$$

$$Af = -13378 + (27750 - 18057) = -3685 \text{ lbs.}$$

$$V = -13285 + (40340 - 25638) = +1417 \text{ lbs.}$$

We see that there is no tension in the columns of the windward side, under any circumstances, but there may be a small tension of 1417 lbs. on the anchorage.

The *longitudinal bracing* in this case would be figured for a force of $68715 \times \phi$, where $\phi = \frac{1}{5}$, or 13743 lbs., acting at the top of the tower.

It causes stresses in the longitudinal ties, which can be easily found by calculation, by multiplying by the sec of the angle α which the ties make with the horizontal. Thus, in the present case, $\sec \alpha = 1.171$ for the first tie, and $\sec \alpha = 1.338$ for the next tie.

The stresses in these ties are then $+13743 \times 1.171 = +16090$ lbs., and $+13743 \times 1.338 = +18388$ lbs.

PONY TRUSSES.—As pony trusses do not admit of over-head bracing, we have only horizontal bracing under the floor. If the floor beams are riveted to the posts, these latter may be continued below the floor beams, and horizontal and vertical sway bracing introduced. The floor beams may also be prolonged and stays or knee braces introduced to support the truss sideways.

WEIGHT OF WIND BRACING.—For preliminary estimates of weight the following formulas will be found useful:

FOR PONY TRUSSES.—*Depth below* 12.5 *feet,*

$$\textit{weight per foot lineal of wind bracing} = 3.6\,N + \frac{540}{p}.$$

FOR THROUGH TRUSSES WITHOUT VERTICAL SWAY BRACING.—*Depth between* 12.5 *and* 24 *feet,*

$$\textit{weight per foot lineal of wind bracing} = 6.4\,N + \frac{672}{p}.$$

FOR THROUGH TRUSSES WITH VERTICAL SWAY BRACING (*depth above* 24 *feet*), OR DECK BRIDGES,

$$\textit{weight per foot lineal of wind bracing} = \frac{6Nl}{170} + \frac{1136}{p},$$

where l = length in feet, N = number of panels, p = panel length in feet.

All these formulæ are for single track, and give the total weight *for both trusses*. For double track multiply by $\frac{b}{15}$, where b = width or breadth of bridge in feet.

FRICTION ROLLERS.—The specifications of the New York, Penn. & Ohio R. R. require all bridges over 70 feet span to have at one end a nest of turned friction rollers of wrought iron, running between planed surfaces.

" The rollers shall not be less than 2 inches in diameter, and shall be so proportioned that the pressure per lineal inch of rollers shall not exceed the product of the square root of the diameter of the roller in inches multiplied by 500," or permissible pressure,

$$p = 500\sqrt{d},$$

where p is the permissible pressure per lineal inch, and d is the diameter in inches.

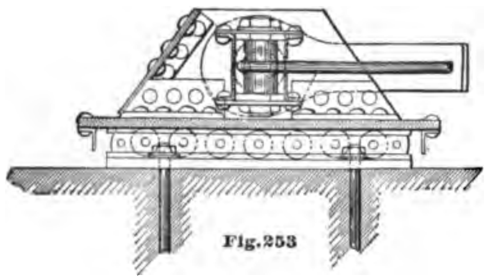


Fig.253

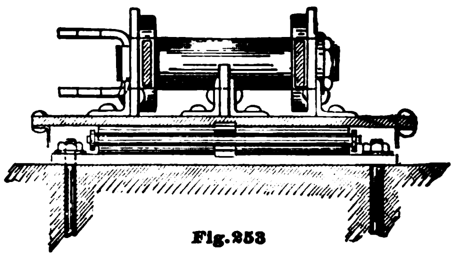


Fig.253

We give, in Figs. 253, the construction of rollers, roller nut, and cover plate for rollers.

A more rational formula has been deduced by Professor Burr (*Stresses in Bridge and Roof Trusses*),

$$p = \frac{4}{3}R\sqrt{\frac{4w^3}{E}},$$

where w = the greatest allowable pressure on a roller, or 12000 lbs. per sq. inch for wrought iron. R = radius of roller in inches. E = coefficient of elasticity = 28000000 for wrought iron.

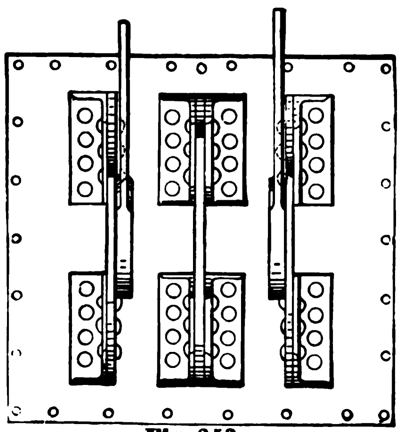


Fig. 253

For R = 1″, this gives p = 662 lbs. per lineal inch, while the first equation gives p = 707 lbs.

We give, in Plates 19, 20, and 21, illustrations of various details which have been referred to in the foregoing pages.

EQUIVALENT LENGTH OF RODS FOR UPSET ENDS, NUTS, SLEEVE NUTS, AND TURN BUCKLES.—We have already given, in Pin Table II. of the preceding chapter, the equivalent length of chord bar required to make the head. For main diagonals and hip verticals it will be sufficient in general to add 3 feet for eyes, and for adjustable rods, such as counters and wind ties, 5 feet for turn buckles. If greater accuracy is required for the latter, we may ascertain what length of

rod is needed at each end for the connection, and how much for adjusting nuts and upset ends, by the following Table:

($\frac{1}{4}''$—1 $''$)	1 upset end and 1 nut	= $1\frac{1}{7}$ feet of rod.
($1\frac{1}{16}''$—$1\frac{1}{2}''$)	" " " "	= $1\frac{5}{8}$ " " "
($1\frac{9}{16}''$—2 $''$)	" " " "	= $1\frac{3}{8}$ " " "
($2\frac{1}{8}$ $''$—$2\frac{1}{2}''$)	" " " "	= $1\frac{7}{10}$ " " "
($\frac{1}{2}''$—$1\frac{1}{4}''$)	2 upset ends and 1 nut	= $2\frac{3}{8}$ " " "
($1\frac{5}{16}''$—$2\frac{1}{2}''$)	" " 1 nut, 1 turn buckle, 3 ft. of rod.	

These equivalent lengths do not include the lengths of the upset ends themselves; they represent simply the extra length to be added to the bar to allow for the weight of the nuts, sleeve nut, or turn buckle, and the extra iron for the enlarged ends, which are generally about 8 inches long.

CAMBER.—In practice the upper and lower chords of bridges are not perfectly horizontal, but are curved upward by such an amount that even when fully loaded they do not quite reach the horizontal.

This upward deflection is called the "*camber.*"

The two chords form, thus, concentric arcs, and since the unit stress is constant, these arcs are circular.

In order to produce the camber the upper chord is made longer than the lower.

The finding of the actual lengths of the members "c to c," or centre to centre of pin holes, is one of the most important points of the design.

Let u' be the allowable working stress per square inch in the upper chord for combined dead and live loads, and u for the lower chord.

Some specifications call for a different unit stress for dead and live loads.

Let $L =$ the live load stress in any member in lbs., and $D =$ the dead load stress in the same member, and let $\sigma =$ the allowed unit stress for the dead load, and σ' for the live load. Then, if U is the combined unit stress for *any* member, for both dead and live loads, we have

$$\frac{D+L}{U} = \frac{D}{\sigma} + \frac{L}{\sigma'}, \text{ or } U = \frac{D+L}{\frac{D}{\sigma} + \frac{L}{\sigma'}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{I.})$$

when, as is most often the case, $\sigma = \sigma'$, we have $u' = \sigma$.

From (1), by introducing the values of L and D, and σ, σ', in any case for the *upper* chord, we can find u', and introducing the values of L and D, σ and σ' for the *lower* chord, we can find u.

Let $s =$ the length of span, and $E =$ the coefficient of elasticity. Then the compression of the upper chord, under the combined dead and live loads, if the truss deflected from a horizontal, would be $\frac{u's}{E}$ (page 243), and its new length would be $s - \frac{u's}{E}$. In the same way the new length of the lower chord, after deflection from the horizontal, would be $s + \frac{us}{E}$.

If we camber the truss upward, in order to just counteract the deflection, we should make the *upper* chord $s + \frac{u's}{E}$, and the *lower* chord $s - \frac{us}{E}$.

Let $d =$ the depth of truss c to c, and r the radius of the lower chord. Then $r + d$ is the radius of the upper chord, and we have

$$r + d : r :: s + \frac{u's}{E} : s - \frac{us}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence,

$$r = \frac{Ed - du}{u + u'},$$

or, since u is small compared to E,

$$r = \frac{Ed}{u + u'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let Δ be the camber. Then, from the right-angled triangle formed by the half span, the radius of the lower chord at end, and the vertical through centre of span, we have

$$(r - \Delta)^2 + \frac{s^2}{4} = r^2, \text{ or } \Delta = r - \sqrt{r^2 - \frac{s^2}{4}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We have, also, if $\frac{I}{2}$ is the increase of the upper chord over the lower in the half span,

$$\frac{I}{2} : d :: \frac{s}{2} : r - \Delta,$$

or, since Δ is very small compared to r, we can neglect it, and hence, $I = \frac{ds}{r}$.

The increase of length of the upper over the lower chord *per unit of length*, that is, in inches for every inch of length, or in feet for every foot of length, is then $i = \frac{d}{r}$, or, substituting the value of r from (2),

$$i = \frac{d}{r} = \frac{u + u'}{E}.$$

To allow for the additional deflection due to the web, let us increase this amount by one-third. We then have

$$i = \frac{4(u + u')}{3E}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{II.})$$

where u is the unit stress for the lower chord, and u' for the upper chord, for combined dead and live loads, to be found from (I.).

A convenient practical rule for this increase of length of the top chord, per unit of length, which is given in Cooper's specifications, may be deduced by taking $u = 10000$ lbs. and $u' = 8000$ lbs., and $E = 24000000$ lbs. Then, from (II.),

$$i = \frac{1}{1000}.$$

For a length of 10 feet this becomes $\frac{1}{100}$, or about $\frac{1}{8}$th of an inch. Hence the rule, "*$\frac{1}{8}$th of an inch for every* 10 *feet of panel.*"

In figuring the lengths of the various members, we give them such lengths that, under the action of the *dead load only*, any lower panel shall have the length p from c to c, any post the length d from c to c, and the remaining camber *shall still be that given by* (*II.*) *for dead and live loads combined.* This insures that, when the bridge is fully loaded, the truss shall still be above the horizontal at the centre, by an amount about equal to the dead load deflection.

The unit stress due to dead load only is from (I.) $\dfrac{D}{\frac{D}{\sigma} + \frac{L}{\sigma'}}$. Hence, the extension or

compression of any member per unit of length, for dead load only, is

$$e = \frac{D}{E\left[\frac{D}{\sigma} + \frac{L}{\sigma'}\right]}; \quad \ldots\ldots\ldots\ldots \quad \text{(III)}.$$

when $\sigma = \sigma'$ this becomes

$$e = \frac{D\sigma}{E(D+L)}.$$

We must allow for this extension or compression due to the dead load, in figuring the lengths, so that, when the dead load only acts, the lower chord panel may be p, the posts d, and the upper chord panel $p + ip$, c to c where i is found from (II.).

Also, since the pin hole is always bored one-fortieth of an inch (0.025″) larger than the pin, we must allow for this clearance.

Equations (I.), (II.), and (III.) completely solve our problem. We recapitulate them here for convenience of reference.

Unit stress for combined dead and live loads in any member.

$$U = \frac{D+L}{\frac{D}{\sigma} + \frac{L}{\sigma'}}, \quad \ldots\ldots\ldots\ldots \quad \text{(I.)}$$

where D is the dead load, L the live load stress in the member, and σ and σ' the unit stresses for dead and live loads. When $\sigma = \sigma'$, this becomes $U = \sigma$.

Increase of length of upper chord per unit of length.

$$i = \frac{4(u+u')}{3E}, \quad \ldots\ldots\ldots\ldots \quad \text{(II.)}$$

where u is the unit stress for lower chord, and u' for upper chord, found from (I.).

Extension or compression per unit of length, of any member, due to the dead load only.

$$e = \frac{D}{E\left[\frac{D}{\sigma} + \frac{L}{\sigma'}\right]}. \quad \ldots\ldots\ldots\ldots \quad \text{(III.)}$$

When $\sigma = \sigma'$ this becomes $e = \dfrac{D\sigma}{E(D+L)}$.

ACTUAL LENGTH OF LOWER CHORD BARS.—Since the lower chord bars are in tension, we must make them a little short, so that, allowing for pin clearance and dead load extension, they will pull out to the length p. We have therefore

$$\textit{actual length of lower chord bars } c \textit{ to } c = p - ep - 0.025, \quad \ldots\ldots \quad (a)$$

when p is the panel length c to c in inches and e is found from (III.).

The length is figured only to the nearest $\frac{1}{32}$d of an inch, as that is the least shop measurement.

ACTUAL LENGTH OF POST.—The post is in compression, and we therefore make it longer than d, to allow for dead load compression and pin clearance. We have therefore

$$\textit{actual length of post } c \textit{ to } c = d + ed + 0.025, \quad \ldots\ldots\ldots \quad (b)$$

where d is the depth c to c in inches, and e is found from (III.).

The length is figured to the nearest $\frac{1}{32}$d of an inch.

ACTUAL LENGTH OF UPPER CHORD PANELS.—Since the chords are in close contact, there is no allowance for pin clearance. We must make it a little long to allow for dead load compression, and also increase it by the amount ip. We have then

$$\textit{actual length of upper chord panel} = p + ip + ep, \quad \ldots \ldots \quad (c)$$

where p is the panel length c to c in inches, e is found from (III.), and i from (I.) and (II.).

The length is figured to the nearest $\frac{1}{32}$d of an inch.

ACTUAL LENGTH OF INCLINED TIES.—The length of the tie is to be

$$l = \sqrt{d^2 + \left(p + \frac{ip}{2}\right)}, \quad \ldots \ldots \ldots \quad (d)$$

where d is the depth c to c in inches, p is the panel length c to c in inches, and $p + \frac{ip}{2}$ is the mean of the upper and lower chord panel lengths. We find i from (I.) and (II.). We have then, allowing for dead load extension and pin clearance,

$$\textit{actual length of ties } c \textit{ to } c = l - el - 0.025, \quad \ldots \ldots \quad (e)$$

where l is found from (d) and e from (III.).

The length is figured to the nearest $\frac{1}{32}$d of an inch.

In a *draw* span each arm may be considered as one span in giving the camber, but whole amount of lengthening of the upper chord must be taken out of the upper chord at centre, or the ends will sink below their original positions.

EXAMPLE.—*Let the span c to c be 200 feet, depth c to c 25 feet, panel length c to c 20 feet. In a given panel we have the following stresses and unit stresses:*

	L	D	σ'	σ
LOWER CHORDS,	240000 lbs.	120000 lbs.	8000 lbs.	16000 lbs.
UPPER CHORDS,	180000 "	90000 "	7000 "	14000 "
TIES,	100000 "	40000 "	8000 "	16000 "
POSTS,	87000 "	35000 "	4000 "	8000 "

Let $E = 24000000$ lbs. *Find the required lengths.*

For the lower chords, we have from (III.) $e = \frac{1}{7500}$, and from (a)

$$\textit{length of chord bars } c \textit{ to } c = 240 - \frac{240}{7500} - 0.025 = 239.943'' = 19 \text{ ft. } 11\tfrac{15}{16} \text{ in.}$$

For the posts, we have from (III.) $e = \frac{1}{17914}$, and from (b)

$$\textit{length of posts } c \textit{ to } c = 300 + \frac{300}{17914} + 0.025 = 300.042'' = 25 \text{ ft. } 0\tfrac{1}{32} \text{ in.}$$

For the upper chords, we have from (III.) $e = \frac{1}{8571}$.

From (I.) we have

$$u = 96000, \ u' = 8400, \text{ and from (II.) } i = \frac{1}{1000}. \quad \text{We have then from } (c)$$

$$\textit{length of upper chord panel} = 240 + \frac{240}{1000} + \frac{240}{8571} = 240.268'' = 20 \text{ ft. } 0\tfrac{9}{32} \text{ in.}$$

For the ties, we have from (III.) $e = \frac{1}{9000}$, and from (I.) and (II.),

$$i = \frac{1}{1000}. \quad \text{Then } p + \frac{ip}{2} = 240.12'', \ l = \sqrt{300^2 + 240.12^2} = 384.262'',$$

and from (d)

$$\textit{length of ties } c \textit{ to } c = 384.262 - \frac{384.26}{9000} - 0.025 = 384.195'' = 32 \text{ ft. } 0\tfrac{3}{16} \text{ in.}$$

EXAMPLE.—*Let the span c to c be 250 feet, depth c to c 45 feet, panel length c to c 25 feet. In a given panel we have the following stresses and unit stresses:*

	L	D	$\sigma = \sigma'$
LOWER CHORDS,	300000 lbs.	150000 lbs.	10000 lbs.
UPPER CHORDS,	340000 "	170000 "	8000 "
POSTS,	100000 "	35000 "	8000 "
TIES,	114000 "	40000 "	10000 "

Let E = 26000000 lbs. *Find the actual lengths.*

For the lower chords we have from (III.) $e = \frac{\sigma}{3E} = \frac{1}{7800}$, and from ($a$),

$$\textit{actual length of chord bars c to c} = 300 - \frac{300}{7800} - 0.025 = 299.94'' = 24 \text{ ft. } 11\tfrac{15}{16} \text{ in.}$$

For the posts we have from (III.) $e = \frac{1}{12536}$, and from (b),

$$\textit{length of posts c to c} = 540 + \frac{540}{12536} + 0.025 = 540.068'' = 45 \text{ ft. } 0\tfrac{1}{16} \text{ in.}$$

For the upper chords, we have from (III.) $e = \frac{1}{7800}$.

From (I.) we have

$$u = 10000,\ u' = 8000, \text{ and from (II.) } i = \frac{24}{26000}.$$

From (c)

$$\textit{length of upper chord panel} = 300 + \frac{300 \times 24}{26000} + \frac{300}{7800} = 300.315'' = 25 \text{ ft. } 0\tfrac{5}{16} \text{ in.}$$

For the ties, we have from (III.) $e = \frac{20\sigma}{77E} = \frac{1}{10010}$.

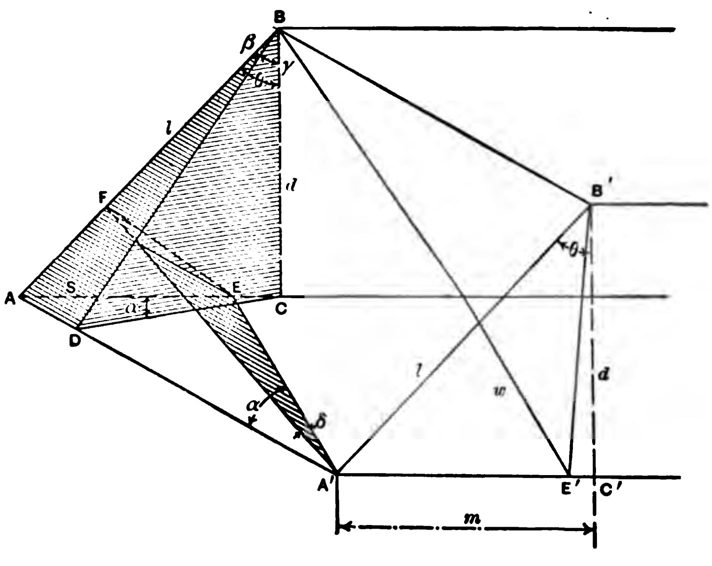
From (I.) and (II.),

$$i = \frac{24}{26000},\ p + \frac{ip}{2} = 300.138,\ l = \sqrt{540^2 + 300.138^2} = 617.804.$$

From (d)

$$\textit{length of ties c to c} = 617.804 - \frac{617.804}{10010} - 0.025 = 617.718'' = 51 \text{ ft. } 5\tfrac{23}{32} \text{ in.}$$

BEVEL ANGLES FOR SKEW PORTALS.—The angles required for laying out a skew portal are the angles $ABD = \beta$, or the amount by which the angle ABB', between the inclined end post and the portal strut, differs from 90°; the angle $DBC = \gamma$, or the angle between the plane of the portal and a vertical plane through BB'; the angle $FA'E = \delta$, or the amount by which the angle between the plane of the portal and the plane of the truss differs from 90°.



In the figure, the line BD lies in the plane of the portal, and is perpendicular to AA'. Therefore, 90 + β gives the angles ABB', and $AA'B'$ and 90 − β gives the angles $BB'A'$ and BAD, all in the plane of the portal. The line DC is

in the plane of the bottom chords and is perpendicular to AA'. Therefore the angle $ACD = \alpha$ is the skew angle, or is equal to $AA'E$, $A'E$ being in the plane of the bottom chords and perpendicular to them. The line BC is vertical and in the plane of the truss, so that the angle $DBC = \gamma$ is the angle between the plane of the portal and a vertical plane through the portal strut BB'. Through $A'E$ we pass a plane perpendicular to AB, the inclined end post. Then the angle $FEA' = 90°$, and the angle $FA'E = \delta =$ the amount by which the angle between the plane of the portal and the plane of the truss differs from 90°.

Let the depth of truss, $BC = B'C' = d$; the width of truss $A'E = w$; the horizontal projection of inclined end posts $AC = A'C' = m$; the length of inclined end posts $= l$; the angle between inclined end posts and vertical, $ABC = A'B'C' = \theta = FEA$; the skew angle $AA'E = \alpha = ACD$; the skew $AE = s$; the length of portal strut $AA' = BB' = \varepsilon$.

Then

$$\left.\begin{array}{lll} \sin\theta = \dfrac{m}{l} = \dfrac{m}{\sqrt{m^2+d^2}}, & \cos\theta = \dfrac{d}{l} = \dfrac{d}{\sqrt{m^2+d^2}}, & \tan\theta = \dfrac{m}{d} \\ \sin\alpha = \dfrac{s}{\varepsilon} = \dfrac{s}{\sqrt{s^2+w^2}}, & \cos\alpha = \dfrac{w}{\varepsilon} = \dfrac{w}{\sqrt{s^2+w^2}}, & \tan\alpha = \dfrac{s}{w} \end{array}\right\} \quad . \quad . \quad . \quad (1)$$

We have also, $l \sin\beta = AD = l \sin\theta \sin\alpha = AC \sin\alpha$.

Hence,

$$\sin\beta = \sin\theta \sin\alpha; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$d \tan\gamma = DC = AC \cos\alpha = d \tan\theta \cos\alpha.$$

Hence,

$$\tan\gamma = \tan\theta \cos\alpha; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$\varepsilon \sin\alpha = AE, \qquad AE \cos\theta = FE, \qquad \varepsilon \cos\alpha = A'E, \qquad A'E \tan\delta = FE.$$

Hence,

$$\varepsilon \sin\alpha \cos\theta = \varepsilon \cos\alpha \tan\delta,$$

or

$$\tan\delta = \tan\alpha \cos\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

Equations (a), (b), and (c) give the required angles β, γ, δ. $90 + \beta$ and $90 - \beta$, give the angles between inclined end posts and portal strut, in the plane of the portal. $90 + \theta$ gives the angle between the top chords and inclined end posts in the plane of the truss. γ gives the angle between the plane of the portal and a vertical plane through portal strut, and gives the bevel for bending plates to connect with top chords. $90 - \delta$ gives the angle between plane of the portal and plane of the truss.

If we substitute in (a), (b), and (c) the values of (1), we have also,

$$\sin\beta = \frac{ms}{\sqrt{m^2+d^2}\ \sqrt{s^2+w^2}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a')$$

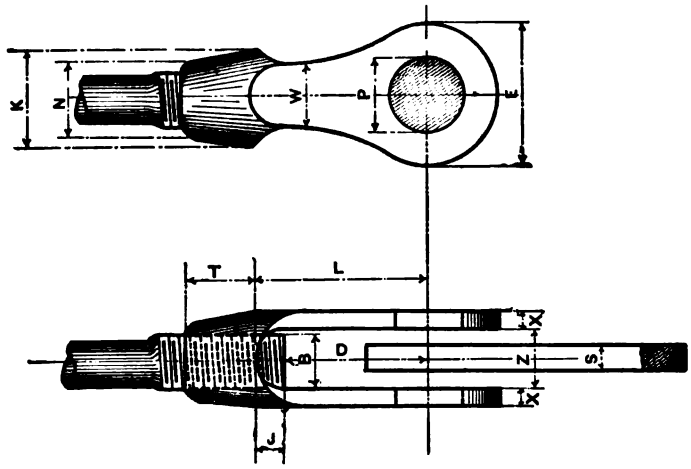
$$\tan\gamma = \frac{mw}{d\sqrt{s^2+w^2}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b')$$

$$\tan\delta = \frac{sd}{w\sqrt{m^2+d^2}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c')$$

From which the required angles can be found in terms of s, w, and m and d.

STANDARD CLEVISES.—For attaching lateral rods, clevises are often used, as illustrated. We give, in the following Table, the dimensions and weight as furnished by the Phœnix Bridge Company.

STANDARD CLEVISES.



Tension (Rod), 12000 lbs. per sq. inch. Bearing, 15000 lbs. per sq. inch. Bending, 15000 lbs. per sq. inch.

Side of Square Bar.	Diameter of Round Bar.	*B* Diameter of Upset.	*W* Width of Strap. × *X* Thickness of Strap.	*P* Diameter of Pin.	*E* Diameter of Eye.	*Z* Width of Fork.	*S* Thickness of Plate.	*L* Length of Fork.	*Y* Projection.	*D* End of Rod to C. of Pin.	*T* Length of Nut.	Nuts. *N & K*	Stock.	Number.	Estimated Weight in Lbs.	Shipping Weight.
"	"	"	" "	"	"	"	"	"	"	"	"	"	"		Lbs.	
		⅞	1½ × 5/16	1	2¼	1	.35	5	½	4½	1⅛		1¾ sq. × 8	1	2¼	
		1	1½ × 5/16	1⅛	2½	1⅛	.38	5	⅝	4⅜	1½	1½ & 1⅞	1¾ " × 8	2	3	
	⅞	1⅛	1½ × ⅜	1¼	3	1¼	.384	5	¾	4¼	1½	1½ & 1⅞	1⅞ " × 8	3	3½	
	1	1¼	2 × ⅜	1⅜	4 & 3¾	1⅜	.45	8	⅞	7⅛	2	2 & 2½	1⅞ " × 10	4	7¼	
	1⅛	1⅜	2 × ⅜	1¾	4½	1½	.46	8	⅞	7⅛	2	2 & 2½	2 " × 12	5	8	
1	1¼	1½	2½ × ⅜	2	5 & 4½	1⅝	.57	8	1	7	2	2 & 2½	2 " × 12	6	9	
1⅛		1⅝	2½ × ⅜	2	5 & 4½	1¾	.57	8	1	7	2	2 & 2½	2 " × 12	7	9¼	
	1⅜	1¾	2½ × ½	2	5 & 4½	1⅞	.64	8	1	7	2½	2½ & 3	2⅜ " × 15	8	10½	
1¼	1½	1⅞	2½ × ⅝	2	5 & 4½	2	.7	8	1⅛	6⅞	2½	2½ & 3	2⅜ " × 15	9	12	
1⅜	1⅝	2	2½ × ⅝	2⅜	5	2⅛	.67	8	1⅛	6⅞	3	2¾ & 3½	2½ " × 16	10	16	
1½	1¾	2⅛	3 × ⅝	2⅜	5–5½–6½	2¼	.84	8	1¼	6¾	3	2¾ & 3½	2½ " × 16	11	17	
	1⅞	2¼	3 × ¾	2⅜	5–5½–6½	2⅜	.96	8	1¼	6¾	3¼	3 & 3⅞	2¾ " × 16	12	20	
1⅝	2	2⅜	3 × ⅞	2⅞	6 & 6½	2½	.88	8	1⅜	6⅝	3¼	3 & 3⅞	2¾ " × 16	13	22	
1¾	2⅛	2½	3 × ⅞	2⅞	6 & 6½	2⅝	1	8	1⅜	6⅝	3½	3¼ & 4	3 " × 16	14	24	
	2¼	2⅝	3½ × ⅞	2⅞	6 & 6½	2¾	1.11	8	1½	6½	3½	3¼ & 4	3¼ " × 17	15	26	
1⅞	2⅜	2¾	4 × ⅞	3	7¼	2⅞	1.18	10	1½	8½	3½	3½ & 4¼	3½ " × 18	16	37	
2	2½	2⅞	4 × 15/16	3⅛	7¼	3	1.25	10	1⅝	8⅜	3½	3½ & 4¼	3½ " × 18	17	39	
	2⅝	3	4 × 1	3¼	7¼	3⅛	1.33	10	1⅝	8⅜	3¾	3¾ & 4½	3½ " × 18	18	43	
2⅛	2¾	3⅛	4 × 1⅛	3½	7¾	3¼	1.36	10	1⅝	8⅜	3¾	3¾ & 4½	3¾ " × 18	19	48	
2¼	2⅞	3¼	4 × 1¼	3¾	7¾	3⅜	1.4	10	1⅝	8⅜	4	4 & 5	3¾ " × 18	20	57	
2⅜	3	3⅜	4 × 1 5/16	3¾	7¾	3½	1.5	10	1⅝	8⅜	4	4 & 5	3¾ " × 18	21	60	

PLATE 19.

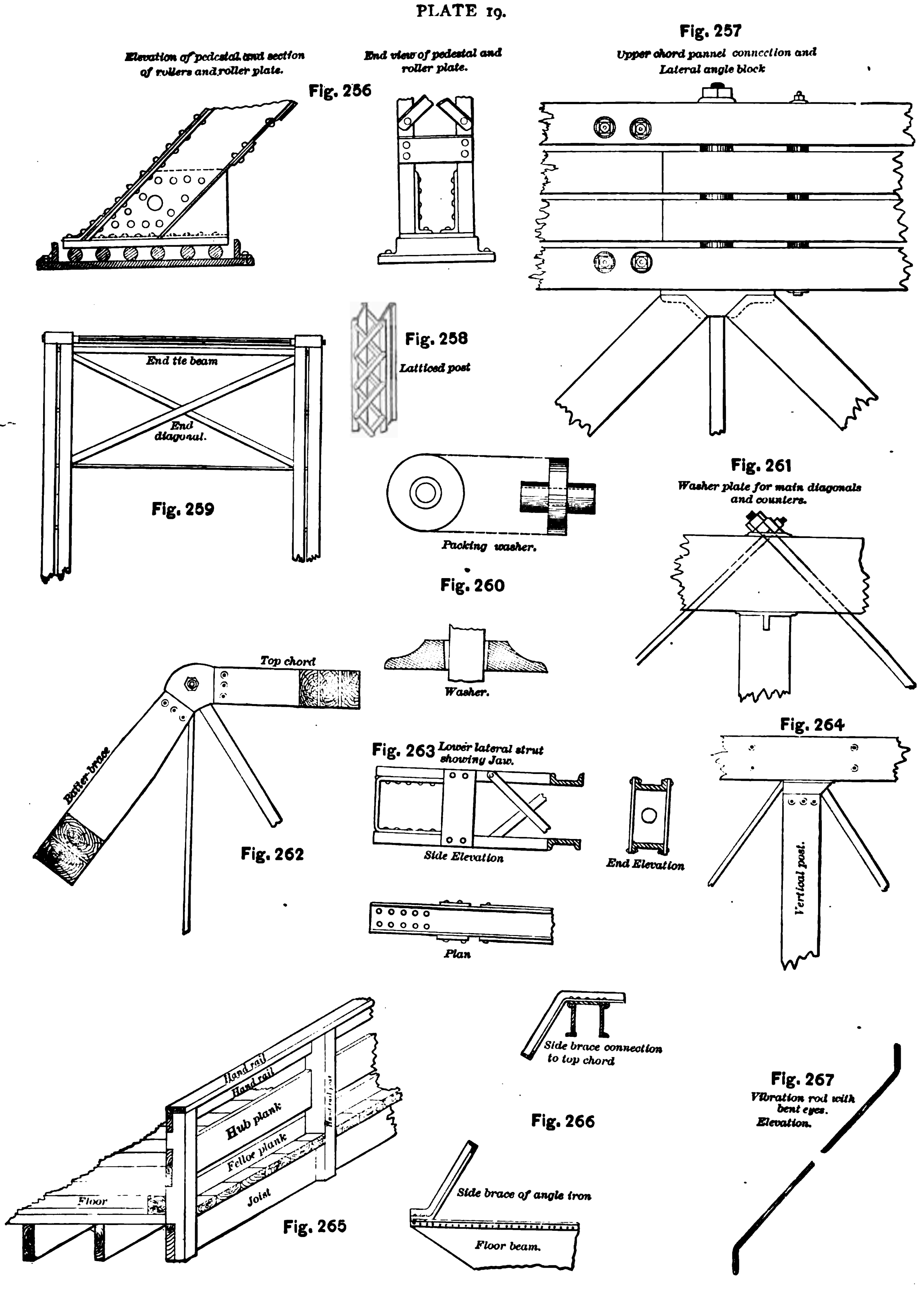


Fig. 256
Elevation of pedestal and section of rollers and roller plate.
End view of pedestal and roller plate.
Fig. 257
Upper chord pannel connection and Lateral angle block
Fig. 258
Latticed post
End tie beam
End diagonal.
Fig. 259
Packing washer.
Fig. 260
Fig. 261
Washer plate for main diagonals and counters.
Washer.
Top chord
Buffer brace
Fig. 262
Fig. 263 Lower lateral strut showing Jaw.
Side Elevation
End Elevation
Plan
Fig. 264
Vertical post.
Hand rail
Hand rail
Hub plank
Felloe plank
Joist
Floor
Fig. 265
Side brace connection to top chord
Fig. 266
Side brace of angle iron
Floor beam.
Fig. 267
Vibration rod with bent eyes.
Elevation.

PLATE 20.

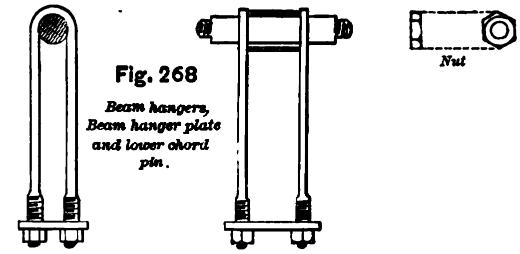


Fig. 268
Beam hangers, Beam hanger plate and lower chord pin.

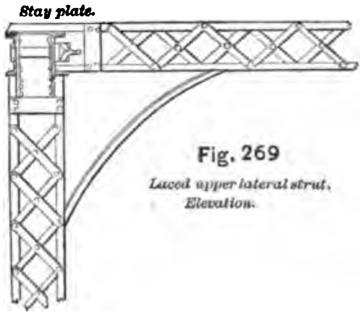


Fig. 269
Laced upper lateral strut. Elevation.

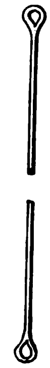


Fig. 270
Hip vertical. Elevation

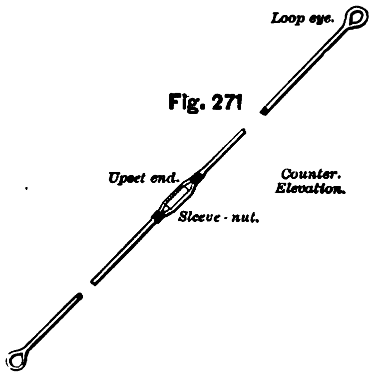


Fig. 271
Counter. Elevation.

Fig. 272
Turn-buckle.

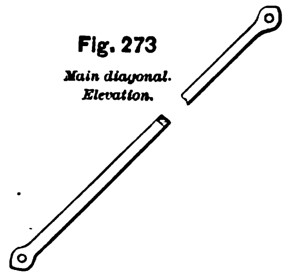


Fig. 273
Main diagonal. Elevation.

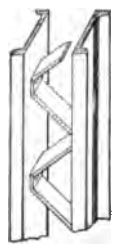


Fig. 274
Trussing

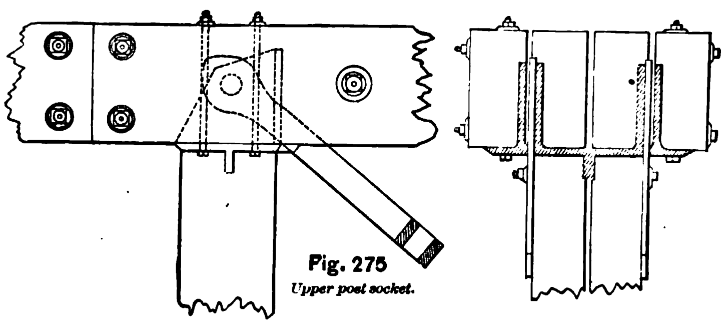


Fig. 275
Upper post socket.

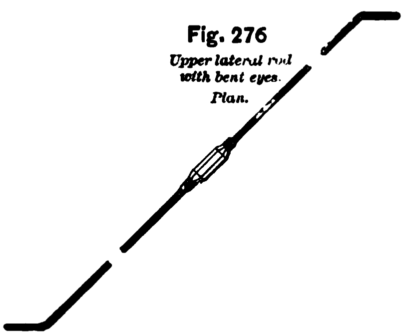


Fig. 276
Upper lateral rod with bent eyes. Plan.

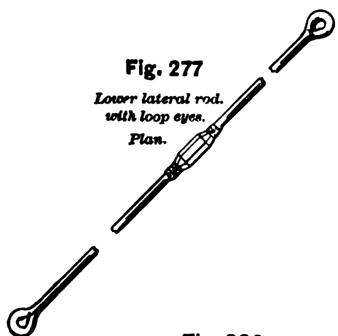


Fig. 277
Lower lateral rod. with loop eyes. Plan.

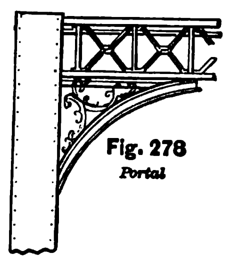


Fig. 278
Portal

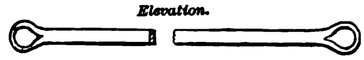


Fig. 281
Chord bar. Elevation.

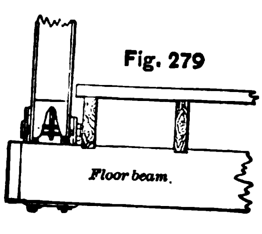


Fig. 279

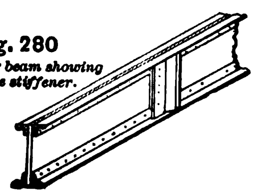
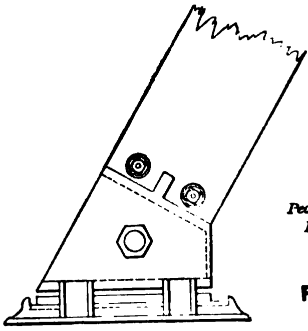


Fig. 280
Built floor beam showing angle stiffener.

PLATE 21.



Pedestal and Bed plate

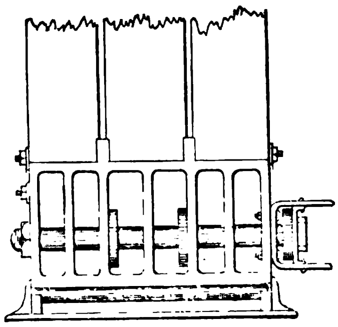
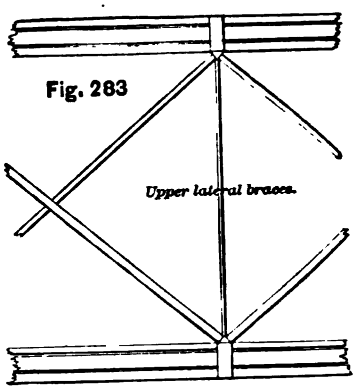
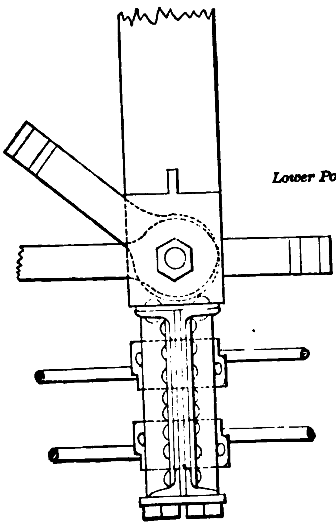


Fig. 282

Fig. 283





Lower Post Socket

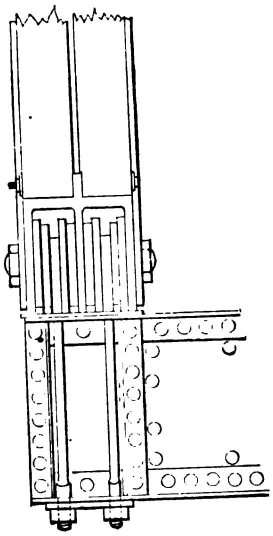


Fig. 284

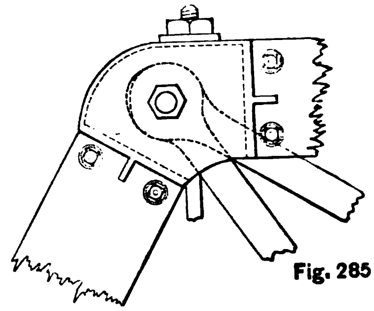


Fig. 285

Hip joint box

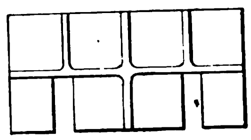
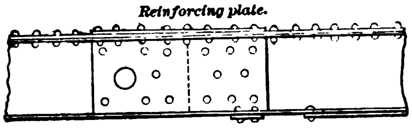


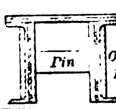
Fig. 286

Reinforcing plate.



Stay plate.

Cover plate.



Outside reinforcing plate.

Filling plate.

Inside reinforcing plate.

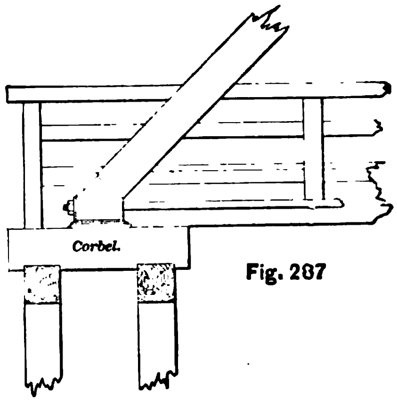


Fig. 287

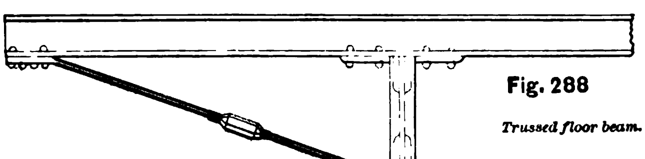


Fig. 288

Trussed floor beam.

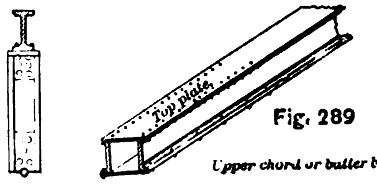


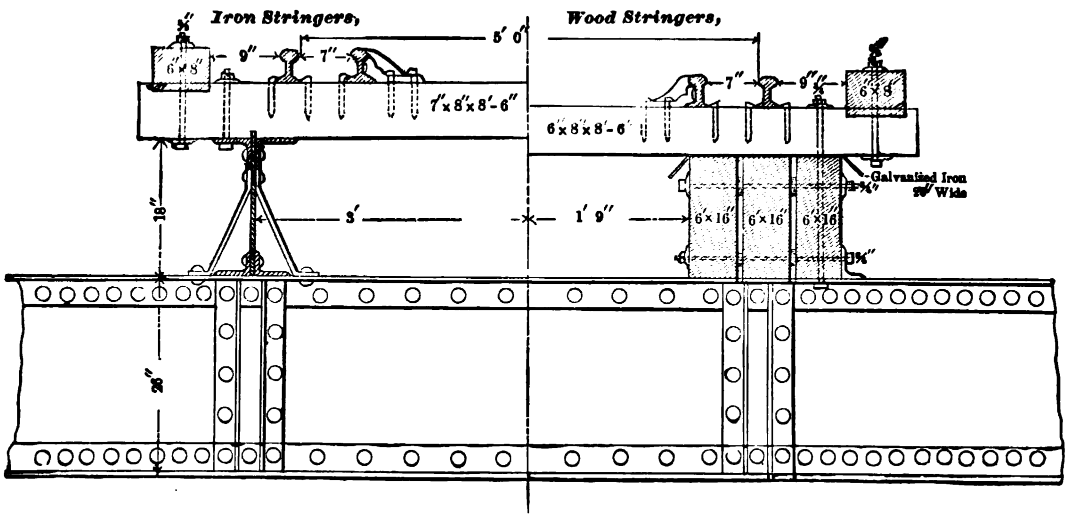
Fig. 289

Upper chord or butter braces

CHAPTER VII.

FLOOR SYSTEM—CROSS GIRDERS—STRINGERS—FLOOR.

FLOOR SYSTEM.—The arrangement of the floor system is shown in Fig. 206, Plate 8, and also by the following Fig.



The cross girder at every panel point is composed of double angle irons for the upper and lower flanges, of such a uniform section as will satisfy the stress at the middle due to the maximum loading. The web consists of a single plate, the thickness of which rarely exceeds $\frac{3}{8}''$, even for the heaviest cross girders, and is never less than $\frac{1}{4}''$ in the lightest. This web is riveted to the flanges above and below, and its edge is very nearly flush with the upper and lower surfaces of the angles. The cross girder is usually of uniform depth and square ends. Sometimes it is of uniform depth only in the central portion, and tapers off at the ends. More rarely still, the cross girder is a trussed frame, as shown in Fig. 288, Plate 21.

The cross girders are either slung from the pin at the panel point by beam hangers, as shown in Fig. 268, Plate 20, or they are riveted to the posts by angle irons, as shown in Fig. 288, Plate 8.

The stringers may be either of wood or of iron, as shown in the Fig. preceding. They either rest upon the cross girders, or are riveted to them as shown in Fig. 206, Plate 8.

Upon the stringers are laid the cross ties, which are usually of white oak, about 8 feet 6 inches long, and 7 inches deep by 8 inches wide, for single track, spaced about $16\frac{1}{2}$ inches from centre to centre, notched on to the stringers about $\frac{3}{4}''$, and bolted to the

stringer flanges. For double track, the ties are about 20 feet 6 inches long, 9 inches deep and 8 inches wide.

Pine guard rails or strips are bolted and notched to the ties, outside of and parallel to the rails, spliced at their ends.

The stringers are usually spaced about 6 feet apart, and for double track usually four stringers are used, so that the load per stringer is the same whether for double or single track.

Upon the ties the rails are laid and secured in the usual manner, and between the rails, a few planks for a foot walk are provided.

The entire weight of rails, spikes, chairs, etc., and also, planking, cross ties and guard strips, is taken at 400 lbs. per ft. lineal, for single track, or 750 for double track.

For highway bridges this weight will be very different according to the style of roadway adopted and the locality and traffic, and must be estimated for the case in hand.

LIVE LOAD.—The live load adopted for railway bridges is that assumed as the basis of our diagram, Part I, page 88. This system of wheel loads is somewhat in excess of the heaviest locomotives now used, thus allowing for future increase, while it approximates closely to the actual loading. By means of the diagram, the stresses may be found with more exactness than by any other method. The train load is small, but the tabular values can easily be increased to suit any given loading.

For *highway bridges*, the live load may be varied according to the situation, as given in the following Table.

TABLE OF LIVE LOADS FOR HIGHWAY BRIDGES.

Span in Feet.	City and Suburban Bridges liable to heavy traffic. Class A.	Bridges in Manufacturing Districts—Ballasted Roads. Class B.	Bridges in Country Districts—Unballasted Roads. Class C.
100 and under	100 lbs. per sq. ft.	90 lbs. per sq. ft.	70 lbs. per sq. ft.
100 to 200	80 " " "	60 " " "	60 " " "
200 to 300	70 " " "	50 " " "	50 " " "
300 to 400	60 " " "	50 " " "	45 " " "
400 and over	50 " " "	50 " " "	45 " " "

The stringers of highway bridges are usually of wood, and the floor beams of iron. The weight of these may be easily estimated, as detailed in what follows. The *flooring* varies too much for any general values to be given. For simple pine flooring, we may take 0.35 lb. for 12 cubic inches, and the flooring is usually 3 inches thick. The weight of railing posts, hand rails, hub rails, guard rails, etc., must be estimated according to the design.

WOOD STRINGERS—TOTAL LOAD, SIZE, WEIGHT.—For highway bridges the load W supported by a stringer will depend upon the weight of roadway and the live load assumed, according to the preceding Table.

For railway bridges, taking a system of wheel loads very similar to Class A of Cooper's *Specifications*, and taking floor, cross ties, rails, bolts, guard strips, etc., at 400 lbs. per lineal foot, we have the equivalent distributed live load for *one stringer*, as given in the following Table.* For double track and 4 stringers, we may take the same loading, because

* Increase the tabular values by about 18 per cent. for the system of loads assumed in our diagram, Part I, page 88. The values given are for a lighter system, and it is scarcely necessary to change them, as they now represent good average practice.

although the load is twice as much, there are twice as many stringers. If to the equivalent distributed live load, we add 200 lbs. per lineal foot for track, and also allow a percentage for impact, we have the total equivalent distributed external load W, not including the weight of the stringer itself, which in the case of wood may be disregarded. The allowance for impact is taken at 30 per cent. of the external load for all spans below 25 feet, and $40 - \frac{2}{5}l$ for spans above 25 feet. (l = span in feet.)

WOOD STRINGERS.*

Equivalent distributed live load and total distributed external load W, upon one stringer, for railway bridges. Allowance for shock 30 per cent. for spans below 25 feet, and 40 — $\frac{2}{5}l$ for spans above 25 feet. Rails, ties, etc., 200 lbs. per ft. per stringer.

Length or panel length in feet.	Live load for one stringer in lbs.	Total external load W in lbs., including allowances for impact and flooring.	Length or panel length in feet.	Live load for one stringer in lbs.	Total external load W in lbs., including allowances for impact and flooring.
5	25000	33800	18	47222	66068
6	25000	34060	19	48685	68230
7	25000	34320	20	50000	70200
8	25000	34580	21	52380	73554
9	25000	34840	22	54545	76628
10	25000	35100	23	56521	79457
11	27272	38314	24	58333	82073
12	33333	43453	25	60000	84500
13	36538	50879	26	61537	86491
14	39286	54711	27	63518	89042
15	41666	58066	28	65355	91390
16	43750	61035	29	67068	93562
17	45588	63684	30	68832	95784

From this Table we can at once take for any given length of stringer, that is, for any given panel length, the corresponding equivalent *total external load W*, including the live load and weight of rails, ties, etc., at 200 lbs. per foot per stringer.

This load W being known, we may take at once from the following Table, the size of beam which will safely carry it.

The table gives the *safe load for one inch in breadth*, for different lengths and depths, on the condition that the deflection shall not exceed $\frac{1}{480}$th of the length, calculated from Trautwine's formula,

$$W = \frac{bd^3}{Bl^2},$$

where d = depth in inches, b = breadth in inches, l = length in feet, and $B = 0.00575$ for white oak, and 0.008 for white or yellow pine, hemlock, and red and black oak. The smaller values in the Table are for white or yellow pine, hemlock, red and black oak, and the larger values for white oak. For a concentrated load at the centre, one half of the tabular values may be taken.

In taking dimensions from this Table, it is well to bear in mind that beams over 14" deep are not readily obtained, also that market sizes are usually even inches in depth and *always* even feet in length. Thus, beams 3" × 8", or 3" × 10", or 3" × 12, are easily procured, while 3" × 9", 3" × 11", etc., are not.

* Increase these values by 18 per cent. for the system of loads assumed in our diagram, Part I, page 88.

WOOD STRINGERS.

Safe Distributed Load for One Inch Breadth, for Different Lengths and Depths, Larger Values for White oak, Smaller Values for White or Yellow Pine, Hemlock, Red and Black oak. For Concentrated Load Half these Values to be Taken.

Length in ft.	5′	6′	7′	8′	10′	12′	14′	16′	18′	20′	22′	24′	26′
Depth in inches. 6″	1080 1502	750 1044	552 766	422 588	270 374	188 262	138 192	106 146	84 116				
7″	1716 2386	1192 1656	876 1218	670 932	428 596	298 414	218 304	168 232	132 184	108 148			
8″	2560 3562	1778 2474	1306 1818	1000 1390	640 890	444 618	326 454	250 348	198 274	160 222	132 184		
9″	3644 5072	2532 3522	1860 2588	1414 1982	912 1268	634 880	466 646	356 496	282 392	228 316	188 262	158 220	
10″	5000 6956	3472 4830	2552 3550	1954 2718	1250 1740	868 1208	638 888	488 680	386 536	312 434	258 358	218 302	186 258
12″	8640 12020	6000 8348	4408 6134	3376 4696	2160 3006	1500 2088	1102 1534	844 1174	666 928	540 752	446 622	376 522	320 444
14″	13720 19088	9528 13256	7000 9740	5360 6456	3430 4772	2382 3314	1750 2434	1340 1864	1058 1472	858 1192	708 986	596 828	508 706
16″	20480 28494	14222 19788	10448 14524	8000 11130	5120 7124	3556 4948	2612 3634	2000 2782	1580 2198	1280 1782	1058 1472	890 1234	758 1054
18″	29160 40570	20250 28174	14878 20698	11390 15848	7290 10142	5062 7044	3720 5174	2848 3962	2250 3130	1822 2536	1506 2096	1266 1762	1078 1500
20″		27778 38648	20408 28394	15626 21740	10000 13914	6944 9662	5102 7098	3906 7436	3086 4294	2500 3478	2066 2874	1736 2416	1480 2058
22″			27164 37792	20796 28934	13310 18518	9244 12860	6792 9448	5200 7234	4108 5716	3328 4630	2750 3826	2312 3214	1970 2738
24″				27000 37566	17280 24042	12000 16696	8816 12266	6750 9392	5334 7420	4320 6010	3570 4970	3000 4174	2556 2556

When the dimensions of stringer have been chosen, the weight may be found from the formula,

$$\text{weight} = bdl\gamma,$$

where b and d are the breadth and depth in inches, l the length in feet, and γ the weight of 12 cubic inches. We may take γ as equal to 0.35 lbs. for ordinary purposes, and hence

$$\text{weight of wood stringer} = 0.35bdl.$$

EXAMPLE.—*A white oak stringer in a railway bridge is 12 feet long. What dimensions should it have, and what is its weight?*

The distributed load W, is from the Table, 43453 lbs. From the last Table, we see that a beam 18" deep and one inch in breadth will carry safely 7044 lbs. Our stringer then may be 18" deep by 6" wide. Other dimensions may be taken from the Table, as for instance 16" deep and 9" wide, etc. If the latter dimensions are adopted, the weight is

$$bdly = 9 \times 16 \times 12 \times 0.35 = 605 \text{ lbs.}$$

We can seldom take more than 16" to 18" depth and 6" to 8" width, as heavier timbers are costly. Where a single beam would be too large, several may be used side by side. Thus instead of one beam 16" by 9", we may have three each 16" by 3". Two beams 14" by 6" would be more easily procured and would be sufficient.

EXAMPLE.—*A white oak stringer in a railway bridge is 16 feet long. What dimensions should it have, and what is its weight?*

Here the distributed load is 61035 lbs. If we take the depth at 14 inches, the safe load is 1864 for one inch width. This would require a width of about 32 inches or 4 beams of 8 inches width. Such beams would be better replaced by iron stringers.

IRON PLATE STRINGERS, THICKNESS, DEPTH AND WEIGHT.—Iron stringers for railway bridges, whether single or double track, of less than 15 feet in length, may usually be made of rolled I beams. Above this length such beams are not heavy enough, and plate girders or built beams of plate and angle irons must be used.

The thickness of plate or web will usually be determined by the size and bearing of rivets. If the web is not thick enough, it will not be possible to have rivets enough in the flanges.

Let the total load, $W' + W$, including therefore the weight W' of the girder itself, be reduced to an equivalent uniformly distributed load, and represented by $W' + W$, and let l be the span in feet and d the depth in inches from outside to outside. Then, with sufficient accuracy for our purposes, the moment at the centre is $\frac{(W' + W)l}{8}$. If we divide this by the depth in feet or by $\frac{d}{12}$ where d is in inches, we get a fair estimate of the stress in one flange. The number of rivets to resist this would be $\frac{6l}{\text{pitch}}$, and the resistance of a single rivet is diameter $\times\ t \times$ bearing resistance per square inch, where t is the thickness of web, and the diameter of rivet is in inches. We have then

$$t = \frac{(W' + W) \times \text{pitch}}{4 \times \text{diameter} \times \text{bearing resistance} \times d}.$$

Taking the bearing resistance per square inch at 12000 lbs. and the diameter of rivet at $\frac{7}{8}$ inch, and the minimum pitch at 3 inches, we have

$$t = \frac{W' + W}{14000d}.$$

The thickness of web must never be less than $\frac{1}{4}$ inch, the least allowable thickness of plate. It will rarely by the above formula be greater than $\frac{3}{8}$ inch.

For stringers, the flanges are usually of uniform cross section. Let the weight of the stringer itself be W'. Then if R is the mean stress per square inch in both flanges, and we disregard the web, the moment at the centre will be, accurately enough for our purposes,

$$\frac{(W + W') \times 12l}{8} = \text{area of one flange} \times R \times d.$$

The area of both flanges then will be

$$\frac{(W + W') \, 12l}{4Rd}.$$

If the thickness of the web is t, its area will be dt. The total area is then about

$$\frac{(W + W') \, 12l}{4Rd} + dt.$$

If we multiply this by $\frac{10}{3}$ we have the weight of one foot in length. The total weight is then about

$$\left[\frac{(W + W') \, 12l}{4Rd} + dt\right] \tfrac{10}{3}l = W'.$$

As the thickness of web is rarely more than $\frac{3}{8}''$, if we take it $\frac{1}{2}''$, we make an allowance to cover connections, etc.; we have then

$$W' = \frac{12Wl^2 + 2Rld^2}{1.2Rd - 12l^2}. \quad \ldots \ldots \ldots \ldots \quad (1)$$

From equation (1) we can make a close estimate of the weight in pounds W' of any plate stringer of uniform depth, when the length in feet l, clear depth in inches d, mean working stress in lbs. per square inch R, and total equivalent load in lbs. W, are known.

Differentiating and putting the first differential equal to zero, we have the depth in inches corresponding to least weight

$$\text{least weight depth} = \frac{10l^2}{R} + \sqrt{\frac{6Wl}{R} + \left(\frac{10l^2}{R}\right)^2}. \quad \ldots \ldots \quad (2)$$

From equation (2), we can find the "least weight depth" in inches for an iron plate stringer, when the total equivalent external load W in lbs., length in feet l, and mean working stress R in lbs. per sq. inch, are known.

The least weight depth is not necessarily the depth for *least cost*, or best depth. Moreover, the depth is usually governed by considerations depending upon the design, so that formulas for depth are of little practical value. If no such considerations apply, the best depth or least cost depth from centre to centre of rivet holes may be taken as not far from $\frac{8}{10}$ths of the least weight clear depth as given by equation (2). As this latter serves then as a basis of estimation, we have thought it well to give it in the Tables which follow. In view of the preceding remarks, the practical value of the equation (2) should not, and probably will not, be over estimated.

TOTAL EXTERNAL EQUIVALENT LOAD W FOR IRON PLATE RAILWAY STRINGERS.—The total external load W, for railway stringers, is composed of the equivalent distributed live load, the allowance for impact and the allowance for weight of rails, ties, etc., *viz.*, 200 lbs. per foot per stringer. It is usual to make allowance for impact by adding to the equivalent live load a certain percentage, depending upon the length of the stringer. We take here in addition, 30 per cent. of the equivalent live load for spans below 25 feet, and $40 - \frac{2}{5}l$ per cent. for spans above 25 feet, where l is the span in feet. The equivalent distributed live load is that found for a system of wheel loads very similar to Class A of Cooper's *Specifications*.

We give, in the following Table, the equivalent distributed live load, and the total

external load W, for different lengths of stringer, for the above allowance for impact and floor and live load. We also give the weight and least weight depth as found from equations (1) and (2), taking $R = 8000$ lbs. per square inch.

IRON PLATE STRINGERS OF UNIFORM DEPTH.*

Equivalent distributed live load and total external load W upon one stringer, for railway bridges. Also weight and least weight depth. Allowance for shock 30 per cent. of external load for all spans below 25 feet, and $40 - \frac{2}{5}l$ for all spans above 25 feet (l = span in feet). Rails, ties, etc., 200 lbs. per ft. per stringer. $R = 8000$ lbs. per square inch.

Length or panel length in feet.	Equivalent live load per stringer in lbs.	Total external load W in lbs., including allowance for impact and flooring at 200 lbs. per ft.	Weight in lbs. W'.	Least weight depth in inches.	Length or panel length in feet.	Equivalent live load per stringer in lbs.	Total external load W in lbs., including allowance for impact and flooring at 200 lbs. per ft.	Weight in lbs. W'.	Least weight depth in inches.
5	25000	33800	188	11.3	18	47222	66068	1816	30.3
6	25000	34060	248	12.4	19	48685	68230	2003	31.6
7	25000	34320	315	13.5	20	50000	70200	2197	33
8	25000	34580	386	14.5	21	52380	73554	2421	34.6
9	25000	34840	463	15.4	22	54545	76628	2652	36.2
10	25000	35100	545	16.4	23	56521	79457	2900	37.7
11	27272	38314	657	18	24	58333	82073	3133	39.2
12	33333	46453	825	20.6	25	60000	84500	3382	40.7
13	36538	50879	974	22.5	26	61537	86491	3633	42
14	39286	54711	1130	24.2	27	63518	89042	3904	43.4
15	41666	58066	1292	25.8	28	65355	91390	4180	44.8
16	43750	61035	1460	27.4	29	67068	93562	4463	46.2
17	45588	63684	1634	28.8	30	68832	95784	4756	47.5

Any depth may of course be taken in designing, which seems desirable. As the weight varies but little with a change of depth, the Table will in all cases give a good estimate of the weight. For the best depth, if no other considerations affect it, $\frac{8}{10}$ths of the least weight clear depth as given by the Table, will not be far from the best or least cost effective depth.

Flanges of Stringers.—From the preceding Table we can find at once the maximum load $W' + W$, sustained by a stringer, and its least weight clear depth. Since W' is small compared to W, the weight W' given in the Table is near enough for any depth which may be taken. The effective depth is the depth from centre to centre of rivet holes. It may be taken as $\frac{8}{10}$ths of the clear depth given in the Table, if no other considerations affect it. The loading assumed in our Table is intended to be large enough to cover future increase of traffic.

The maximum load $W' + W$ being thus known, the moment at the centre in inch pounds will be $\frac{(W' + W)l}{8}$, where l is the length in inches. If d is the effective depth in inches, the moment of resistance of the web is $\frac{RI}{v} = \frac{Rtd^2}{6}$, where R is the allowable stress in lbs. per square inch, and t is the thickness of the web in inches. The area of the upper flange at the centre is then

$$\frac{(W' + W)l}{8Rd} - \frac{td}{6},$$

where $(W' + W)$ is taken from the Table in lbs., d is $\frac{8}{10}$ths of the value for d in inches given in the Table, if no other considerations determine the depth, and l is the span in inches.

* Increase values for W by 18 per cent. for the system of loads of our diagram, Part I. page 88. The depths remain the same.

The nearest angle iron which will suit can then be taken from Carnegie's Pocket Book. The bottom flange should be calculated from net section or area, with rivet holes deducted. The rivets are usually taken at from $\frac{5}{8}$ to $\frac{7}{8}$ inches.

Web Plate of Stringers.—The web is composed of plate, not less than $\frac{1}{4}$ inch and rarely more than $\frac{5}{8}$ths inch. The upper limit may be found by the formula already given, $t = \dfrac{W' + W}{14000d}$. The shear at any point ought not to exceed 8000 lbs. per sq. inch. The shear is of course greatest at the ends, where it is equal to half the total load or $\dfrac{W' + W}{2}$. The web must also be prevented from buckling.

This condition is attained when the shear per square inch of cross section at any point does not exceed the

$$\textit{safe resistance to buckling per square inch} = \frac{10000}{1 + \dfrac{d^2}{3000t^2}},$$

where d and t are the depth and thickness of web in inches.

Stiffeners.—Ordinarily this formula gives a lower stress per square inch than 8000 lbs., so that when it is fulfilled, the web is safe against shearing also. When, however, the web is safe against shearing, at 8000 lbs. per square inch, but not safe against buckling, as tested by the preceding formula, instead of increasing the thickness of the whole web, "*stiffeners*" are used.

These stiffeners consist of vertical strips or angle irons, riveted to the web at intervals. The intervals between stiffeners, in girders over 3 feet in depth, should not exceed the depth of girder, with a maximum limit in any case of 5 feet. Under 3 feet depth, they may be spaced every 3 feet when needed. They should be calculated as columns by the formula, $\textit{safe resistance to buckling per square inch of cross section} = \dfrac{10000}{1 + \dfrac{d^2}{3000t^2}} \geqq$ *the shear per square inch at the point where the stiffener is placed*, where $d =$ depth in inches, and $t =$ thickness of web and stiffener in inches.

Stiffeners should always be placed at the ends, wherever the web plate is spliced, and at any point where a concentrated load acts, as the point of attachment of the stringers to the cross girders. Splicing of the web sheets is unnecessary in stringers and cross girders, as sheets of the requisite depth and length can be supplied in one piece.

EXAMPLE.—*Required to design a railway track stringer 17 feet long.*

From our Table, the weight of such a stringer is about 1634 lbs. and the least weight clear depth about 29 inches. If no other considerations influence our choice of depth, we may then take about $\frac{8}{10} \times 29 = 23$ inches for the least cost or best effective depth. $W' + W$ is then $63684 + 1634 = 65318$, or about 65000 lbs.

Flanges.—If we take the web at $\frac{1}{4}$ inch, the area of the top flange is

$$A = \frac{65000 \times 17 \times 12}{8000 \times 8 \times 23} - \frac{23}{4 \times 6} = \text{about 8 square inches.}$$

This requires angles weighing $\dfrac{8 \times 10}{3 \times 2} = 13.3$ lbs. per ft. From Carnegie we see that angles $4\frac{1}{2} \times 3 \times \frac{9}{16}$ will answer. For the bottom flanges we must have 8 sq. inches net. Taking $\frac{5}{8}''$ rivets, the gross section should be $8 + 2 \times \frac{9}{16} \times \frac{5}{8} = 8.70$ sq. inches. This calls for angles weighing $\dfrac{8.7 \times 10}{3 \times 2} = 14.5$ lbs. per foot. From Carnegie we have angles $5 \times 3 \times \frac{9}{16}$ for the lower flanges.

The application of our rule for thickness of web gives $\frac{65000}{14000 \times 23} = 0.20$ inch.

Web.—Let us therefore take the web plate at $\frac{1}{4}''$ thick, and see if this is safe against shear. The shear at the end is $\frac{65000}{2} = 32500$ lbs.; at 8000 lbs. per sq. inch, this requires 4.06 square inches. But the actual cross section is $23 \times \frac{1}{4} = 5.75$ sq. inches. The thickness is then more than sufficient to resist the shear.

Stiffeners.—At the ends we always need stiffeners, unless the stringers are riveted at the ends to the web of the cross girders, when the rivet angles will answer the purpose. We must also have stiffeners every three feet if found necessary. The shear at the end is $\frac{32500}{5.75} = 5652$ lbs. per square inch. But as the web is $t = \frac{1}{4}''$ thick, its safe resistance to buckling is $\frac{10000}{1 + \frac{d^2}{3000t^2}} = \frac{10000}{1 + \frac{529 \times 16}{3000}} = 2617$ lbs. per sq. inch. As this is less than 5652, we need stiffeners at the end. If we take two filling plates $2'' \times \frac{9}{16}''$, giving an area of 2.25 sq. inches, and two angle irons $2 \times 2 \times \frac{1}{4}$, area 1.86 sq. inches, the total area, including the web, is $2.25 + 1.86 + 0.5 = 4.61$ sq. ins., and the total thickness is $\frac{9}{8} + \frac{1}{2} + \frac{1}{4} = \frac{15}{8}$ inches. The resistance to buckling is then $\frac{10000}{1 + \frac{529 \times 64}{3000 \times 225}} = 9524$ lbs. per square inch of cross section, or $9524 \times 4.61 = 43905$ lbs. As this is greater than the end shear of 32500 lbs., the stiffeners are ample.

At 3 feet from the end, the shear is $32500 - 3 \times \frac{65000}{17} = 21030$ lbs. or $\frac{21030}{5.75} = 3657$ lbs. per square inch. As this is greater than the safe resistance of the web to buckling, 2617 lbs., we need stiffeners here also. Let us take here simply two filling plates, $2 \times \frac{9}{16}$, area 2.25 sq. ins., or total area, including the web, 2.75 sq. ins., and total thickness $\frac{9}{8} + \frac{1}{4} = \frac{11}{8}$ ins. Then the resistance to buckling is $\frac{10000}{1 + \frac{529 \times 64}{3000 \times 121}} = 9150$ lbs. per square inch of cross section, or $9150 \times 2.75 = 25162$ lbs. As this is greater than the shear, 21030 lbs., it is sufficient.

At 6 feet from the end, the shear is $32500 - 6 \times \frac{65000}{17} = 9560$ lbs., or $\frac{9560}{5.75} = 1662$ lbs. per sq. inch. As this is less than the safe resistance of the web to buckling, 2617 lbs., no stiffeners are needed.

Rivets and Rivet Spacing.—The size of rivets may be found by the rule $d = 1\frac{1}{4}t + \frac{3}{16}$, except that if this rule in any case gives a less diameter than $\frac{3}{4}$ or $\frac{5}{8}$ at least, the latter diameter is to be taken.

We have already illustrated the method of determining the number of rivets quite fully, page 440. In the present case our rule gives $d = \frac{5}{4} \times \frac{9}{16} + \frac{3}{16} = \frac{7}{8}''$ rivets. The distributed load is 65000 lbs. The bearing resistance for $\frac{1}{4}''$ plate and $\frac{7}{8}''$ rivet is, from Rivet Table I., 2730 lbs. The horizontal stress at any distance x from end is $\frac{65000x}{3.83}\left(1 - \frac{x}{17}\right)$, see page 400. If we take $x = 2.5$, 5 and 8.5 feet, we have the horizontal stresses 14.35 tons, 23.75 tons, 28.6 tons. Subtracting each from the one following, we have 14.35 tons, 9.4 tons, 4.85 tons, for the horizontal stresses to be taken by the rivets in the different lengths. The load on the first length of 2.5 feet is $\frac{65000}{17} \times 2.5 = 4.78$ tons; on the next 2.5 feet, 4.78 tons; on the last 3.5 feet, 6.7 tons. The resultant stress for the first division of 2.5 feet is then $\sqrt{14.35^2 + 4.78^2} = 15.12$ tons or 30240 lbs. In the next division of 2.5 feet it is $\sqrt{9.4^2 + 4.78^2} = 10.54$ tons or 21080 lbs. In the last division of 3.5 feet it is $\sqrt{4.85^2 + 6.7^2} = 8.27$ tons or 16540 lbs. We require for bearing then $\frac{30240}{2730} = 11$ or 12 rivets; in the next 2.5 feet, $\frac{21080}{2730} = 8$ rivets; and in the last 3.5 feet $\frac{16540}{2730} = 6$ rivets. If we take a pitch of 2.5 inches in the first 2.5 feet, which is just 3 times the diameter, and therefore the least allowable, 4 inches in the next 2.5 feet, and 5 inches in the last 8.5 feet, we shall have rivets enough.

We should always arrange to have rather more than less rivets as calculated. We see also that if the depth is taken too small, the flange stresses will be so great that it may be impossible to get in rivets enough without overcrowding.

FLOOR BEAMS OR CROSS GIRDERS.—EXTERIOR LOADING, THICKNESS, WEIGHT, DEPTH.—Equations (1) and (2) apply to plate cross girders also. The total external load W upon a cross girder consists of the greatest live load, the weight of the stringers, weight of rails, ties, etc., and the allowance for impact. We may take W therefore, for double track, at about twice what it is for single track.

Since the stringers are attached to the cross girders at or near the quarter points, the total load upon a cross girder may be taken as uniformly distributed, so far as the moment

at the centre is concerned. The thickness of web is never to be less than $\frac{1}{4}$ inch. The other limit is, as for stringers, already found

$$t = \frac{W' + W}{14000d},$$

where W is the equivalent distributed load.

The same remarks as to depth hold here as to stringers. The least weight clear depth given in the Tables which follow, multiplied by $\frac{8}{10}$ths, will give the best effective depth near enough, if no other considerations limit the depth.

We give, in the following Table, the live load on a cross girder for different panel lengths, based upon a load system very similar to Class A of Cooper's *Specifications*. Also the total external load W, including the live load, the weight of two stringers, the weight of rails, ties, etc., taken at 400 lbs. per ft. and the allowance for impact, taken at 30 per cent. of the load. The Table is for single track. For double track, double the tabular values may be taken.

IRON PLATE CROSS GIRDERS OF UNIFORM DEPTH.*

Live load and total external load W for single track. For double track take double these values. Rails, ties, etc., 400 lbs. per ft., allowance for impact 30 per cent.

Panel length in feet.	Live load in lbs.	Total external load W in lbs., including live load, weight of two stringers and floor, and allowance for impact.	Panel length in feet.	Live load in lbs.	Total external load W in lbs., including live load, weight of two stringers and floor, and allowance for impact.
5	25000	35590	18	78055	115560
6	33333	47100	19	81578	121140
7	39285	55530	20	84750	126290
8	43750	62040	21	87619	131400
9	47222	67270	22	90226	135630
10	50000	71620	23	93260	140740
11	54545	78340	24	96041	145480
12	58332	84220	25	98400	150040
13	62690	90790	26	100960	154220
14	66428	96575	27	103147	158280
15	69666	101725	28	105714	162860
16	72500	106370	29	108103	167220
17	75000	110590	30	110333	171400

The total external load W being known, we can easily find the least weight clear depth and the weight W' of the cross girder for any given length and loading from equations (1) and (2), page 474.

We give in the following Table, the least weight clear depth and the corresponding weight for cross girders 15 and 25 feet long, for single and double track respectively. Any depth may of course be taken in designing, which may be desired, and as the weight varies but little with a change of depth, the Table will in all cases give a good estimate of the weight, for the lengths assumed. About $\frac{8}{10}$ths of the depth given in the Tables will be the best effective depth, if no other considerations affect it.

* Increase these values by 18 per cent. for the system of loads given by our diagram, Part I, page 88.

IRON PLATE CROSS GIRDERS.*

WEIGHT AND ECONOMIC DEPTH FOR SINGLE TRACK, 15 FEET WIDE, AND DOUBLE TRACK, 25 FEET WIDE. RAILS, TIES, ETC., 400 LBS. PER FT. $R = 8000$ LBS. PER SQUARE INCH. ALLOWANCE FOR IMPACT 30 PER CENT.

Panel length in feet.	Single track 15 feet wide.		Double track 25 feet wide.		Panel length in feet.	Single track 15 feet wide.		Double track 25 feet wide.	
	Depth in inches.	Weight in lbs.	Depth in inches.	Weight in lbs.		Depth in inches.	Weight in lbs.	Depth in inches.	Weight in lbs.
5	20	1014	37	3110	18	36.3	1811	66.5	5551
6	23	1165	43	3567	19	37.2	1860	68.2	5682
7	25	1263	46	3868	20	38	1898	69.6	5743
8	27	1358	49	3985	21	38.7	1937	71	5915
9	28	1390	51	4270	22	39	1967	72	6008
10	29	1433	52.6	4384	23	40	2003	73.4	6060
11	30	1500	55	4582	24	40.7	2036	74.6	6161
12	31	1553	57	4748	25	41.4	2068	75.8	6257
13	32	1612	59	4886	26	42	2096	76.8	6345
14	33	1662	61	5080	27	42.4	2124	77.8	6427
15	34	1706	62.5	5212	28	43	2154	79	6564
16	35	1744	64	5328	29	43.6	2172	80	6664
17	35.5	1772	65	5400	30	44.2	2209	81	6746

The designing of the cross girder is the same as that of a stringer, as already illustrated, page 476.

EXAMPLE.—*A single track railway bridge has a width of 15 feet and panel length of 17 feet. What is the best depth and weight of the cross girders? Also, if the stringers are attached at 4 feet from the ends, required to design the girder.*

The best effective depth by the preceding Table is $\frac{13}{16} \times 35.5 = 28.5$ inches. The total external load is by our Table 110590 lbs. The weight by the preceding Table is about 1772 lbs.

We can now design the cross girder just as in the case of a stringer.

Flanges.—Thus the total load is $110590 + 1772 = 112362$ lbs. $= W' + W$. If the stringers are attached at say 4 feet, or 48 inches, from the ends, the moment at the centre is $\frac{W' + W}{2} \times 48 = 2696688$ inch lbs. If the web is $\frac{1}{4}$ inch, the moment of the web is $\frac{Rtd^2}{6}$. Subtracting this from the moment at the centre, we have the moment for the upper flange. Dividing by R and by d, we have the area

$$A = \frac{2696688}{8000 \times 28.5} - \frac{28.5}{4 \times 6} = 11 \text{ sq. inches.}$$

The upper angles should weigh then $\frac{11 \times 10}{2 \times 3} = 18.3$ lbs. per ft. each. From Carnegie, this calls for angles $5 \times 4 \times \frac{3}{8}$ for the upper flanges.

For the lower flange we must have 11 sq. inches net. Our rule $1\frac{1}{4}t + \frac{3}{16}$ gives $\frac{5}{4} \times \frac{3}{8} + \frac{3}{16} = \frac{11}{16}$ for the rivets. The gross area then, should be $11 + 2 \times \frac{3}{8} \times \frac{11}{16} = 12.17$ square inches. The bottom angles weigh then $\frac{12.17 \times 10}{2 \times 3} = 20.3$ lbs. per ft. From Carnegie this calls for angles about $6 \times 4 \times \frac{3}{8}$.

Web.—For the web we have the thickness by our rule $\frac{112362}{14000 \times 28.5} = 0.28$. If we take the web $\frac{5}{16}$ of an inch thick, then there will be more bearing than the rivets require. The area at end then is $\frac{5}{16} \times 28.5 = 8.9$ sq. ins. at 8000 lbs. per square inch, this gives 71200 lbs. safe resistance. The shear at end is $\frac{112362}{2} = 56181$ lbs. There is therefore ample resistance to shear.

* Increase the values for weight by 18 per cent. for the system of loads given by our diagram. Part I, page 88. The depth remains the same.

If the cross girder is riveted at the ends to the posts, no stiffeners will be needed at the ends, and if the stringers are riveted to the web of the cross girder, no stiffeners will be needed there. If, however, the girder is hung by beam hangers from the chord pin, and if the stringers are laid on top, stiffeners may be needed.

In such case, the safe resistance of web per square inch to buckling is

$$\frac{10000}{1 + \frac{d^2}{3000t^2}} = \frac{10000}{1 + \frac{28.5^2 \times 256}{3000 \times 25}} = 2652 \text{ lbs.}$$

The shear per square inch at the end is $\frac{56181}{8.9} = 6312$ lbs. We need then stiffeners at the end and also at the points where the stringers cross.

If we use for the end stiffeners, two filling plates $\frac{5}{8}''$ thick, the total thickness is $\frac{25}{16}$ inches. The resistance to buckling is then $\frac{10000}{1 + \frac{28.5^2 \times 256}{3000 \times 625}} = 9009$ lbs. per sq. inch. If the filling plates are 4 inches wide, the area, including the web, will be 6.25 square inches, and the safe resistance $9009 \times 6.25 = 56306$ lbs. As the shear at the end is 56181 lbs., these plates will be sufficient. The same stiffeners may be used under the stringers.

As to the rivets, the size already determined is $\frac{15}{16}''$. The bearing resistance for this size and $\frac{5}{16}''$ plate is, from Rivet Table 1, 3660 lbs. The horizontal stress at the first stringer, which is 4 feet from the end, is $\frac{56181 \times 48}{28.5}$ = about 94620 lbs. The horizontal stress beyond this point at any point is the same. The vertical load is 56181 lbs. The resultant stress is then *for the half span*, $\sqrt{47.3^2 + 28^2} = 55$ tons $= 110000$ lbs. This requires $\frac{110000}{3660}$ = about 30 rivets. If we pitch the rivets then at 3 inches for the entire length, which is allowable, as this pitch is greater than 3 times the diameter, we shall have about 30 rivets in the half span.

BEAM HANGERS.—When the cross girder is not riveted to the post, it is hung from the pin by beam hangers, as represented in Fig. 268, Plate 20. The hangers go in pairs, and each one takes therefore $\frac{1}{4}$ and each leg $\frac{1}{8}$ of the total load $W + W'$, on a cross girder. Owing to impact, the unit stress is taken very low, about 5000 lbs. per square inch. The allowance for upset ends and nuts will be found on page 459.

EXAMPLE.—*In the preceding example, what should be the size of the beam hangers?*

The total load $W' + W$ has been found to be 112362 lbs. The tension on each leg is therefore 14045 lbs. At 5000 lbs. per square inch, this gives 2.809 sq. ins, or about $1\frac{7}{8}''$ diameter. The length of rod required to make a beam hanger, since the cross girder is 35.5 inches deep, will be about 74 inches. To this add 2 feet (page 408) for upset ends, and we have about 8 feet. The weight will be, from Carnegie, about 73 lbs. for each hanger.

PLATE GIRDER BRIDGES, LIVE LOAD.—Below 80 feet, plate girder bridges are usually preferred to pin connected trusses with open web.*

The designing of a plate girder is similar to that of a track stringer or cross girder. except that the flanges cannot usually be made of angle irons alone, but must be re-enforced by cover plates laid on top of the angles and riveted to them. The flange area can thus be adjusted to the stress at different points, by increasing the number of cover plates or their thickness from end to centre of girder.

Total External Load.—Our equations (1) and (2), will give us a good estimate of the weight of girder and least weight clear depth, provided the total external load W is known. About $\frac{8}{10}$ths of this depth may be taken as the least cost effective depth. The total external load is composed of the flooring, rails, ties, etc., which for single track railway may be taken at 400 lbs. per foot; of the weight of the track stringers and cross girders; of the wind bracing; and of the live load. We have just learned how to design the track

* Plate girders are practically limited to lengths which do not require more than two ordinary flat cars 33 feet long for transport, *i.e.*, 65 feet span. The length is more rarely extended to three car lengths, or about 100 feet maximum. They are riveted at the shops, and are preferable to lattice girders, being cheaper, costing less for maintenance, and having greater security; as faulty rivets produce less reduction of strength. They are also more free from corners and recesses, and are therefore cleaner and less exposed to oxidation.

stringers and cross girders and find their weight. The formulæ for weight of wind bracing, page 457, will give a good estimate. The equivalent distributed live load for any span can be found from our diagram, Part I, page 88. We give in the following Table the equivalent distributed live load thus found for a system very similar to Class A of Cooper's *Specifications*. The values given are for single track, and for all the girders. Thus, if *two* plate girders are used, one half the tabular values should be taken. For double track, double the tabular value gives the total live load, which is to be divided among the girders according to their number. The allowance for impact is 30 per cent. of the load for spans under 25 ft., and $40 - \frac{2}{5}l$ for greater spans, where l is the span in feet.

EQUIVALENT DISTRIBUTED LIVE LOAD FOR SINGLE TRACK PLATE GIRDER BRIDGES.*

Span in feet.	Live load.	Span in feet.	Live load.	Span in feet.	Live load.	Span in feet.	Live load.
10	50000	28	130700	46	192860	64	235520
11	54550	29	134140	47	195190	65	237280
12	66670	30	137670	48	198420	66	239130
13	73080	31	140310	49	200340	67	240850
14	78580	32	143130	50	203500	68	242500
15	83340	33	152700	51	205270	69	244750
16	87500	34	154950	52	207970	70	246770
17	91180	35	157000	53	209790	71	248530
18	94450	36	158900	54	212480	72	250470
19	97370	37	163000	55	213960	73	252380
20	100000	38	170000	56	217300	74	253940
21	104760	39	172800	57	218770	75	256070
22	109090	40	176280	58	222040	76	258470
23	113040	41	179190	59	224900	77	260760
24	116670	42	181890	60	227770	78	262770
25	120000	43	184580	61	229860	79	264490
26	123080	44	186940	62	231820	80	267120
27	127040	45	189300	63	233700		

Girder Spacing.—Single track plate girder deck bridges usually have the girders spaced 6′ 6″ from centre to centre, and double track deck bridges have usually 3 girders spaced 9′ 3″ apart, so that each girder will carry an equal share of the total load on both tracks.

Single track plate girder through bridges should be at least 15 feet from centre to centre of girders, and double track have usually 3 girders likewise 15 feet apart, in which case the outer girders carry one-half the total load on one track, and the middle girder the entire load of one track. If only two girders are used, they can be spaced 28 feet from centre to centre, each one carrying the entire load for one track.

The spacing of the main girders determines the length of panel for the wind bracing, which may be taken a little longer than the width, and so as to make an even division of the length. The number of panels being thus chosen, the *total* weight per ft. lineal of the wind bracing may be found from the formula of page 417, viz.:

$$\textit{total weight per ft. lineal of wind bracing} = 3.6N + \frac{540}{p},$$

where l = length in feet of span, N = number of panels, p = panel length in feet.

For double track, multiply by $\frac{b}{15}$, where b is the width in feet.

The length of cross girders will be also determined by the width, and the length of track stringers by the panel length just found. The cross girders and stringers may therefore be calculated as already illustrated. The flooring, rails, ties, etc., being then esti-

* Increase these values by 18 per cent. for the system of loads given by our diagram, Part I, page 88.

mated, and finally the live load taken from the preceding Table, we can find the total external load W.

Weight and Depth.—This load can then be divided among the girders according to their number and spacing. We can then find the weight and least weight clear depth of the girders, from the equations,

$$\text{weight} = \frac{12\ Wl^2 + 2\ Rld^2}{1.2\ Rd - 12l^2},$$

$$\text{depth} = \frac{10l^2}{R} + \sqrt{\frac{6Wl}{R} + \left(\frac{10l^2}{R}\right)^2},$$

where $W =$ the total external load per girder; $R =$ allowable unit stress $= 8000$ lbs., $l =$ length in feet, $d =$ depth in inches.

About $\frac{8}{10}$ths of the least weight clear depth may be taken as the least cost effective depth.

Floor System.—For single track deck plate girders the cross ties are notched to the girder flanges and secured to them through the guard strips by bolts. The rails are laid over the cross ties in the usual way. No stringers or floor beams are required. For the girder spacing already given, the ties are of sawed white oak, about 8′ 6″ long, 7″ deep and 8″ wide, spaced about 16″ from centre to centre. For double track, for the girder spacing as given, the cross ties may be about 20′ 6″, 9″ deep and 8″ wide, spaced 16″ between centres. The pine guard strips are 6″ by 8″, laid outside of and parallel to the rails, and spliced and bolted at joints.

For through plate girder bridges, iron floor beams and stringers should be used. The ties and rails are laid upon the stringers precisely as in the case of deck bridges. The iron track stringers are usually spaced 6′ 6″ between centres. They may rest upon the floor beams or be riveted between them. In the first case they must be strongly spliced at the joints, and continued over the piers, and rest at the pier ends upon bearing plates so as to allow of contraction and expansion. In the second case they may be supported by bracket angles riveted to the floor beams, and the stringer ends should be riveted to the web of floor beams by angle irons.

Web and Flanges.—The web for the heaviest bridge is rarely over $\frac{5}{8}$″ thick, and never less than $\frac{1}{4}$″. Within these limits it may be proportioned by our rule $\frac{W' + W}{14000d}$.

Crippling or buckling of the web is to be guarded against by the formula

$$\text{safe resistance per sq. inch} = \frac{10000}{1 + \dfrac{d^2}{3000t^2}},$$

where d is the depth and t the thickness in inches, and the stiffeners are to be calculated by the same formula, and spaced, for girders over three feet in depth, at distances apart not exceeding the depth, with a maximum limit of five feet. Under three feet depth, 3 feet apart.

The flange angles are of equal section throughout and re-enforced by cover plates as the stress increases towards the centre. These cover plates should project slightly beyond the outer edges of the horizontal legs of the angles, and are riveted to them by rivets of proper size and number.

Web and flange angles can nearly always be ordered in one length. If the web is ordered in sections, it can be so arranged that the splices come at the stiffeners. The flange angles at least should be always in one length.

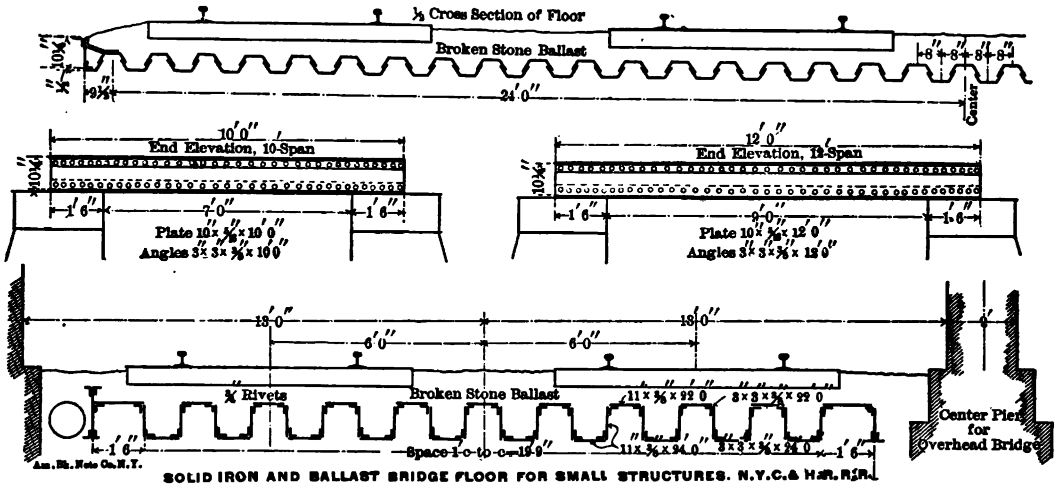
In through bridges knee braces may be introduced at every floor beam for lateral support to the girders. The wind bracing offers no special points of difference from ordinary bridges.

Rivets.—The size of rivet may be taken from our rule,

$$d = 1\tfrac{1}{4}t + \tfrac{3}{16},$$

where d is the diameter and t the greatest thickness of plate, in inches, provided that the result is not less than $\frac{3}{4}''$. The pitch should never exceed 6″, or to prevent buckling, 16 times the thinnest outside plate, nor be less than 3 diameters. The distance from edge to centre of rivet hole should not be less than $1\frac{1}{2}''$, and if practicable at least 2 diameters. When the flange plates are over 12 inches wide, or more than 3″ project beyond the angles, an extra line of rivets with a pitch of not over 9″ should be driven along the edges to draw the plates together and keep out water.

SOLID FLOOR PLATE GIRDER.—Instead of the style of floor consisting simply of cross girders, stringers, and ties, there is a decided tendency on the part of some of our leading railroads to "solid floors," the ballast and roadway being continued on the bridge itself. The accompanying illustration, taken from the *Engineering News* for November 16, 1889, shows the practice in this respect of the N. Y. C. & H. R. R.R., as given by George H. Thomson, C.E., Bridge Engineer of the Company.



SOLID IRON AND BALLAST BRIDGE FLOOR FOR SMALL STRUCTURES. N.Y.C.& H.R.R.R.

The top section shows a floor built of Pencoyd standard heavy trough sections, fastened by rivets. This form of floor is for small spans. The lower section shows half of a four track bridge, length, 24 feet, clear span, 21 feet, 2 feet depth of floor.

The two elevations given between the two sections show short spans of similar floors.

In the next illustration, we have the system for larger spans.

Mr. Thomson, in an abstract of a paper on "Railway Structural Economics," summarized in *Engineering News*, November 23, 1889, classifies floors under the following heads:

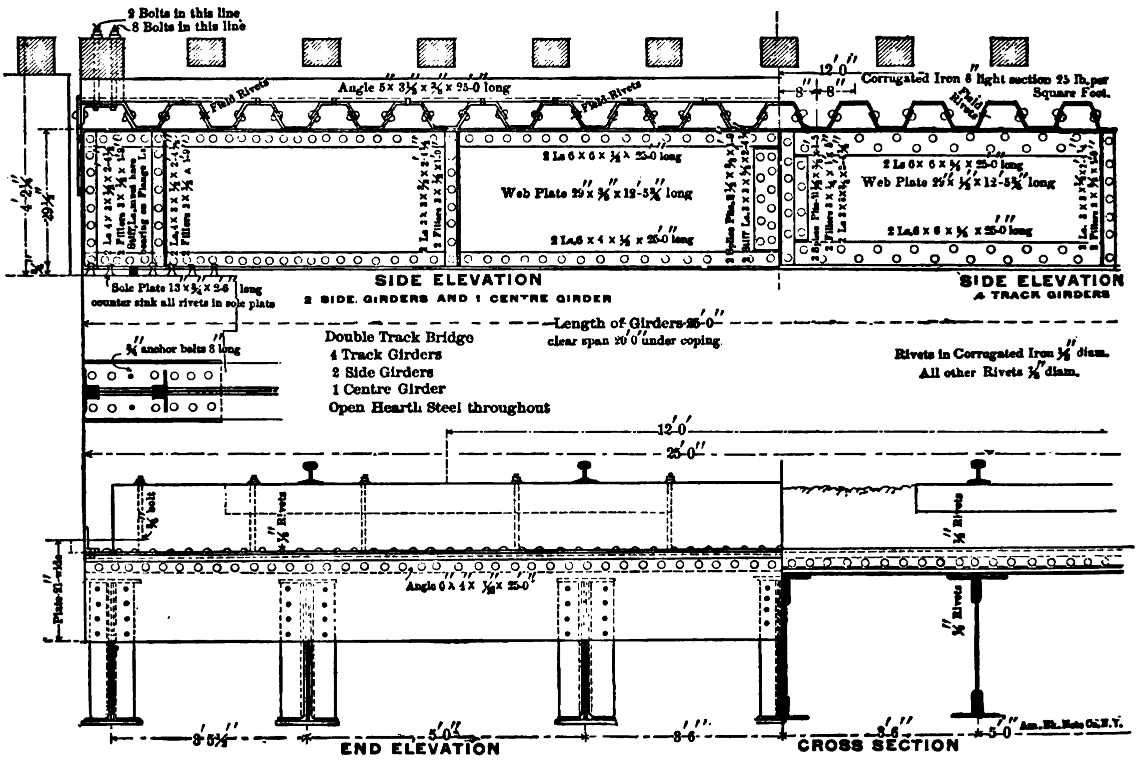
By a first-class floor is meant a solid floor of the type illustrated.

By a second-class floor is meant the usual system of cross girders and stringers.

By a third-class floor is meant wooden floor beams and cross ties.

The phenomena of "bunching" and "scooping," which occur with second and third, cannot obtain with first-class floors.

A derailed train is admirably provided for in the first class. The danger of fires is *nil.* The cost of masonry is less than for second and third class.



First class are often cheaper than the other classes, especially where the thickness of door is limited.

There is no shock on entering and leaving the bridge, such as always occurs when the ballast is followed on the bridge by cross ties resting on the back walls.

Bed Plates and Rollers.—Under 50 feet one end may be left free to slide on planed surfaces. The pressure of bed plates on masonry should not exceed 250 lbs. per square inch.

Over 50 feet, nests of turned friction rollers should be used at one end as described on page 458.

Much valuable information upon plate girders may be found in a paper by M. J. Becker, Eng. P. C. & St. L. R. R., published in the Sixth Annual Report of the Ohio Soc. of Civil Engineers, from which many of the points above given are taken.

EXAMPLE.—*Required to design a single track through plate girder bridge, 63 feet long and 15 feet wide, centre to centre.* In this case we may take the distance between the floor beams at $\frac{63}{4} = 15.75$ ft., and we thus have 4 panels and 5 floor beams. The stringer length will be 15.5 feet. The tails, cross ties, etc., we may take at 400 lbs. per foot, or 200 lbs. for each girder.

The total weight per foot for the wind bracing is then

$$3.6N + \frac{540}{p} = 48 \text{ or say } 50 \text{ lbs.}$$

For each girder then, we have 25 lbs. per ft. for wind bracing. The weight of stringer is given by our table at about 1292 lbs. As there are 8 stringers, the total weight is 10336 lbs., or 5168 lbs. for each girder.

The economic depth of floor beams is, from our Table for floor beams, $\frac{8}{10}$ths of 34 = 27 inches, and weight about 1706 lbs.

Weight and Depth.—We have found, then, for each girder, the wind bracing 25 lbs. per foot, or $25 \times 63 = 1575$ lbs. per girder. The stringers give 5168 lbs. per girder. The floor beams give $\frac{1706 \times 5}{2} = 4265$ lbs. per girder. The rails, ties, floor, etc., give $200 \times 63 = 12600$ lbs. per girder. The live load from our Table is $\frac{233700}{2} = 116850$ lbs. per girder. The total is 140458 lbs. per girder. The allowance for impact is $40 - \frac{2}{5}l$ per cent., or 14.8 per cent., or 20787. The total external load for one girder then, including impact, is $W = 161245$ lbs.

Taking this value for W, we have the depth for our case,

$$d = \frac{10 \times 63^2}{8000} + \sqrt{\frac{6 \times 161245 \times 63}{8000} + \left(\frac{10 \times 63^2}{8000}\right)^2} = 92.5 \text{ inches or } 7.7 \text{ feet.}$$

Taking $\frac{8}{10}$ths of this, we have about 6 feet for the effective economic depth, from centre to centre of rivet holes The weight of one girder is then

$$W' = \text{weight} = \frac{12 \times 161245 \times 63^2 + 2 \times 8000 \times 63 \times 72^2}{1.2 \times 8000 \times 72 - 12 \times 63^2} = 20050 \text{ lbs.}$$

The total load per girder, including weight of girder, is then

$$W' + W = 161245 + 20050 = 181295 \text{ lbs.}$$

We may take the thickness of web at $\frac{3}{8}$ inch.

Flanges.—If we take the effective depth in calculation of the flanges, the web should be taken into account. If the clear depth, the web may be neglected. The moment of the stress in the web is $\frac{RI}{v} = \frac{Rtd^2}{6}$, where t is the thickness of the web in inches, and d is the depth in inches, and R is the allowable stress per square inch.

The moment in inch lbs. at any point distant x feet from the left end due to the loading is

$$\frac{(W + W')\,12x}{2}\left(1 - \frac{x}{l}\right),$$

where x is in feet and l in feet. If we subtract the moment due to the web, we have the moment to be resisted by the flanges. Dividing then by d in inches, we have the stress in the upper flange, taking the web into account,

$$\frac{12(W + W')x}{2d}\left(1 - \frac{x}{l}\right) - \frac{Rtd}{6},$$

or if x, l and d are all in feet, and t in inches,

$$\frac{(W + W')x}{2d}\left(1 - \frac{x}{l}\right) - 2\,Rtd.$$

The last term is omitted if the web is to be disregarded in the calculation.

In the present case, $W + W' = 181295$, $d = 6$, $R = 8000$, $t = \frac{3}{8}$, $l = 63$, hence we have for any distance of x feet from the left, the upper flange stress

$$15108x\left(1 - \frac{x}{63}\right) - 36000.$$

Let us take for the angle irons in the top flange, angles $6 \times 6 \times \frac{5}{8}$, area 14.52 sq. ins. This is nearly the largest size given by Carnegie. At 8000 lbs. per sq. inch, such angles will sustain a stress of $14.52 \times 8000 = 116160$ lbs.

Putting then $116160 = 15108x\left(1 - \frac{x}{63}\right) - 36000$, we find $x =$ about 13 feet for the point at which the angles need to be re-enforced by a top plate. For the sake of security and to allow for the net area of the lower flange, let us take $x = 10$ feet.

The first top plate then must be $63 - 20 = 43$ feet long. Let us take this plate 13 inches wide by $\frac{1}{2}$ inch thick. Its area then is 6.5 sq. ins. and the total area of flange is now $14.52 + 6.5 = 21.02$ sq. ins. At 8000 lbs. per sq. inch, this will give a resistance of $21.02 \times 8000 = 168160$ lbs. We have then $168160 = 15108x\left(1 - \frac{x}{63}\right) - 36000$,

or $x =$ about 20 feet, for the distance from the end at which the angles and top plate cease to be sufficient. Taking $x = 18$, we have the second top plate $63 - 36 = 27$ feet long.

Let us take this plate 13″ wide by $\frac{3}{8}$″ thick, area 4.87. The total area is then $21.02 + 4.87 = 25.89$, or at 8000 lbs. per square inch, $25.89 \times 8000 = 207120$ lbs. But the stress at the centre is $15108 \times \frac{63}{2}\left(1 - \frac{63}{2 \times 63}\right) - 36000 = 201951$, which is less than the flange resistance. No other plate is needed.

The size of rivets for the angles is by our rule $1\frac{1}{4}t + \frac{3}{16} = \frac{5}{4} \times \frac{5}{8} + \frac{3}{16} = \frac{31}{32}$, or about 1″ diameter, and for top plates $\frac{5}{4} \times \frac{3}{8} + \frac{3}{16} = \frac{21}{32}$″. The net area of lower flange is then, at 10 feet from end, $14.52 - \frac{10}{8} = 13.27$ sq. ins. At 8000 lbs. this gives 106160 lbs., while the stress at 10 feet from the end is $15108 \times 10\left(1 - \frac{10}{63}\right) - 36000 = 91100$ lbs. The net area of lower flange is therefore sufficient here.

At 18 feet from the end the net area is $21.02 - \frac{10}{8} - \frac{13}{16} = 18.96$, and resistance $18.96 \times 8000 = 151680$ lbs. The flange stress is $15108 \times 18\left(1 - \frac{18}{63}\right) - 36000 = 158245$ lbs. The net area is therefore a little small. If we take the second lower flange plate as commencing at 17 feet from the end, instead of 18, and therefore $63 - 34 = 29$ ft. long, it will be sufficient.

If the top plates are in two lengths, a splice plate at centre will be required. If in one length, no splice is necessary. The angles should be in one length always.

Web.—The shear at the end is $\frac{181295}{2} = 90647$ lbs. If we take the web $\frac{3}{8}$″ thick, its area is $72 \times \frac{3}{8} = 27$ sq. ins. At 8000 lbs. this gives a resistance of 216000, or much greater than the shear.

Stiffeners.—At the end the resistance of the web to buckling is

$$\frac{10000}{1 + \dfrac{d^2}{3000\, t^2}} = \frac{10000}{1 + \dfrac{60^2 \times 64}{3000 \times 9}} = 1052 \text{ lbs. per square inch.}$$

$$\text{The shear is } \frac{90647}{27} = 3357 \text{ lbs. per square inch.}$$

As this is more than the web will stand without buckling, we need stiffeners. These we must space at intervals of 5 feet, and calculate them for the shear at each point.

At the end, if we take two angles $4 \times 4 \times \frac{3}{8}$, area 5.7 sq. ins., and two filling plates $4 \times \frac{5}{8}$, area 5 sq. ins., the total area, including the web, is $5.7 + 5 + 1.5 = 12.25$ sq. ins., and the thickness is $t = \frac{3}{4} + \frac{5}{4} + \frac{3}{8} = 2\frac{3}{8}$″. The safe resistance per square inch of cross section is then

$$\frac{10000}{1 + \dfrac{60^2 \times 64}{3000 \times 19^2}} = 8264.$$

$$\text{The total resistance is then } 8264 \times 12.25 = 101234 \text{ lbs.}$$

As the shear at the end is 90647 lbs., the resistance of these stiffeners is sufficient.

For the other stiffeners, let $x =$ any distance from end, and u the static load, and w the live load per foot. Then the shear at any point distant x feet from the end is

$$\frac{ul}{2} - ux + \frac{w\,(l - x)^2}{2l}.$$

In the present case, the live load per foot is

$$w = \frac{116850 + 116850 \times 0.148}{63} = 2130 \text{ lbs.},$$

and the static load per foot is

$$u = \frac{181295}{63} - 2130 = 748 \text{ lbs.}$$

In the present case, therefore, the maximum shear at any point is

$$23562 - 748x + 16.9\,(63 - x)^2.$$

At 5 feet from the end, the maximum shear is then 76673 lbs. If we take 2 angles $3\frac{1}{2} \times 3 \times \frac{3}{8}$, area 4.62, and two filling plates $3 \times \frac{5}{8}$, area 3.75, the total area is $4.62 + 3.75 + 1.12 = 9.5$, and since the thickness is still $2\frac{3}{8}$″, the safe resistance is as before 8264 lbs. per sq. inch. The total safe resistance is then $8264 \times 9.5 = 78508$, or greater than the maximum shear.

At 10 feet from the end, the maximum shear is 63554 lbs. If we take here two angles $2 \times 2 \times \frac{3}{8}$, area 2.88, and two filling plates $3 \times \frac{5}{8}$, area 3.75, the total area is $2.88 + 3.75 + 1.12 = 7.75$; the thickness is as before, and the resistance $8254 \times 7.75 = 64046$, or greater than the maximum shear.

At 15 feet from the end, the maximum shear is 51279 lbs. If we take here two angles $2 \times 2 \times \frac{3}{8}$, area 2.88, and two filling plates $2.5 \times \frac{5}{8}$, area 3.125, the total area is $2.88 + 3.125 + 0.75 = 6.75$; the thickness is as before, and the resistance $8264 \times 6.75 = 55782$, or greater than the maximum shear.

At 20 feet from the end, the maximum shear is 39850 lbs. If we take here two filling plates $3.5 \times \frac{5}{8}$, area 4.375, the total area is 5.68, the thickness is $1\frac{5}{8}''$, the safe resistance $\dfrac{10000}{1 + \dfrac{60^2 \times 64}{3000 \times 13^2}} = 6900$, and the resistance is 6900×5.68 $= 39192$, or about the same as the shear.

At 25 feet from the end, the maximum shear is 29265 lbs. If we take here two filling plates $3 \times \frac{5}{8}$, area 3.75, the total area is 4.87, the thickness as before $1\frac{5}{8}''$, the resistance $6900 \times 4.87 = 33603$ lbs.

At the centre the maximum shear is 16770 lbs. Two filling plates $2 \times \frac{5}{8}$ will give a total area of 3.25 and a resistance of $3.25 \times 6900 = 22425$ lbs.

Rivets.—The size of rivets, as already found, is about 1'' for the flange angles, and $\frac{7}{8}''$ for the flange plates. In the top plate, we may have a pitch of 4 inches. As the top plate is 13'' wide, we can run an extra line of rivets along the edge with a pitch of 8 inches, to draw the plates together. The bearing resistance of a flange rivet for $\frac{3}{8}''$ web is, from Rivet Table I., 4690 lbs. The total load is 181295 lbs. We have found the horizontal stress at 13 feet from the end to be 116160 lbs. or 58 tons. At 20 feet, 168160 lbs. or 84 tons. At the centre, 201951 lbs. or 100 tons. We have then for the first 13 feet 58 tons, for the next 7 feet $84 - 58 = 26$ tons, for the next 11.5 feet $100 - 84 = 16$ tons, to be taken by the rivets. The load per ft. is 2878 lbs. or 18.7 tons on the first 13 feet, 10 tons on the next 7 feet, and 16.5 on the next 11.5 feet. The resultant stresses are

$$\sqrt{58^2 + 18.7^2} = 61 \text{ tons} = 122000 \text{ lbs.}, \quad \sqrt{26^2 + 10^2} = 28 \text{ tons} = 56000 \text{ lbs.}, \quad \sqrt{16^2 + 16.5^2} = 23 \text{ tons} = 46000 \text{ lbs.}$$

We have then for the number of rivets in the first 13 feet $\frac{122000}{4690} = 27$, in the next 7 feet $\frac{56000}{4690} = 13$, in the remaining 11.5 feet $\frac{46000}{4690} = 10$. If we space the rivets at 5 inches for the first 20 feet, and at 6 inches, which is the largest pitch allowable, for the rest of the way to centre, we shall have more than are called for.

WEB SPLICES FOR PLATE GIRDERS—FLOOR BEAMS AND STRINGERS.—The web of a plate girder, floor beam, or stringer is usually a single plate. If the web is in sections, it should be spliced, and the splice should come where a stiffener is placed if possible.

The splice may be a single plate on each side, as shown in Fig. 1.

Fig. 1.

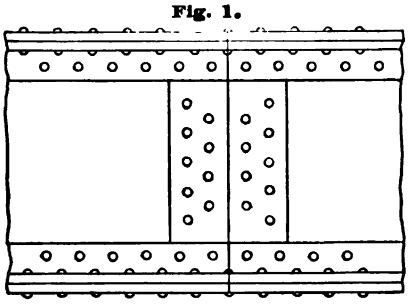
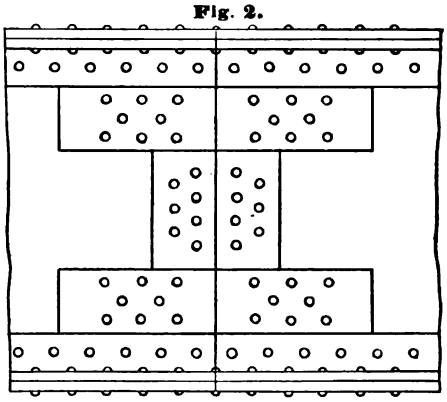


Fig. 2.



For a $\frac{3}{8}''$ web $\frac{5}{16}''$ splice plates would be used.

In Fig. 1 the splice plates must transmit the shearing and bending stresses at the section.

If such a splice as represented in Fig. 1 is too heavy or expensive, we may splice as in Fig. 2. Here the splice plates at top and bottom transmit the stress due to bending in the web, while the vertical splice plate transmits the shear at the section.

Design.—Let A = the area of flange section, top or bottom;

h = height of girder between centres of mass of flanges;

σ = allowed unit stress for flanges.

Then the resisting moment for the flanges is

$$M_F = A\sigma h.$$

Let t = thickness of web.
Then the resisting moment for the web is, by the theory of flexure,

$$M_w = \frac{\sigma t h^2}{6}. \quad \dots \dots \dots \dots \quad (1)$$

If M is the moment at the splice, we have then

$$M = A\sigma h + \frac{\sigma t h^2}{6}. \quad \dots \dots \dots \dots \quad (2)$$

From (2) we obtain

$$\sigma = \frac{M}{\left(A + \frac{th}{6}\right)h}.$$

Inserting this value of σ in (1), we have for the resisting moment of the web

$$M_w = \frac{Mth}{6\left(A + \frac{th^2}{6}\right)}. \quad \dots \dots \dots \dots \quad (3)$$

Let r = the stress on the extreme rivet of the splice in Fig. 1;
v = the distance from the neutral axis to the extreme rivet of the splice in Fig. 1;
d = the distance of any rivet from the neutral axis, above or below.
Then the stress on any rivet of the splice at a distance d is

$$\frac{r}{v}d.$$

The moment of this stress is

$$\frac{r}{v}d^2.$$

The sum of the moments of the stresses in all the rivets is then

$$\Sigma\frac{r}{v}d^2.$$

This should be equal to M_w, or the resisting moment of the web. Hence

$$\Sigma\frac{r}{v}d^2 = M_w, \quad \text{or} \quad \Sigma d^2 = \frac{M_w v}{r},$$

where M_w is given by (3).

If p is the pitch and n is the number of rivet spaces above and below the neutral axis, then

$$\Sigma d^2 = 2 \times p^2(1 + 2^2 + 3^2 + \dots n^2) = \frac{2p^2 n(n+1)(2n+1)}{6}.$$

Substituting, since $np = v$, we have

$$(n+1)(2n+1) = \frac{3M_w n}{rv}.$$

Since n is practically equal to $n + 1$ when n is large, and $2n + 1$ is equal to the number N of rivets on one side of the joint, we have practically

$$N = \frac{3M_w}{rv} \quad \dots \dots \dots \dots \quad (4)$$

for the number N of rivets on each side of the splice in Fig. 1.

In Fig. 2, since the top and bottom splice plates transmit the stress due to bending in the web, we have, if $d =$ the distance between the centres of these plates and r is the stress carried by a rivet, for the number of rivets on each side

$$N = \frac{M_w}{rd}. \quad \dots \dots \dots \dots \quad (5)$$

If the shear at the section is S, then the number of rivets on each side of the vertical splice plate in Fig. 2 is

$$N = \frac{S}{r}. \quad \dots \dots \dots \dots \quad (6)$$

EXAMPLE.—*Let the moment M at the splice be* 10000000 *inch pounds, the depth h be* 60 *inches, the area of flange $A = 14$ sq. inches, and of web $th = 20$ sq. inches. Let the distance v from the neutral axis to the extreme rivet in Fig.* 1 *be* 24 *inches. If the rivet resistance is $r = 4000$ lbs., find the number of rivets required on each side of the splice in Fig.* 1.

We have from (3)

$$M_w = \frac{10000000 \times 20}{6\left(14 + \frac{20}{6}\right)} = 1923077 \text{ inch pounds,}$$

and from (4)

$$N = \frac{3 \times 1923077}{4000 \times 24} = 60.$$

That is, we must have 60 rivets on each side of the joint in Fig. 1.

If we make the splice as in Fig. 2, we have from (5), taking $d = 40$,

$$N = \frac{1923077}{4000 \times 40} = 12.$$

We have then 12 rivets on each end of each upper and lower plate in Fig. 2. If the shear is 52000 lbs., we have from (6)

$$N = \frac{52000}{4000} = 13,$$

or 13 rivets on each side of the vertical plate in Fig. 2. In Fig. 1, then, we have 120 rivets. In Fig. 2, 74 rivets. The latter splice is then to be preferred.

CHAPTER VIII.

ROOF AND BRIDGE TRUSSES—DEAD WEIGHT—ECONOMIC DEPTH.

FOR highway trusses the allowable live load has been given on page 470. The weight of flooring, etc., will depend upon the circumstances of the case, and must be estimated in accordance with such circumstances. The weight of stringers and floor beams, and of lateral bracing, can be estimated as in Chap. VII., page 474, and Chap. VI., page 457. It remains to find the dead weight of the truss itself. When this is known, the maximum stresses due to loading can be found, and the various members designed in accordance with the preceding rules and principles.

The same remarks hold good for railway bridges, except here the weight of flooring, rails, cross ties, etc., may be taken at once at 400 lbs. per foot for single track and 750 per foot for double track. The entire external load of a truss can then be easily found. It remains to estimate the dead weight of the truss itself.

For roof trusses, we have already taken the horizontal wind pressure at 50 lbs. per ft. and have given in Part I., page 64, a Table giving the normal pressure upon an inclined surface due to the wind force, and shown how to find the stresses due to it. The only other loads to which a roof truss is subjected are the snow load and the weight of roof covering. The total external load being known, it remains to estimate the dead weight of the truss itself. The maximum stresses may then be found and the various members proportioned.

ROOF TRUSSES—SNOW LOAD AND ROOF COVERING.—The snow load for roofs may be taken at 30 lbs. per square foot *as a maximum*. Locality should be considered of course in the design, as also pitch of roof.

The wind pressure can be found as in Part I., page 65, and the pressure at the end supports due to it can be found.

The weight of roof covering can be estimated from the following Table:

WEIGHT OF VARIOUS ROOF COVERINGS IN LBS. PER SQUARE FOOT.

Shingles, 16 inch	2	Cast iron plates ($\frac{3}{8}''$)	15
Shingles, long	3	Sheet iron ($\frac{1}{16}''$)	3
Thatch	6.5	Slates (ordinary)	5 to 9
Felt and asphalt	1	Slates (large)	9 to 11
Felt and gravel	8 to 10	Tiles (average)	12
Tin	0.7 to 1.25	Tiles (large)	7 to 20
Sheet lead	5 to 8	Tiles (with mortar)	25 to 30
Copper	0.8 to 1.25	Slates and iron laths	10
Zinc	1 to 2	Sheathing, pine, 1 inch thick	3
Iron, galvanized	1 to 3	Sheathing, chestnut or maple	4
Iron, corrugated	1 to 3.75	Sheathing, ash, hickory, pine, oak	5
Sheet iron and laths	5	Laths and plaster	9 to 10

To the weight of roof covering thus estimated, must be added the weight of the "*purlins*" or stringers, whether wood or iron, which are laid across from truss to truss at the apices, to support the roof and covering. In any case, then, we may make a close estimate of the total external load, due to wind, snow, roof covering and purlins. Call this total external load W. It is now required to estimate the dead weight W' of the truss.

ROOF TRUSSES—DEAD WEIGHT.—Let the length of span in feet be l, and rise in feet be r, and the allowable stress per square inch be σ.

Then the stress in the tie will be $\frac{(W+W')}{2}\tan\theta$, where θ is the angle of the rafter with the vertical. Since $\tan\theta = \frac{l}{2r}$, we have the tie stress $\frac{(W+W')l}{4r}$. The cross section of tie is then $\frac{(W+W')l}{4\sigma r}$. Let γ be the weight of 12 cubic inches of material. Then the weight of the tie per foot is $\frac{\gamma(W+W')l}{4\sigma r}$, and the weight of the entire tie is $\frac{\gamma(W+W')l^2}{4\sigma r}$.

The rafter stress is

$$\frac{(W+W')}{2}\sec\theta = \frac{(W+W')}{2}\frac{\sqrt{\frac{l^2}{4}+r^2}}{r}.$$

Divide by σ and we have the cross section. Multiply by γ and we have the weight per foot. Multiply by $\sqrt{\frac{l^2}{4}+r^2}$ and we have the weight of one rafter. For two rafters then the weight is

$$\frac{\gamma(W+W')\left(\frac{l^2}{4}+r^2\right)}{\sigma r}$$

The total weight of both rafters and the tie is then, disregarding the bracing, approximately the weight of the truss, or

$$W' = \frac{\gamma(W+W')}{\sigma r}\left(\frac{l^2}{2}+r^2\right),$$

hence,

$$W' = \frac{W}{\frac{\sigma r}{\gamma\left(\frac{l^2}{2}+r^2\right)} - 1}.$$

For iron, we have $\gamma = \frac{10}{3}$, $\sigma = 10000$. For wood $\gamma = 0.35$, $\sigma = 1200$. We have neglected the web in this formula, but for short spans its influence is small. On the other hand, we have treated the rafter as of constant cross section, which for long spans gives an excess and tends to balance the error in disregarding the web.

EXAMPLE.—*An iron roof truss, with corrugated iron covering, has a span of 100 feet and a rise of 20 feet. It is spaced 7 feet from the adjacent trusses on each side. Each rafter is divided into 4 equal panels, and the purlins are rolled iron beams. What is the dead weight?*

Here $l = 100$, $r = 20$, $\gamma = \frac{10}{3}$, $\sigma = 10000$. It remains to estimate the total external load W.

The maximum snow load is $100 \times 7 \times 30 = 21000$ lbs. The angle of roof with horizon is 21° 48'. From our

Table, Part I., page 65, the normal pressure per square ft. of wind is 24.77 lbs. The length of rafter is 53.85 ft. The exposed area is $53.85 \times 7 = 376.95$ sq. ft. The normal wind pressure is $376.95 \times 24.77 = 9326$ lbs. The vertical component of this pressure is $9326 \cos 21° 48' = 9326 \times 0.9285 = 8660$ lbs.

The weight of roof covering is say 3 lbs. per sq. ft. The whole weight is $53.85 \times 7 \times 3 \times 2 = 2262$ lbs. One-eighth of this acts at each apex. So also for the snow load. For the wind load $\frac{1}{4}$ of $8660 = 2165$ lbs. acts at an apex. The total apex load is then

$$\frac{2262}{8} + \frac{21000}{8} + 2165 = 5070 \text{ lbs.}$$

This is the load on a purlin.

From Carnegie, we see that a 5 inch I beam, 10 lbs. per foot, will be required for the purlins. Each purlin weighs then 70 lbs. There are 8 purlins, and their weight is 560 lbs.

The total external load W is now

$$W = 21000 + 8660 + 2262 + 560 = 32482 \text{ lbs.}$$

The dead weight of the truss is therefore

$$W' = \frac{32482}{\dfrac{10000 \times 20}{\frac{10}{3}\left(\frac{100^2}{2} + 20^2\right)} - 1} = \frac{32482}{10.11} = 3212 \text{ lbs.}$$

Now that we know the dead weight of the truss itself, also the snow load, the weight of roof covering and of purlins, we can find the stresses due to total static loading. Then, as detailed in Part I, page 65, we can find the wind stresses, and can then make out the maximum stress for each member. The various members can then be proportioned in accordance with preceding principles.

BRIDGE TRUSSES—DEAD WEIGHT.—For highway bridges we must estimate the flooring and roadway according to the design and case in hand. No general estimate can be given. For railway bridges, we may take the rails, ties, planking, etc., at 400 lbs. per ft. for single track, and 800 lbs. per ft. for double track.

We can then find the weight of the stringers, as detailed in the preceding chapter, whether the stringers are of wood or iron. Next we can find the weight of the cross girders or floor beams.

We can then estimate the weight of the lateral system or wind bracing by the formulas of page 457. If we denote by w_3 the weight per ft. *per truss* of the wind bracing, these formulæ may be written as follows:

For *single track*,

$$\text{for pony trusses—depth below 12.5 feet, } w_3 = 1.8\,N + \frac{270}{p};$$

for through trusses, without vertical sway bracing—depth between 12.5 and 24 feet,

$$w_3 = 3.2\,N + \frac{336}{p};$$

for through trusses, with vertical sway bracing, depth above 24 feet, or for deck bridges,

$$w_3 = \frac{3Nl}{170} + \frac{568}{p},$$

where l = span in feet, N = the number of panels, and p = panel length in feet.

For double track, multiply by $\frac{b}{15}$ where b = width in feet.

We represent the weight per foot *per truss* of the stringers, cross girders, and of the rails, ties, planking, etc., by w_2, and the weight per foot *per truss* of the uniformly distributed load, equivalent to the live load assumed, by w_1. This equivalent load can easily be found from our diagram, Part I, page 88, for any span. Let the weight per foot of one main

truss be w_4, and let w_0 be the weight per foot of lattice bars, pins, eye bar heads, splice and cover plates, rivets, etc. Then the total load per foot per truss, is $w_1 + w_0 + w_2 + w_3 + w_4$. Let the length of panel be p, then $(w_1 + w_0 + w_2 + w_3 + w_4)\, p$ will be the total panel load for one truss. Let N be the number of panels, $d =$ the depth in feet, and $l =$ the span in feet.

Let us consider first the Warren girder, Fig. 88, Part I, page 103. The reaction at end for full load is, according to our notation,

$$\frac{(w_1 + w_0 + w_2 + w_3 + w_4)\,(N - 1)\,p}{2}.$$

The stress in the 1st lower panel is the reaction multiplied by the half panel length $\frac{p}{2}$, and divided by the depth d. If this stress is divided by the stress per square inch for tension, R_t, we have the area in square inches. The area multiplied by the length of panel, p, will be the volume, and this multiplied by $\frac{10}{3}$ will give the weight. We have then for the weight of the first lower panel,

$$\frac{10\,(w_1 + w_0 + w_2 + w_3 + w_4)\,p^2}{12 R_t d}\,[N - 1].$$

In a similar way we can easily find the weight of each lower panel, and thus obtain the following:

$$
\begin{array}{llll}
\text{Wt. of 1st lower panel,} & \dfrac{10\,(w_1 + w_0 + w_2 + w_3 + w_4)\,p^2}{12 R_t d} & [N - 1]. \\
\quad\text{"}\quad \text{2d}\quad\text{"}\quad\text{"} & \qquad\qquad\text{"} & [3\,(N - 1) - 2]. \\
\quad\text{"}\quad \text{3d}\quad\text{"}\quad\text{"} & \qquad\qquad\text{"} & [5\,(N - 1) - 8]. \\
\quad\text{"}\quad \text{4th}\quad\text{"}\quad\text{"} & \qquad\qquad\text{"} & [7\,(N - 1) - 18],
\end{array}
$$

and so on.

Summing up by series, we have for the weight of N lower panels,

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,Np^3\,(N^2 - 1)}{18 R_t d}.$$

Since $Np = l =$ the span, the weight *per ft.* per truss of the lower chord will be given by

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,p^2\,(N^2 - 1)}{18 R_t d}.$$

In a precisely similar manner, we find for the weight of the upper chord per ft. per truss,

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,p^2\,(N^2 - 1)}{18 R_c d},$$

where R_c is the stress per square inch for compression.

$$\text{For the braces, the sec } \theta = \frac{\sqrt{\dfrac{p^2}{4} + d^2}}{d} = \frac{\sqrt{p^2 + 4d^2}}{2d}.$$

We have then for full loading, the stress in the first tie =

$$\frac{p\,(w_1 + w_0 + w_2 + w_3 + w_4)\,\sqrt{4\,d^2 + p^2}}{4d}\,[N - 1].$$

We have, then, multiplying by the length $\frac{\sqrt{4d^2 + p^2}}{2}$, dividing by R_t and multiplying by $\frac{10}{8}$:

$$\text{Weight of 1st tie, } \frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,(4d^2 + p^2)\,p}{12R_t d}\,[N - 1].$$

$$\text{" " 2d " " } [(N - 1) - 2].$$

$$\text{" " 3d " " } [(N - 1) - 4].$$

$$\text{" " 4th " " } [(N - 1) - 6],$$

and so on.

The weight of N ties is then $\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N^2 p}{24R_t d}$.

The weight per foot per truss of the ties is then

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N}{24R_t d},$$

and for the struts we have, in precisely similar manner,

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N}{24R_c d},$$

where R_c is the stress per square inch for compression.

The whole weight per foot is therefore, exclusive of details, $w_4 =$

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)}{18d}\left[\frac{(N^2 - 1)\,p^2}{R_t} + \frac{(N^2 - 1)\,p^2}{R_c} + \frac{0.75\,N\,(p^2 + 4d^2)}{R_t} + \frac{0.75N(p^2 + 4d^2)}{R_c}\right].$$

For the sake of brevity let us put

$$w_4 = \frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)}{18d}\left[\frac{T}{R_t} + \frac{C}{R_c} + \frac{S}{R_s}\right],$$

where T refers to the lower chord and ties, C to the upper chords, and S to the struts; hence $T = (N^2 - 1)\,p^2 + 0.75N\,(p + 4d^2)$, $C = (N^2 - 1)\,p^2$, and $S = 0.75N\,(p^2 + 4d^2)$.

We have then,

$$w_4 = \frac{w_1 + w_0 + w_2 + w_3}{\dfrac{3.6d}{\dfrac{T}{R_t} + \dfrac{C}{R_c} + \dfrac{S}{R_s}} - 1}.$$

Rankine's formula for long struts is, for the upper chords, $R_c = \dfrac{\mu}{1 + \dfrac{p^2}{250r_1^2}}$, and for the struts $R_s = \dfrac{\mu}{1 + \dfrac{p^2 + 4d^2}{4 \times 125\,r_2^2}}$, where $\mu = 8000$ and r_1, r_2 are the least radii of gyration of the cross section. We have then,

$$w_4 = \frac{w_1 + w_0 + w_2 + w_3}{\dfrac{3.6\mu d}{\dfrac{\mu}{R_t}\,T + C + S + \dfrac{Cp^2}{250r_1^2} + \dfrac{S\,(p^2 + 4d^2)}{500r_2^2}} - 1}.$$

Now R_t is on the average about 9000 lbs., and $\mu = 8000$ lbs. We shall make but slight error in taking $\frac{\mu}{R_t} = 1$. For r_1^2 the simple expression $r_1^2 = \frac{(N-1)\,p^2}{100}$ gives very close values as compared with practice. For the struts we take $r_2^2 = \frac{N-1}{50}$ multiplied by the square of the length, or, in this case,

$$r_2^2 = \frac{(N-1)\,(p^2 + 4d^2)}{200}.$$

If, then, we put, for the sake of brevity,

$$T + C + S = p^2\left(2N^2 + \frac{3N}{2} - 2\right) + 6Nd^2 = \alpha p^2 + \beta d^2,$$

we have

$$w_4 = \frac{w_1 + w_0 + w_2 + w_3}{\dfrac{3.6\mu d}{\alpha p^2 + \beta d^2 + \dfrac{\alpha p^2 + 4\beta d^2}{5\,(N-1)}} - 1}.$$

The form of this equation is entirely rational, and only the constants α and β and w_0 remain to be determined.

For w_0 we have the empiric formula $w_0 = \frac{Nd}{3} + A$, where $A = 0.875N(12 - N) + 6$. We have finally, then, the following formula for the weight per foot of one truss:

Formula for the Dead Weight of one Main Truss.

Let w_1 = the weight per ft. per truss of the equivalent uniform load.
w_0 = the weight per ft. per truss of details.
w_2 = the weight per ft. per truss of the stringers, cross girders, and of the rails, ties, planking, etc.
w_3 = the weight per ft. per truss of the wind bracing.
w_4 = the weight per ft. of one main truss.
p = the panel length in feet.
d = the depth in feet.
N = the number of panels.
μ = the numerator of Gordon's formula = 8000 for iron. Then

$$w_4 = \frac{w_1 + w_0 + w_2 + w_3}{L - 1}; \qquad \text{(I.)}$$

$$\text{where } L = \frac{3.6\mu d}{\alpha p^2 + \beta d^2 + \dfrac{\alpha p^2 + 4\beta d^2}{5\,(N-1)}},$$

and

$$w_0 = \frac{Nd}{3} + A, \quad A = 0.875N(12 - N) + 6.$$

We have, then, *total weight of iron per foot* $= 2(w_2 - 200 + w_0 + w_3 + w_4)$; also for the values of α and β, we have

For Warren girder,

$$\alpha = (2N^2 + 1.5N - 2); \quad \beta = 6N.$$

In precisely the same way as for the Warren girder we may deduce for

Single intersection Pratt truss,

$$\alpha = (2N^2 + 3N - 2); \quad \beta = 6N - 12 + \frac{33}{N}.$$

Double intersection Whipple,

$$\alpha = 2N^2 + 6N - 20 + \frac{24}{N}; \quad \beta = 3N - 6 + \frac{48}{N}.$$

For Post truss,

$$\alpha = 2N^2 + 3.75N - 21.5 + \frac{30}{N}; \quad \beta = 3N - 6 + \frac{24}{N}.$$

For parabolic bow-string,

$$\alpha = 3N^2p; \quad \beta = 16Np.$$

For double parabolic bow-string,

$$\alpha = 3N^2p; \quad \beta = 24Np.$$

TABLES FOR FACILITATING CALCULATION OF DEAD LOAD.—For ready application of the formula I., for dead weight of truss, we recapitulate here the formulas for weight per ft. per truss of wind bracing, w_3, and also give Tables for the value of the equivalent uniformly distributed live load per ft. per truss, or w_1, for the value of the weight of stringers, floor beams, flooring, rails, ties, etc., per ft. per truss, or w_2, and for the values of A for single and double intersection Pratt truss, Post truss, and Warren girder. This Table can easily be extended to the other systems for which the value of A is given, if desired.

FORMULAS FOR w_3.—For ***single track,*** width 15 feet,

$$\text{for pony trusses, depth below 12.5 feet, } w_3 = 1.8N + \frac{270}{p};$$

for through trusses, without vertical sway bracing, depth between 12.5 and 24 feet,

$$w_3 = 3.2N + \frac{336}{p};$$

for through trusses, with vertical sway bracing, depth above 24 feet, or for deck bridges,

$$w_3 = \frac{3Nl}{170} + \frac{568}{p};$$

where l = span in feet, N = number of panels, p = panel length in feet. For any width, divide by 15 and multiply by the width.

TABLE I.

VALUES OF α AND β FOR DIFFERENT TRUSSES.

N	Single Intersection.		Double Intersection.		Warren.		Post.	
	α	β	α	β	α	β	α	β
2	12	16.5	12	24	9	12	9	12
3	25	17	24	19	20.5	18	17.75	11
4	42	20.25	42	18	36	24	33	12
5	63	23.6	64.8	18.6	55.5	30	53.25	17.8
6	88	29.5	92	20	79	36	76.33	16
7	117	34.714	123.48	21.857	106.5	42	107.036	18.45
8	150	40.125	159	24	138	48	139.875	21
9	187	45.666	198.666	26.333	173.5	54	177.584	23.67
10	228	51.3	242.4	28.8	213	60	219	26.4
11	273	57	290.18	31.36	266.5	66	264.477	29.15
12	322	62.666	342	34	304	72	314	32
13	375	68.538	397.846	36.69	355.5	78	367.557	34.8465
14	432	74.357	457.714	39.4285	411	84	425.143	37.715
15	493	80.2	521.6	42.2	470.5	90	486 75	40.6
16	558	86.0626	589.5	45	534	96	551.4375	43.5
17	627	91.941	661.412	47.8235	601.5	102	622.018	46.412
18	700	97.833	737.333	50.666	673	108	695.666	49.333
19	777	103.737	817.263	53.5263	748.5	114	773.329	52.263
20	858	109.65	901.4	56.4	828	120	855	55.2

TABLE II.—VALUES OF w_1.*

EQUIVALENT UNIFORM LOAD w_1, IN LBS. PER FOOT, PER TRUSS, FOR THE LOAD SYSTEM SIMILAR TO "CLASS A" OF COOPER'S SPECIFICATIONS.

Table gives values of w_1 for single track for one truss. For double track take double these values.

Span =	50	55	60	65	70	75	80	90	100	110	120	130 feet.
w_1 =	1848	1733	1718	1677	1618	1570	1546	1552	1560	1569	1579	1586 lbs.
Span =	140	150	160	170	180	190	200	210	220	230 feet and over.		
w_1 =	1574	1562	1556	1546	1533	1516	1511	1506	1502	1500 lbs.		

The table gives w_1 for one truss, single track, on the assumption of two trusses to the bridge.

TABLE III.—VALUES OF $w_0 = \frac{Nd}{3} + A$.

For the weight per foot per truss of details, $w_0 = \frac{Nd}{3} + A$, where $A = 0.875N(12 - N) + 6$ *we have the following values of A.*

$N=$	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$A=$	+ 34	+36.6	+ 37.5	+ 36.6	+ 34	+ 29.6	+ 23.5	+ 15.6	+ 6	− 5.37	− 18.5	− 33.4	− 50	− 68.4	− 88.5	− 110.4	− 134.

For the values of w_2, we have the following Table, based upon the Tables for weight of stringers and cross girders already given, the weight of rails, ties, etc., being taken at 400 lbs. per foot for single track, and 800 lbs. for double track.

* Increase these values by 18 per cent. for the system of loads given by our diagram, Part I, page 88.

TABLE IV.*

Panel length in feet.	Single track—15 feet wide.				Panel length in feet.	Double track—25 feet wide.			
	½ cross gird'r	1 stringer.	½ floor.	w_2		½ cross gird'r	2 stringers	Floor.	w_2
5	507	188	1000	339	5	1555	376	2000	786
6	582	248	1200	338	6	1783	496	2400	780
7	631	315	1400	335	7	1934	630	2800	766
8	679	386	1600	333	8	1992	772	3200	745
9	695	463	1800	327	9	2135	926	3600	740
10	716	545	2000	326	10	2192	1090	4000	728
11	750	657	2200	328	11	2291	1314	4400	727
12	776	825	2400	333	12	2374	1650	4800	735
13	806	974	2600	337	13	2443	1958	5200	738
14	831	1130	2800	340	14	2540	2260	5600	743
15	853	1292	3000	343	15	2606	2584	6000	746
16	872	1460	3200	345	16	2664	2920	6400	748
17	886	1634	3400	348	17	2700	3268	6800	751
18	906	1816	3600	351	18	2775	3632	7200	755
19	930	2003	3800	354	19	2841	4006	7600	760
20	949	2197	4000	357	20	2871	4394	8000	763
21	968	2420	4200	361	21	2957	4840	8400	771
22	983	2652	4400	365	22	3004	5304	8800	777
23	1001	2900	4600	369	23	3030	5800	9200	783
24	1018	3133	4800	373	24	3080	6266	9600	789
25	1034	3382	5000	376	25	3128	6764	10000	795
26	1048	3633	5200	380	26	3172	7266	10400	801
27	1062	3904	5400	384	27	3213	7808	10800	808
28	1077	4180	5600	387	28	3282	8360	11200	816
29	1086	4463	5800	391	29	3332	8926	11600	822
30	1104	4756	6000	395	30	3373	9512	12000	829

With the aid of these Tables and Formula I., we can readily and easily compute the weight of any R.R. bridge.

EXAMPLE I.—*Let us take a single track R. R. bridge, 150 feet span, 9 panels, double intersection, 27.8 feet deep.*

Since the depth is greater than 24 feet, we have the weight per foot per truss for wind bracing, from the formula, page 440, $w_3 = 57$ lbs. per foot per truss.

If there are 9 panels, each panel is 16⅔ feet long. We have then, from Table IV., $w_2 = 347$ lbs. per foot per truss.

From Table II., we have $w_1 = 1562$ lbs. per foot per truss. From Table III., $w_0 = 113$. Hence $w_1 + w_2 + w_3 + w_0 = 2079$ lbs. per foot per truss. From Table I., for 9 panels, double intersection, we have $\alpha = 198\frac{2}{3}$, $\beta = 26\frac{1}{3}$, and from Formula I., taking $\mu = 8000$, $w_4 = 327$ lbs. per foot per truss.

The weight of each main truss is then 327 lbs. per foot. For the total weight of iron in the structure we have for the trusses $327 \times 2 = 654$ lbs. per foot. For the details $113 \times 2 = 226$ lbs. per foot. For the floor and wind bracing we have $347 - 200 = 147$ lbs. per foot per truss, for the floor, and 57 lbs. per foot per truss for wind bracing, or $147 + 57 = 204$ lbs. *per truss*, or 408 lbs. per foot for the structure. We subtract 200 lbs. from 347, because the rails, ties, planking, etc., weigh 400 lbs. per foot for both trusses, and this portion is not part of the structure. The structure weighs then $408 + 654 + 226 = 1288$ lbs. per foot, or $1288 \times 150 = 193200$ lbs. The panel dead load is $\frac{193200}{2 \times 9} + 200 \times 16\frac{2}{3} = 14066$ lbs. per truss. The stresses can now be found for this loading and for the live load assumed.

In the same way we can find the weight of truss and of structure for any other kind of truss, single or double track.

ECONOMIC DEPTH AND BEST NUMBER OF PANELS.—If our Formula (I.) is reliable, and gives even with tolerable accuracy the weight of truss, then, since it is rational in form, the least weight depth, or the depth which gives the least weight, can also be deter-

* Increase the values for cross girder and stringer by 18 per cent. for the system of loads given by our diagram, Part I, page 88. The floor remains the same.

mined. The least cost depth, or economic depth, ought to be somewhat less than this, usually by about $\frac{1}{8}$th.*

Differentiating and putting the first differential equal to zero, we have

$$\frac{d}{l} = \frac{1}{N}\sqrt{\frac{\alpha\left[1 + \frac{1}{5\,(N-1)}\right]}{\beta\left[1 + \frac{4}{5\,(N-1)}\right] + \frac{1.2\,\mu N}{(w_1 + w_2 + w_3 + w_0) + A}}} \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

The values of α and β are taken from Table I. and of A from Table III. For standard specifications and the locomotive system adopted, we may take $w_1 + w_2 + w_3 + w_0 = 2000$ lbs. without noticeable error.

If we use this value of $w_1 + w_2 + w_3 + w_0$, we can make at once the following tabulation, which will enable us to find directly the best depth for any span, single or double intersection, or Warren.

TABLE V.

LEAST WEIGHT DEPTH, $d = Cl$. VALUES OF C GIVEN IN TABLE.

N	Warren. C	Single Intersection. C	Double Intersection. C
4	0.2207	0.2510	0.2592
5	0.1978	0.2258	0.2436
6	0.1805	0.2018	0.2272
7	0.1669	0.1846	0.2122
8	0.1559	0.1708	0.1992
9	0.1467	0.1596	0.1875
10	0.1389	0.1502	0.1773
11	0.1348	0.1420	0.1684
12	0.1263	0.1350	0.1605
13	0.1211	0.1290	0.1534
14	0.1164	0.1235	0.1470
15	0.1123	0.1196	0.1413
16	0.1084	0.1142	0.1360
17	0.1049	0.1102	0.1312
18	0.1016	0.1065	0.1267
19	0.0986	0.1031	0.1226
20	0.0958	0.0999	0.1187

For constructive reasons it is well to limit p to about 30 feet and d to 50 feet. Within these limits we can find best depth from Table V., and best number of panels N by trial. The total weight per foot of all the iron is $2\,(w_2 + w_3 + w_4 + w_0 - 200)$. That value of N which gives this a minimum is the best.

Thus, for span 104 feet, single intersection, we have for $N = 4$,

$N = 4$, $w_1 = 1564$, $d = 0.251$, $l = 26$ feet, $p = 26$ feet, $w_2 = 380$, $w_3 = 29$, $w_0 = 68$, $\alpha = 42$, $\beta = 20\frac{1}{4}$.

* Least weight does not necessarily mean least cost. The relative amount of the various kinds of iron, the cost of manufacturing the various shapes and members, whether riveted, rolled, or forged, facility of transportation and erection, all influence the cost. The influence of these factors varies from time to time, and the factors themselves may even vary at the same time at different manufactories. Constant employment in the preparation of competitive designs and alternate plans is necessary to enable a designer to choose best proportions. But even to such, the determination of proportions for least weight will be valuable, and to others most important as a guide to the judgment.

Therefore,

$w_1 + w_2 + w_3 + w_0 = 2041, \quad 3.6\mu d = 748800, \quad \alpha p^2 = 28392, \quad \beta d^2 = 13689, \quad L = 15.72.$

We have, then, from (I.), $w_4 = 138$, and total weight per foot of iron = 830 lbs.

For $N = 5$, we have,

$w_1 = 1564, \quad d = 23.5, \quad p = 20.8, \quad w_2 = 360, \quad w_3 = 36, \quad w_0 = 75, \quad \alpha = 63, \quad \beta = 23.6,$

$w_1 + w_2 + w_3 + w_0 = 2035, \quad 3.6\mu d = 676800, \quad \alpha p^2 = 27256, \quad \beta d^2 = 13033, \quad L = 15.26,$

and $w_4 = 142$; total weight per foot of iron = 826 lbs.

For $N = 6$,

$w_1 = 1564, \quad d = 21, \quad p = 17\frac{1}{2}, \quad w_2 = 349, \quad w_3 = 39, \quad w_0 = 79, \quad \alpha = 88, \quad \beta = 29.5,$

$w_1 + w_2 + w_3 + w_0 = 2031, \quad 3.6\mu d = 604800, \quad \alpha p^2 = 26365, \quad \beta d^2 = 13005, \quad L = 14.23,$

and $w_4 = 152$; total weight per foot of iron = 838 lbs.

We see at once that, as the number of panels diminishes, or the panel length increases, the truss grows lighter, but at the same time the floor grows heavier, as shown by Table IV. There is, then, a best number of panels, in this case *five*, for which the total weight is a minimum. The best panel length is, then, 20.8 feet.

The corresponding best depth is 23.5 feet. Formula I., however, shows that for the best number of panels *a considerable change in depth affects the weight of truss but little.* This may then be taken more or less than the value from Table V., without much effect on weight. In any case the best value of N is easily found by trial, and in case p is greater than 30 feet, it would be well to limit it to that value.

For span 150 feet, double intersection, we have, $N = 5$,
$w_1 = 1562, \quad d = 36, \quad p = 30, \quad w_2 = 395, \quad w_3 = 32, \quad w_0 = 96, \quad \alpha = 64.8, \quad \beta = 18.6,$
$w_4 = 198$; total weight of iron per foot = 1042 lbs.

$N = 6, \quad w_1 = 1562, \quad d = 34, \quad p = 25, \quad w_2 = 376, \quad w_3 = 38, \quad w_0 = 105, \quad \alpha = 92, \quad \beta = 20,$
$w_4 = 201$; total weight of iron per foot = 1040 lbs.

$N = 7, \quad w_1 = 1562, \quad d = 28, \quad p = 21.43, \quad w_2 = 363, \quad w_3 = 37, \quad w_0 = 102, \quad \alpha = 123.428,$
$\beta = 21.857, \quad w_4 = 221$; total weight of iron per foot = 1046 lbs.

We thus establish $N = 6$ as the best number of panels.

For span 320 feet, double intersection, we find that the total weight decreases as N decreases, until we have $p = 29$ for $N = 11$. As it is not advisable to have a longer panel than this, we may take $N = 11$ or more.

As we have repeatedly said in the proper connection, double intersection trusses are no longer built. The single advantage, that for large depths the long posts can be pinned at centre, is equally well obtained by some modification of the Baltimore Truss (Part I, page 124), such as the "sub-Pratt," illustrated in Part I, page 124.

LIMITING LENGTH OF GIRDER.—If we denote by L the limiting length of girder, or that length for which the girder will just support its own weight, we have

$$w_4 L = (w_1 + w_2 + w_3 + w_4)\, l, \text{ or } w_4 = \frac{w_1 + w_2 + w_3}{\frac{L}{l} - 1}.$$

This expression is precisely similar in form to Formula I.

We have, therefore, by reference to Formula I.,

$$L = \frac{3.6\ \mu d}{\alpha p^2 + \beta d^2 + \dfrac{\alpha p^2 + 4\beta d^2}{5\,(N-1)}},$$

and this equation will give for any case the limiting length. Thus, for a span of 104 feet, single intersection, $N = 6$, $p = 17\frac{1}{3}$, $d = 24$, if we take $w_1 + w_2 + w_3 + w_0 = 2031$, we have $L = 1480$ feet.

HIGHWAY BRIDGES—DEAD LOAD.—Our method is equally applicable to highway bridges. We have only to figure up the value of w_1, w_2 and w_3 for the case in hand.

EXAMPLE.—*A single intersection iron Pratt truss highway bridge, of "Class A," page 420, is 160 ft. long and 14 ft. wide, and has 8 panels. What should be the depth? Also if the flooring is 3 inch pine, what is the weight of each main truss, what is the total weight of iron, and what is the total static load? The cross girders are to be of iron, and the stringers of white pine.*

The 3 inch flooring will weigh $14 \times 12 \times 3 \times 0.35 = 176.4$ lbs. per ft. lineal, or 3528 lbs. per panel. From our Table, page 470, we have for the live load for "Class A," 80 lbs. per sq. ft. or 11200 lbs. per panel per truss. The total panel floor load is then $3528 + 22400 = 25928$ lbs. If this is carried by 6 joists or stringers, each one must carry $\frac{25928}{6} = 4320$ lbs. From our Table, page 472, we see that joists 6″ × 14″ will carry, if 20 ft. long, $858 \times 6 = 5148$ lbs., and will therefore be sufficiently strong. Each joist will weigh $0.35 \times 6 \times 14 \times 20 = 588$ lbs.

The load on each cross girder is $3528 + 588 \times 6 + 22400 = 29456$ lbs. The least weight depth of cross girder is by our formula, page 434,

$$\frac{10 \times 14^2}{8000} + \sqrt{\frac{6 \times 29456 \times 14}{8000} + \left(\frac{10 \times 142}{8000}\right)^2} = 17.6 \text{ inches.}$$

The weight of such a cross girder, is from the formula, page 434,

$$\frac{12 \times 29456 \times 14^2 + 2 \times 8000 \times 14 \times 17.6^2}{1.2 \times 8000 \times 17.6 - 12 \times 14^2} = 832 \text{ lbs.}$$

There are seven such cross girders, the total weight being $832 \times 7 = 5824$ lbs. or $\frac{5824}{160} = 36$ lbs. per ft., or 18 lbs. per ft. per truss. The wind bracing is, from page 417, $6.4 \times 8 + \frac{672}{20} = 84$ lbs. per ft., or 42 lbs. per ft. per truss $= w_3$. We have now for the live load per ft. per truss, $w_1 = \frac{11200}{20} = 560$ lbs. For the flooring we have $\frac{176.4}{2} = 88.2$, for the joists $\frac{588 \times 6}{20 \times 2} = 88.2$, for the cross girders 18, and hence $w_2 = 194.4$, and $w_1 + w_2 + w_3 = 791$ lbs.

The best depth from our Table V. is then about 27 ft., $w_0 = 106$, and the weight of one truss is, from our formula, page 496, 219 lbs. per ft.

Add to this 18 for the cross girders and 42 for the wind bracing, and 106 for details, and we have $385 \times 2 = 770$ lbs. per ft. of iron in the entire structure. The total static load is $w_2 + w_3 + w_4 + w_0 = 561$ lbs. per ft. per truss. The lumber weighs 342 lbs. per ft. for the entire structure.

We can now find the stresses and design the structure. The total weight of iron will be 123200 lbs., and of lumber 54720 lbs.

RESULTS OF APPLICATION OF FORMULAS.—We give here the tabulated results of our formulas for dead weight, for single and double intersection railway bridges, on the assumption of two trusses, and in accordance with the specifications assumed in this chapter. For the system of loads assumed in our diagram, Part I., page 88, the weight should be increased 18 per cent. The best number of panels and best depth as determined by our formulas are also given. A change of depth of a few feet will not materially affect the weights given. The total weight of iron given does not include shoe-plates, rollers, etc.

The general formulas can be adapted to any practice and specifications, by proper Tables for w_1, w_2, and w_3.

Dead Weight of Iron Railway Bridges, according to our Formulæ, with Economic Dimensions.

Table gives the weight per foot of iron, exclusive of shoe-plates, rollers, etc.
For single track add 400 *lbs. per foot for ties, rails, chairs, spikes, etc.*
For double track add 800 " " " " " " "

Span in ft., l	Single Intersection.				Double Intersection.			
	Best Number of Panels, N	Best Depth in Ft., d	Single Track, 15 ft. Total Weight of Iron per ft.	Double Track, 25 ft. Total Weight of Iron per ft.	Best Number of Panels, N	Best Depth in Ft., d	Single Track, 15 ft. Total Weight of Iron per ft.	Double Track, 25 ft. Total Weight of Iron per ft.
60	4	15	628	1194	5	15	632	1186
70	4	18	658	1238	5	17	660	1234
80	5	18	695	1310	5	20	696	1288
90	6	18	740	1438	5	22	730	1342
100	6	20	822	1504	5	24	784	1436
110	6	23	872	1594	5	27	840	1532
120	6	24	924	1686	5	29	892	1612
130	5	29	968	1770	5	26	947	1700
140	5	32	1020	1864	6	32	996	1780
150	5	34	1076	1956	6	34	1044	1866
160	6	32	1138	2056	6	36	1100	1938
170	6	34	1212	2188	7	36	1160	2048
180	6	36	1264	2278	7	38	1206	2130
190	7	35	1340	2398	7	40	1258	2216
200	7	37	1404	2512	7	42	1316	2318
210	7	40	1468	2614	7	46	1370	2398
220	9	35	1560	2776	8	44	1460	2546
230	8	39	1634	2916	8	46	1504	2618
240	8	41	1722	3046	8	48	1570	2726
250	9	40	1816	3220	9	47	1656	2870
260	9	41	1924	3406	9	49	1730	2982
270	9	43	1984	3506	9	50	1782	3086
280	10	42	2120	3770	10	50	1906	3260
290	10	46	2210	3884	11	49	2006	3432
300	10	45	2304	4056	12	48	2120	3636

Empiric Formulas for Total Weight.—As variations in depth do not greatly affect the weight of truss, it would seem possible to construct an empiric formula, which shall contain the span as the only variable, and give at once, with little calculation, and with sufficient accuracy, the entire weight of iron.

We have seen that the total weight of iron in lbs. per foot is given by

$$2\,(w_2 - 200 + w_3 + w_0 + w_4),\ \text{or putting for } w_4 \text{ its value } \frac{w_1 + w_2 + w_3 + w_0}{L - 1},$$

we have

$$2\left[\frac{(w_1 - 200 + w_3 + w_0)\,L + w_1 + 200}{L - 1}\right].$$

Now we find that $L = \dfrac{\text{constant}}{l}$ very nearly. We have, then, at once, for the form of empiric formula,

$$\text{total weight per foot} = \frac{a + bl}{c - l}.$$

This formula, we find, gives very excellent results when the proper values of a, b, and c are used, and these values will vary according to specifications and kind of bridge.

For the specifications of this chapter and live load similar to Class A of Cooper's *Specifications*, we have,

For SINGLE INTERSECTION,

single track, weight per foot of iron in lbs. $= \frac{276250 + 1890l}{666 - l}$;

double track, weight per foot of iron in lbs. $= \frac{536900 + 3294l}{676 - l}$.

For DOUBLE INTERSECTION,

single track, weight per foot of iron in lbs. $= \frac{271230 + 1630l}{654 - l}$;

double track, weight per foot of iron in lbs. $= \frac{566340 + 3010l}{704 - l}$.

For DECK PLATE GIRDERS, 8 feet wide, ties on top chord,

single track, weight per foot of iron in lbs. $= \frac{228612 + 7774l}{1110 - l}$.

For double track, about 70 per cent. greater.

For *Iron Highway Bridges*, of Class A (page 430),

$$\text{weight of iron in lbs. per foot} = \frac{7600 + 124l}{1100 - l} w;$$

weight per foot of lumber $= 120 + 12w$, where $w =$ width of roadway in feet.

For the live load of our diagram, **Part I**, page 88, add 18 per cent. to weight.

PRACTICAL FORMULÆ FOR WEIGHT OF TRUSS.—The dead load is made up of the weight of the track, which ranges from 300 to 500, usually taken at 400 lbs. per linear foot, the floor system, and the trusses and lateral system.

As the weight of the floor has no connection with the weight of the rest, it is in practice designed first, and its correct weight is then always known. There remains, therefore, only to estimate the weight of the trusses and lateral system.

For this purpose the following empiric formulas are in general use:

For SINGLE TRACK PLATE GIRDER SPANS,

weight per foot of girders and lateral system $= 10l$.

For AVERAGE SINGLE TRACK PRATT TRUSS,

weight per foot of trusses and lateral system $= 5l$,

where $l =$ span in feet.

For lattice girder spans take the weight intermediate between plate girders and Pratt truss.

For AVERAGE SINGLE TRACK PIN-CONNECTED PIVOT SPANS,

weight per foot of trusses and laterals $= 6$ to $7l$,

where $l =$ length of one arm.

For double track double these values.

These formulas are purely empiric, and the coefficients must be varied according to judgment, to suit different specifications and live loads.

EXAMPLE.—*Single track through Pratt truss $l = 153$ feet. To be designed for the live load of our diagram, page 89, and by Cooper's Specifications.*

From our Table, page 502, we have, for best proportions, that is, for $N = 5$ and d about 34 feet, the weight of iron per foot $= 1100$ lbs. for live load, similar to Class A of Cooper's *Specifications*. We also find, page 496, $w_1 = 57$, from Table IV., page 498, $w_2 = 348$, and from Table III., $w_0 = 108$. The floor and laterals weigh, therefore, $2(w_0 + w_1 + w_2 - 200) = 626$ lbs. per foot.

If we wish weight for the live load of our diagram, Part I, page 88, we add 18 per cent., and have weight = 1300 lbs. per foot for best dimensions. From our empiric formula, page 503, we have,

$$\frac{276250 + 1890 \times 153}{666 - 153} = 1100 \text{ lbs.},$$

agreeing perfectly with our Table, page 502.

If we use the formula, page 495, and take $N = 9$, $d = 26$, we have weight of iron per foot = 1186 instead of 1100. This shows the result of departing from the best dimensions. For the live load of our diagram, Part I, page 88, we add 18 per cent., and have weight of iron per foot = 1400 lbs.

From the practical formula, page 503, we have for weight of trusses and lateral system $5l = 765$ lbs. per foot. If the floor is found by actual design to weigh 340 lbs., we would have weight = 1105 lbs. per foot, agreeing with preceding results.

ECONOMICAL SPAN.—When there are a number of spans and piers, the question arises, what length of span, taking into account the cost of the piers, will be the best, that is, corresponds to the least cost.

We have seen, page 503, that the weight of iron per foot is given by $\frac{a + bl}{c - l}$, where l is the span in feet, and a, b, and c are constants for which we have already given the values. Let there be n spans of length l, and let $L = nl$ be the total length. Let C be the cost in cents per pound of the iron, including manufacture, freight, erection, etc. Then $\frac{nC(al + bl^2)}{100(c - l)}$ will be the cost in dollars of all the spans, or, inserting $l = \frac{L}{n}$, $\frac{C(aLn + bL^2)}{100(cn - L)}$.

Now let the average cost of a pier be P, then the total cost will be

$$y = \frac{C}{100}\left[\frac{aLn + bL^2}{cn - L}\right] + P(n + 1).$$

Differentiating, and placing the first differential equal to zero, we have for minimum cost, after reduction,

$$P = \frac{Cl^2}{100}\left[\frac{cb + a}{(c - l)^2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From this formula, when the estimated average cost of a pier is known, the economical span l is easily determined.

If we take, for instance, $C = 5$ cents, and as already given, page 503, $a = 276250$, $b = 1890$, $c = 666$, then for single track, single intersection,

$$P = 0.05l^2\left(\frac{1534990}{(666 - l)^2}\right).$$

Suppose that in any case the estimated average cost of piers is \$5000. Then we have, $4.91l = 666$, or $l = 135$ feet. If the distance to be spanned were from 500 to 600 feet we should then have four spans.

The formula (1) can be adapted to any cost C, and any form of span, by giving to a, b, and c proper values, as given, page 503.

It will be seen that the usually accepted rule, that the economical span is that which costs the same as one pier, is not strictly correct. For the same values of C, and a, b, and c, the cost of a span of 135 feet would be \$6755, or 1.35 times as much as the average pier.

In case of a long structure, where the erection of piers offers no special difficulty, and the cost of a pier can be accurately estimated, our formula may give valuable information as to the length of span which should be selected.

The Bismarck Bridge, for instance, consists of three spans, single track, double intersection, each 400 feet long. The cost of the spans was about eight cents per lb., the freight being very high. The cost of the piers was actually as follows:

1st pier	\$54144
2d "	171123
3d "	155800
4th "	65372
Total	\$446439

The cost of a span was \$84,000. Total cost, \$698,439 for piers and spans.

By comparison with the actual weight of a span, we find that we should take only $\frac{7}{10}$ of the weight given by our formula, page 503, for double intersection, the difference being due to the use of steel and different train load.

For the present case we have, then, $P = \$111,609$, and,

$$P = \frac{56l^2}{1000}\left(\frac{1337250}{(654 - l)^2}\right).$$

This gives for economical span $l = 360$ feet. As the distance to be spanned is 1200 feet we should have either three spans of 400 feet each, as actually built, or two spans of 350 and two of 250 feet, or three spans of 350 and one of 150 feet. The extra pier can be taken at \$170,000, and the cost of these different suppositions easily estimated.

In the present case the actual choice of three spans of 400 feet is justified by the calculation.

The practical difficulty of estimating the average cost of pier may, in many cases, prevent our formula from being used. Where this difficulty does not exist, its use may be a guide in the selection of length of spans.

CHAPTER IX.

SPECIFICATIONS—LIST OF BRIDGE MEMBERS.

WHEN a bridge is to be built, either for a railway or a city, the work is generally advertised and let to some responsible company, bridge-builder, or contractor, who gives bonds for the satisfactory performance of the work within a certain specified time and for a certain specified price. In such case, it is the duty of the engineer of the city or railway to draw up a "specification," which shall give precisely and in sufficient detail the requirements as to construction and finish of the work. The contractor must execute the work in exact accordance with these specifications, and it is the duty of the engineer and his assistants to see that he does so.

The drawing up of a complete list of specifications, then, is a labor implying thorough knowledge on the part of the engineer who draws them up, not only of all the principles which enter into the construction, of the strength of the materials employed and the best way of utilizing them, of the processes of erection, etc., but also of those practical difficulties and sources of disagreement which often arise between the engineer and the contractor. A complete list of specifications is therefore an epitome of the science of bridge construction.

There are many such specifications in use, and the student can easily obtain them by application to the engineers of our leading railroads and bridge companies. They differ in many minor points of more or less importance. Indeed, in this respect no two are alike, embodying as they do the special experience and personal preferences of the authors. No exercise will be more profitable to the student than the careful and intelligent comparison of such points of difference.

In the preceding chapters we have covered one by one nearly all the points of construction, and an orderly *résumé* of these points would constitute the specifications of this work, and the practice here illustrated and endorsed. This practice varies as intimated, and other specifications would show points of difference as well as of agreement. The designer must be prepared to work to any given specification, and must follow it closely in his work.

We give here, by permission of the author, the *Specifications* of Theodore Cooper, C. E., which are deservedly well known and widely adopted. We shall, in future chapters, design a bridge entirely according to these specifications, referring to the preceding chapters upon construction, already given, for principles and illustration of methods.

The *Specifications* of Mr. Cooper are published by the Engineering News Publishing Company, Tribune Building, New York, and are easily obtainable.* We give them here for convenience of reference. The portion in ordinary type comprises the specifications *verbatim* as given by Mr. Cooper. We have given on each page, in fine print, in connection with each article, such explanatory remarks as seem desirable for the student. For many of these remarks we are indebted to Morgan Walcott, C. E., formerly with the Phœnix Bridge Company.

* By the same author can be obtained, *General Specifications for Iron and Steel Highway Bridges and Viaducts.*

GENERAL SPECIFICATIONS

FOR

STEEL RAILROAD BRIDGES AND VIADUCTS.

NEW AND REVISED EDITION, 1896.

BY THEODORE COOPER, CONSULTING ENGINEER.

ENGINEERING NEWS PUBLISHING COMPANY.

TRIBUNE BUILDING, NEW YORK.

BY PERMISSION OF THE AUTHOR.

GENERAL DESCRIPTION.

1. All parts of the structures shall be of steel, except ties and guard rails. Cast iron or cast steel may be used in the machinery of movable bridges and in special cases for bed-plates.

2. The following kinds of girders shall preferably be employed:

Kind of Girders. Spans, up to 20 feet.... Rolled beams.
" 20 to 75 " ...Riveted plate girders.
" 75 to 120 " ....Riveted plate or lattice girders.
" 120 to 150 " ....Lattice or pin-connected trusses.
" over 150 " ...Pin-connected trusses.

Generally "double track through" bridges will have but two trusses, to avoid spreading the tracks at bridges.

Length of Span. In calculating strains the length of span shall be understood to be the distance between centres of end pins for trusses, and between centres of bearing plates for all beams and girders.

Spacing of Girders. 3. The girders shall be spaced, with reference to the axis of the bridge, as required by local circumstances, and directed by the Engineer of the Railroad Company. (§ 5.) Longitudinal floor girders shall in no case be less than three feet and three inches from centre line of tracks. (§ 6.)

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1. The flooring, floor joists, ties, and guard rails are of wood. The machinery of movable bridges, of course, allows of the use of cast iron. But it is not allowed in any part of the structure proper. It is of no value in tension, and is not so good as wrought iron in compression. It is considered as unreliable by reason of brittleness and want of homogeneousness.

For bed-plates, a special case where it might be allowed is when a space occurs between the bottom of the pedestal and the masonry, of, say 3 inches. This must be filled up, and as wrought iron is not rolled so thick, it might be cheaper to use a cast plate rather than build up a wrought gridiron.

Again, if the span is on a grade, and the bed-plate has to be made with a slant or bevel, it is cheaper to cast it, as, if it were of wrought iron, it would have to be faced down.

2. The tracks are generally 13 feet apart, c. to c. on straight lines. If we had a "double track through bridge," with three trusses, one in centre, we should have to allow about 2 feet for width of centre truss, and 7 feet clearance from centre of each track, making 16 feet from c. to c. on the bridge. This would require the tracks to be spread, which the railroad company would wish to avoid.

The length c. to c. of girders is their "effective length," and should be distinguished from actual length, or "length over all."

3. To space the stringers nearer than 6′ 6″ makes the cross-girders heavier. The moment for a cross-girder is its reaction at end multiplied by the distance from end to the stringer. The less this distance the smaller the moment for the cross-girder.

On the other hand, the track is 4′ 8½″, and if the stringers are spaced much farther than this, there is large bending in the ties.

4. For all through bridges and overhead structures there shall be a clear head-room of 21 feet above the base of the rails, for a width of six feet over each track. **Head-room.**

5. In all through bridges the clear width from the centre of the track to any part of the trusses shall not be less than seven (7) feet at a height exceeding one foot above the rails where the tracks are straight, and an equivalent clearance where the tracks are curved. **Clear width.**

6. The standard distance, centre to centre of tracks on straight lines, will be thirteen (13) feet.

7. Each trestle bent shall, as a general rule, be composed of two supporting columns, and the bents united in pairs to form towers; each tower thus formed of four columns shall be thoroughly braced in both directions, and have struts between the feet of the columns. Transversely the columns shall have a batter of not less than one horizontal to six vertical. The feet of the columns must be secured to an anchorage capable of resisting double any possible uplifting. (§ 25.) **Trestle Towers.**

8. Each tower shall have sufficient base, longitudinally, to be stable when standing alone, without other support than its anchorage. (§§ 25, 26.)

9. Tower spans for high trestles shall not be less than 30 feet. **Trestle Spans.**

10. Unless otherwise specified, the form of bridge trusses may be selected by the bidder; for through bridges the end vertical suspenders and two panels of the lower chord, at each end, will preferably be made rigid members. In general, all spans shall have end floor beams for supporting the stringers; such end floor beams may have one intermediate bearing on the masonry. **Form of Trusses.**

11. Preference will in all cases be given to those designs using stiff lateral and portal bracing of angles and shapes, and to those designs having the least possible number of adjustable members.

12. The wooden floors will consist of transverse ties or floor timbers; their scantling will vary in accordance with the design of the supporting steel floor. (§ 15.) They shall be spaced with openings not exceeding six inches, and shall be secured to the supporting girders by ¾-inch bolts at distances not over six feet apart. For deck bridges the ties will extend the full width of the bridge, and for through bridges at least every other tie shall extend the full width of bridge for a footwalk. **Wooden Floor.**

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4. This clear head-room is only requisite at the centre of the bridge, for a space of about 6 feet for single track. The brackets or knee-braces reduce this clear depth at the sides.

5. The cover-plate on the inclined end-post is usually the widest part, so that the distance c. to c. of trusses, on a straight line, is 14 feet in clear, plus the width of a cover-plate.

"Equivalent clearance" means that, on a curve, the circle *tangent* to the sides of the cars must have the clearance specified. This requires that the circle through the corners of the cars shall have an equivalent clearance.

6. This is to agree with the railroad company.

7. A trestle bent consists of two columns, one on each side of track, each inclined or battered toward the axis in a vertical plane, and connected by transverse bracing. A tower consists of two trestle bents united by longitudinal bracing. Every other pair is thus united, making every other span an expansion span, with a fixed span between. The usual transverse batter is 6 vertical to 1 horizontal; often, however, 8 vertical to 1 horizontal.

8. That is, the fixed or tower spans must be stable when standing alone with the maximum wind force, and no dependence is placed on the girder connections at the cap.

9. This is to secure stability.

12. It is always necessary, especially in deck spans, to figure the sizes of ties required. If P is the weight of the heaviest single driver and a the distance from rail to end bearing of tie, then Pa is the moment, and (page 292) $Pa = \frac{RI}{v}$, where R is the allowed fibre stress and v is the distance from centre of gravity of cross-section to outer fibre. For a rectangular cross-section $I = \frac{bd^3}{12}$ and $v = \frac{d}{2}$, where b = breadth and d = depth in inches. There-

Guard Timbers. 13. There shall be a guard timber (scantling not less than 6 x 8") on each side of each track, with its inner face parallel to and not less than 3 feet 3 inches from centre of track. Guard timbers must be notched one inch over every floor timber, and be spliced over a floor timber with a half-and-half joint of six inches lap. Each guard timber shall be fastened to every third floor timber and at each splice with a three-quarter (¾) inch bolt. All heads or nuts on upper faces of ties or guards must be countersunk below the surface of the wood. (§ 57.)

14. The guard and floor timbers must be continued over all piers and abutments.

Allowed Strain on Timber. 15. The maximum strain allowed upon the extreme fibre of the best yellow pine or white oak floor timbers will be 1,000 pounds per square inch. The weight of a single engine wheel may be assumed as distributed over three ties spaced as per § 12.

16. The floor timbers from centre to each end of span must be notched down over the longitudinal girders so as to reduce the camber in the track, as directed by the Engineer.

17. All the floor timbers shall have a full and even bearing upon the stringers; no open joints or shims will be allowed.

18. On curves the outer rail must be elevated, as may be directed by the Engineer.

Proposals. 19. In comparing different proposals, the relative cost to the Railroad Company of the required masonry or changes in existing work will be taken into consideration.

20. Contractors in submitting proposals shall furnish complete strain sheets, general plans of the proposed structures, and such detail drawings as will clearly show the dimensions of all the parts, modes of construction, and the sectional areas.

21. Upon the acceptance of the proposal and the execution of contract, all working drawings required by the Engineer must be furnished free of cost.

Approval of Plans. 22. No work shall be commenced or materials ordered until the working drawings are approved by the Engineer in writing; if such working drawings are detained more than one week for examination, the Contractor will be allowed an equivalent extension of time.

LOADS.

23. All the structures shall be proportioned to carry the following loads:

---

fore $d = \sqrt{\frac{6Pa}{Rb}}$, where d can be found for any assumed value of b. As the rails are stiff it is customary to assume the load as carried by three ties.

EXAMPLE.—Tie of white oak, weight of driver $P = 25,600$ lbs., $a = 1' 4''$. Take $R = 800$ lbs. per square inch. Then for one tie we have $\frac{1}{3} P = 8,533$ lbs. instead of P. If we take $b = 9$ inches, we have $d = 10$ inches.

16. The camber causes the centre of the truss to be higher than its ends. This notching down reduces the track on bridge to the desired grade.

18. This is the same as is done on all curves.

19. One proposal may require entirely new masonry, or great changes in the existing masonry. Another may utilize the existing masonry without material change, by taking dimensions for the truss to suit. If in such case the truss is more costly, it is but fair to consider the saving of masonry. Again, one plan may call for more costly masonry than another, even when there is none already existing.

20. In many cases a complete strain sheet is considered sufficient without detail drawings.

21. These drawings are required for use by the inspectors.

22. Many roads do not require the working drawings at all.

23. The dead load can be estimated as directed, page 495. Whatever the system adopted, the maximum

1st. The weight of metal in the structure. 2d. A floor weighing 400 pounds per linear foot of *track*, to consist of rails, ties, and guard timbers only.

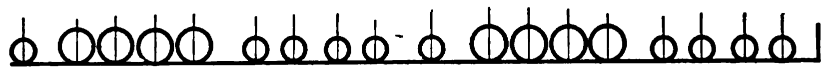
These two items, taken together, shall constitute the "dead load." **Dead Load.**

3d. A "live load," on each track, supposed to be moving in either direction, consisting of two "consolidation" engines, coupled and followed by a train load, distributed as shown on diagram E....; or 100,000 pounds equally distributed on two pair of driving wheels, spaced seven and a half feet, centre to centre. **Live Loads.**

NOTE.—As all the wheel loads in each diagram are made of the same percentages of the driving wheel loads, the strains due to the different engine diagrams will be proportionate to the numerical classes of the engines.

Any intermediate numbers may be selected, with the understanding that this rule of proportion applies.

DISTANCE IN FEET.

Class	8	5	5	5	9	5	6	5	8	8	5	5	5	9	5	6	5	5	Uniform Load.
																			
E 27	13500	27000	27000	27000	27000	17550	17550	17550	17550	13500	27000	27000	27000	27000	17550	17550	17550	17550	2700 lbs. per lin. ft.
E 30	15000	30000	30000	30000	30000	19500	19500	19500	19500	15000	30000	30000	30000	30000	19500	19500	19500	19500	3000 lbs, per lin. ft.
E 35	17500	35000	35000	35000	35000	22750	22750	22750	22750	17500	35000	35000	35000	35000	22750	22750	22750	22750	3500 lbs. per lin. ft.
E 40	20000	40000	40000	40000	40000	26000	26000	26000	26000	20000	40000	40000	40000	40000	26000	26000	26000	26000	4000 lbs. per lin. ft.
E..																			

The maximum strains due to all positions of either of the above "live loads," of the required class, and of the "dead load," shall be taken to proportion all the parts of the structure.

24. To provide for wind strains and vibrations, the top lateral bracing in deck bridges, and the bottom lateral bracing in through bridges, for all spans **Wind Bracing.**

stresses are found as illustrated, page 88, by the use of a diagram prepared for each system. The introduction of this method by diagram, and its invention, are due to Mr. Cooper, and also, independently, Mr. Robert Escobar, C. E., of the Union Bridge Company.

24. The exposed area of the train is about 10 square feet for every foot in length. At 30 lbs. per square foot this gives 300 lbs. for every foot of length, which should be treated as a moving load.

The truss would probably not have more than about 10 square feet of exposed surface for every foot in length; this also, at 30 lbs. per square foot, would give 300 lbs. per foot of length for wind pressure on the whole truss. Taking one-half of this on each chord, upper and lower, we have a fixed load of 150 lbs. per linear foot on each chord. We have thus, as specified, 150 lbs. fixed load, per linear foot, for the unloaded chord, and 450 lbs. per linear foot for the loaded chord, of which 300 is live, and 150 fixed. We have used these values in the example of page 449.

up to 300 feet, shall be proportioned to resist a lateral force of 450 pounds for each foot of the span; 300 pounds of this to be treated as a moving load, and as acting on a train of cars, at a line 8.5 feet above base of rail.

The bottom lateral bracing in deck bridges, and the top lateral bracing in through bridges for all spans up to 300 feet, shall be proportioned to resist a lateral force of 150 pounds for each foot of the span.

For spans exceeding 300 feet, add, in each of the above cases, 10 pounds for each additional 30 feet of span.

25. In trestle towers the bracing and columns shall be proportioned to resist the following lateral pressures, in addition to the strains from dead and live loads:

1st. With either one track loaded with cars only, or with both tracks loaded with maximum train load, the lateral forces specified in § 24; and a lateral pressure of 100 pounds for each vertical lineal foot of the trestle bents; or

2d. With both tracks unloaded, a lateral force of 500 pounds for each longitudinal lineal foot of the structure, acting at the centre line of the girders; and a lateral pressure of 200 pounds for each vertical lineal foot of the trestle bents.

Longitudinal Forces. 26. The strains produced in the bracing of the trestle towers, in any members of the trusses, or in the attachments of the girders or trusses to their bearings, by the greatest tractive force of the engines or by suddenly stopping the maximum trains on any part of the work, must be provided for; the coefficient of friction of the wheels on the rails being assumed as 0.20.

Temperature. 27. Variation in temperature, to the extent of 150 degrees, shall be provided for.

Centrifugal Force. 28. When the structures are on curves, the additional effects due to the centrifugal force of trains shall be considered as a live load. It will be assumed to act 5 feet above base of rail, and will be computed for a speed of $50 - 2d$ miles per hour; d being the degree of curve.

29. All parts shall be so designed that the strains coming upon them can be accurately calculated.

PROPORTION OF PARTS.

Tensile Strains. 30. All parts of the structures shall be proportioned in tension by the following allowed unit strains:

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25. Generally, the true wind forces are taken at their actual points of application, as we have done in the example, page 449.

26. The stresses from traction should always be figured for viaducts. If W is the weight on a bent, and ϕ the coefficient of friction, the tractive force, F, acting longitudinally at the top of the bent, is $F = \phi W$.

27. A bar of iron 1 foot long will lengthen about 0.000006 foot for a rise of temperature of 1 degree. For 150 degrees this gives 0.0009 foot per foot, or, for a bar 100 feet long, 0.09 foot, or about 1 inch. Hence the rule, "*one inch per one hundred feet.*" In designing the roller bed plates, allowance should be made so that the checks for the rollers shall permit of the expansion and contraction of the truss.

28. The centrifugal force for curves has been given page 448.

29. If it is impossible to avoid an indeterminate member, the member should be designed for the maximum stresses which can occur, whichever way the stresses go.

30. The floor-beam hangers are liable to sudden loading and impact, and this is allowed for by taking a small unit stress. The lateral bracing is called in play only at long intervals, perhaps never to its full extent, and the stress is applied slowly. The unit stress is therefore taken large. For the main chords, the dead load forms quite a large percentage of the live load, and hence the unit stress is large for the dead load, and reduced for the live load.

By net section is meant the section after rivet-holes are deducted. The stresses are reduced for swing bridges to allow for effects of motion.

For Medium Steel.

Medium Steel.

	Pounds per square inch.
Floor beam hangers, and other similar members liable to sudden loading (bars with forged ends)	7,000
Floor beam hangers, and other similar members liable to sudden loading (plates or shapes), net section	6,000
Longitudinal, lateral, and sway bracing for wind and live load strains (§§ 24–28)	18,000
Solid rolled beams, used as cross floor beams and stringers	10,000
Bottom flanges of riveted cross girders, net section	10,000
Bottom flanges of riveted longitudinal plate girders, used as track stringers, net section	10,000

	For live loads.	For dead loads.
Bottom chords, main diagonals, counters, and long verticals (forged eye-bars)	10,000	20,000
Bottom chords and flanges, main diagonals, counters, and long verticals (plates or shapes), net section	9,000	18,000

For swing bridges and other movable structures, the dead load unit strains, during motion, must not exceed three-fourths of the above allowed unit strains for dead load on stationary structures.

The areas obtained by dividing the live load strains by the live load unit strains will be added to the areas obtained by dividing the dead load strains by the dead load unit strains to determine the required sectional area of any member. (§ 45.)

Soft steel may be used in tension with unit strains ten per cent. less than those allowed for *medium steel.* Soft Steel.

31. Angles subject to direct tension must be connected by both legs, or the section of one leg only will be considered as effective

32. In members subject to tensile strains full allowance shall be made for reduction of section by rivet-holes, screw-threads, etc. (§ 56.) Net Section.

33. Compression members shall be proportioned by the following allowed unit strains : Compressive Strains.

For Medium Steel.

Chord segments, $P = 10000 - 45\frac{l}{r}$ for live load strains.

$$P = 20000 - 90\frac{l}{r} \text{ for dead load strains.}$$

31. Thus, if the diagonal ties of a Warren girder are angles, and are riveted to the chords by one leg only, the section of one leg only is to be considered as effective. To make both legs effective, the other leg must be also attached to the chords by means of connecting angles. It is, however, sometimes considered allowable, in the first case, to take the *gross* section of the leg, under the assumption that the metal taken out by rivet-holes is balanced by the metal in the other leg.

32. The diameter of hole multiplied by thickness of plate gives area to be taken out. In compression there is evidently no deduction to be made.

33. The chords are usually considered as having fixed ends, while the posts have pin ends. Mr. Cooper reduces the stresses in the posts on account of their liability to blows from derailment, and not because of their end conditions.

Medium Steel. All posts of through bridges, $P = 8500 - 45\frac{l}{r}$ for live load strains.

$P = 17000 - 90\frac{l}{r}$ for dead load strains.

All posts of deck bridges and trestles, $P = 9000 - 40\frac{l}{r}$ for live load strains.

$P = 18000 - 80\frac{l}{r}$ for dead load strains.

End posts are not to be considered chord segments.

Lateral struts and rigid bracing, $P = 13000 - 70\frac{l}{r}$ for wind strains; for live load strains use two-thirds of the above.

Lateral struts, with adjustable bracing, will be proportioned by the above formula to resist the maximum due either to the wind and load or to an assumed initial strain of 10,000 pounds per square inch on all the rods attached to them. (§ 39.)

$P =$ the allowed strain in compression per square inch of cross section, in pounds.

$l =$ the length of compression member, in inches.

$r =$ the least radius of gyration of the section, in inches.

No compression member, however, shall have a length exceeding 125 times its least radius of gyration.

Soft Steel. *Soft steel* may be used in compression with unit strains fifteen per cent. less than those allowed for *medium steel.*

For swing bridges and other movable structures, the dead load unit strains during motion must not exceed three-fourths of the above allowed unit strains for dead load on stationary structures.

34. For long span bridges, when the ratio of the length and width of span is such that it makes the top chord, acting as a whole a longer column than the segments of the chord, the chord will be proportioned for this greater length.

Alternate Strains. 35. Members subject to alternate strains of tension and compression shall be proportioned to resist each kind of strain. Both of the strains shall, however, be considered as increased by an amount equal to $\frac{8}{10}$ of the least of the two strains, for determining the sectional areas by the above allowed unit strains. (§§ 30, 33.)

Effect of Wind on Chord Strains. 36. The strains in the truss members or trestle posts from the assumed wind forces need not be considered except as follows:

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35. If a member has tension of 130,000 lbs., and compression of 90,000 lbs., $\frac{8}{10}$ of the latter is 72,000 lbs. The increased stresses are, therefore, tension 202,000 lbs., compression 162,000 lbs. If the allowable unit stress is 10,000 lbs. per square inch for tension, and 7,000 for compression, the member should have 20.2 square inches, net, for the tensile, or 23.14 square inches, gross, for the compressive stress, whichever comes out largest.

36. If the dead load stress on a chord is 60,000 lbs., the live load 140,000 lbs., and the wind stress 80,000 lbs., the total stress from live and dead is 200,000 lbs. One quarter of this is 50,000 lbs., which the wind stress exceeds.

Now if σ' is the allowable unit stress for live load L, and σ for dead load D, and u is the unit stress for dead and live loads combined, we have $\frac{L}{\sigma'} + \frac{D}{\sigma} = \frac{L + D}{u}$, or $u = \dfrac{L + D}{\frac{L}{\sigma'} + \frac{D}{\sigma}}$. If, in our present case, $\sigma = 16,000$ lbs., $\sigma' = 8,000$ lbs., then $u = 9,400$ lbs. Increasing this by $\frac{1}{4}$, we have 10,750 lbs. as the allowable unit stress for combined

1st. When the wind strains on any member exceed one-quarter of the maximum strains due to the dead and live loads upon the same member. The section shall then be increased until the total strain per square inch will not exceed by more than one-quarter the maximum fixed for dead and live loads only.

2d. When the wind strain alone or in combination with a possible temperature strain can neutralize or reverse the strains in any member.

Rivets, Bolts, and Pins.

37. The rivets in all members, other than those of the floor and lateral systems, must be so spaced that the shearing strain per square inch shall not exceed 9,000 pounds; nor the pressure on the bearing surface (diameter × thickness of the piece) of the rivet-hole exceed 15,000 pounds per square inch.

The rivets in all members of the floor system, including all hanger connections, must be so spaced that the shearing strains and bearing pressures shall not exceed 80 per cent. of the above limits.

The rivets in the lateral and sway bracing will be allowed 50 per cent. increase upon the above limits.

In the case of field riveting (and for bolts as per § 57) the above allowed shearing strains and pressures shall be reduced one-third.

Rivets and bolts must not be used in direct tension.

38. Pins shall be proportioned so that the shearing strain shall not exceed 9,000 pounds per square inch; nor the crushing strain on the projected area of the semi-intrados of any member (other than forged eye-bars, see § 80) connected to the pin be greater per square inch than 15,000 pounds; nor the bending strain exceed 18,000 pounds, when the applied forces are considered as uniformly distributed over the middle half of the bearing of each member.

Combined Strains.

39. When any member is subjected to the action of both axial and bending strains, as in the case of end posts of through bridges (§ 36), or of chords carrying distributed floor loads, it must be proportioned so that the greatest

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dead, live, and wind stresses of 280,000 lbs. This calls for 26 square inches, while, if the wind were disregarded, only 21.2 square inches would be needed.

If the compressive wind stresses in the lower chord are greater than the tensile due to dead load, it will be necessary to stiffen the lower chord to make the difference.

37. We must therefore test the rivets for both shear and bearing (page 436). If rivets were used in direct tension, the heads would tear off.

38. Main pins are only figured for bending and bearing (page 428). If large enough for these they are also large enough for shear. Bolts and small pins should be figured for shear also.

39. For combined flexure and direct stress see page 378. Let $M =$ the maximum bending moment in the member. Let $S =$ the direct stress, tension, or compression. Let $\sigma_1 =$ the allowable unit stress for direct stress, and σ_2 for bending. Let $a =$ the area of the member, and $I =$ the moment of inertia of its cross-section $= ar^2$, where r is the radius of gyration. Let $\sigma = \sigma_1 + \sigma_2$. Then, from theory of flexure (page 292),

$$M = \frac{\sigma_2 I}{v}, \text{ where } v \text{ is the distance from neutral axis to extreme fibre.}$$

Hence

$$\sigma_2 = \frac{Mv}{I} = \frac{Mv}{ar^2}. \quad \text{But } \sigma_1 = \frac{S}{a}. \quad \text{Therefore}$$

$$\sigma_1 + \sigma_2 = \sigma = \frac{S}{a} + \frac{Mv}{ar^2}, \text{ or } a = \frac{1}{\sigma}\left(S + \frac{Mv}{r^2}\right)$$

If the fibre stress due to weight of member were just 10 per cent. of the allowed unit stress, it would add $\frac{1}{10}$ of a square inch to the cross-section. If it exceeds this, the specification requires it should be considered. This is rarely the case. The inclined end-posts are the most apt to exceed the limit. For very large bridges the limit may be exceeded.

fibre strain will not exceed the allowed limits of tension or compression on that member.

If the fibre strain resulting from the weight only of any member exceeds ten per cent. of the allowed unit strain on such member, such excess must be considered in proportioning the areas.

Compression Flanges. 40. In beams and plate girders the compression flanges shall be made of same *gross* section as the tension flanges.

Depth of Girders. 41. Riveted longitudinal girders shall have, preferably, a depth not less than $\frac{1}{10}$ of the span.

Rolled beams used as longitudinal girders shall have, preferably, a depth not less than $\frac{1}{12}$ of the span.

Plate Girders. 42. Plate girders shall be proportioned upon the supposition that the bending or chord strains are resisted entirely by the upper and lower flanges, and that the shearing or web strains are resisted entirely by the web-plate; no part of the web-plate shall be estimated as flange area.

The distance between centres of gravity of the flange areas will be considered as the effective depth of all girders.

Web Plates. 43. The webs of plate girders must be stiffened at intervals, about the depth of the girders, wherever the shearing strain per square inch exceeds the strain allowed by the following formula:

$$\text{Allowed shearing strain} = \frac{12{,}000}{1 + \dfrac{H^2}{3{,}000}}$$

where $H =$ ratio of depth of web to its thickness; but no web-plates shall be less than three-eighths of an inch in thickness.

Rolled Beams. 44. Rolled beams shall be proportioned (§§ 30, 40) by their moments of inertia.

Counters. 45. The areas of counters shall be determined by taking the difference in areas due to the live and dead load strains considered separately (§ 30); the counters in any one panel must have a combined sectional area of at least three square inches, or else must be capable of carrying all the counter live load in that panel.

DETAILS OF CONSTRUCTION.

Details. 46. All the connections and details of the several parts of the structures shall be of such strength that, upon testing, rupture will occur in the body of the members rather than in any of their details or connections.

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42. This is contrary to some specifications, which allow $\frac{1}{8}$ of the web to aid each flange, or $\frac{1}{4}$ of the web in all available for flange section. The web undoubtedly does assist the flanges. The "effective depth" is to be used in figuring all stresses. The web is omitted by Mr. Cooper partly on account of splicing, partly as an allowance for the indefinite impact stresses.

43. Web plates are not made less than $\frac{3}{8}$ inch thick, in order to resist the action of rust and to prevent the web from being unsteady, and to enable it to resist impact as well as to reduce the stffening angles.

The practice of stiffening the webs of plate girders differs widely. Some specifications require many stiffeners. Some require them spaced closer at the ends, others at equal distances throughout the length.

44. For rolled beams we have $M = \frac{\sigma_2 I}{v}$, where M is the maximum moment, I the moment of inertia of the cross-section, σ_2 the allowable fibre stress, v the distance from neutral axis to extreme fibre. If $a =$ the area of the cross-section, then $I = ar^2$, where r is the radius of gyration, and $a = \frac{Mv}{\sigma_2 r^2}$.

If $v = r$, as is the case for a pin, we have $a = \frac{M}{\sigma_2 r}$, where r is the half depth. If d denote the depth, $a = \frac{2M}{\sigma_2 d}$, or, total area of both flanges $= a\sigma_2 = \frac{2M}{d}$, or, area of one flange $= \frac{1}{2} a\sigma_2 = \frac{M}{d} = \frac{\text{Bending Moment}}{\text{Depth}}$.

46. This makes the main members limit the safety of the span.

47. Preference will be had for such details as shall be most accessible for inspection, cleaning, and painting; no closed sections will be allowed.

48. The pitch of rivets in all classes of work shall never exceed 6 inches, or sixteen times the thinnest outside plate, nor be less than three diameters of the rivet. Riveting.

49 The rivets used shall generally be $\frac{3}{4}$ and $\frac{7}{8}$ inch diameter.

50. The distance between the edge of any piece and the centre of a rivet-hole must never be less than $1\frac{1}{4}$ inches, except for bars less than $2\frac{1}{2}$ inches wide; when practicable it shall be at least two diameters of the rivet.

51. For punching, the diameter of the die shall in no case exceed the diameter of the punch by more than $\frac{1}{16}$ of an inch, and all holes must be clean cuts without torn or ragged edges.

52. All rivet-holes must be so accurately spaced and punched that when the several parts forming one member are assembled together, a rivet $\frac{1}{16}$ inch less in diameter than the hole can generally be entered, hot, into any hole, without reaming or straining the metal by "drifts;" occasional variations must be corrected by reaming.

53 The rivets when driven must completely fill the holes. The rivet-heads must be round and of a uniform size for the same-sized rivets throughout the work. They must be full and neatly made, and be concentric to the rivet-hole, and thoroughly pinch the connected pieces together.

54. Wherever possible, all rivets must be machine driven. The machines must be capable of retaining the applied pressure after the upsetting is completed. No hand-driven rivets exceeding $\frac{7}{8}$ inch diameter will be allowed.

55. Field riveting must be reduced to a minimum or entirely avoided, where possible.

56. The effective diameter of a driven rivet will be assumed the same as its diameter before driving In deducting the rivet-holes to obtain net sections in tension members, the diameter of the rivet-holes will be assumed as $\frac{1}{8}$ inch larger than the undriven rivets. Net Sections.

The rupture of a riveted tension member is to be considered as equally probable, either through a transverse line of rivet-holes or through a diagonal line of rivet-holes, where the net section does not exceed by 30 per cent. the net section along the transverse line.

The number of rivet-holes to be deducted for net section will be determined by this condition.

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48. A greater pitch than 6 inches in compression might allow the plate to "buckle." For this reason, if 16 times the thickness of the plate is less than 6 inches, that should be the limit. A less pitch than 3 diameters renders the holes liable to tear out, as well as injures the metal when punched.

49. For girders and main compression members, $\frac{7}{8}''$ is the size generally used.

50. This for the same reason as §48.

51. If the clearance between the punch and the die is over $\frac{1}{16}''$, there is a tendency to draw and bunch the iron and make a ragged hole.

52. Reaming is expensive, and forcing holes into opposition by driving through a steel drifting-pin is injurious to the metal. On the shop drawings the rivet-holes are always ordered to be punched $\frac{1}{16}''$ larger than the rivet.

53 Rivets which do not fill the hole when driven are called "loose rivets. The inspector should require them to be replaced. The rivets, by pinching the plates, develop friction which increases their value.

54. The rivet spacing should be so designed that all may be machine driven. It is sometimes impossible to avoid driving some by hand, owing to the locality. Field rivets are usually driven by hand.

If the machine is not capable of retaining the applied pressure after the upsetting is completed, the plates will not remain thoroughly pinched together.

In the case of a large rivet exceeding $\frac{7}{8}'$ it would be impossible to properly upset it by hand driving.

56. For a $\frac{7}{8}''$ rivet the hole would be punched $\frac{15}{16}''$. If this hole is reamed it may easily reach $1''$, and the net section is assumed on this basis.

Bolts. 57. When members are connected by bolts, the holes must be reamed parallel and the bolts turned to a driving fit. All bolts must be of neat lengths, and shall have a washer under the heads and nuts where in contact with wood. Bolts must not be used in place of rivets, except by special permission.

58. The several pieces forming one built member must fit closely together, and when riveted shall be free from twists, bends, or open joints.

Splices. 59. All joints in riveted tension members must be fully and symmetrically spliced.

Abutting Joints. 60. In compression members, abutting joints with planed faces must be sufficiently spliced to maintain the parts accurately in contact against all tendencies to displacement.

61. In compression members, abutting joints with untooled faces must be fully spliced, as no reliance will be placed on such abutting joints. The abutting ends must, however, be dressed straight and true, so there will be no open joints.

Web Splices. 62. The webs of plate girders must be spliced at all joints by a plate on each side of the web.

Stiffeners. 63. All web-plates must have stiffeners over bearing points and at points of local concentrated loadings; such stiffeners must be fitted at their ends to the flange angles, at the bearing points.

64. All other angles, filling and splice plates on the webs of girders and riveted members must fit at their ends to the flange angles, sufficiently close to be sealed, when painted, against admission of water.

Web Plates. 65. Web-plates of all girders must be arranged so as not to project beyond the faces of the flange angles, nor on the top be more than $\frac{1}{16}$ inch below the face of these angles, at any point.

66. Wherever there is a tendency for water to collect, the spaces must be filled with a suitable waterproof material.

Flange Plates. 67. In girders with flange plates, at least one-half of the flange section shall be angles or else the largest sized angles must be used.

68. In lattice girders and trusses, the web members must be double and connect symmetrically to the webs of the chords. The use of plates or flats, alone, for tension members must be avoided, where it is possible; in lattice trusses, the counters, suspenders, and two panels of the lower chord, at each end, must be latticed. (See Arts. 85, 86, and 87.)

69. The compression flanges of beams and girders shall be stayed against transverse crippling when their length is more than twenty times their width.

70. The unsupported width of plates subjected to compression shall not

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60 The faced joints are relied upon to transmit the strain, but there should be enough splice plates to prevent displacement from jars, etc. Tension joints must, of course, be fully spliced to take the entire stress.

61. This is a "full splice." Open joints admit rain, and are hard to paint or protect from rust.

65. This clearance allows cover plates to fit closely against the backs of the flange angles. If there is no cover plate, a clearance of more than $\frac{1}{16}''$ would collect and hold water, and would be difficult to protect.

67. This corresponds pretty well with the rule that the flange angle shall be $\frac{1}{16}''$ thicker than the cover plate. The object of the clause is to prevent the using of small angles and piling up cover plates. It also favors the concentration of the flange material as near as possible to the web connection.

68. To avoid eccentricity of stress.

69. If a girder has a top flange 12″ wide, it would be necessary to brace it against transverse crippling if its length was over 30 feet. In a deck-plate girder this would be done by transverse bracing to the other girder. In a through plate girder knee-braces can be used at every ten or fourteen feet, depending upon the distance c. to c. of girder.

70. A cover-plate for a top chord $\frac{1}{2}''$ thick should not have an unsupported width exceeding 20 inches. The unsupported width would be the distance between the lines of rivets in the flanges. This clause is to prevent the cover-plate from buckling transversely.

exceed thirty times their thickness; except cover plates of top chords and end posts, which will be limited to forty times their thickness.

71. The flange plates of all girders must be limited in width so as not to extend beyond the outer lines of rivets connecting them with the angles more than five inches or more than eight times the thickness of the first plate. Where two or more plates are used on the flanges, they shall either be of equal thickness or shall decrease in thickness outward from the angles.

72. Where the floor timbers are supported at their ends on one flange of an angle, such angle must have two rows of rivets in its vertical leg, spaced not over 4 inches apart.

73. No metal shall be used less than $\frac{5}{16}$ inch thick, except for lining or filling vacant spaces.

Eye Bars.

74. The heads of eye-bars shall be so proportioned and made that the bars will preferably break in the body of the original bar rather than at any part of the head or neck. The form of the head and the mode of manufacture shall be subject to the approval of the Engineer of the Railroad Company. (Art. 132.)

75. The bars must be free from flaws and of full thickness in the necks. They shall be perfectly straight before boring The holes shall be in the centre of the head, and on the centre line of the bar.

76. The bars must be bored to lengths not varying from the calculated lengths more than $\frac{1}{64}$ of an inch for each 25 feet of total length.

77. Bars which are to be placed side by side in the structure shall be bored at the same temperature and of such equal length that upon being piled on each other the pins shall pass through the holes at both ends without driving.

78. The lower chord shall be packed as narrow as possible.

Pins.

79. The pins shall be turned straight and smooth; chord pins shall fit the pin-holes within $\frac{1}{50}$ of an inch, for pins less than $4\frac{1}{2}$ inches diameter; for pins of a larger diameter the clearance may be $\frac{1}{32}$ inch. Lateral pins shall fit the pin-holes within $\frac{1}{16}$ of an inch.

80. The diameter of the pin shall not be less than three-quarters the largest dimension of any eye-bar attached to it. The several members attaching to the pin shall be so packed as to produce the least bending moment upon the pin, and all vacant spaces must be filled with wrought-iron filling rings.

Upset Ends.

81. All rods with screw ends shall be upset at the ends, so that the diameter at the bottom of the threads shall be $\frac{1}{16}$ inch larger than any part of the body of the bar.

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71. If the cover-plates extended over 5 inches beyond the outer line of rivets, there would be a tendency to buckle along their outer edges.

73. Metal, less than $\frac{5}{16}''$ thick, mig after a little exposure, become unfit to perform its duty.

77. When bars are side by side, it is still more necessary that their lengths should be the same, otherwise they are strained unequally.

78. In order to concentrate the material as near as possible to the centre line of stress.

79. It is customary to turn off from the rough pins, $\frac{1}{16}$ to $\frac{3}{16}$ of an inch, according to the size of the pin, in order to get a smooth and straight pin surface. If the pin-hole and pin were of exactly the same size, the erectors would be unable to drive the pin without injury to it.

80. Eye-bars cannot have pin-plates riveted to them in order to get sufficient pin bearing. It is therefore necessary to have enough pin bearing without any pin plates. Too small a pin would not give sufficient bearing. The ratio we have deduced, page 420, is $\frac{3}{4}$. For a less diameter the head must be thicker than the bar, in order to get sufficient bearing. Vacant spaces on pins must always be filled with filling rings, to prevent displacement of the members on the pin. Cast filling rings are liable to be broken.

81. This is to make the screw ends at least as strong as the body of the bar. The process of "upsetting" consists in making the member larger at a particular point than it is elsewhere. This is done by forging.

82. All threads must be of the United States standard, except at the ends of the pins.

Hangers. 83. Floor beam hangers shall be made without adjustment and so placed that they can be readily examined at all times.

84. All the floor beams must be effectually stayed against end motion or any tendency to rotate from the action of the lateral system.

Compression Members. 85. Compression members shall be of steel, and of approved forms.

86. The pitch of rivets at the ends of compression members shall not exceed four diameters of the rivets for a length equal to twice the width of the member.

87. The open sides of all compression members shall be stayed by batten plates at the ends and diagonal lattice-work at intermediate points. The batten plates must be placed as near the ends as practicable, and shall have a length not less than the greatest width of the member or $1\frac{1}{2}$ times its least width. The size and spacing of the lattice bars shall be duly proportioned to the size of the member. They must not be less than $2 \times \frac{5}{16}$ inches for posts 6 inches wide, nor $2\frac{1}{2} \times \frac{7}{16}$ inches for posts 15 inches wide. They shall be inclined at an angle not less than 60° to the axis of the member for single latticing, nor less than 45° for double latticing with riveted intersections. The pitch of the latticing must not exceed the width of the channel plus nine inches.

88. Where necessary, pin-holes shall be re-enforced by plates, some of which must be of the full width of the member, so the allowed pressure on the pins shall not be exceeded, and so the strains shall be properly distributed over the full cross section of the members. These re-enforcing plates must contain enough rivets to transfer their proportion of the bearing pressure, and at least one plate on each side shall extend not less than six inches beyond the edge of the batten plates. (§ 87.)

89. Where the ends of compression members are forked to connect to the pins, the aggregate compressive strength of these forked ends must equal the compressive strength of the body of the members.

90. In compression chord sections, the material must mostly be concentrated at the sides, in the angles and vertical webs. Not more than one plate, and this not exceeding $\frac{1}{2}$ inch in thickness, shall be used as a cover plate, except when necessary to resist bending strains, or to comply with § 70. (§ 39.)

91. The ends of all square-ended members shall be planed smooth, and exactly square to the centre line of strain.

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83. Owing to their severe duty, importance, and position, the hangers should be specially examined, and such examination should be aided by making every part accessible.

86. It is usually the custom to space the rivets 3″ apart for two or three feet at the ends of compression members, and in the centre 6″ apart, unless this distance is not greater than 16 times the thickness of the thinnest plate, in which case the rivets in centre would be spaced, say $4\frac{1}{2}$″ apart.

87. The battens or stay plates cannot be put, in general, directly at the ends, because of the inclined ties and counters, which would interfere. Practice varies somewhat as to size, and stay plates about as long as wide are common. For main members the angle of lattice bars should not be less than 60°; but for lateral struts and small members it is allowable to take the angle somewhat less.

88. The last sentence is to prevent a single channel from being overstrained at the end before the channel is united to the other channel by the batten, after which the stress runs through the section as a whole. The radius of gyration for a single channel is less than for the whole section, and the allowed unit stress in the jaw would therefore be less than for the whole member. Hence there should be an excess of section until the stress has reached the main member proper.

90. The cover plate of a top chord unites the two channels forming its webs, but it would seem doubtful whether it takes its share of its stress, as it does not directly touch the pin, while the webs do. Therefore Mr. Cooper keeps the proportion of cover plate to total section as small as is consistent with a firm union of the two channels.

91. This is to get the full value out of the abutting top chord joints.

92. All members must be free from twists or bends. Portions exposed to view shall be neatly finished.

93. Pin-holes shall be bored exactly perpendicular to a vertical plane passing through the centre line of each member, when placed in a position similar to that it is to occupy in the finished structure.

94. Where rods are used in the lateral, longitudinal, or sway bracing (§ 11), they shall be square bars, but in no case shall they have a less area than one square inch. Rods with bent eyes must not be used. **Lateral Bracing.**

95. All through bridges shall have latticed portals, of approved design, at each end of the span, connected rigidly to the end posts and top chords. They shall be as deep as the specified head-room will allow. (§ 4.) (§ 11.) **Transverse Diagonal Bracing.**

96. When the height of the trusses exceeds 25 feet, an approved system of overhead diagonal bracings shall be attached to each post and to the top lateral struts.

97. All bars and rods in the web, lateral, longitudinal, or sway systems must be securely clamped at their intersections to prevent sagging and rattling.

98. Pony trusses and through plate or lattice girders shall be stayed by knee braces or gusset plates attached to the top chords at the ends and at intermediate points, and attached below to the cross floor beams or to the transverse struts.

99. All deck girders shall have transverse braces at the ends. All deck bridges shall have transverse bracing at each panel point. This bracing shall be proportioned to resist the unequal loading of the trusses.

100. All bed-plates must be of such dimensions that the greatest pressure upon the masonry shall not exceed 250 pounds per square inch. **Bed-Plates.**

101. All bridges over 75 feet span shall have at one end nests of turned friction rollers running between planed surfaces. These rollers shall not be less than 2⅝ inches diameter for spans 100 feet or less, and for greater spans this diameter shall be increased in proportion of 1 inch for 100 feet additional. **Friction Rollers.**

The rollers shall be so proportioned that the pressure per lineal inch of roller shall not exceed the product of the diameter in inches by 300 pounds (300 *d.*).

The rollers must be of machinery steel and the bearing plates of medium steel.

The rollers and bearings must be so arranged that they can be readily cleaned and so that they will not hold water.

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93. This is to insure that the pin-hole in one channel of a built member shall come directly opposite that in the other.

95. There are a great variety of designs for these portals, but those in which the metal is curved are no longer "approved design." The weight of the portal bracing and its arrangement depend more on the width of the bridge than anything else.

96. If there is sufficient head-room, deep bracing, with either a stiff lattice system or rods, is generally used, preferably the former, as in this case knee-braces can also be used, running to a panel point of the lattice. If the head-room does not allow a deep strut, a shallow one with knee-braces is used.

99. In deck-plate girder spans which are long enough, it is good practice to put in, besides the end transverse bracing, intermediate transverse cross-braces. The transverse bracing must be figured to transmit the panel wind load to the lower chord, and, if on curves, for unequal loading and the centrifugal force. The transverse bracing between the inclined end-posts of deck spans should carry the entire wind load which may come to the abutment through the top chord.

100. The quality of masonry is apt to vary considerably. If it is known that the masonry is particularly good, 300 lbs. per square inch will not be too great a pressure.

101. The rollers are kept in position by straps uniting their centres. They roll thus together *en masse* on the planed surface of the bed-plate. Angle-iron checks riveted to the bed-plate keep them from rolling too far. These checks should allow for change of length of the truss due to temperature.

102. Bridges less than 75 feet span shall be secured at one end to the masonry, and the other end shall be free to move longitudinally upon planed surfaces.

103. Where two spans rest upon the same masonry, a continuous plate, not less than $\frac{5}{8}$ inch thick, shall extend under the two adjacent bearings, or the two bearings must be rigidly tied together.

Pedestals and Bed-Plates. 104. Pedestals shall be made of riveted plates and angles. All bearing surfaces of the base plates and vertical webs must be planed. The vertical webs must be secured to the base by angles having two rows of rivets in the vertical legs. No base plate or web connecting angle shall be less in thickness than $\frac{3}{8}$ inch. The vertical webs shall be of sufficient height and must contain material and rivets enough to practically distribute the loads over the bearings or rollers.

Where the size of the pedestal permits, the vertical webs must be rigidly connected transversely.

105. All the bed-plates and bearings under fixed and movable ends must be fox-bolted to the masonry; for trusses, these bolts must not be less than $1\frac{1}{4}$ inches diameter; for plate and other girders not less than $\frac{7}{8}$ inch diameter. The Contractor must furnish all bolts, drill all holes, and set bolts to place with sulphur or Portland cement.

106. While the roller ends of all trusses must be free to move longitudinally under changes of temperature, they shall be anchored against lifting or moving sideways.

Camber. 107. All bridges shall be cambered by giving the panels of the top chord an excess of length in the proportion of $\frac{1}{8}$ of an inch to every ten feet.

Trestle Towers. 108. The lower struts in trestle towers must be capable of resisting the strains due to changes of temperature or of moving the tower pedestals under the effects of expansion or contraction.

For high or massive towers, these lower struts will be securely anchored to intermediate masonry piers, or the tower pedestals will have suitably placed friction rollers, as may be directed by the Engineer.

109. All joints in the tower columns shall be fully spliced for all possible tension strains, and to hold the parts firmly in position.

Bed-Plates. 110. Tower footings and bed-plates must be planed on all bearing surfaces, and the holes for anchor bolts slotted to allow for the proper amount of movement. (§ 27.)

Workmanship. 111. All workmanship shall be first-class in every particular.

112. All eye-bars must be made of medium steel.

113. Eye-bars, all forgings, and any pieces which have been partially heated or bent cold must be wholly annealed. Crimped stiffeners need not be annealed.

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102. Such short spans will slide on the bed-plate, and rollers are unnecessary. The holes for the foundatior bolts are oblong at the expansion ends, thus leaving room for play.

103. To protect the masonry.

105. Foundation bolts are "swedged' and "fox-bolted," as shown page 445. Melted sulphur run into the holes when the bolts are in place, cools, expands, and hardens, and grips the bolt firmly.

106. For this it is necessary to run the foundation bolts through the pedestal plate in oblong holes, as well as through the bed-plate.

107. We have treated camber fully, page 462.

108. If these struts were not able to move the tower columns, the diagonal rods would slacken, owing to the movement of the girders resting on the top of the columns. For, as these expand or contract, the columns are pulled out of the vertical.

109. Sometimes the wind may cause tension in the windward columns.

114. No reliance will be placed upon the welding of steel.

115. No sharp or unfilleted angles or corners will be allowed in any piece of metal.

116. Riveted work of medium steel will be subject to the following conditions:

All sheared edges of plates and angles will be planed off to a depth of $\frac{1}{4}$ of an inch, and all punched holes will be reamed to a diameter $\frac{1}{8}$ of an inch larger so as to remove all the sheared surface of the metal; unless the material is such that any rivet-holes punched as in ordinary practice (§§ 49, 50, 51) will stand drifting to a diameter one-third greater than the original holes, without cracking either in the periphery of the holes or on the external edges of the piece, whether they be sheared or rolled.

Medium steel may be used in compression in chords, posts, flanges, and bearing plates without reaming for any thicknesses of metal which will stand the above drifting test. Medium steel may be used in tension without reaming up to a thickness of $\frac{9}{16}$ of an inch, if the metal of this thickness will stand the above drifting test and the adjacent edges of the pieces be rolled or planed off, as above required.

117. Soft steel need not be reamed, if it satisfies the above drifting test (116).

118. All parts of any tension or compression flange or member must be of the same kind of steel, but webs of plate girders and the tension members of all girders, plate or lattice, may be made of soft steel in connection with compression members of medium steel.

119. All splices must be of the same kind of steel as the parts to be joined.

120. Pilot nuts must be used during the erection to protect the threads of the pins.

QUALITY OF MATERIAL.

Steel.

121. The steel must be uniform in character for each specified kind. The finished bars, plates, and shapes must be free from cracks on the faces or corners, and have a clean, smooth finish. No work shall be put upon any steel at or near the blue temperature or between that of boiling water and of ignition of hard-wood sawdust.

122. All tests shall be made on samples cut from the finished material after rolling. The samples to be at least 12 inches long, and to have a uniform sectional area not less than $\frac{1}{2}$ square inch.

123. Material which is to be used without annealing or further treatment is to be tested in the condition in which it comes from the rolls. When material is to be annealed or otherwise treated before use, the specimen representing such material is to be similarly treated before testing.

The elongation shall be measured on an original length of 8 inches. Two test pieces shall be taken from each melt or blow of finished material, one for tension and one for bending. (Art. 137.)

124. All samples or full-sized pieces must show uniform fine-grained fractures of a blue steel-gray color, entirely free from fiery lustre or a blackish cast.

125. *Medium steel* shall have an ultimate strength, when tested in samples **Medium Steel.**

120. A pilot nut is a rounded cap screwed on the thread of the pin, so that when the pin is driven into place the thread is protected. The pilot is then removed, and the pin nut screwed on. The pilot nut has a hole in the end, so that by putting a rod through, it may be screwed and unscrewed.

of the dimensions above stated, of 60,000 to 68,000 pounds per square inch, an elastic limit of not less than one-half of the ultimate strength, and a minimum elongation of 22 per cent. in 8 inches. Steel for pins may have a minimum elongation of 15 per cent.

126. Before or after heating to a low cherry red and cooling in water at 82 degrees Fah., this steel must stand bending to a curve whose inner radius is one and a half times the thickness of the sample, without cracking.

Soft Steel

127. *Soft steel* shall have an ultimate strength, on same-sized samples, of 54,000 to 62,000 pounds per square inch, an elastic limit not less than one-half the ultimate strength, and a minimum elongation of 25 per cent. in 8 inches.

128. Before or after heating to a light yellow heat and quenching in cold water, this steel must stand bending 180 degrees, to a curve whose inner radius is equal to the thickness of the sample, without sign of fracture.

129. Rivet steel shall have an ultimate strength of 50,000 to 58,000 pounds per square inch and an elongation of 26 per cent.

130. The steel for rivets must, under the above bending test (128), stand closing solidly together without sign of fracture.

131. Eye-bar material, 1½ inches and less in thickness, shall, on test pieces cut from finished material, fill the above requirements. For thicknesses greater than 1½ inches, there will be allowed a reduction in the percentage of elongation of 1 per cent. for each ⅛ of an inch increase of thickness, to a minimum of 20 per cent. (Art. 112.)

132. Full sized eye-bars shall show not less than 10 per cent. elongation in the body of the bar, and an ultimate strength not less than 56,000 pounds per square inch. Should a bar break in the head, but develop 10 per cent. elongation and the ultimate strength specified, it shall not be cause for rejection, provided not more than one-third of the total number of bars tested break in the head.

133. A variation of cross-section or weight in the finished members of 2½ per cent. from the specified size may be cause for rejection.

Steel Castings.

134. Steel castings will be used for drawbridge wheels, track segments, and gearing. (Art. 1.)

They must be true to form and dimensions, of a workmanlike finish, and free from injurious blowholes and defects.

When tested in specimens of uniform sectional area of at least ½ square inch for a distance of 2 inches, they must show an ultimate strength of not less than 67,000 pounds per square inch, an elastic limit of one-half the ultimate, and an elongation in 2 inches of not less than 10 per cent.

The metal must be uniform in character, free from hard or soft spots, and capable of being properly tool finished.

Cast Iron.

Cast Iron.

135. Except where chilled iron is required, all castings must be of tough, gray iron, free from cold shuts or injurious blowholes, true to form and thickness, and of a workmanlike finish. Sample pieces, 1 inch square, cast from the same heat of metal in sand moulds, shall be capable of sustaining, on a clear span of 12 inches, a central load of 2,400 pounds, when tested in the rough bar. A blow from a hammer shall produce an indentation on a rectangular edge of the casting without flaking the metal

TIMBER.

Timber.

136. The timber, unless otherwise specified, shall be strictly first-class Southern yellow pine or white oak bridge timber, sawed true, and out of wind, full size, free from wind shakes, large or loose knots, decayed or sap wood, worm holes, or other defects, impairing its strength or durability. It will be subject to the inspection and acceptance of the Engineer.

INSPECTION.

Inspection.

137. All facilities for inspection of the materials and workmanship shall be furnished by the contractor. He shall furnish without charge such specimens (prepared) of the several kinds of iron or steel to be used, as may be required to determine their character.

138. The contractor must furnish the use of a testing machine capable of testing the above specimens at all mills where the iron or steel may be manufactured, free of cost.

139. Full-sized parts of the structure may be tested at the option of the Engineer of the Railroad company, but if tested to destruction, such material shall be paid for at cost, less its scrap value to the contractor, if it proves satisfactory. If it does not stand the specified tests, it will be considered rejected material, and be solely at the cost of the contractor.

PAINTING.

Painting.

140. All iron-work, before leaving the shop, shall be thoroughly cleansed from all loose scale and rust, and be given one good coating of pure raw linseed oil, well worked into all joints and open spaces.

141. In riveted work the surfaces coming in contact shall each be painted before being riveted together. Bottoms of bed-plates, bearing-plates, and any parts which are not accessible for painting after erection, shall have two coats of paint; the paint shall be a good quality of iron-ore paint, subject to approval of the Engineer.

142. After the structure is erected, the iron-work shall be thoroughly and evenly painted with two additional coats of paint, mixed with pure linseed oil, of such color as may be directed. All recesses which will retain water, or through which water can enter, must be filled with thick paint or some waterproof cement before receiving the final painting.

143. Pins, bored pin-holes, and turned friction rollers shall be coated with white lead and tallow before being shipped from the shop.

ERECTION.

Erection.

144. The contractor shall furnish all staging and false work, shall erect and adjust all the metal-work, and put in place all floor timbers, guards, etc., complete, ready for the rails.

145. The contractor shall so conduct all his operations as not to impede the operations of the road, interfere with the work of other contractors, or close any thoroughfare by land or water.

146. The contractor shall assume all risks of accidents to men or material prior to the acceptance of the finished structure by the Railroad Company.

The contractor must also remove all false work, piling, and other obstructions, or unsightly material produced by his operations.

FINAL TEST.

147. Before the final acceptance the Engineer may make a thorough test by passing over each structure the specified loads, or their equivalent, at a speed not exceeding 45 miles an hour, and bringing them to a stop at any

point by means of the air or other brakes, or by resting the maximum load upon the structure for 12 hours.

After such tests the structures must return to their original positions without showing any permanent change in any of their parts.

SUPPLEMENTARY.

The following special clauses shall apply, in addition to previous general clauses, to the special work included in the attached contract:

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Proposals for building and erecting complete, ready for the ____________ a bridge over ____________ near ____________, on the ____________ Division ____________ Railroad, in accordance with the attached specifications and accompanying profile, will be received up to ____________. The live load to be adopted for this bridge will be Class E ____________ paragraph 23.

LIST OF THE DIFFERENT MEMBERS IN A BRIDGE.—From a paper read by Prof. J. A. S. Waddell, before the "*Pi Eta*" Scientific Society, Rensselaer Polytechnic Inst., Troy, N. Y., we extract the following complete lists of the different members which go to make up a bridge.

I. HIGHWAY BRIDGE.—COMBINATION OF WOOD AND IRON.

WOOD.

Top Chords.
Batter Braces.
Vertical Posts.
End Tie Beams.
End Diagonals.
Floor Beams.
Flooring.
Batter-Brace Stiffeners.
Lateral Braces.
Joints.
Hand-rail Cap.
Hub Plank.
Hand-rail Post.
Felloe Plank.
Corbels.
False Caps.
Wall Plates.
Cover Boards for Chords and Batter Braces.
Lath for same.
Cross Diagonals in Deck Bridge.
Lower Lateral Struts in Deck Bridge.

WROUGHT IRON.

Main Portions.

Main Diagonals.
Counters.
Hip Verticals.
Upper Lateral Rods.
Lower Lateral Rods.
Bottom-Chord Bars.
End Lateral Struts.
Batter-Brace Ties.
Star Iron Side Braces.
Cross Diagonals in Deck Bridge.
Lower Lateral Struts in Deck Bridge.
*Floor Beams.
Beam Truss Rods.

DETAILS.

BOLTS.
- Chord Bolts.
- Batter-Brace Bolts.
- Post Bolts.
- Bracket Bolts.
- Hand-rail Post Bolts.
- Name-Plate Bolts.
- Bed-Plate Bolts.
- Expansion Pedestal Fastening to Bed Plate.
- Lower Lateral-Rod Bolts.
- Drift Bolts.
- Floor-Beam Packing Bolts.

Beam Hangers.
Beam-Hanger Plates.
Hip Vertical Plates on Castings.
Lacing on Hip Verticals.
Side-Brace Connections to Chord Pins.
Side-Brace Connections to Floor Beams.
Lateral-Rod Connections to Floor Beams.
Rollers and Roller Frames.
Jaws on End Struts.
Dowels for Upper Laterals.
Fillers for Pins.

SPECIAL WROUGHT-IRON DETAILS.

Hip-Joint Boxes.
Upper-Chord Panel Connections.
Lower Post Sockets.
Pedestals.
Bed Plates.

CORRUGATED OR GALVANIZED IRON.

Cover for Top Chords and Batter Braces.

CAST IRON.

Bed Plates.
Hip-Joint Boxes or Hoods.
Pedestals.
Upper Post Sockets.
Upper-Chord Panel Connection.
Lower Post Sockets.
Lateral Angle Blocks.
Name Plates.
Brackets.
Washer Plates for Main Diagonals and Counters.

WASHERS.
- Chord-Bolt Washers.
- Batter-Brace Bolt Washers.
- Post-Bolt Washers.
- Upper Lateral-Rod Washers.
- Lower Lateral-Rod Washers.
- Beam-Hanger Washers.
- Name-Plate Bolt Washers.
- Bracket-Bolt Washers.
- Hand-rail Post Bolt Washers.
- Bed-Plate Bolt Washers.
- Bevel Washers.
- Floor-Beam Bolt Washers.

PACKING WASHERS.
- Chord-Bolt Packing Washers.
- Lateral-Rod Packing Washers.
- Batter-Brace Bolt Packing Washers.
- Tie-Bar Packing Washers in Batter Braces.
- Post-Bolt Packing Washers.
- Bracket-Bolt Packing Washers.
- Floor-Beam Bolt Packing Washers.

* For details of built floor beams, see list of members in Iron Highway Bridge.

II. HIGHWAY BRIDGE.

WROUGHT IRON.

Main Portions.

CHANNEL BARS.
- Top Chords.
- Batter Braces.
- Posts.
- Lateral Struts.
- Portal Braces.

PLATE.
- Top Chords.
- Batter Braces.

BARS.
- Main Diagonals.
- Counters.
- Hip Verticals.
- Upper Lateral Rods.
- Lower Lateral Rods.
- Cross Diagonals on Batter Braces.
- Cross Diagonals on Posts.
- Lower Chord Bars.

T IRON. Lower Lateral Struts.

I BEAMS.
- Floor Beams.
- Intermediate Struts.
- Upper Lateral Struts.
- Lower Lateral Struts.
- Top Chords.
- Batter Braces.

STAR IRON.
- Side Bracing.
- Hip Verticals.

IRON HAND-RAILING.

FLOOR BEAMS.

BEAM TRUSS RODS.

DETAILS.

STAY PLATES.
- Top Chords.
- Ends of Posts.
- Middle of Posts.
- Ends of Lateral Struts.
- Batter Braces.
- Portal Braces.

FILLING PLATES.
- At Panel Points of Top Chord.
- Floor Beams.

COVER PLATES.
- Shoe.
- Hip Joint.
- Intermediate Panel Points Top Chord.

REINFORCING PLATES.
- Hip Inside.
- Hip Outside.
- Top Chord, Intermediate Panel Points Inside.
- Top Chord, Intermediate Panel Points Outside.
- Bottom Chord, Intermediate Panel Points Inside and Outside for Channel Bottom Chords.
- Shoe Inside.
- Shoe Outside.
- Lower Ends of Posts Inside.
- Lower Ends of Posts Outside.
- Middle of Posts Inside.
- Middle of Posts Outside.
- Floor Beam at holes for Beam Hangers.
- Floor Beam Lateral Connections.

CONNECTING PLATES.
- Batter Brace to Top Chord.
- Post to Top Chord.
- Lateral Struts to Top Chord.
- Intermediate Struts to Posts.
- Portal Braces to Batter Braces.

OTHER PLATES.
- Shoe Under Lateral Connection to Floor Beams.
- Roller Plates. Name Plates.
- Beam Hanger Plates. Top Plate in Floor Beam.

LACING OR LATTICING.
- Top Chord Upper.
- Top Chord Lower.
- Batter Brace Upper.
- Batter Brace Lower.
- Posts.
- Lateral Struts.
- Portal Braces.

TRUSSING.
- Posts.
- Lateral Struts.
- Portal Braces.

PINS.
- Bottom Chord.
- Top Chord.
- Middle of Posts.
- Upper Lateral Connection.
- Lower Lateral Connection.
- Cross Diagonal Connection.

BOLTS.
- Bracket Bolts.
- Name-Plate Bolts.
- Cross Diagonal Bolts in Batter Braces.
- Cross Diagonal Bolts in Posts.
- Bed-Plate Bolts.
- Expansion Pedestal Fastening to Bed Plate.
- Upper Lateral-Rod Connection to Chords.
- Lower Lateral-Rod Connection to Floor Beam.
- Hand-rail Post Bolts.
- Lateral Struts Connection to Chord.
- T-Iron Brace Bolts.

BRACKETS FOR PORTALS, INCLUDING ORNAMENTAL WORK.

T-IRON BRACES.
- Posts to Lateral Struts.
- Stiffeners in Built Floor Beams.

BEAM HANGERS.

TURN BUCKLES.

FILLERS FOR PINS.

ROLLER FRAMES.

EXPANSION ROLLERS.

SLEEVE NUTS.

JAWS
- Upper Lateral Struts.
- Intermediate Lateral Struts.
- Lower Lateral Struts.

PIECES OF CHANNELS.
- Upper Lateral Strut Connection.
- Lower Lateral Strut Connection.
- Batter-Brace Channel Connection to Shoe under Plates.

ANGLE IRON.
- Intermediate Struts to Posts.
- Upper Lateral Struts to Chord.
- Lower Lateral Struts to Pedestal.
- Lower Lateral Struts to Chord (Channel Lower Chords).
- Batter Braces to Shoe Under Plates.
- Side and End Angles for Roller Plates.
- Angles in Built Beams.

WASHERS FOR HAND-RAIL POST BOLTS.

RIVET HEADS.
- Top Plate to Chord and Batter-Brace Channels.
- Latticing or Lacing to Channels in Top Chords, Posts and Struts.
- Stay Plates to Channels.
- Reinforcing Plates to Channels.
- Cover Plates to Channels.
- Connecting Plates to Channels.
- Lateral Connection to Floor Beam.
- Trussing to Channels on Bars.
- Ornamental Work in Brackets.
- T-Iron Braces to Posts and Struts.
- Jaws to Lateral Struts.
- The Various Angle Irons to the Parts which they Connect.
- The Various Pieces of Channels to the Parts which they Connect.

DETAILS OF BUILT BEAMS
- Web.
- Top Plate.
- Upper Angles.
- Lower Angles.
- Stiffening Angles.
- T Stiffeners.
- Filling Plates.
- Lateral-Rod Connections.
- Reinforcing Plates at Beam Hanger Holes.
- Rivet Heads.

TIMBER.
- Joist.
- Flooring.
- Hand-rail Cap Pieces.
- Hand-rail Posts.
- Hub Plank.
- Felloe Plank.

III. WOODEN HOWE TRUSS RAILROAD BRIDGE.

WOOD.

Lower Chords.
Clamps and Keys in same.
Upper Chords and Keys for same.
Upper Lateral Braces.
Lower Lateral Braces.
Cross Diagonal Braces in Deck Bridge.
Batter Braces and Keys for same.
Main Braces.
Counter Braces.
Tie Beams at Ends of Top Chords.
Spreaders at Ends of Bottom Chord.
End Diagonals at Portals.
Track Stringers and Packing.
Batter-Brace Stiffeners.
Floor Beams.
Guard Rails.
Corbels.
Wall Plates.
Keys—Corbels to Wall Plates
Track Ties.

WROUGHT IRON.

Truss Rods.
Upper Lateral Rods.
Lower Lateral Rods.
Batter-Brace Ties.
Camp Bars.
Rods for Batter-Brace Stiffeners.

Dowels for Lateral Braces.
SPIKES. { Ties to Stringers.
Guard Rails to Ties.
Truss-Rod Plates at Top and Bottom.

BOLTS. { Upper Chord Bolts.
Batter-Brace Bolts.
Lower Chord Bolts.
Intersectional Bolts.
Track Stringer Bolts.
Floor Beams to Chords.
Track-Stringers to Floor Beams.
Guard Rails to Ties.
Corbels to Chords.
Brackets to Tie Beams and Batter Braces.
Name-Plate Bolts.
Anchor Bolts.
Drift Bolts.

CAST IRON.

Top-Chord Angle Blocks.
Bottom-Chord Angle Blocks.
End-Chord Angle Blocks.
Top-Chord Lateral Angle Blocks.
Bottom-Chord Lateral Angle Blocks.

WASHERS. { Upper Lateral-Rod Washers
Lower Lateral-Rod Washers.
Chord Bolt Washers.
Intersectional Bolt Washers.
Track-Stringer Bolt Washers.
Batter-Brace Bolt Washers.
Bracket-Bolt Washers.
Batter-Brace Stiffening Rod Washers.
Name-Plate Bolt Washers,
Corbel or Anchor-Bolt Washers.
Guard-Rail Bolt Washers.

Brackets.
Name Plates.
Clamp Heads.
Lower Chord Keys.

PACKING WASHERS. { Chord-Bolt Packing Washers.
Batter-Brace Bolt Packing Washers.
Lateral-Rod Packing Washers.
Track-Stringer Bolt Packing Washers.
Bracket-Bolt Packing Washers.
Tie-Bar Packing Washers in Batter Braces.

IV. COMBINATION PRATT TRUSS RAILROAD BRIDGE.

WOOD.

Top Chords.
Batter Braces.
Lateral Braces.
Vertical Posts.
End Tie Beams.
End Diagonals on Batter Braces.
Floor Beams.
Track Stringers.
Track Stringer Packers.
Ties.
Guard Rails.
Chord and Batter-Brace Covering.
Lath for Same.
Batter-Brace Stiffeners.
Corbels.
False Caps.
Cross Diagonals in Deck Bridge.
Lower Lateral Struts in Deck Bridge.

WROUGHT IRON.

Main Portions.

Main Diagonals.
Counters.
Hip Verticals.
Upper Lateral Rods.
Lower Lateral Rods.
Bottom Chord Bars.
Bottom Chord Channels for Stiffened End Panels.
End Lateral Struts.
Batter-Brace Ties.
Cross Diagonals in Deck Bridge.
Lower Lateral Struts in Deck Bridge
*Floor Beams.
Track Stringers.
Side Braces in Pony Trusses.
Batter-Brace Stiffening Rods.
End-Post Bracing Ties.
Beam Truss Rods.

DETAILS.

BOLTS.
- Chord Bolts.
- Batter-Brace Bolts.
- Post Bolts.
- Bracket Bolts.
- Name-Plate Bolts.
- Bed-Plate Bolts.
- Expansion Pedestal Fastening to Bed Plate.
- Lower Lateral-Rod Bolts.
- Stringer Packing Bolts.
- Joint Boxes to Top Chord.
- Guard Rail to Ties.
- Side Brace Bolts.
- Drift Bolts.
- Floor-Beam Packing Bolts,
- Track Stringers to Floor Beams.
- Corbels to Foundations.

Beam Hangers.
Beam-Hanger Plates.
Hip Vert. Plates on Castings.
Lacing on Hip Verts. in Pony Trusses.
Side-Brace Connection to Chord.
Side-Brace Connection to Floor Beams.
Lateral-Rod Connection to Floor Beams.
Pins.
Rollers and Roller Frames.
Jaws on End Struts.
Dowels for Upper Laterals.
Rods for Trussing Beams.
Boat Spikes.
Lacing or Latticing, Stay Plates, Reinforcing Plates and Rivets for Bottom Chord Channels.
Fillers for Pins.
Turn-buckles.
Sleeve-nuts.

SPECIAL WROUGHT-IRON DETAILS.

Hip-Joint Boxes.
Upper Chord Panel Connection.
Lower Post Sockets.
Pedestals.
Bed Plates.
Jaws for Lower Lateral Struts.

CORRUGATED OR GALVANIZED IRON.

Cover for Top Chords and Batter Braces.

CAST IRON.

Bed Plates.
Hip-Joint Boxes or Hoods.
Pedestals.
Upper Post Sockets.
Upper Chord Panel Connection.
Lower Post Connection.
Castings for Trussing Wooden Beams.
Lateral Angle Blocks.
Name Plates.
Brackets.

WASHERS.
- Chord-Bolt Washers.
- Batter-Brace Bolt Washers.
- Post-Bolt Washers.
- Upper Lateral-Rod Washers.
- Lower Lateral-Rod Washers.
- Beam-Hanger Washers.
- Name-Plate Bolt Washers,
- Bracket-Bolt Washers.
- Track-Stringer Bolt Washers.
- Bed-Plate Bolt Washers.
- Joint-Box Bolt Washers.
- Guard-Rail Bolt Washers.
- Side-Brace Bolt Washers.
- Batter-Brace Stiffening-Rod Washers.
- Floor-Beam Bolt Washers.

PACKING WASHERS.
- Chord-Bolt Packing Washers.
- Lateral-Rod Packing Washers.
- Batter-Brace Bolt Packing Washers.
- Tie-Bar Packing Washers in Batter Braces.
- Post-Bolt Packing Washers.
- Bracket-Bolt Packing Washers, in Batter Braces.
- Stringer-Bolt Packing Washers.
- Floor-Beam Bolt Packing Washers.

* For details of built floor beams, see list of members in Iron Highway Bridge.

V. WROUGHT-IRON RAILWAY BRIDGE.

MAIN PORTIONS.

CHANNEL BARS.
- Top Chords.
- Batter Braces.
- Posts.
- Lateral Struts.
- Portal Braces.
- Bottom Chords.
- Track-Stringer Bracing Struts.

I BEAMS.
- Floor Beams.
- Intermediate Struts.
- Upper Lateral Struts.
- Lower Lateral Struts.
- Top Chords.
- Batter Braces.
- Track-Stringer Bracing Struts.

PLATE.
- Top Chords.
- Batter Braces.

BARS.
- Main Diagonals.
- Counters.
- Hip Verticals.
- Upper Lateral Rods.
- Lower Lateral Rods.
- Portal Bracing Diagonals.
- Track-Stringer Bracing Diagonals.
- Vibration Rods.
- Lower Chord Bars.

T IRON.
- Lower Lateral Struts.
- Side Bracing.
- Hip Verts.
- Track-Stringer Bracing Struts.

FLOOR BEAMS. **TRACK STRINGERS.** **RAILS.**

DETAILS.

PLATES.
- **STAY PLATES.**
  - Top Chords.
  - Ends of Posts.
  - Middle of Posts.
  - Ends of Lateral Struts.
  - Batter Braces.
  - Portal Braces.
  - Stiffened Bottom Chords.
- **REINFORCING PLATES.**
  - Hip Inside.
  - Hip Outside.
  - Top Chord Intermediate Panel Points Inside.
  - Top Chord Intermediate Panel Points Outside.
  - Bottom Chord Intermediate Panel Points Inside and Outside for Channel Bottom Chords.
  - Shoe Inside.
  - Shoe Outside.
  - Lower Ends of Posts Inside.
  - Lower Ends of Posts Outside.
  - Middle of Posts Inside.
  - Middle of Posts Outside.
  - Floor Beam at Holes for Beam Hangers.
  - Floor Beam Lateral Connection.

PLATES.
- **FILLING PLATES.**
  - At Panel Points of Top Chord.
  - At Panel Points of Stiffened Bottom Chords.
  - Floor Beams.
- **COVER PLATES.**
  - Shoe.
  - Hip Joint.
  - Intermediate Panel Points Top Chords.
- **CONNECTING PLATES.**
  - Batter Brace to Top Chord.
  - Posts to Top Chord.
  - Lateral Struts to Top Chord.
  - Intermediate Struts to Top Chord.
  - Portal Braces to Batter Braces.
  - Track-Stringer Splice Plates on Web.
  - Track Stringer Splice Plates on Flanges
  - Iron Stringer Connection to Floor Beams.
  - Wooden Stringer Connection to Floor Beams.
  - Track-Stringer Bracing Connection to Stringers
- Pedestal Plates.
- Roller Plates.
- Beam Hanger Plates.
- Lateral Connection to Floor Beam.
- Name Plates.
- Top Plate in Floor Beam.
- Bottom Plate in Floor Beam.
- Top Plate in Track Stringer.
- Bottom Plate in Track Stringer.
- Bed Plates for Track Stringers.

LACING OR LATTICING.
- Top Chord Upper.
- Top Chord Lower.
- Bottom Chord Upper.
- Bottom Chord Lower.
- Batter Brace Upper.
- Batter Brace Lower.
- Posts.
- Lateral Struts.
- Portal Baces.
- Track-Stringer Bracing Struts.

TRUSSING | Verts in Pony Trusses.

PINS.
- Bottom Chord.
- Top Chord.
- Middle of Posts.
- Upper Lateral Connection.
- Lower Lateral Connection.
- Vibration Diagonal Connection.
- Track-Stringer Bracing Diagonal Connection.

BOLTS.
- Bracket Bolts.
- Name-Plate Bolts.
- Vibration Diagonal Bolts in Batter Braces.
- Vibration Diagonal Bolts in Posts.
- Bed-Plate Bolts.
- Expansion Pedestal Fastening to Bed Plates.
- Upper Lateral-Rod Connection to Chords.
- Lower Lateral.Rod Connection to Floor Beams
- Lateral Strut Connection to Chords.
- T-Iron Brace Bolts.
- Track-Stringer Bracing Connection.
- Rail Splice Bolts.
- Track-Stringer Packing Bolts.
- Guard Rails to Ties and Track Stringers
- Shim Bolts.

BRACKET CONNECTION FOR POSTS TO FLOOR BEAMS IN PONY TRUSSES.
BRACKETS ATTACHING IRON TRACK-STRINGERS TO BEAMS.
BRACKETS FOR PORTALS, INCLUDING ORNAMENTAL WORK.

T-IRON BRACES.
- Posts to Lateral Struts.
- Stiffeners in Built Floor Beams and Track Stringers.

BEAM HANGERS.
EXPANSION ROLLERS.
ROLLER FRAMES.

FILLERS FOR PINS.
SPLICE PLATES FOR RAILS.
SPIKES FOR TIES AND GUARD-RAIL FACING.

JAWS
- Upper Lateral Struts.
- Intermediate Lateral Struts.
- Lower Lateral Struts.
- Track-Stringer Bracing Struts.

PIECES OF CHANNELS.
- Upper Lateral Strut Connection.
- Lower Lateral Strut Connection.
- Batter-Brace Channel Connection to Pedestal Plates.

WASHERS FOR STRINGER BOLTS.

ANGLE IRON.
- Intermediate Struts to Posts.
- Upper Lateral Struts to Chords.
- Lower Lateral Struts to Pedestals.
- Lower Lateral Struts to Chords (Channel Lower Chords).
- Batter Braces to Pedestal Plates.
- Side and End Angles for Roller Plates.
- Angles in Built Beams and Track Stringers.
- Wooden Track.Stringer Side Fastening to Beams.
- Wooden Track-Stringer Supporting Angles.
- Iron Track.Stringer Supporting Angles.
- Facing on Guard Rails.

RIVET HEADS.
- Top Plate to Chord and Batter-Brace Channels.
- Latticing or Lacing to Channels in Chords, Posts and Struts.
- Intersection of Lattice.
- The Various Stay Plates to Channels.
- The Various Reinforcing Plates to Channels.
- Cover Plates to Channels.
- Connecting Plates to Channels, etc.
- Lateral Connection to Floor Beams.
- Trussing to Channels, Bars, or T iron.
- Ornamental Work in Brackets.
- T-iron Braces to Posts and Struts.
- Jaws to Lateral Struts.
- The Various Angle Irons to the parts which they connect.
- The Various Pieces of Channels to the parts which they connect.
- Brackets to Floor Beams, Track Stringers and Posts.
- Track-Stringer Splice Plates to Stringers.
- Iron Stringers to Floor Beams.
- Floor Beams to Posts.

DETAILS OF BUILT BEAMS.

- Web.
- Top Plate.
- Bottom Plate.
- Upper Flange Angles.
- Lower Flange Angles.
- Stiffening Angles.
- T Stiffeners.
- Filling Plates.
- Lateral-Rod Connections.
- Reinforcing Plates at Beam Hanger Holes.
- Rivet Heads.
- Stringer Supports.
- Stringer Side Connection.

DETAILS OF BUILT TRACK STRINGERS.

- Web.
- Top Plate.
- Bottom Plate.
- Upper Flange Angles.
- Lower Flange Angles.
- Stiffening Angles.
- T Stiffeners.
- Filling Plates.
- Connection for Bracing.
- Connection to Floor Beams.
- Rivet Heads.

LUMBER.

SHIMS FOR TRACK STRINGERS.
TRACK STRINGERS AND PACKING.
GUARD RAILS.
TIES.

LIST OF MEMBERS IN A DECK PLATE GIRDER BRIDGE.

Webs,
Top Plates,
Bottom Plates,
Upper Flange Angles,
Lower Flange Angles,
Vertical Stiffening Angles,
Inclined Stiffening Angles,
Filling Plates,
Bed Plates,
Web Splice Plates,
Anchor Bolts with Nuts,
Cross Frames at ends,
Intermediate Cross Frames,
Connecting Plates for same,
Rivets,
Tie Bolts,
Spikes for rails.
Guard Rail Angles,
Washers for Tie Bolts.

In plates 19, 20, and 21, will be found illustrations of most of the members included in the preceding lists, so that the student need be at no loss to understand precisely what the terms used signify.

CHAPTER X.

COMPLETE DESIGN FOR AN IRON RAILWAY BRIDGE.

In the first part of this work we have shown how to find the stresses, in the second part how to design the various members to resist these stresses. The student is now prepared to learn the art of designing.

We have given at the end of this work the working drawings for an actual bridge, as furnished by the bridge company that designed and erected it. We shall now give the figuring necessary to design this bridge on the basis of Cooper's specifications, except that we shall assume for our live load the system of our diagram, Part I, page 88. As the bridge was actually designed according to other specifications and live load, we shall not get precisely similar results. This is not our object. But by comparison it will be seen what differences in design we obtain. Then, by careful study of the working drawings given, the student should be able to make his own working drawings to suit the new results.

Finally, he can obtain, at slight expense, blue prints of working drawings from some of our leading bridge companies, and can check, by actual calculation, the design, according to the specifications and live load adopted. He can obtain such drawings in great variety, for plate girder spans, square and skew, as well as for swing spans, highway bridges, etc. It is therefore unnecessary to multiply illustrations here. Having brought the student to this point, his further progress must be left largely to himself.

We shall, therefore, only give in detail the calculations for this single example.

REQUIRED TO DESIGN A SINGLE TRACK THROUGH SPAN PRATT TRUSS BRIDGE, 153 FT. C. TO C. OF END PINS; 9 PANELS; DEPTH, 26 FT. C. TO C. OF PINS; WIDTH, 16 FT. 3 INCHES C. TO C.; STRINGERS, 7 FT. 6 INCHES C. TO C., RIVETED BETWEEN FLOOR BEAMS. FLOOR BEAMS RIVETED BETWEEN POSTS. COOPER'S SPECIFICATIONS AND LIVE LOAD ACCORDING TO OUR DIAGRAM, PART I, PAGE 88. TRACK, 400 LBS. PER FT.

We first proceed to design the floor system by itself, and commence with the stringers.

STRINGERS.—The end stringers which rest on the masonry are longer than the intermediate stringers. These latter are 17 ft. "o. a.," over all, and this length is also effective. The end stringers we shall take as 21 ft. o. a. and 18 ft. effective, from c. to c. of bearing.

By Cooper's specifications (§ 41),* we must take a depth of not *less* than $\frac{1}{10}$ of the span, or 20 inches. By our table, page 476, the least weight depth over all is 29 inches, and about $\frac{8}{10}$ of this gives for the effective depth 23 inches, for least cost. We shall take 22 inches effective and 24 inches o. a.

From our table, page 476, the weight for live load is 1,634 lbs. For our assumed live load we add 18 per cent., and have 1,928 lbs. This gives for weight per ft. $\frac{1928}{17} = 115$ lbs. nearly, for each stringer. The track is 400 lbs., or 200 lbs. per ft. for one stringer, and the dead load per ft. is, therefore, 315 lbs.

* We shall hereafter always refer to Cooper's specifications by giving the clause number in this manner.

Our live load concentrates 128,000 lbs. in 17 ft., which is 32,000 lbs. at end of each stringer. The end shear therefore is $32000 + 2677 = 34677$ lbs.

The maximum moment due to the dead load is $\frac{wl^2}{8} = \frac{315 \times 15^2}{8} = 11380$ ft. lbs. The maximum moment due to the live load is when the centre of gravity of the loading is as far on one side of centre as a driver is on the other side, and as much load as possible is on (page 245). We find for this position, second driver at $1\frac{1}{16}$ ft. on left of centre, and maximum moment 225,300 ft. lbs. Half of this for one stringer gives 112,650 ft. lbs. The maximum moment then is $112650 + 11380 = 124030$ ft. lbs.

This gives for the chord stress $\dfrac{124030}{\frac{22}{12}} = 67650$ lbs. (§ 42.) For the lower flange this calls for $\frac{67650}{7000} = 9.66$ square inches *net*, taking for σ the values of page 369.

The web must not be less than $\frac{34677}{4000} = 8.67$ square inches.

We shall take our web plate, $24'' \times \frac{3}{8} = 9$ square inches. This weighs 30 lbs. per foot. (NOTE.—*Area in square inches multiplied by* 10 *and divided by* 3 *gives weight per foot for iron. For steel add* 2 *per cent.*) For 17 feet long, we have weight of web plate 510 lbs.

We take, for the lower flange, two angles each $6'' \times 4''$, 18 lbs. per foot. This gives a thickness of about $\frac{9}{16}''$ (*Carnegie*). The area of each angle is then 5.4, or for both, 10.8 square inches *gross*. For $\frac{7}{8}''$ rivets we have rivet-hole $1''$. (§ 56.) Deduct two rivet-holes $2 \times 1 \times \frac{9}{16} = 1.13$ square inches, and we have 9.67 square inches net. (§ 56.)

We take the same top angles as bottom. The weight of top angles is $2 \times 18 \times 17 = 610$ lbs., and bottom the same.

We must have fillers at the ends, two at each end, or four in all, which fit in between the flange angles, so that the connecting angles which fasten the stringer to the floor beams can be riveted on. They must have same thickness as the flange angles, or about $\frac{9}{16}''$. Taking them $6''$, their area is about 3 square inches, or 10 lbs. per foot. The weight of four is 40 lbs.

We have four connecting angles, two at each end, each 2 feet long. Taking them $6'' \times 4''$, 12.5 lbs., they weigh 25 lbs. apiece, or 100 lbs.

The allowable shear (§ 43), since $H = 64$, is 5,074 lbs. per square inch. As the web at ends is safe for 4,000 lbs. unit strain, no intermediate stiffeners are required.

If we pitch the rivets at $3''$ throughout the top flange, and $6''$ for centre $8\frac{1}{2}$ feet of bottom flange, and $3''$ at ends, we have 140 rivets. Weight from *Carnegie*, 43.1 lbs. per 100. Hence, rivets weigh 60 lbs.

the intermediate, viz., 24″ over all, and 22″ effective. We take the weight a little larger than for the intermediate, say 120 lbs. per foot. The track makes the total dead-load 320 lbs. per foot.

Our live-load gives end shear 33,780 lbs., and dead-load 2,880 lbs., total end shear = 36,660 lbs.

The maximum moment for live-load is 249,555 ft. lbs., for dead-load 12,960 ft. lbs., total 137,760 ft. lbs.

The chord stress is then $\dfrac{137760}{\frac{22}{12}} = 75140$ lbs., and hence for the lower flanges, at 7,000 lbs. per square inch, we have 10.73 square inches, net, taking for σ the value of page 369. The area of web plate should not be less than $\dfrac{36660}{4000} = 9.16$ square inches. This is so close to 9 square inches that we take web plate as before, viz., $24'' \times \frac{3}{8}'' = 9$ square inches, 30 lbs. per foot, or 630 lbs. in all.

For the lower flange we take two angles $6'' \times 4''$, 20 lbs. per foot. This gives a thickness of about $\frac{5}{8}''$ (*Carnegie*). The area is then 12 square inches gross. Deduct for rivets $2 \times 1 \times \frac{5}{8} = 1.25$ square inch, and we have 10.75 square inches, net. (§ 56.)

Taking same top angles as bottom, we have weight of top angles $2 \times 20 \times 21 = 840$ lbs., and bottom the same.

At the cross-girder end we have two end fillers 1 foot long, $6'' \times \frac{5}{8}'' = 3.75$ square inches, or 12.5 lbs. per foot, weight 25 lbs. We have also two end connecting angles 2 feet long, $6'' \times 4''$, 12.5 lbs. per foot, weight 50 lbs. At the masonry end we take four end fillers 1 foot long, $3'' \times \frac{5}{8}'' = 1.87$ square inches, or 6.25 lbs. per foot, weight 25 lbs., and four end angles 2 feet long, $3\frac{1}{2}'' \times 3''$, $7\frac{3}{8}$ lbs. per foot, weight 60 lbs.

No intermediate stiffeners are necessary.

If we pitch the rivets as before, we have 180 $\frac{7}{8}''$ rivets, weight 43.1 lbs. per 100, or 80 lbs. (*Carnegie*).

In addition we have a foundation or wall plate, say $24'' \times 6\frac{1}{8}'' \times \frac{3}{4}''$, weight 30 lbs., and two foundation bolts 1″ diameter and 10″ long, weight 10 lbs.

We have, then, for end stringer,

1 web plate $24'' \times \frac{3}{8}''$, area 9 square inches	630 lbs.
2 top angles $6'' \times 4'' \times \frac{5}{8}''$, 20 lbs., 12 square inches gross	840 "
2 bottom angles $6'' \times 4'' \times \frac{5}{8}''$, 20 lbs., 10.75 square inches net	840 "
2 end fillers $6'' \times \frac{5}{8}''$	25 "
2 end angles $6'' \times 4''$, 12.5 lbs.	50 "
4 end fillers $3'' \times \frac{5}{8}''$	25 "
4 end angles $3\frac{1}{2}'' \times 3''$, $7\frac{3}{8}$ lbs.	60 "
180 $\frac{7}{8}''$ rivets	80 "
1 wall plate $24'' \times 6\frac{1}{8}'' \times \frac{3}{4}''$	30 "
2 foundation bolts 1″, 10″ long	10 "
	2,590

Weight per foot $\dfrac{2590}{21} = 123$ lbs., assumed 120 lbs.

There are four of these end stringers, and their weight is $2590 \times 4 = 10360$.

Finally we have, at the masonry ends, between end stringers, two sets of end cross-frames, as shown in Plate 27 at end of this work, at 140 lbs. per set, weight 280 lbs.

CROSS-GIRDERS.—The width of bridge c. to c. is 16′ 3″. Allowing for posts, we take

for the floor beams or cross-girders a length of 15′ 6″ o. a. and effective. From our Table page 478, we see that the depth is about 34″. But the stringers have been taken at 24″. In order that they may be riveted to the floor-beam webs without interference of the angles, we take the depth of floor beams at 36″ o. a., or 34″ effective. From Table, page 478, the weight is 1,725 for live load. For our assumed live load add 18 per cent., and we have for weight of a cross-girder 2,035 lbs. This gives for weight per ft. $\frac{2035}{15.5} = 130$ lbs., nearly.

The half weight of an intermediate stringer is 965 lbs., of an end stringer, 1,295. Hence, load concentrated on floor beam at points where stringers are attached, taking in the track, is $965 + 1295 + 200 \times 17 = 5660$ lbs. The concentration at each of these points due to the assumed live load is 46,680 lbs. Total, 52,340 lbs. The half weight is 1,018 lbs., and hence end shear is $52340 + 1018 = 53360$ lbs., nearly.

The stringers are attached 4 ft. from ends, hence the moment due to external loading is $52340 \times 4 = 209360$ ft. lbs., and due to own weight of girder, $\frac{130 \times 15.5^2}{8} = 3900$ ft. lbs., nearly. Total bending moment $= 209360 + 3900 = 213260$ ft. lbs.

The chord stress is then $\frac{213260}{\frac{34}{12}} = 75270$ lbs., and hence, for the area of lower flanges at 8,000 lbs. (§ 30), we have 9.4 sq. in. net, taking for σ the values of page 369. The area of web plate should not be less than $\frac{53360}{4000} = 13.34$ sq. in. We take web plate $36'' \times \frac{3}{8}''$, area 13.5 sq. ins., weight 45 lbs. per ft., or $15.5 \times 45 = 700$ lbs., nearly.

For the lower flange we take two angles, $6'' \times 4''$, $17\frac{3}{4}$ lbs. per ft. This gives a thickness of about $\frac{9}{16}''$ (*Carnegie*). The area is 10.6 sq. ins. gross. Deduct for rivets, $2 \times 1 \times \frac{9}{16} = 1.13$, and we have 9.47 sq. ins. net. (§ 56.)

We take the same top angles as bottom, 10.6 sq. ins. gross, or $35\frac{1}{3}$ lbs. per ft. Weight of top angles, $35\frac{1}{3} \times 15.5 = 550$ lbs., nearly, and bottom flanges the same.

At each end we have two end fillers, $6'' \times \frac{9}{16}''$, area, 3.375 sq. ins., and weight, 11.25 lbs. per ft. Each of these is 2 feet long, and weighs 22.5 lbs. Weight of the four, 90 lbs.

We also have four connecting angles, $6'' \times 4''$, 45 lbs. per ft., or weight $= 15 \times 3 \times 4 = 180$ lbs.

If we pitch the rivets 6″ for 7 feet in centre, and 3″ at ends, and allow for rivets in stringer connecting angles,* we have 130 rivets. Weight at 43.1 lbs. per 100 (*Carnegie*), about 60 lbs.

We have, then, for one cross-girder,

1 web plate, $36'' \times \frac{3}{8}''$, area 13.5 sq. ins.	700 lbs.
2 top angles, $6'' \times 4'' \times \frac{9}{16}''$, area 10.6 sq. ins., gross	550 "
2 bottom angles, $6'' \times 4'' \times \frac{9}{16}''$, area 9.47 sq. ins., net	550 "
4 end fillers, $6'' \times \frac{9}{16}''$	90 "
4 end angles, $6'' \times 4''$, 15 lbs.	180 "
130 $\frac{7}{8}''$ rivets	60 "
	2,130 lbs.

* Value of $\frac{7}{8}''$ rivet in double shear, 2,256 lbs. (Table I., page 436). Hence, $\frac{52340}{2256} = 24$ rivets, stringer to floor beam, $\frac{53360}{2256} = 24$ rivets, floor beam to post.

This gives for weight per ft., $\frac{2130}{15.5} = 138$ lbs., assumed 130 lbs. There are eight of these floor beams, and their weight is $2130 \times 8 = 17040$ lbs.

In Fig. 206, Plate 8, page 397, we have illustrated the connection of floor beam to post, and stringers to floor beam.

It will be seen that there are plates, or "diaphragms," between the post channels, just as though the web of the floor beam ran straight through the inside channel. These plates are fastened by angles on inside of post channels. We consider these diaphragms as continuation of the floor-beam web, and hence consider them with their angles as part of the floor system.

We take the diaphragms, each $8'' \times \frac{3}{8}'' \times 36''$, weight 30 lbs. Also four angles, $5'' \times 3'' \times 36''$, $8\frac{1}{2}$ lbs. per ft., or 100 lbs. And including rivets for floor beam to post, 80 $\frac{7}{8}''$ rivets, weight 40 lbs.

Hence diaphragm, angles, and rivets weigh 170 lbs. There are sixteen of these, or $170 \times 16 = 2720$ lbs.

We can now recapitulate the results for the floor.

FLOOR.

14 Intermediate stringers @ 1930 lbs.	27020 lbs.
4 End stringers @ 2590 lbs.	10360 "
2 Sets of end cross frames @ 140 lbs.	280 "
8 Floor beams @ 2130 lbs.	17040 "
16 Diaphragms @ 170 lbs.	2720 "
Total for floor	57420 lbs.

Weight per ft. of floor, $\frac{57420}{153} = 376$ lbs.

These results are entirely independent of length of span, and can be obtained for given width and panel length and live load, without reference to any special span. In the office, such designs are numerous, and in any special case a floor system can generally be found to suit, already estimated, so that this portion of the design need take but little time, especially if a close estimate of weight for a bid is all that is needed.

We now proceed to design the main trusses. We have the track 400 lbs. per ft., and the floor, as just found, about 380 lbs. per ft.

We must estimate the weight of trusses and laterals. This we can do as illustrated in the example, page 501, according to any of the methods there given. For the case in hand, we have there found the total weight of iron 1,400 lbs. per ft. We have just found the floor about 380 lbs. per ft., and if we subtract this from 1,400, we have 1,020 lbs. for trusses and laterals.

We have, then,

$$\text{Dead load,}\left\{\begin{array}{lll} \text{Track,} & 400 & \text{lbs. per ft.} \\ \text{Floor,} & 380 & \text{"} \quad \text{"} \\ \text{Trusses, etc.,} & 1020 & \text{"} \quad \text{"} \\ \hline & 1800 & \end{array}\right.$$

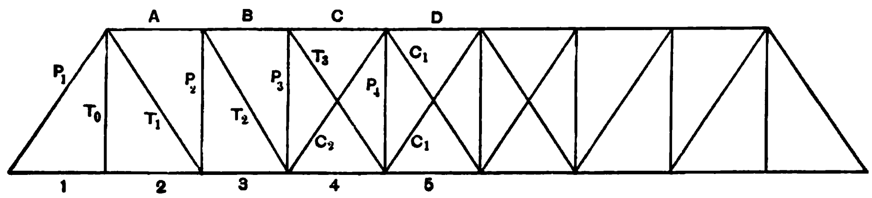
About half of the weight of trusses, etc., is taken as acting on the unloaded chord, or, in this case, the upper. The rest on the loaded, or lower, chord. We have thus 500 lbs. per ft. for upper chords, and 1,300 lbs. per ft. for lower chords. We must take one-half of these for one truss, or 4,250 lbs. upper apex dead load, and 11,050 lower apex dead load, per truss.

We can now find the stresses for dead load and for live load, by use of our diagram, as illustrated on page 88.

We give these stresses here, and the student should check them. A comparison with those given on Plate 22, at the end of this work, will show the differences caused by our live load and specifications, from that of the Bridge Company. The results of Plate 22 represent the practice of several years ago. We have changed the notation of Plate 22 to one which seems more convenient.

We allow, for estimating, 3 ft. additional length for chord bars and ties, in order to make the eye-bar heads. This makes length of chord bars, for estimate, 20 ft., and of ties, 34 ft. The length of posts over all is taken at 27 ft., of inclined end-posts, at 32.5 ft.

For the hip vertical T_0 we add 1.5 feet for length over all. The end upper panel, A, we take 17.5 feet long, the rest 17 feet.



	Stresses.	Unit Stresses (§ 30).	Area □″	Total area required.	Sizes.	Area □″	Total length.	Weight.
T_1	Live, 135250	8000	16.91	20.33	4 Bars 5″ × 1″	20	136 ft.	9070 lbs.
	Dead, 54840	16000	3.42					
T_2	Live, 100620	8000	12.58	14.86	2 Bars 5″ × $1\frac{1}{2}$″	15	136 ft.	6800 "
	Dead, 36560	16000	2.28					
T_3	Live, 70500	8000	8.81	9.95	2 Bars 5″ × 1″	10	136 ft.	4530 "
	Dead, 18280	16000	1.14					
C_1	Live, 45380	8000	5.67	5.67	2 Bars $1\frac{3}{4}$″ square	6.12	144 ft.	2940 "
	Dead, 0	16000	0					
C_2	Live, 8340	8000	1.04	1.04	1 Bar $1\frac{1}{8}$″ square	1.27	144 ft.	610 "
	Dead, 0	16000	0					
1	Live, 95760	8000	11.97	14.47	2 Bars 6″ × $1\frac{3}{16}$″	14.26	80 ft.	3800 "
	Dead, 40000	16000	2.50					
2	Same as 1							3800 "
3	Live, 163680	8000	20.46	24.84	2 Bars 6″ × $2\frac{1}{16}$″	24.75	80 ft.	6600 "
	Dead, 70040	16000	4.38					
4	Live, 207940	8000	26.0	31.63	4 Bars 6″ × $1\frac{5}{16}$″	31.52	80 ft.	8400 "
	Dead, 90040	16000	5.63					
5	Live, 233580	8000	29.20	35.45	4 Bars 6″ × $1\frac{1}{2}$″	36	40 ft.	4800 "
	Dead, 100040	16000	6.25					
T_0	Live, 46680	7500	6.22	6.96 net	Two 12″ channels, 20 lbs. per ft., 12 sq. inches gross. Deduct for rivets 4 × 1″ × $\frac{5}{16}$″ = 1.25 and 2 × 1″ × $\frac{3}{8}$″ = 0.75 . . 10 net.		110 ft.	4400 "
	Dead, 11050	15000	0.74					

In designing the built sections for posts and upper chords, we shall make use of "Osborn's Tables" (*Tables of Moments of Inertia*, etc., by Frank C. Osborn, Engineering

News Publishing Co., New York, 1889). These can be readily obtained by the student, and are, together with *Carnegie*, necessary in checking our results. Bridge companies have, of course, their own tables of built sections. We take built sections because they can be made, at present prices, cheaper than rolled.

Thus, for P_1 we have live load stress, 174,970 lbs.; dead load, 73,120 lbs., $l = 372$ inches. Taking $r = 6.2$,* we have (page 402) for the unit stresses allowable for live load, 4,600 lbs.; for dead, 9,200 lbs. Hence area $= 30 + 6.3 = 36.3$. From Osborn's *Tables*, page 61, we see that No. 106 very nearly fills the requirements. If we make the top plate $20'' \times \frac{1}{2}''$, the area will be 46.04 sq. ins. As the eccentricity is 1.25, this will add to the moment of inertia $1 \times (8 - 1.25)^2 = 45.56$ inch lbs. We have, then, $I = 1770.56$, and $r^2 = \dfrac{1770.56}{46.04} = 38.46$, or $r = 6.2$, which agrees with what we assumed.

In this way we get the following results:

	Stresses.		Unit Stresses (§ 33).	Area required.					
P_1	Live, 174970	$l = 372''$	4600	38.03	45.97	1 Cover Plate $20'' \times \frac{1}{2}''$,	10.0 sq. ins.	130 ft.	19950 lbs.
	Dead, 73120	$r =$ 6.2	9200	7.94		2 Webs $16'' \times \frac{11}{16}''$,	22		
						2 Angles $3'' \times 3''$, 9.6 lbs.	5.76		
						2 " $3'' \times 4''$, 13.8 lbs.	8.28		
							46.04		
A	Live, 163680	$l = 204$	7060	23.18	28.13	1 Cover Plate $20'' \times \frac{3}{8}''$,	7.5	70 ft.	6615 lbs.
	Dead, 70040	$r =$ 6.5	14120	4.95		2 Webs $16'' \times \frac{5}{16}''$,	10.0		
						2 Angles $3'' \times 3''$, 6.8 lbs.	4.0		
						2 " $3'' \times 4''$, 11.6 lbs.	6.9		
							28.4		
B	Live, 207940	$l = 204$	7060	29.45	35.8	1 Cover Plate $20'' \times \frac{1}{2}''$, area	10.0	68 ft.	8160 lbs.
	Dead, 90040	$r =$ 6.5	14120	6.35		2 Webs $16'' \times \frac{7}{16}$,	14.0		
						2 Angles $3'' \times 3''$, 6.4 lbs.	3.84		
						2 " $3'' \times 4''$, 13.6 lbs.	8.16		
							36		
C	Live, 233580	$l = 204$	7013	33.30	40.43	1 Cover Plate $20'' \times \frac{1}{2}''$, area	10.0	68 ft.	9290 lbs.
	Dead, 100040	$r =$ 6.2	14026	7.13		2 Webs $16'' \times \frac{5}{8}$,	20.0		
						2 Angles $3'' \times 3''$, 6.3 lbs.	3.78		
						2 " $3'' \times 4''$, 12 lbs.	7.2		
							40.98		
D	Same as for C							34 ft.	4645 lbs.
P_2	Live, 84200	$l = 312$	4056	20.76	25.06	Two 12″ channels, $41\frac{2}{3}$ lbs., 25 sq. ins.		108 ft.	9010 lbs.
	Dead, 34850	$r =$ 4.24	8112	4.30					
P_3	Live, 59000	$l = 312$	4144	14.24	16.60	Two 12″ channels, $27\frac{2}{3}$ lbs., 16.6 sq. ins.		108 ft.	5980 lbs.
	Dead, 19550	$r =$ 4.4	8288	2.36					
P_4	Live, 37980	$l = 312$	4200	9.04	9.54	Two 12″ channels, 20 lbs., 12 sq. ins.		110 ft.	4400 lbs.
	Dead, 4250	$r =$ 4.46	8400	0.5					

These are the lightest 12″ channels we can take.

Total weight of trusses .... 123800 lbs.

We take for the pins.

8 End Pins, $5\frac{1}{16}''$, 14 ft. .... 940 lbs.
28 Intermediate, $4\frac{9}{16}$, 44 feet .... 2400 "
Nuts for same .... 400 "

3740 lbs.

Total weight of trusses and pins .... 127540 lbs.

* An approximate rule for assuming r, is to take r, $\frac{4}{10}$ of the depth of web desired. In this case, for 16″ web, we have $r = 6.4$. With this to guide us we use the Table.

LATERALS AND DETAILS.—In the example, page 448, we have already calculated the stresses in the lower lateral ties for this case of 153 feet span. Taking the unit stress, 8,000 lbs. (§ 30), we have the following sizes, referring for notation to the figure, page 448. The areas and weights of rods for different diameters are given in *Carnegie*, The length of a panel diagonal is about 23 feet. But we shall attach the lateral rods at bottom by clevises, so that the length of each rod is only about 20 feet. As we have two rods in each panel, one for wind on one side and one for wind on the other side, we shall want 40 feet of rod in each panel, on *each side of centre*, and 40 feet in centre panel.

We have then:

	Strain.						
End panel (1),	44,280 lbs.,	2.95 sq. ins.,	1 rod	2″	diam.,	80′ long,	840 lbs.
" (2),	34,028 "	2.27 " "	1 "	$1\frac{3}{4}$″	"	80′ "	640 "
" (3),	24,600 "	1.64 " "	1 "	$1\frac{1}{2}$″	"	80′ "	470 "
" (4),	16,000 "	1.07 " "	1 "	$1\frac{1}{4}$″	"	80′ "	320 "
Centre panel (5),	8,200 "	0.55 " "	1 "	$1\frac{1}{8}$″	"	40′ long,	130 "
	* 36 clevises for these rods..................						500 "
	Pin plates....................................						700 "
	Bolts...						200 "
							3,800 lbs.

The clevises are attached by a bolt passing through a pin plate riveted to the bottom flange of the cross girder. There are, therefore, no bottom lateral struts except at the ends between end-posts.

We make these struts of two angles each, 6″ × 4″, $13\frac{1}{3}$ lbs. per foot, or 434 lbs. the pair, adding angle rests and rivets, 450 lbs. each strut, or, for both struts, 900 lbs.

For the top laterals we take all rods, $1\frac{1}{8}$″ diameter, 3.313 lbs. per foot, and 350 feet of rod gives 1,160 lbs. There are twenty-eight angle brackets for these rods, at 20 lbs. each, making 560 lbs., or, total, 1,720 lbs.

For the top intermediate struts we take two angles, 3″ × $2\frac{1}{2}$″, $4\frac{1}{3}$ lbs. per foot, and a plate 4″ × $\frac{3}{8}$″, as represented in Plate 11, Fig. 221, page 400. Weight of plate 5 lbs. per foot. The struts are 16 feet c. to c. of flange angles; weight of angles and plate, 218 lbs. Taking 64 rivets at 43 lbs. per 100, we have 27 lbs. for rivets. Total weight of strut about 250 lbs. There are six of these struts, and weight = 250 × 6 = 1,500 lbs.

For the portal struts, we take four angles $3\frac{1}{2}$″ × 3″, $7\frac{2}{3}$ lbs. per foot, latticed; weight, including lattice bars and rivets, 600 lbs. Two of these make 1,200 lbs.

We have twelve knee braces, each 2 angles, 3″ × $2\frac{1}{2}$″, $4\frac{1}{3}$ lbs. per foot, each weighing 75 lbs., or 900 lbs. for all.

Also, four portal knee braces, consisting of 2 angles, $3\frac{1}{2}$″ × 3″, $7\frac{2}{3}$ lbs. per foot, at 150 lbs. apiece, or 600 lbs. for all.

Total for laterals:

Lower lateral ties..................................	3,8co lbs.
2 lower end struts @ 450............................	900 "
Top lateral ties with brackets......................	1,720 "
6 top intermediate struts @ 250.....................	1,500 "
2 portal struts @ 600...............................	1,200 "
12 intermediate knee braces @ 75....................	900 "
4 portal knee braces @ 150..........................	600 "
Total weight of laterals............................	10,620 "

* For weight of clevises see page 425.

DETAILS.—*Of top chord.*

2400 $\frac{7}{8}''$ Rivets	1,070 lbs.
12 Intermediate web splices, $9'' \times \frac{3}{8}'' \times 12''$	130 "
6 " cover splices, $20'' \times \frac{3}{8}'' \times 21''$	270 "
14 Bottom splice and battens, $24'' \times \frac{5}{16}'' \times 24''$	700 "
* $3'' \times \frac{3}{8}''$ Lattice	480 "
Pin plates at hip	150 "
2 Hood plates, $20'' \times \frac{7}{16}'' \times 21''$	100 "
	2,870 × 2....5,740 lbs.

DETAILS.—*Of inclined end-posts.*

560 $\frac{7}{8}''$ Rivets	250 lbs.
2 Battens, $24'' \times \frac{5}{16}'' \times 24''$	100 "
$3'' \times \frac{3}{8}''$ Lattice	160 "
Pin plates	300 "
	810 × 4.....3,240 lbs.

DETAILS.—*Of intermediate vertical posts and suspenders.*

120 $\frac{7}{8}''$ Rivets	60 lbs.
4 Battens, $14'' \times \frac{5}{16}'' \times 15''$	70 "
$2\frac{1}{4}'' \times \frac{3}{8}''$ Lattice	300 "
Jaw plates	200 "
	630 × 16..10,080 lbs.
Total for laterals and details	29,680 lbs.

Subtract from this 900 for lower end struts, and we have 28,780 lbs. We have then already found,

Floor	57,420 lbs.
Laterals and details	28,780 "
Trusses and pins	127,540 "

$$213{,}740 \text{ lbs., or, } \frac{213740}{153} = 1397 \text{ lbs. per foot.}$$

We assumed for our calculation 1,400 lbs. per foot.

MASONRY MEMBERS.—For the pedestals, we have, from Table I., page 427, for the lineal bearing on pin $5\frac{1}{16}''$, 0.031 inches per ton. The end shear is 146,420 live, 61,200 dead, total, 207,620 lbs., and hence lineal bearing is $\frac{207620}{2000} \times .03 = 3.1$ inches. At 250 lbs. per square inch, we require $\frac{207620}{250} = 830$ square inches of wall plate.

We take, for the pedestal,

2 $12'' \times \frac{3}{8}''$ Webs, 32″ long	150 lbs.	
2 $6'' \times 6''$ angles, 29 lbs., 32″ long	150 "	
2 $6'' \times \frac{3}{4}''$ fillers, 28″ long	60 "	
For fixed pedestal, 1 base plate, $30'' \times \frac{7}{8}'' \times 32''$	240 "	
For roller pedestal, 1 base plate, $30'' \times \frac{3}{4}'' \times 32''$		200
	600	560

* For weight of lattice see page 406.

We have then,

2 Fixed pedestals @ 600	1,200 lbs.
2 Roller " " 560	1,120 "
2 Sets of rollers " 520	1.040 "
2 Roller wall plates, 30″ × $\frac{7}{8}$″ × 32″	520 "
8 Foundation bolts, $1\frac{1}{4}$″, 18″	70 "
	3,950 lbs.

Our total weight is then as follows:

Masonry members	3,950 lbs.
Laterals and details and end struts	29,680 "
Floor	57,420 "
Trusses and pins	127,540 "
Total net weight of span	218,590
Add 3%	6,560
Gross weight of span	225,150 lbs.

The excess of this weight over that given for same span at end of this work is due to the very heavy live load, and to the proportions, as well as to the specifications adopted.

As we have seen, page 501, by taking 5 or 6 panels instead of 9, and a depth of about 32 feet instead of 26 feet, we could reduce the weight to 1,300 lbs. instead of 1,400 lbs. per foot. This shows the use of our formula for weight, page 453.

The allowance of 3 per cent. is to cover waste, corners of plates clipped off, holes punched out, etc.

ESTIMATE OF COST.—We can now estimate the cost of the bridge, somewhat after the following manner:

Iron, say	2.1¢ per lb.
Labor	1.1¢ "
Freight—for a haul of 100 miles	0.1¢ "
Engineering	0.3¢ "
Profit	0.4¢ "
Erection (varies according to local circumstances)	1.0¢ "
	5 cts. per lb.

For a plate girder span labor would be less, say 0.7 cent per lb. Erection varies more widely than any of the other items. Local freight rates can always be ascertained. The cost of the iron, " f. o. b.," that is, " free on board," or loaded on cars ready for shipment, would be, in the above case, 3.9 cents per lb., after deducting freight and erection.

The total cost of our span would now be $225150 \times .05 = \$11257.50$, and on this basis a bid can be made, offering to deliver and erect the bridge for so much, the masonry, of course, to be supplied by other parties. Accompanying this, a stress diagram is furnished, which consists of a skeleton outline of the truss, with the live load, dead load, and all other data on it, and also all the sections. In short, all the results we have just figured out, similar to Plate 22, at the end of this work.

THE MEMORANDUM. CAMBERED LENGTHS, AND SKETCHES OF DETAILS.—Before the working drawings can be made, and the work put into the shop, the actual length of

the various members must be carefully figured as detailed in the Example, page 462. After these lengths are found, the engineer must carefully sketch the details at each joint, and get the data so arranged that the draughtsmen can commence on the shop drawings.

All these data and results should be noted by the engineer, and constitute the "MEMORANDUM."

In our case, taking $E = 26000000$ lbs., we have, page 461, for the length of lower chord bars, taking panel 5,

$$e = \frac{100040}{26000000\left[\frac{100040}{16000} + \frac{233580}{8000}\right]} = 0.000108, \quad \text{and}$$

length of lower chord bars c. to c. $= 204'' - 0.022 - 0.025 = 16$ feet $11\frac{15}{16}$ inches.

For the other panels we would get the same result, but as no difference is ever made in the lengths of chords, or posts, we take, in applying our method, the heaviest member of each kind, and find the cambered length for it, and make the others the same.

Thus, for the posts, we have, taking P_2,

$$e = \frac{34850}{26000000\left[\frac{34850}{8112} + \frac{84200}{4056}\right]} = 0.000053, \quad \text{and}$$

length of post c. to c. $= 312'' + 0.016 + .025 = 36$ feet $0\frac{13}{32}$ inch.

For the upper chord panels we have, taking D,

$$u' = 8307, \qquad u = 9411, \qquad i = 0.000908, \qquad e = 0.000096.$$

We have, then, for A,

length of $A = 210'' + 0.19 + 0.02 = 17$ feet $6\frac{7}{32}$ inches.

For the other panels,

length $= 204'' + 0.19 + 0.02 = 17$ feet $0\frac{7}{32}$ inch.

For the inclined ties we have, for T_1,

$$i = 0.000908, \qquad e = 0.000103, \qquad p + \frac{ip}{2} = 204.0926, \qquad l = 372.82,$$

length of ties c. to c. $= 372.82'' - 0.028 - 0.025 = 31$ feet $0\frac{3}{4}$ inch.

Sketches of the details for top and bottom chord packing at every joint, giving the exact distances, clearances, thickness of pin plates, width of jaws, arrangement of top chord splices, etc., should now be made. Also list of all the eye bars, with data for ordering the same. The pins can now be refigured exactly, to see that they are not overstrained (page 424). This completes the memorandum.

CHAPTER XI.

SHOP DRAWINGS.

By MORGAN WALCOTT, C. E.

TO make a shop drawing well requires some little skill and practice. The constant aim should be to make everything clear and plain for the men in the shops. All necessary dimensions should be plainly marked on the drawings in shop units, that is, in feet, inches, and halves, quarters, eighths, sixteenths, and thirty-seconds of an inch; this latter being the smallest measurement used in bridge engineering. Unnecessary dimensions should be avoided. End views, or sections, should be placed at the ends which they represent. For the sake of clearness, any brackets or other details on one end of a piece, which would show in a true mechanical drawing or projection of the other end, are nevertheless not shown in this projection; but a special view of their end is made, on which they are shown.

With beginners, the drawings should first be made with pencil on paper, as there will probably be alterations which can more readily be made on paper than on tracing linen. Experienced draughtsmen, however, generally make simple drawings directly on the tracing linen. In order to "take" the ink, the surface of the tracing linen must be perfectly clean. To secure this, rub the surface thoroughly with a clean towel, and if this does not answer, rub a very little powdered chalk on it. If it becomes necessary to erase, and afterwards to draw over the spot, the ink will probably blot, unless the spot has been rubbed with soapstone. When the work to be erased is of any magnitude, nothing but a prepared rubber ink eraser should be used. Small points or short lines can often be picked out with the sharp point of a penknife or ink scratcher.

It is usual to use the dull or unglazed side of the tracing linen. The advantage of using the smooth or glazed side, is that ink lines are more easily erased than on the dull side. The advantages of the dull side are: (1) If it is desired to make pencil sketches on the finished drawings, the pencil marks will show better on this side. (2) If the ink lines are on the dull side of the cloth, the drawings will lie flat, while, if they are on the glazed, the drawings will curl, or roll up. The reason of this is, that the preparation on the glazed side, and the ink lines, both tend to shrink the sides that they are on, and thus make the drawing roll up. If the glazing and the ink lines are on opposite sides of the cloth, their tendencies to roll the cloth up neutralize each other.

All drawings should be made in black ink; red ink is rarely used even for dimensions. Black ink is preferred because it takes better blue prints than any other color. Outlines are made heavy, and the dimension lines fine.

A good scale for the shop drawings is one inch to the foot; sometimes a scale of three-quarters of an inch to the foot, and sometimes a scale of an inch and a half to the foot, may be used advantageously. The drawings should be on sheets of tracing linen usually about 3 feet long by 20 inches wide. Frequently long posts and other sections can be shortened up by omitting the central portions, and indicating the length by some such

device as "10 Panels @ $2'-0''$ each $= 20'-0''$." If there are any brackets or pin-holes in the centre portion of the piece, it may be impossible to indicate the length in this manner. Or, it may be possible by making two breaks in the piece instead of one. It is well to make the drawings to scale, as this serves as a check in designing. The exact scale, however, is not of such importance as it might seem at first sight, as every needed dimension should be clearly marked on the drawing, and the men in the shops are not allowed to scale distances. If any dimension is lacking, it must be supplied by the draughtsman who made the drawing. Some of the general data which should go on every shop drawing are: sizes of rivets, sizes of open holes, number of pieces wanted and their mark, title, scale, date, and initials of draughtsman.

The rivets on one drawing are quite apt to be all of the same size, so that a general remark, such as "All rivets $\frac{7}{8}''$O," will often be all that is needed. In like manner the sizes of the open holes can generally be covered by some such remark as "All open holes $\frac{15}{16}''$O, unless marked otherwise." If there are any pin-holes or bolt-holes of a different size, their size is then specially marked near them on the drawing, with an arrow running to them.

In giving the number of pieces wanted, and their marks, they can be given thus: "2 pcs. wanted, mark P_1R.'

The only title necessary is something of the following nature:

INCLINED END-POSTS

FOR

1—153′—0″ S. Tr. Thro' Span,

FOR

SHEFFIELD SCIENTIFIC SCHOOL.

It is a waste of time to print titles for such work. They should be legibly written in a large, plain hand. Script writing should be avoided, however. Each letter should be distinct, and separate from the others. The scale, date, and initials of the draughtsman should be written in small letters in the extreme lower right-hand corner of the drawing. It is often customary, after the word "scale," to put a dash, and omit giving the scale on the drawing. Writing the word "scale" shows that the draughtsman has not forgotten it, while the dash after it warns any one not to take distances from the drawing by scale. When the two halves of a member are alike, it is only necessary to show one-half in full, and, at most, the general outlines of the other half, placing on the drawing some such note as "This half exactly like other half." Or, if the two halves differ slightly, the note would be something like this: "All dimensions on this half, not marked otherwise, same as for other half."

Wherever it is possible to make two pieces alike, or only differing in right and left, it should always be done, as then the punching of the two pieces is alike, and a complete set of templets is saved. Having the pieces alike may also facilitate erection. In drawing lattice bars, it is only necessary to draw their centre lines, except for one or two at the ends, which should be drawn in full. If there is reason to fear rough handling of the iron in transit, it may be necessary to ship pieces loose, which could otherwise be shipped fast, but the more loose pieces the more field riveting, and field riveting is expensive, not so good as shop riveting, and delays erection.

Rivets are denoted either by a cross or by a circle of the same size as the head. The latter method is about as quick and easy as the first, and shows more clearly what it is intended to represent.

Open holes through which rivets are to go in the field, are denoted by a blackened hole of the same size as the rivet.

A countersunk rivet is one which has either one or both of its heads flush with the plate. A flat-head rivet has either one or both of its heads flat, generally $\frac{3}{8}''$ high. Countersunk rivets are used only when it is necessary to get sufficient clearance, or in the bottom of a plate which rests on masonry, or another plate. If it is possible to substitute a $\frac{3}{8}''$ flat-head for a countersunk rivet it should always be done.

Pin-holes are too large to blacken, and should be hatched, to indicate that they are open.

The following Table gives Osborn's notation for rivets. This notation has now been very generally adopted by all the large bridge companies:

Two full heads.

Countersunk inside.

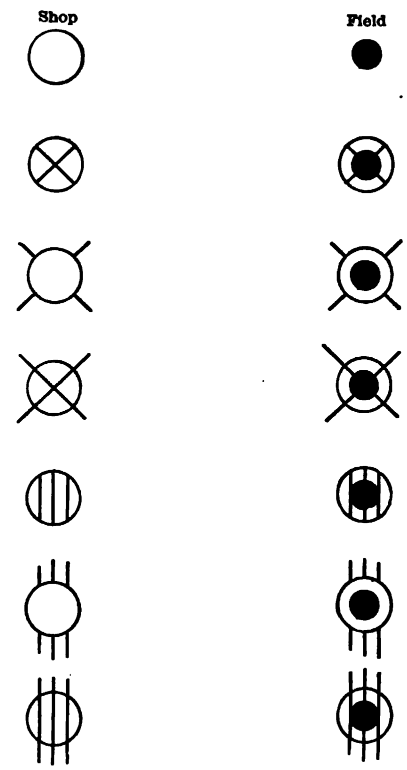
Countersunk outside.

Countersunk both sides.

$\frac{3}{8}''$ Flat-head inside.

$\frac{3}{8}''$ Flat-head outside.

$\frac{3}{8}''$ Flat-head both sides.



The foundation of the system is the diagonal cross to represent a countersink, the blackened circle for a field rivet, and the vertical stroke to represent a flattened head. The position of the cross with respect to the circle (inside, outside, or both sides) indicates the location of the countersink, and the number and position of the vertical strokes indicates the height and position of the flattened head. Any combination of field, countersunk, and flat-head rivets, liable to occur, may be readily indicated by the proper combination of these signs.

A point which comes up in the notation for rivets is, "Which side of the piece is inside and which outside?" About as good a way as any other is to let the outside be the near side, or side shown in the view in question; and to let the inside be the far side, or the side not shown in the view in question.

After laying out a complete system of rivets for any member, the draughtsman may check his addition by seeing that the sum of the rivet spaces and end distances are equal to the length of the member.

Allowing the rivets in the webs of girders, posts, chords, etc., to come opposite the rivets in the flanges should be carefully avoided. First, because it may necessitate hand driving the rivets; and, secondly, because if the member is in tension it will take out more section in a given line than if the rivets were staggered. When there are more than two consecutive rivet spacings alike, instead of giving them separately they should be given thus: "9 spaces @ 3″ each = 2′ 3″." This also applies to panels of lattice bars, which may be given thus: "11 panels @ 17″ = 15′ 7″."

Instead of giving the exact sizes of the pin-holes, it is preferable to give the sizes of the pins which are to go through them, thus: "Bored for $4\frac{13}{16}$″○ turned pin."

When angles are turned off, they should be given in feet and inches, not in degrees. This is done by giving the slope; that is, so many feet horizontal to so many vertical. Thus, a 53° angle may be given by a distance of 1′ $11\frac{1}{2}$″ horizontal to 2′ $7\frac{3}{16}$″ vertical. The templet makers can then lay the angle off directly from measurements. In some cases it is permissible to give the angle 45° in degrees. Thus, when there is a projecting corner, it may be ordered "clipped at 45°." But in all other cases angles should be given by their slopes in feet and inches.

In giving the sizes of pin-plates, battens, and other small plates, it is better to give these sizes in the nearest clear space to the plate, and draw an arrow to the plate, rather than to put the size directly on the plate, if in so doing it is necessary to crowd it in with rivet spacing and other data. The size of a plate should always be given thus, "10″ × $\frac{7}{16}$″ pl., $14\frac{1}{2}$″ lg.," the first being the width of the plate, that is, the direction at right angles to the fibres. A plate may thus have a greater width than length, as "24″ × $\frac{3}{8}$″ pl., 22″ lg." When lattice bars are in a position where they will be seen, they should have rounded ends. In giving their lengths, give them both from centre to centre of rivet holes and over all, thus, "All lattice 3″ × $\frac{3}{8}$″, $18\frac{19}{32}$″ c. to c., $22\frac{3}{32}$″ o. a." If the lattice is in a position where it will not be seen, the ends may be cut bevel.

In splicing the top chord the splice plates should be so arranged that all the field-priven rivets do not come in the same member.

For the sake of appearance, projecting corners of gussets and brackets should be clipped off.

When cover plates or stiff lateral bracing is used on plate-girder spans, care must be taken that the rivets in the flange do not come opposite those in the web, and also that the flange rivets do not come opposite the leg of a stiffener, otherwise the stiffener must be clipped or the rivet countersunk. To countersink one or two holes in a long plate requires a special handling of that plate, and is expensive. To clip the leg of the stiffener would be cheaper, but looks badly.

The number of views necessary to show a piece depends upon the piece and the amount of detail there is on it.

Generally a top view, an elevation, a sectional plan, and a couple of end views or sections will be all that is necessary.

The reason that a sectional plan is preferred to a bottom view is that, in the sectional plan, details are shown in the same relative position as in the top view. When the member has a system of lattice on both sides, in the top view the top lattice should be shown with full, and the bottom lattice with broken, lines. The bottom lattice should be shown in the top view, as the relative position of the two systems of lattice is thus clearly shown.

The following is a list of the shop drawings which would probably be required for an ordinary 150 foot pin-connected through span:

1. Inclined end-posts.
2. End chord sections.

3. Intermediate chord sections.
4. Intermediate posts.
5. Vertical suspenders, if other than forged eye-bars.
6. Portal bracing.
7. Intermediate upper bracing.
8. End lower struts.
9. Intermediate stringers.
10. End stringers.
11. Floor beams.
12. Pedestals.
13. Roller frames and rollers.
14. Wall plates.
15. Castings, such as filling rings.
16. Pilots for pins.

The last four items are often only sketched on the lists ordering the iron, instead of making a regular drawing.

The eye-bars, counters, lateral rods, pins, and blacksmith work are also in general only sketched on the order lists. For longer bridges, the number of drawings will increase, as several sheets will be necessary for the chords and posts. If the bridge is on a skew, the number of shop drawings needed will be nearly doubled.

To sum up: Make all drawings clear and distinct. Do not crowd any of the dimensions or data so they cannot be read. Do not waste time on fancy lettering. Make figures, especially, plain; words may be guessed at, but not figures. Aim to simplify the work of the men in the shops.

NOTE.—The drawings at the end of this work can now be carefully inspected, referring to the data of the strain sheet. Then, with the data already made out in the preceding chapter, the student will be prepared to make his own working drawings for the span figured out. By application to our large bridge companies, he can also undoubtedly obtain blue print copies of working drawings in great variety. A careful study of these will do much for the student.

CHAPTER XII.

THE ORDER BOOK, SHIPPING, AND INSPECTING.

AFTER the shop drawings have been made, or while they are in course of preparation, all the iron needed must be ordered of proper dimensions for the shop. The orders are grouped according to some convenient system on sheets properly ruled and headed, and these sheets when bound together constitute the "Order Book." The Order Book thus contains a list of every piece of iron which goes into the bridge. The forms for the Order Book are various. Without professing to give the actual practice of any company, we shall give in this chapter a series of forms which will illustrate sufficiently well how such a book may be made, and the information it should contain.

The different forms may be classified as follows:

Form A. Castings.
Form B. Built members.
Form C. Eye-bars and upset rods.
Form D. Pins, pin-nuts, and pilots.
Form E. Bolts and small forgings.
Form F. List of shop rivets.
Form G. List of field rivets.

As an example of Form A, we give the following:

FORM A.

CASTINGS.

FOR 117′ 6″ D. TR. TRO. SKEW SPAN, *for*..........______________.

	NO. OF PCS.	MARK.	DESCRIPTION.	FACINGS.	PIN-HOLE BORED FOR.	DIA. OF TENON TURNED.	PATTERN NO.	DRAWING NO.	SHIPPER'S NO. OF PCS.	GEN'L MARK.
1	2	C_1 R	Check washers.	Rough.	Core	$2\frac{13}{16}''$	New.	5000	2	C_1 R
2	8	L P_1	$7\frac{1}{2}'' \times 2\frac{1}{4}''$ collars.	Rough.	Core	6″	New.	5000	8	L P_1
3	1	W D_5	Wall plates, $13\frac{3}{8}''$ between lugs.	Rough.	$15_8'' \times 1\frac{25}{32}''$	× 18″	New.	5010	1	W D_5
4	2	28 N	Bed plates, 29″ between lugs.	1	$37'' \times 1\frac{7}{8}''$	× 30″	New.	5011	2	28 N
5										
6										
7										
8										
9										
10										

The span is 117′ 6″, double track, through skew.

In the first column is the number of the item. The sheet may be of any length, to accommodate any desired number of items, as for instance 30, on a page. In the second column the number of pieces wanted is given, and in the third the mark which is to be put on each. In the fourth is a description, and when necessary sketches may be made in it. In the fifth is given the number of facings. In the sixth the size of pin-hole, if the hole is bored. In the seventh the diameter of any turned tenons which the casting may

have. In the eighth the pattern number, and in the ninth the drawing number, which enables the working drawing for the piece to be found.

In the tenth column is given the shipper's number of pieces, which will generally be the same as the number of pieces in the second column; but if two pieces cast separately are bolted together for shipment, the shipper's number would be different. In the last column the general mark of the piece is given, which may differ from that in the third column for the same reason.

We have filled in a few items for illustration merely. The first item is two check washers which are not faced smooth, and are therefore marked "rough." As they have no bored pin-hole nor a tenon, it is simply noted that the core is $2\frac{1}{8}''$ diameter. The patterns being new, shows that there are no old patterns of the size required.

The next item is 8 collars, $7\frac{1}{2}''$ outside diameter and $2\frac{1}{4}''$ thick, with a core 6″ diameter, no facings, bored pin-holes, or tenons.

The next item is a wall plate $13\frac{3}{8}''$ between the "lugs" or projections for confining the rollers, faced on one side.

All the castings required are thus entered, one by one, and the iron required can be furnished, put into the shop, and finished according to the drawing for each piece.

FORM B.

BUILT MEMBERS.

FOR 117′ 6″ D. TR. TRO. SKEW SPAN, *for*........

	NO. OF PCS.	MARK.	SHAPE AND SIZE.	TOTAL LENGTH.	HOW CUT.	MAY VARY.	BEVEL.	TOTAL NO. AND DESCRIPTION OF FINISHED PIECES.	DRAWING NO.	NO. OF PCS.	GEN'L MARK.
			″ ″	′ ″		″					
1	8		12 × $\frac{5}{16}$ Pls. Web.	26 0	Sq.	± $\frac{1}{4}$					
2	8		$11\frac{1}{4}$ × $\frac{5}{16}$ Pls. Pin.	23	Sq.						
3	8		$11\frac{1}{4}$ × $\frac{5}{16}$ Pls. Pin.	2 1	Sq.					2	P_2 R
4	8		6 × $\frac{5}{16}$ Fillers.	23	Sq.			Riveted up into 4 Int. Posts, 26′ $0\frac{1}{4}''$ lg. o. a., 25′ $0\frac{1}{32}''$ c. to c. of pin-holes, $12\frac{1}{4}''$ o. to o. of angles.			
5	8		6 × $\frac{5}{16}$ Fillers.	2 1	Sq.				7081		
6	16		$17\frac{1}{2}$ × $\frac{5}{16}$ Pls. Battens.	21	Sq.					2	P_2 L
7	208		$2\frac{1}{4}$ × $\frac{3}{8}$ Pl. Lattice Bars.	$20\frac{1}{4}$	Temp.	$17\frac{1}{2}$ c. to c.					
8	16		3 × 3 angles, 18 p. y.	26 0	Sq.	± $\frac{1}{4}$					
9	4		5 × 3 angles, 28 p. y.	12	Sq.						
10	4		$3\frac{1}{2}$ × 3 angles, 23 p. y.	$11\frac{1}{4}$	Sq.						
11	4		3 × $\frac{5}{16}$ Fillers.	6	Sq.						
12											
13											
14	12		36 × $\frac{3}{8}$ Sh. Pl. S. S. Web.	23 5	Sq.	± $\frac{1}{8}$	2 corners clipped, 4 kinds.			3	I D_1
15	48		6 × $\frac{5}{8}$ Pl. Fillers.	2 3	Sq.			Riveted up into 12 Int. Track Stringers, 23′ $5\frac{1}{2}''$ back to back of end stiffs., 36″ deep.	9158	3	I D_2
16	24		6 × 4 angles, 68 p. y. Top Fl.	23 $3\frac{1}{4}$	Sq.	± $\frac{1}{8}$	4 kinds.				
17	24		6 × 4 angles, 62 p. y. Bot. Fl.	23 5	Sq.	± $\frac{1}{8}$	All alike.			3	I D_3
18	48		6 × 4 angles, 39 p. y. End Stiff.	2 $10\frac{1}{4}$	Temp.	R.& L.				3	I D_4
19	96		3 × $2\frac{1}{2}$ angles, 13 p. y. Int. Stiff.	2 $10\frac{1}{4}$	after	bend'g					

FORM B. BUILT MEMBERS.—Under the heading "Total Length," is given the length over all. Under "How Cut," square denotes that the ends are cut perpendicular to the length of the piece, and can therefore be sheared off without a templet. A templet is made of wood of the exact shape desired, and is laid on the iron, its ends and edges marked, and the iron is then cut by these marks. Under the heading "May Vary," the margin of variation of length is put. Thus $\pm \frac{1}{4}''$ means that the piece must not be longer or shorter than the required length by more than $\frac{1}{4}''$, while $-\frac{1}{4}''$ would indicate that it must not be shorter than $\frac{1}{4}''$, and must not be longer than order.

In the 7th item, for lattice bars, two lengths are given, $20\frac{1}{4}''$ length over all, and $17\frac{1}{2}''$ length c. to c. of rivet-holes.

There is no "bevel" in the items given. A piece whose ends are cut slanting to its length, is beveled, and the bevel given is the length of the projection, in the direction of the length, of the slant side. It is the base of the right triangle, of which the slant side is the hypothenuse, and the width of piece the other side.

In item 14, "Sh. Pl." stands for "Sheared Plate." This directs the mill to supply a sheared plate instead of a universal rolled plate. The letters "S. S." stand for "Strain Shear." This indicates the character of stress the plate is subjected to, and may influence the manner in which the iron is piled for rolling in the mill. The intermediate stiffeners, which are given in the last item, are bent around the flange angles, to avoid putting fillers underneath the stiffeners. The length of these stiffeners is given $2'\ 10\frac{1}{4}''$ *after bending*. A sketch can be made to illustrate.

FORM C.

EYE BARS AND UPSET RODS.

FOR 117′ 6″ D. TR. TRO. SKEW SPAN, *for*

	NO. OF PCS.	MARK.	SHAPE AND SIZE.	ROUGH LENGTH FOR M. O.	FINISHED LENGTH. O. A.	FINISHED LENGTH. C TO C. C. TO END.	PIN-HOLE BORED FOR	SIZE OF RING OR UPSET.	NO. OF DIE.	SHIPPERS NO. OF PCS.	GEN'L MARK.
				′ ″		′ ″	″	″ ″			
1	2	L T_2	$6'' \times 1\frac{3}{4}''$ eye bar.	38 6		35 $5\frac{23}{32}$	$5\frac{11}{16}$ $4\frac{15}{16}$	$14 \times 1\frac{7}{8}$ $14 \times 1\frac{7}{8}$	179 179	2	L T_2
2											
3	4	S_3	$5'' \times 1\frac{1}{2}''$ eye bar.	26 6		23 $5\frac{15}{16}$	$4\frac{15}{16}$	$12 \times 1\frac{5}{8}$	93	4	S_3
4											
5	4	S C	$2\frac{1}{4}''$ sq. eye and upset rod.	33 3		29 $11\frac{3}{8}$	$4\frac{15}{16}$	9×3	Rt. Th'd.		
6	4	S C	$2\frac{1}{4}''$ sq. eye and upset rod.	7 9		4 0	$4\frac{15}{16}$	9×3	Left Th'd.	4	S C
7	4	S C	Cleveland Turn buckles.	2d length	9″ clear		3″ th'd	R & L			
8											
9											
10											
11											
12											

FORM C. EYE BARS AND UPSET RODS.—In the illustration given, item 5 is a counter, made of square bar iron in two pieces, these pieces united, before shipment, by a Cleveland Turn buckle. As drawings are seldom made for counters, a sketch of counter can be made to give any dimensions not provided for in the columns. The lengths $29'\ 11\frac{3}{8}''$ and $4'\ 0''$ are from the centre of pin-hole to end of rod, not from c. to c., as is the case for eye bars. In item 1, the pin-holes at each end are of different sizes, as they fit different-sized pins. In item 2, both ends go on same-sized pins.

FORM D.

PINS, PIN-NUTS, AND PILOTS.

FOR 117′ 6″ D. TR. THRO. SKEW SPAN, *for*..

	NO. OF PCS.	MARK.	SHAPE AND SIZE.		TOTAL LENGTH.	SKETCH.	DESCRIPTION.	SHIPPER'S NO. OF PCS.	GEN'L MARK.
			ROUGH.	FINISHED.					
1	4	L E	6″	$5\frac{11}{16}$″	2′ $2\frac{3}{4}$″			4	L E
2									
3									
4									
5	1	L E P	6″	$5\frac{11}{16}$″	7	Pilot with $4\frac{5}{8}$″ th'd to	fit on L E	1	L E P
6									
7									
8									
etc.									

MALLEABLE NUTS FOR ABOVE PINS.

	NO. OF PIECES.	MARK.	HOLE.	DIA. OF THREAD.	SHORT DIAM.	THICKNESS.	THREAD.	RECESS.
24	32	P_{10}	$3\frac{3}{4}$″	$3\frac{7}{8}$″	$6\frac{1}{4}$″	$1\frac{1}{2}$″	1″	$\frac{1}{2}$″
25	8	P_{13}	$4\frac{1}{2}$″	$4\frac{5}{8}$″	7″	$1\frac{1}{2}$″	2″	$\frac{1}{2}$″
26								
27								
28								
etc.								

FORM D. PINS, PIN-NUTS, AND PILOTS.—In the columns headed "Shape and Size," the rough diameter gives the size of iron as rolled, the finished diameter is that to which the pin is turned down. The pilot protects the thread of the pin while it is being driven. The pin-nuts have a recess on inside, so as to fit over the head of the pin. Item 1 should have a sketch, giving length of pin between shoulders, length of pin over all, length of threaded ends, and diameter of thread.

FORM E.

BOLTS AND SMALL FORGINGS.

FOR 117′ 6″ D. TR. THRO. SKEW SPAN, *for*..

	NO. OF PIECES.	MARK.	SHAPE AND SIZE.	LENGTH FOR M. O.	FINISHED LENGTH.	SKETCH.	SHIPPER'S NO. OF PIECES.	NOTE FOR SHIPPER.	GEN'L MARK.
1	14	①	$1\frac{1}{4}$″ Foundation Bolts.		18″		14		①
2	14	①	Stand. Hex. Nuts for	$1\frac{1}{4}$″	th'd	$1\frac{1}{4}$″ thick.		Fast on.	①
3									
4									
5									
6									
7									
8									
9									
etc.									

FORM E. BOLTS AND SMALL FORGINGS.—We can use this form for bolts and blacksmith work, such as loop swivels, clevises, and other small forgings. The example given is for a swedged foundation bolt (page 444). A sketch should be made, giving length over all, length of thread on end, distances between indentations or shoulders, etc.

FORM F.

LIST OF SHOP RIVETS.

FOR 117′ 6″ D. TR. THRO. SKEW SPAN, *for*________________________________.

	NO. OF PIECES.	MARK OF MEMBER.	SIZE OF RIVET.	LENGTH UNDER HEAD.	LOCATION.	DRAWING NO.
1	408	1 Pc.	⅞″	2⅜″	button. Cov. Pl. + angles, also Top Fl. angles + Web. Also Splice Pl. + Web.	
2		S C				
		and				
3	12	1 Pc.	⅞″	3⅝″	countersunk. Ins. Pl. + Web + Filler + Outs. Pl. + Jaw Pl.	
4		S A				
5						
6						
7						
8						
9						
10						
etc.						

FORM F. LIST OF SHOP RIVETS.—We have given an illustration of round-head rivet, and also of a countersunk rivet. The length of round-head and flat-head rivets should be given from underneath the head, the other head is made when the rivet is put in. The length of a rivet with one round and one countersunk head should be given from underneath the round head, the countersunk head being made when the rivet is put in. A rivet with two countersunk heads should have its length over all given. Sketches should be inserted for items 1 and 3, showing the rivet with length marked.

The following Table gives the additional length for making head, to be added to length of metal passed through.

DIAMETER OF RIVET.	ADDITIONAL LENGTH REQUIRED TO FORM ONE HEAD IN PASSING THROUGH THE FOLLOWING THICKNESSES OF METAL.			
	¾″ and below.	1½″ and below to ¾″.	2½″ and below to 1½″.	above 2½″
″	″	″	″	″
½	⅞	1	1	1
⅝	1	1⅛	1⅛	1⅛
¾	1⅛	1¼	1⅜	1½
⅞	1¼	1⅜	1½	1⅝

No percentage for waste need be added to the number of shop rivets ordered.

FORM G.

LIST OF FIELD RIVETS.

FOR 117′ 6″ D. TR. THRO. SKEW SPAN *for*________________________________.

	NO. OF PIECES.	SIZE OF RIVETS.	LENGTH.	LOCATION.	SHIPPER'S NO. OF PCS.
		″	″		
1	50	¾	2⅜ button.	Int. knee brace + Gusset, also Int. knee brace + Bracket.	50
2					
3	100	⅞	3¼ "	Hood + Pl. on Portal + Flange End Post,	100
4				also Stiff. Bracing + Loose Pl. + Stringer Flange.	
5					
6					
etc.					

FORM G. LIST OF FIELD RIVETS.—Anywhere from 5 % to 25 % should be added for waste, due to burning of rivets in the field, etc. The greater the number of short rivets the less the percentage allowed, and the greater the number of long rivets the greater the percentage allowed, because a short rivet can be made from a long one, if the short rivets run out. It is a help to the erectors to order each item separately, even if the rivets are the same size and length, as they can thus see the number required for any particular joint.

SHIPPING.—Every piece of iron shipped, except rivets and bolts, should have its mark for the erectors.

Rivets and bolts are shipped in boxes, and have, in the case of rivets, their size marked on the outside. In the case of bolts the bolt mark is on the outside, if the bolt has a mark, if not, their size is given on outside of box.

The mark of a member should not consist of more than three figures or letters if possible, and it is well to have the marks have some meaning so far as may be, as P_2 R, for intermediate post, " right." A member is right or left when it can only be used on one side, and is not reversible, so as to be used on the other. Two members, as posts, may be exactly alike in all respects, except that the addition of a bracket, or some similar addition, on one side, may prevent it from being used on the other truss, as in that case the brackets would come on the wrong side.

A list of all the iron ordered can be sent to the erectors, and with an erection plan, consisting of a skeleton outline of the truss, with the mark and location of every piece, they can erect the bridge without the shop drawings.

In general a piece over ten or twelve feet high cannot be shipped as a whole, but must be spliced in the field. Girders seldom exceed this height. They can be shipped on two or three flat cars coupled together, properly braced by wooden braces.

Chord sections, posts, eye bars, etc., can be shipped in one piece. A deep portal, over, say, twelve feet high, will have to be shipped loose and riveted up in the field, as it cannot be taken on its side, or vertical.

A short deck girder span would have the girders shipped separately, as, if the transverse bracing were riveted on in the shop, and the whole shipped together, it would be clumsy to manage in erection, and the transverse bracing liable to injury.

INSPECTING.—When, as is sometimes the case, iron bridges have been standing for years, which were originally designed for much lighter loads than those in present use, a careful examination is necessary. A judicious strengthening of such a structure, based on such examination, may prolong its life for many years. A neglect of such examination may result in disaster and loss of life. The fact that a structure has fulfilled its duty for many years is no evidence of its present efficiency, and sometimes is quite the contrary.

The examination should consist in a careful external inspection for external evidence of weakness, and calculations of the strains to which each member is subjected, based upon the present traffic and the actual dimensions, as given by the working drawings or by actual measurement. Both of these investigations are necessary, as a bridge may be weak and give no external evidence of its condition; and, on the other hand, there may be defects of construction, material, manufacture, or injuries, which can only be discovered by actual inspection. All bridges should have such a field examination at least once a year.

As rolling-stock increases in weight and heavier locomotives are built, many iron bridges carry daily loads in excess of those assumed in the original design. Such structures, however, are often sufficiently strong to serve their purpose for years, or may be made so by proper strengthening. Others possibly require immediate removal. Constant and thorough examination thus becomes more imperative every year.

Of equal importance is the inspection of structures in process of construction, to

insure that the requirements of the specifications are complied with. (See Cooper's specifications under the head of "Inspection.")

The following paragraphs are from the Atlantic Coast Line's specifications: "For wrought iron a set of specimens shall include one specimen tensile, transverse or compressive test, and one specimen bending tests. A set of specimens for channels and beams shall be understood to include one set, as above specified, from the web, and one set from the flange. The test specimens and the pieces from which they are taken shall be marked with the same stamp, so that these pieces can be found if they prove defective. The test specimens shall be prepared from pieces selected by the inspector, and shall be sufficient in number to fairly represent, in his judgment, the material furnished, not to exceed the following, however, at the option of the inspector:

"On any contract for wrought iron a minimum number of ten sets of specimens shall be tested, and when the contract is for an amount exceeding 100,000 pounds, one set of specimens shall be tested for each additional 20,000 pounds, provided that each order is completed at one rolling; when this is not the case, the requirements of the preceding sentence may be applied to each rolling.

"To determine the strength of the eyes, full-sized eye-bars and rods with eyes may be tested to destruction. Notice will be given in advance of the number and size required, so that the material can be rolled at the same time as that for the structure.

"The following tests shall be made at the option of the inspector:

"One full-sized bar or rod for each 25 bars or rods of wrought iron, unless a lot contains less than that number, in which case a like number may be required for each lot. Any lot of bars or rods from which full-sized members are tested shall be accepted, provided:

"First. That the bar or rod tested does not break in the head or neck.

"Second. That its quality is not inferior to that required by the specifications.

"All full-sized built members taken for tests, and which prove to be good and acceptable material, shall be paid for by the railroad company, at the net cost less its scrap value; but no payment shall be made for any material, workmanship, or testing of any member which proves defective.

"Test specimens from universal mill plates shall not be taken from the edge of the plate. No greater deficiency than 2½ per cent will be allowed between the estimated and the actual weight of any piece of material.

"The acceptance of any material by the inspector or his assistants shall not prevent its subsequent rejection, if found defective, after delivery; and such material shall be replaced by and at the expense of the contractor."

The above gives some idea of the number of tests required, and of the responsible duties of the inspector. It is also his duty to detect all shop errors before shipment. Thus, through error, track stringers might be made too long, so as not to fit between posts, or the posts may be riveted up so that the rivet-holes for the floor beams come in the outside web. Such errors affect the erection. Other errors may even endanger the structure, as when a plate girder should have a ¾" cover plate on the top flange, and a ½" cover plate on the bottom flange, and the plates get reversed in the shop. Mistakes affecting erection, when not corrected in the shop, cause delay in the field, and are an indirect expense to the purchaser. Even if the manufacturer is the erector also, there is annoyance and expense to the purchaser. Correction in the field of shop errors is also liable to be done hurriedly and incompletely, to the detriment of the structure. Members having errors which reduce strength but do not affect erection are not so likely to be sent back for correction.

These facts show the importance, to the purchaser, of having an inspector not only for the mill, but also for the shop.

The specifications quoted show that the railroad companies as purchasers appreciate this importance. As guardians of the public safety it is in some cases perhaps to be regretted that they do not seem to equally appreciate the importance of thorough and regular inspection of their bridges after erection.

CHAPTER XIII.

THE ERECTION OF ENGINEERING STRUCTURES.

BY JOHN STERLING DEANS, M. AM. SOC. C. E., CHIEF ENGINEER THE PHŒNIX BRIDGE COMPANY.

WITHIN the past few years the subject of the final "Erection of Engineering Structures" has become a much more important branch of engineering work, and this department, which has until lately been somewhat slighted by most construction companies, has been found to demand the same careful supervision and attention as are called for in the designing of the permanent structures themselves.

In the past it was not so much what was an economical "false work" and "traveller," as what was "strong enough," and the competition amongst contractors, and the cost of materials *then*, demanded nothing different; now these conditions have changed, and the margins upon which contracts are secured or lost are daily becoming less.

These temporary structures, therefore, must be of the most economical design, and their principal members designed and proportioned for the exact loads which will come upon them. In view of these facts, it seems eminently proper to give this subject of "Erection" a place in the text-books on "The Strains in Framed Structures," that students and others interested may become more familiar with this important branch of engineering work.

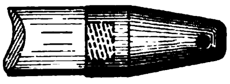
To some the present short chapter on this subject may seem to be written too much in detail, and contain points which are so well and generally known as to hardly warrant insertion in such an article; but it must be borne in mind that this is written primarily for students, and those who have not, as yet, been engaged in the active work of the profession.

POINTS TO BE CONSIDERED IN DESIGNING PERMANENT STRUCTURES.—From holding the final erection in view the present type of "American Pin-connected Truss" owes its design, as much as to any other single fact. This truss requires the least amount of field work; its joints being pin-connected there is no field riveting except that for the floor system and minor details, and field rivets should be as few as possible, since, owing to the poorer facilities for doing the work, they cannot be driven so well, nor so cheaply, as in the shops; most specifications therefore require as high as 20 per cent. excess for field-driven rivets. In many instances, where rivets are hard to drive, it is much better to use turned bolts in drilled or reamed holes. In a large bridge lately built in the West, the contractors used turned bolts for all the floor connections, believing there was a saving in so doing.

Another important matter to consider, especially when the structure is to span a stream subject to sudden rises, is to so design the connections that the trusses may be swung and be self-supporting in the least possible time, leaving the floor beams, stringers, outer chord bars, and most of the bracing to be put in later, reducing the risks of loss from washouts.

Aside from the economy in the shops, the number of pieces composing a structure should be as few as possible, by making long panels and concentrating the metal, since it requires about as much time and power to handle and connect a large piece as a small one.

Allow plenty of clearance, at least $\frac{1}{4}''$, after due allowance for packing of plates and rivet heads, between the jaws of built members, and also in packing members between the jaws of built sections; to keep a whole gang waiting in the field, while chipping is being done, is a very expensive piece of experience.



Always furnish pilot nuts for each size of pin, with an easy draught, to facilitate the centring and connection of members at panel points. These pilots should have a draught of at least $1\frac{1}{2}$ inches; in the rough assembling of panels the bars and other members comprising the joint are often over 1 inch short, and the pilot on the end of the pin will catch and centre these members.

Carefully mark the individual pieces, composing each riveted joint, which have been assembled, faced, and reamed, or drilled together, in the shops, with some distinguishing letter, so that the same pieces may be put together in the field, saving all unnecessary fitting.

Anchor bolt holes should be so arranged that drilling of masonry can be done after the span has been connected and swung, and its exact location on the supports established.

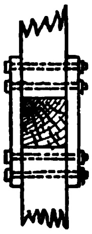
Have as few adjustable members as possible, since such members work loose, and the adjustment is usually allotted to those who fail to realize its importance in the proper working of the structure under load. Many other points which should be borne in mind to facilitate erection might be mentioned, but those indicated are the ones which most frequently occur, and which should be especially considered in the designing of details.

MATERIALS AND TOOLS USED IN ERECTION.—The principal timber used in these temporary structures is "Long leaf Southern Yellow Pine," owing to its more uniform strength and reliability.

Where lightness is an item to be considered, and where it is necessary to do considerable framing, "White Pine" is used. "Oak" is used rarely, owing to its weight and expense. "Hemlock" should be discarded except for the most insignificant work, owing to its unreliability. "Spruce" is better than hemlock, but rarely used.

For "piling" yellow pine is most generally used, and these piles can be had in perfect straight lengths up to 70 feet. In localities where yellow pine is scarce and expensive, chestnut, oak, beech, hickory, or any of the hard woods may be used; the latter, however, being harder to remove after the work is finished, when it is only necessary to break off the piles. No pile should be used less than 8 inches full diameter at the small end, and it should be straight throughout its length.

It should never be left out of sight that false works are simply temporary structures, and only intended to answer as a support for a very limited period, and therefore the material used should be of suitable quality to answer such a purpose, with due regard to safety; and the least possible expense should be expended upon it in framing, etc. In



most cases good round timber may be used for legs instead of square stuff; and where it is necessary to make trestles of two or more stories, it is rarely necessary to "dap" the legs into the caps; abutting the ends of legs or abutting the legs against the cap and splicing the joint with a piece on each side, answering every purpose. This same idea should be held uppermost in the framing of all joints, only expending the amount of work on each, which the actual safety of the structure demands—nothing more.

For stringers and other members subject to bending, the strain on the extreme fibres for good yellow pine may be taken as high as 1,600 lbs. per square inch; and upon this assumption the following table is figured; this table shows the capacity in bending moments (foot-pounds) for 1,600 lbs. strain on the extreme fibres:

TABLE I.

Width Inches.	Depth of Beam in Inches.											
	6	7	8	9	10	12	14	16	18	20	22	24
3	2400	3267	4266	5397	6666	9600	13066	17066	21600	26666	32266	38300
4	3200	4355	5688	7200	8888	12800	17421	22755	28800	35555	43020	51200
5	4000	5444	7101	9000	11112	16000	21778	28441	36000	44444	53770	64000
6	4800	6533	8533	10800	13333	19200	26133	34133	43200	53333	64533	76800
7	5600	7623	9956	12600	15623	22400	30489	39756	50400	62223	75289	89600
8	6400	8711	11378	14400	17777	25600	34844	45511	57600	71110	86044	102400
9	7200	9800	12800	16200	20000	28800	39200	51200	64800	80000	96800	115200
10	8000	10889	14222	18000	22222	32000	43556	56890	72000	88889	107556	128000
12	9600	13066	17067	21600	26667	38400	52266	68266	86400	106666	129066	153600

TIMBER COLUMNS.—In members subject to compression good yellow pine may be strained as high as 1,100 lbs. per square inch for columns, when the ratio of least side to length does not exceed 20; for columns over this length the unit strains should be reduced by the following formula, $U = 1500 - 18\frac{l}{d}$, where U equals unit strain, l equals length in inches, and d equals the least side in inches. White pine should be strained about 30 per cent. less than above. No column should be used longer than fifty times its least width. The following table has been figured for timber columns, using the formula given:

TABLE II.

$\frac{l}{d}$	Yellow Pine. $1500 - 18\frac{l}{d}$	White Pine. $1000 - 18\frac{l}{d}$	$\frac{l}{d}$	Yellow Pine. $1500 - 18\frac{l}{d}$	White Pine. $1000 - 18\frac{l}{d}$
20	1140	640	32	924	424
22	1104	604	34	888	388
24	1068	568	36	852	352
26	1032	532	38	816	316
28	996	496	40	780	280
30	960	460	42	744	244

For structures where traffic is carried during the erection, the strains per square inch, in those members supporting the live load, should be reduced by 20 per cent. from the unit strains derived by using Tables I. and II.

PILING.—In the driving of piles of sizes usually used in false work, viz., 8″ to 10″ diameter at small end and 60 feet long, a hammer should be used weighing about 2,400 pounds, and should have a final fall of 30 feet. After driving piles on this plan and into the ground 15 to 18 feet, if the pile does not penetrate more than two inches under the last blow, a load of 18 tons may safely be placed upon it. Formulæ for obtaining the safe bearing values of piles are numerous, but none are entirely satisfactory, nor can they be relied upon, as so much depends upon the particular soil into which they are driven, and other attendant circumstances. The formula proposed by Sanders will answer for most cases, and by using a factor of 10 a safe result is obtained,

$$T = \frac{wh}{S};$$

where T = ultimate bearing value; w = weight of hammer in pounds, h = height of last fall in inches, S = penetration of pile under last blow.

PRICES AND SPECIFICATIONS.—Good long leaf Southern yellow pine can be bought in the northern market of suitable sizes for $25 per M., and at the mills in the South as low as $11 per M. The following specification answers for false work lumber:

"To be long leaf southern yellow pine, cut from sound, untapped trees; to be free from large or loose knots and other material defects; straight, well manufactured, true and full to sizes given."

Good oak costs from $15 to $30 per M., according to location. Specification as follows:

"To be cut from sound, live trees, straight-grained, free from knots, wind shakes, and other imperfections."

The specifications for material for permanent structures of course are more severe; generally calling for three corners to show heart lumber throughout the length of the piece, and the remaining corner may show sap wood for $\frac{1}{10}$ the width of its face. Yellow-pine piling costs at the site from 6 to 10 cents per lineal foot.

"Piles must be cut from sound trees, not less than 8 inches in diameter at small end, and straight throughout their length."

ROPE.—For hoisting and rigging purposes the best manilla rope should be used. It is sold in coils containing from 950 feet in the larger sizes to 1,100 feet in the smaller sizes. The following table shows the weight per foot and the ultimate strength of the usual sizes. The "working strain" is usually taken at $\frac{1}{4}$ of the ultimate. Present price of rope, 8— cents per pound.

TABLE III.

DIAMETER.	WEIGHT PER FOOT.	ULTIMATE STRENGTH.	DIAMETER.	WEIGHT PER FOOT.	ULTIMATE STRENGTH.
$\frac{3}{4}''$	.17	3900 lbs.	$1\frac{1}{4}''$	.47	10600 lbs.
$\frac{7}{8}''$	.25	5700 "	$1\frac{1}{2}''$	.75	16900 "
1″	.30	6750 "	2″	1.30	29300 "

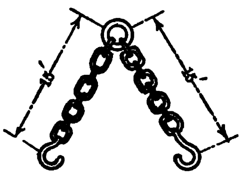
Wire rope is also used for guys, etc., and occasionally for hoisting purposes, but for general practice the manilla is better and cheaper. Only when the wire rope is used for hoisting and running rapidly, making it liable to become heated, is it necessary to use a wire centre; in all other cases the wire rope should be laid with a hemp centre. When it is necessary to use metal, it is generally better to select steel rope, as it is much lighter and stronger than the iron. The following table gives the weight per foot, ultimate strength, and cost of the usual sizes. Working strain taken at $\frac{1}{4}$ ultimate.

TABLE IV.

DIAMETER.	WEIGHT PER FOOT, *Iron.*	ULTIMATE IN TONS, 2000.	WEIGHT PER FOOT, *Steel.*	ULTIMATE IN TONS, 2000.	PRICE.
$\frac{3}{4}''$	.88	8.8	.88	17.	$1\frac{1}{2}''$ Steel, 40 cts. per ft.
$\frac{7}{8}''$	1.12	12.3	1.12	22.	$\frac{3}{4}''$ " 19 " "
1″	1.50	16.0	1.50	30.	
$1\frac{1}{4}''$	2.28	25.0	2.28	44.	$1\frac{1}{2}''$ Iron, 30 cts. per ft.
$1\frac{1}{2}''$	3.37	36.0	3.37	62.	$\frac{3}{4}''$ " 9 " "

CHAIN.—Nothing but the best material should be used in chains, and they should be of the most approved manufacture and nothing smaller than $\frac{3}{4}''$ diameter of iron in links should be allowed. These chains are used principally in hoisting, and made usually about 25 or 30 feet long, with a hook at each end and a large ring in the centre, called double chains. Single chains are from 5 feet to 15 feet long, with hook at one end and ring at the other end.

CRABS.—For hand power hoisting "A" crabs are used (see Sketch 1) having a drum around which the rope is wound. In some cases, as derricks, or when attached to the legs of the traveller, a "square-framed" crab is more convenient.



BLOCKS.—All blocks should be of the most approved pattern, extra heavy strapped, and metalline bushed sheaves (see Sketch 2).

HYDRAULIC AND SCREW JACKS.—Jacks are indispensable where it is necessary to raise or lower heavy weights, and are especially useful when ready to swing the permanent truss free of the false work, in raising the truss to relieve the pressure on the blocking, so that it can be removed, and then lowering the span (see Sketch 3).

ENGINES.—For light hoisting, the 4-spool engine is usually used (see Sketch 4), the hoisting-rope being wrapped around the spool. For pile driving the double-drum engine is used (see Sketch 5). For heavy hoisting the engine shown in Sketch 6 is used. The engine and boiler are fastened to a single solid frame, making it possible to move the engine easily from place to place. These engines are made with 6 or 8 spools, permitting the use of more hoisting lines at one time.

LIST OF TOOLS.—The preceding are the main materials and tools used in the erection of structures; the following will be found an average list of the tools required for the erection of particular classes of work, but not including falsework bolts.

SPANS UP TO 100′ AND WORKING 30 MEN.	SPANS 100′ TO 300′ AND WORKING 75 MEN.	SPANS 300′ TO 600′ AND WORKING 200 MEN.
4 Sets 10" Double Blocks.	4 Sets 16" Triple Blocks.	20 Sets 16" Triple Blocks.
4 Sets 8" Double Blocks.	6 Sets 14" Double Blocks.	20 Sets 14" Double Blocks.
2 10" Single Blocks.	6 Sets 12" Double Blocks.	20 Sets 12" Double Blocks.
4 10" Snatch Blocks.	8 Sets 8" Double Blocks.	10 Sets 10" Double Blocks.
	8 12" Single Blocks.	20 Sets 8" Double Blocks.
4 1" Lines, 160′ each.	8 12" Snatch Blocks.	16 14" Snatch Blocks.
4 ¾" Lines, 160′ each.	8 1½" Lines, 300 feet each.	10 12" Snatch Blocks.
6 1" Hand Lines, 40′ each.	6 1¼" Lines, 200 feet each.	20 12" x 14" Single Blocks.
20 Lashings, 30′ each.	10 1¼" Hand Lines, 60 feet each.	10 Coils 1½" Rope.
8 Rope Slings.	40 1½" Lashings, 40 feet each.	15 Coils 1¼" Rope.
	20 Slings.	10 Coils 1" Rope.
4 "A" Crabs.	2 Coils ¾" Rope.	10 Coils ¾" Rope.
1 Axe.	2 Coils 1" Rope.	20 1¼" Hand Lines, 80′.
1 Cross-cut Saw.	1 Derrick.	40 1¼" Lashings.
1 Man Saw.	4 "A" Crabs.	40 Slings.
4 ⅞" Crank Augers.	2 Square Crabs.	4 Derricks.
2 Iron Crowbars.	1 4-spool Hoisting Engine.	10 "A" Crabs.
2 Steel Crowbars.	3 Complete Riveting Kits.	4 Square Crabs.
4 Steel Connecting Bars.	1 Blacksmith Kit.	5 Hoisting Engines.
4 8-lb. Sledges.	1 Dolly Car.	1 Pile Engine and Driver.
	6 10-lb. Sledges.	1 Blacksmith Kit.
1 Complete Riveting Kit.	6 8-lb. Sledges.	4 Dolly Cars.
3 Flat Chisels.	4 Axes.	15 10-lb. Sledges.
3 Round-nose Chisels.	4 Cross-cut Saws.	10 8-lb. Sledges.
3 Cold Cutters	4 Man Saws.	10 Axes.
2 Chipping Hammers.	10 ⅞" Crank Augers.	15 Cross-cut and Man Saws.
2 Timber Jacks.	6 Iron Crowbars.	20 ⅞" Crank Augers.
2 Fork Wrenches for ¾" Bolts.	6 Steel Crowbars.	6 Iron Crowbars.
2 Fork Wrenches for ⅞" Bolts.	6 Steel Connecting Bars.	10 Steel Crowbars.
2 Monkey Wrenches, 18" long.	12 Flat and Round-nose Chisels.	15 Steel Connecting Bars.
	6 Cold Cutters.	6 Complete Riveting Kits.
2 Stone Drills.	4 Chipping Hammers.	20 Chisels and Cold Cutters.
12 1⅜" and ¾" Drift Pins.	6 Timber Jacks.	6 Chipping Hammers.
2 Button Sets, ¾" rivets.	12 Fork Wrenches, ¾" and ⅞".	15 Timber Jacks.
2 Button Sets, ⅞" rivets.	2 Key Wrenches.	25 Fork Wrenches, ¾" to 3"
	4 Monkey Wrenches, 18" and 21"	4 Key Wrenches.
	long.	6 Monkey Wrenches.
50 ¾" Fitting-up Bolts, 3" long.	2 Stone Drills.	15 Cant Hooks.
50 ⅞" Fitting-up Bolts, 3" long.	12 1⅜" and 1⅝" Drift Pins.	4 Stone Drills
	10 Button Sets, ¾" and ⅞".	20 1⅜" and 1⅝" Drift Pins.
150 Washers.	6 Cant Hooks.	20 Button Sets, ¾" and ⅞".
	200 ¾" Fitting-up Bolts, 2¼" to 3½".	400 ¾" Fitting-up Bolts.
	200 ⅞" Fitting-up Bolts, 2½" to 4".	400 ⅞" Fitting-up Bolts.
	500 Wrought Washers.	1,000 Wrought Washers.

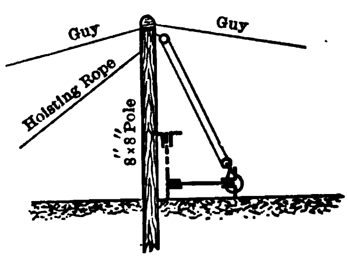
Having reviewed the points which should be borne in mind when designing the details of permanent structures, the materials required, and the tools used for the erection of the temporary structures, we will now proceed to give the methods and plans pursued in the erection of various sizes and types of engineering structures. In all cases where railway spans are considered, they are assumed to be for "single track," as the same plans would hold good for "double-track" spans, differing only in the width and strength of the false work, and the heavier rigging necessary to support and handle the increased weight.

SPANS UP TO 25'.

Girders for spans of this length are usually made of double I beams for the shorter lengths, and plate girders for those approaching 25'. These girders are usually connected with stiff lateral bracing. Girders of this length can be handled directly from the car to the bridge seat, by skidding down on rails to a temporary wooden stringer thrown across the opening, and then pulled directly over the final supports and up-ended. (See Plate I.) This is assuming the road is a new one; if it is necessary to provide for traffic, only slight interruption to which is allowed, it would be better to put the whole span together near the site, tear up the track between trains, and put it in position by sliding it on to the bridge seats from the side, or skidding it from a dolly-car at the end of the track. A span like this should be put in position and finished completely by ten men in two days; the heaviest single piece to handle being 4,500 pounds.

PLATE OR LATTICE GIRDERS, 25' TO 85'.

The same plan is to be pursued as in the case of the shorter spans, but owing to the increased lengths, one or more bents of the false work should be put in to support the girders as they are being launched out from the car; bents made of 2 — 8" × 8" legs spaced 15' apart longitudinally, and supporting 4 lines of 8" × 12" stringers, would be ample when no traffic is to be carried. These large girders should be launched from the car lying on the side, to prevent accident from upsetting, and after being set directly over the bridge seat, turned up to a vertical position by means of a pole rigged to one side; or by placing a temporary wooden frame over the girders and attaching hoisting blocks to it.



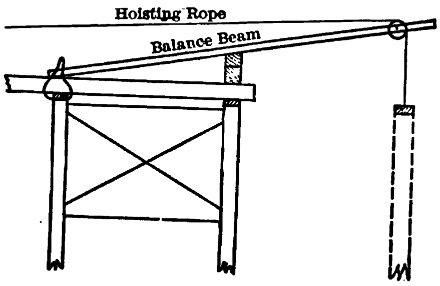
The longer span lattice girders are often shipped in two or more pieces to facilitate handling, in which case great care must be exercised to see that the girder is in perfect line and has a proper camber before riveting is commenced. The same care should be used in all cases, to have the girders in perfect line before riveting the lateral bracing.

Girders are also often put in position from the track by being supported from the side of the car, run out over the bridge seats, and lowered into position; or if the seat is directly under the rail, lower girders to one side, remove the track, and slide them into position. (See Plate II.) This is probably the cheapest plan for placing girders where such a method can be used, and a span should be finished completely by fifteen men in three days; the heaviest piece to handle being 15,000 pounds.

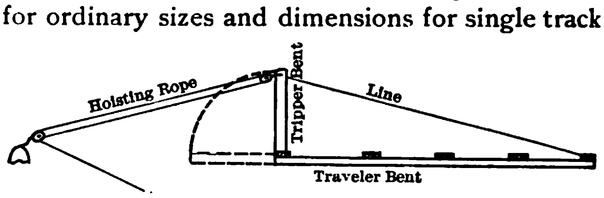
As the spans approach the longer lengths, and become very heavy, it is better to erect two or three bents of upper false work, depending on the length of girder; so that the girder can be picked directly off of the car and the car run from under it, and the girder then lowered into position. (See Plate III.)

THROUGH SPANS, 85′ TO 150′.

For the erection of single track through spans, the false work is usually made in bents of 3 legs each, spaced about 20 feet c. to c., and capped. On these bents are placed 4 lines of stringers. The sizes given in Plate IV. are for ordinary height and weight of span.



These bents of false work are usually framed and put together on shore, and floated to position and up-ended in place by the means of balance beams; or if it is not practicable to float the whole bent out, bolted together, it is put in place by the same means piece by piece. The top of the false work is so designed as to be at least 12″ below the lowest iron to be erected, so that there may be plenty of room to block up. After the false work is ready, the "traveller," or top movable staging, is put up; this traveller runs on rails spaced sufficiently far apart to allow it to span the new truss. (See Plate IV. for ordinary sizes and dimensions for single track spans.) The bents of the traveller are framed complete and bolted together lying down on the false work, and raised to a vertical position on the sills by means of a "tripper bent," and the two bents are then braced together.



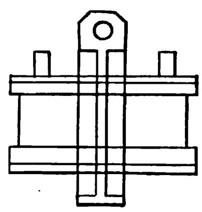
On the stringers on top of the traveller, 4 "A" crabs are placed, one near each corner, if the hoisting is to be done by man power. In many cases the crabs are placed on false work near end of span, to have the men at hand for other work.

After the false work and traveller are ready, the next proceeding is to lay out the longitudinal centre line of the trusses on the false work and locate the position of the panel points; at each of these points a sufficient amount of blocking is placed, upon which the iron rests, to give the new truss, when first placed in position and before swinging clear of the false work, an increased camber, varying from about 3″ for 100′ spans to 9″ for 550′ spans; this increased camber is put on to facilitate the connection of the new trusses by shortening the distance, as it does, between the diagonal panel points. The end wall-plates and shoes are then placed in position, beginning with the fixed end, and on centres furnished by the engineer; the lower chord bars are then distributed at their proper panels, as well as the pins and washers.

The erection of the trusses begins at the centre panel, for at this point we find diagonal members running in each direction, and the panel is therefore held both ways. The section of upper chord of the centre panel is first hoisted slightly above its final position and lashed to the traveller; this latter relieves the "A" crabs, and the two interior posts are taken hold of and hoisted into position, and the lower pins, connecting the lower chord bars, tie bars, and interior posts, are then driven; only sufficient chord bars, however, being put on at this time to complete the connection, as those on the outside of the post can be put on later. The section of the upper chord is then lowered slightly into its position over the interior posts, and, the upper ends of the diagonal bars and counters being hoisted into position, the upper pins are driven, thus completing one panel of the truss. The same plan is pursued simultaneously with the centre panel of the opposite truss, and the upper lateral and transverse bracing joining the trusses is put in place. Great care must be exercised in the adjustment of this first panel of the bracing to see that the panel points are exactly opposite and square with the centre line of the bridge, and that the interior posts are perpendicular. The facility of final connection depends very much on the careful adjustment of the first panel; if it is started square and true, and the others adjusted to it as they are erected, the chances are there will be little trouble at the end. During the

erection the false work should be watched closely, and if it settles—especially when it settles unevenly—the blocking should be increased at such points, so that the relative elevations of the centre line of the lower chord pins above a level line are kept as intended, and in a regular, increased camber curve.

After the centre panel is complete the traveller is moved one panel toward the "fixed" end of the span, and this panel is put in position and connected, pursuing exactly the same course as with the centre, and so on to the end; the traveller is then run back to the panel beyond the centre and toward the "roller" end, and these panels are put up in order. The last pin to be driven is usually the pin at the top of the leaning end-post. It is often necessary to raise or lower the truss to make connections, especially the final ones, and for this purpose good, powerful hydraulic jacks should be kept at hand. In applying these jacks to move any point, place them under the bars or pins, but close to the support in the jaws of the post; otherwise, through the liability of unequal loading, members would probably get seriously bent. After the span is all connected and all splices well filled with good bolts, it should be swung clear of the false work by starting at the highest point; this point is shown plainly by the excessive buckling of the diagonal members; and lowering this point until the buckling is nearly taken out, and then lower the panel points adjacent, working toward each end, and lowering each point about 1″ to 1½″, and taking the greatest care not to overload any point by permitting it to remain high and those adjacent too low; this undue loading is plainly seen, as before stated, by the diagonal members buckling. Before starting to "swing" the span, the counters should be slackened thoroughly, and after the span is swung and complete, including ties and rails and any other dead load it is to carry, they should be tightened, and brought simply to a good square bearing on the pins.



If the floor is so designed that it must be connected at the same time as the lower chord bars and web members—that is, if the floor beams have riveted end-hangers through which the pin passes—the floor beams are placed directly on the blocking first, the stringers put in between them, and then the remainder of the erection proceeds exactly as outlined above.

DECK SPANS, 85′ TO 150′.

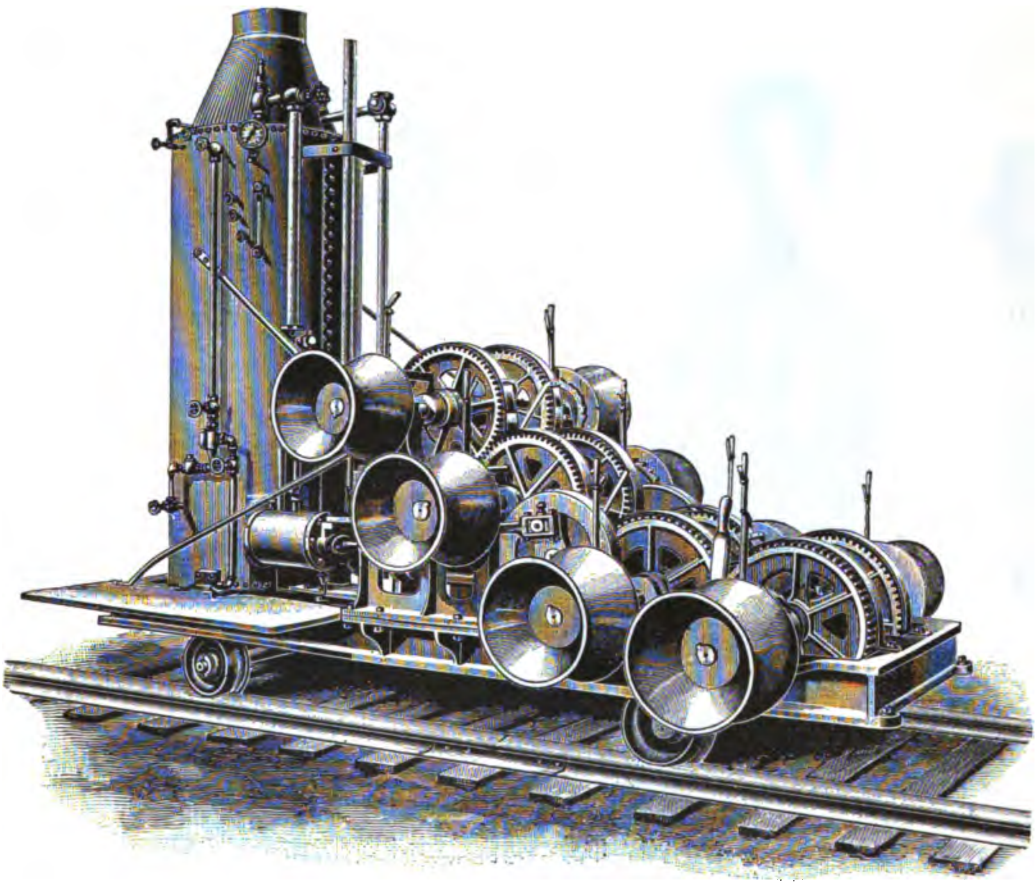
For the erection of "Deck Structures" up to 150′ in length the same character of false work is used as for "Through" spans of the same length and weight; and when the false work is only run up to the lower chord of the iron truss, the erection of the latter is proceeded with in precisely the same manner as for "Through" spans. At times, however, it is advisable to continue the false work up to a short distance below the upper chord. (See Plate V.) In this plate the false work is also shown arranged to carry traffic and to remove the old truss. The iron in this case is run out on a derrick car and *lowered* into position, the upper chord being first lined out, starting from the fixed end and from points given on the masonry by the engineer. At about the level of the lower chord the bracing on the false work is arranged with some additional planking (see plate), so as to support temporarily the lower chord and the lower ends of the interior posts, until the connections are made by driving the upper chord pins first and then those for the lower chord, at the centre panel, and working from here toward the "fixed" end, and then from the centre toward the "roller" end of the span. The same remarks apply to the adjusting of counters, the thorough bolting of joints before swinging off, and the care to be exercised in the adjustment of bracing, and squaring and lining up of trusses, particularly of the first panel.

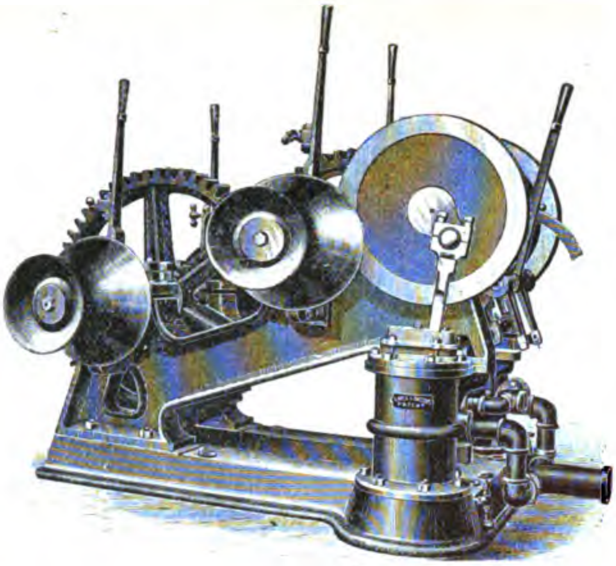
SPANS, 150′ TO 350′.

In considering the longer spans we will at the same time assume that it is necessary to use high false works, at least 60′ high. At this height it is advisable to increase the panel lengths, making the stringers correspondingly heavier, thereby reducing the number of bents required, the cost "in place" of these bents increasing rapidly as this height is approached. For ordinary structures 12″ × 12″ stringers can be used for panel lengths of 20′; over this 12″ × 16″ up to 24′ panels, and if necessary or advisable to increase the panels to 25′ or even 30′, the stringers can be trussed simply. (See Plate VI.) This plate also shows false work arranged to carry traffic and to remove the old structure. These long panels are put in where the crossing is over streams subject to sudden freshets, accompanied with a heavy run of drift, in order to leave as much open water-way as possible. The braced towers in such cases are made less in width, say 12′ to 16′, according to the most economical panel division. When the false work is 50′ high or more, it is always best to use steam power for hoisting, as man power is too slow for the necessary long lifts.

The hoisting engine is either placed at some convenient place on the bank and a lead-line run from there to sheaves in the ends of balance beams, or the engine is set up on a travelling crane or derrick traveller (see Plate XIV.), and the false work is lowered or raised in place from a swinging boom, which boom is long enough to command the position of all legs of the bent ahead of the last one erected. In our discussion so far we have assumed that false work legs could be set directly on the bottom, the formation being rock or other material sufficiently hard to preclude the possibility of scour or settlement under the loads which are to come upon them. When the bottom is of a medium solid character, it is advisable to use a sill at the foot of the trestle legs running transversely and receiving all the legs of the bent; and if the bottom is very soft, in addition to this sill short mud-sills must be placed at right angles, to still further distribute the pressure. If even with these additions, it is probable settlement or scour will take place, piles must be driven. Piles should be selected so that they may be well driven, and sawed off at some distance above the ordinary stage of water. Piles should be straight throughout, and before driving are pointed, and the butts roughly dressed square. They are driven at the panel points of the trestle, and as near as possible in line transversely and longitudinally. After all the piles in one bent are driven, they are sawed off, pulled still further into line by being braced to the previously finished bent, and then capped and braced. The false work is then put on this cap, in the same manner as previously outlined for ordinary cases, where the false work bent is placed directly on the bottom. In ordinary material piles are usually driven 12′ to 18′ with a 2,600-lb. hammer, dropping at the last blow 40′, and the penetration of the pile under this blow should not be more than 2″. Piles driven in such a manner can sustain safely a load of 18 tons. Forty piles should be driven each day of ten hours, under ordinary conditions, with one driver. (See Plate VII., showing driver, boat, and engine complete.)

The travellers for these longer spans are usually made of three bents, as the panels of trusses are generally made at least 25′ long, and the traveller must extend at least 2′ 6″ beyond the centre of the panel, making it 28′ centre to centre of the end legs (see Plate VIII.), or the traveller may be designed with two regular end braced bents and a centre cap supporting the upper stringers, which cap is supported by two leaning posts in the same plane as the inner legs of the bents. (See Plate VI.) This last traveller is probably the cheaper design, containing less lumber and framing, and yet answering the purpose. Great care must be exercised in the framing and raising of these large travellers, to see that every bent is square and plumb, and kept so when hoisting; and if necessary to insure this, good guys should be used. We have thus briefly outlined the false work and travellers required





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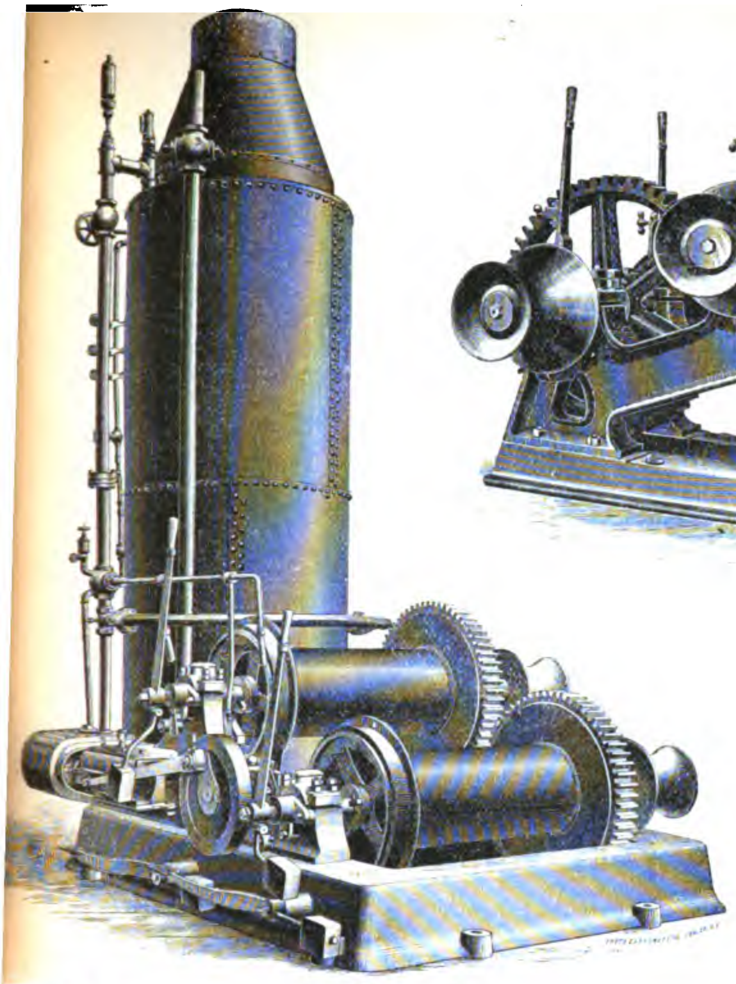

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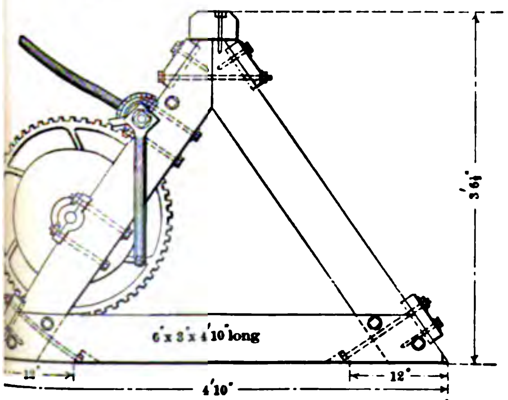
2

3

2

2

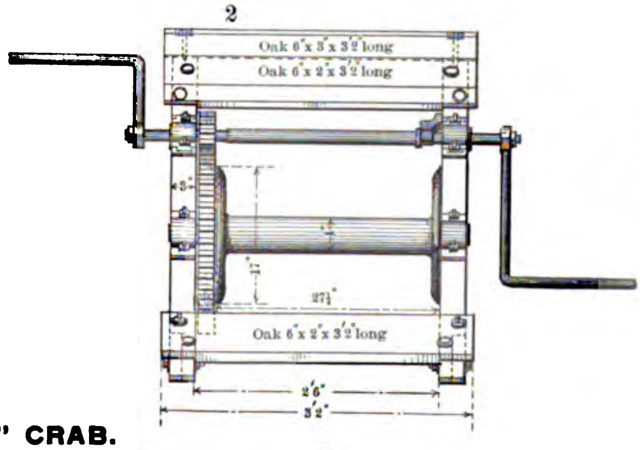
2



1

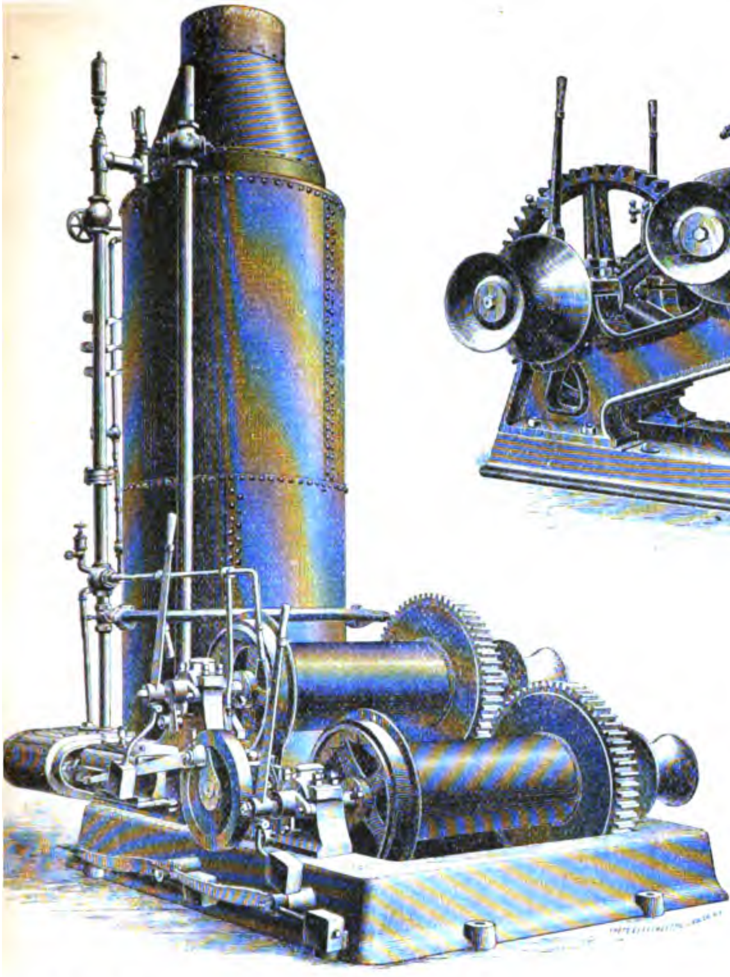
"A" CRAB.

SCALE, ¼ INCH TO A FOOT.



1

2



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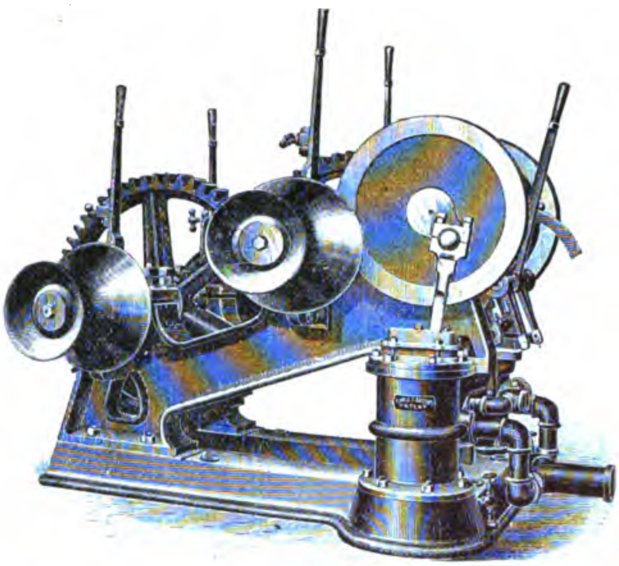

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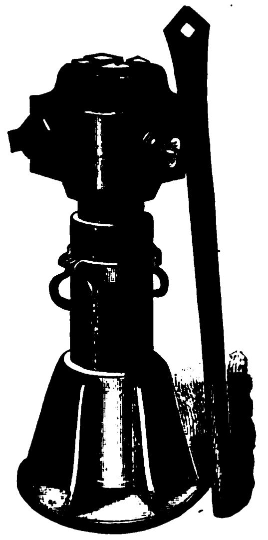
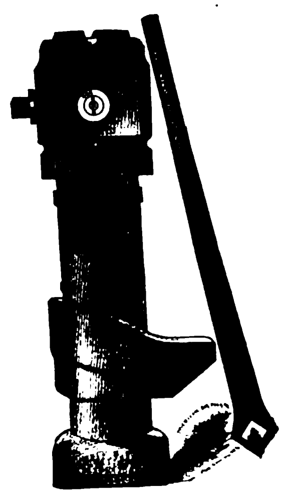


3

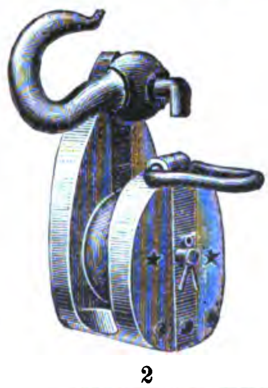
2



3

2

2



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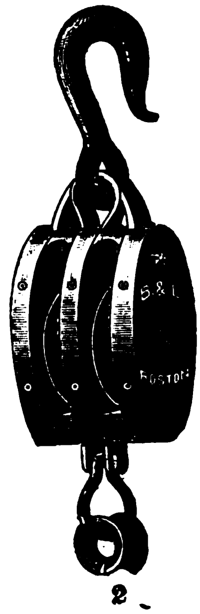

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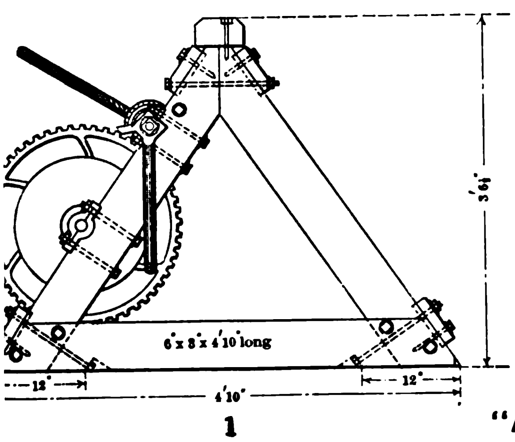
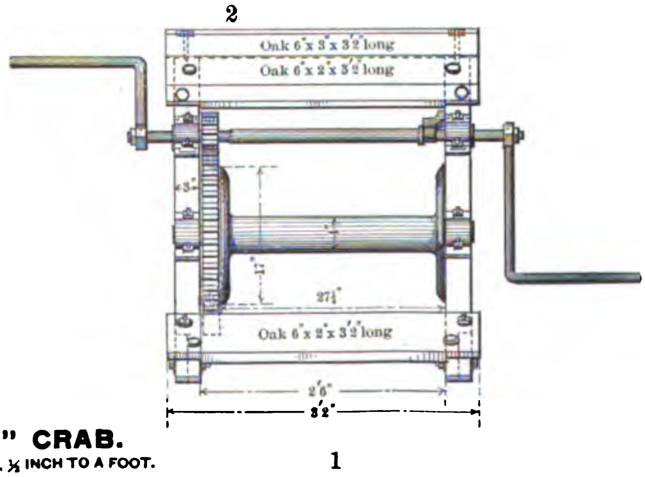


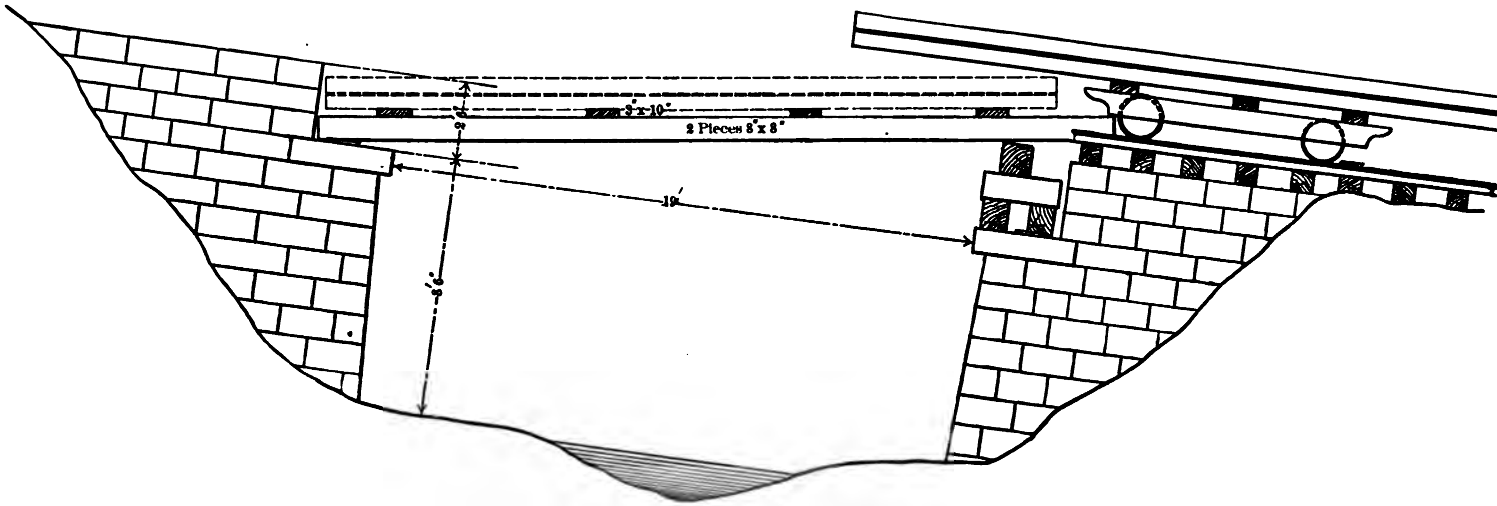
1

"A" CRAB.
SCALE, ¼ INCH TO A FOOT.



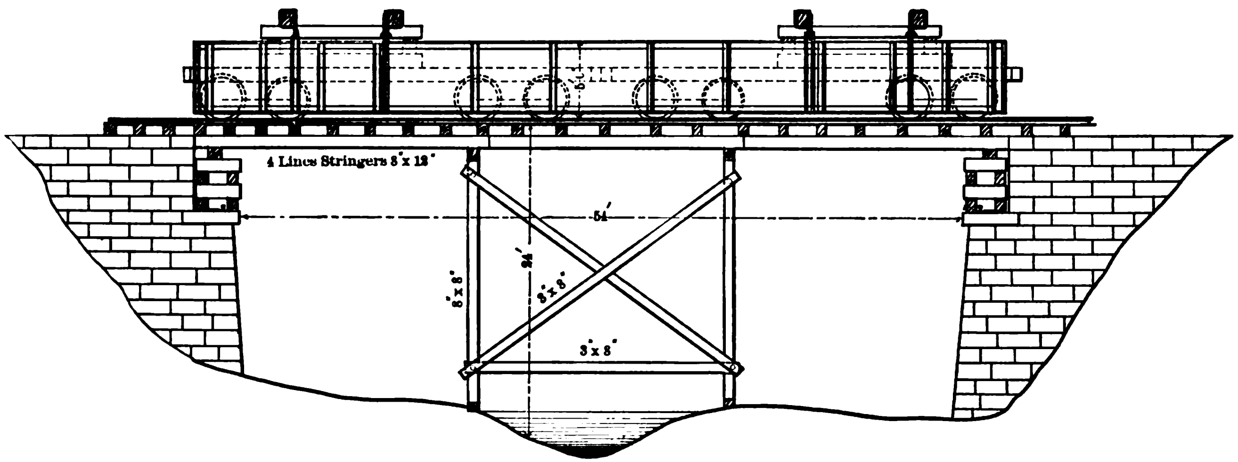
1

Plate I.

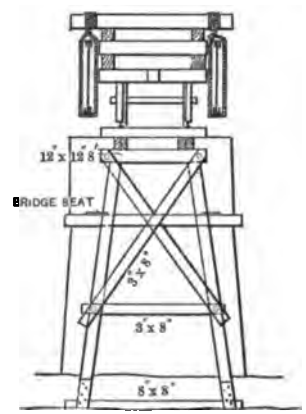


SPANS UP TO 25 FEET.

Plate II.

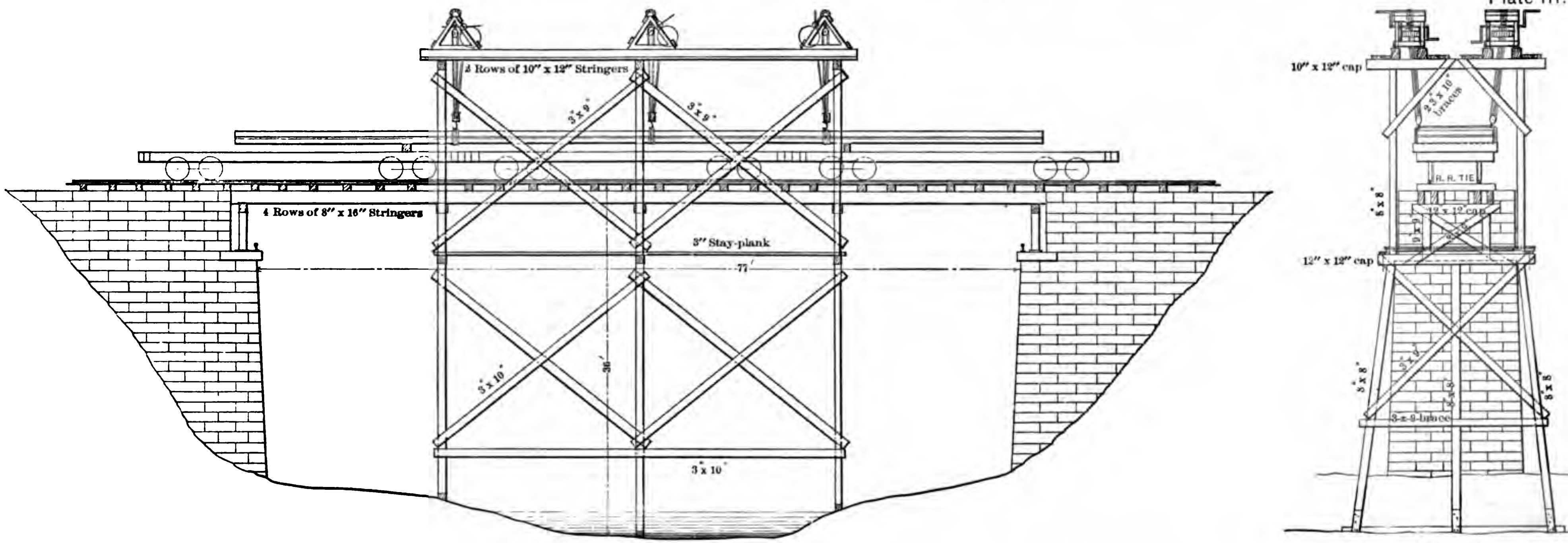


SPANS 25 FEET TO 60 FEET.



SECTION.

Plate III.

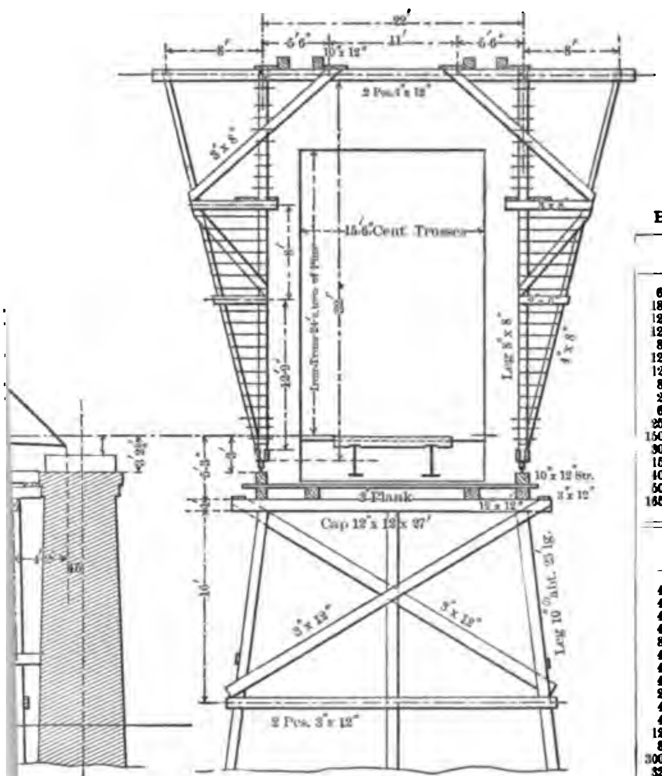


SPANS 60 FEET TO 80 FEET.

SECTION.

SCALE, 16 FT. TO 1 IN.

Plate IV.



BILL OF MATERIAL.

FALSEWORK.

6	Caps,	12 x 12 in.	27 ft.
18	Legs, 10 in. small end di.		23 "
12	Braces,	3 x 12 in.	32 "
12	"	" "	28 "
8	"	" "	24 "
12	"	" "	22 "
12	Stringers,	12 x 12 "	24 "
8	"	" "	25 "
2	"	10 x 12 "	23 "
6	"	" 20 ft.	10 in.
25	Floor plank, 3 in. thick,		25 ft.
150	Oak Wedges.		
30	Bolts,	3-4 in. diam.	17 in.
15	"	" "	18 "
40	"	" "	20 "
50	Drift bolts,	" "	24 "
165	Washers.		

TRAVELLER.

4	Caps,	4 x 12 in.	40 ft.
4	Legs,	8 x 8 "	34 "
4	Braces,	4 x 8 "	33 "
4	"	3 x 8 "	17 "
8	"	" "	9 "
4	"	2 x 6 "	5 "
4	"	4 x 8 "	32 "
4	"	" "	30 "
2	"	3 x 8 "	27 "
4	Sills,	4 x 12 "	28 "
4	Stringers,	10 x 12 "	29 "
12	Floor plank,	3 x 12 "	29 "
8	Cleats,	3 x 8 "	2 "
300	Lin. ft. 1 1-4 in. x 4 ft.		
32	Bolts,	3-4 in. diam.	13 in.
12	"	" "	15 "
24	"	" "	16 "
8	"	" "	19 "
4	"	" "	27 "
8	"	" "	17 "
4	Wheels with Journals.		

Plate V.

FALSE WORK FOR DECK SPAN,

ARRANGED TO REMOVE OLD STRUCTURE
AND CARRY TRAFFIC.

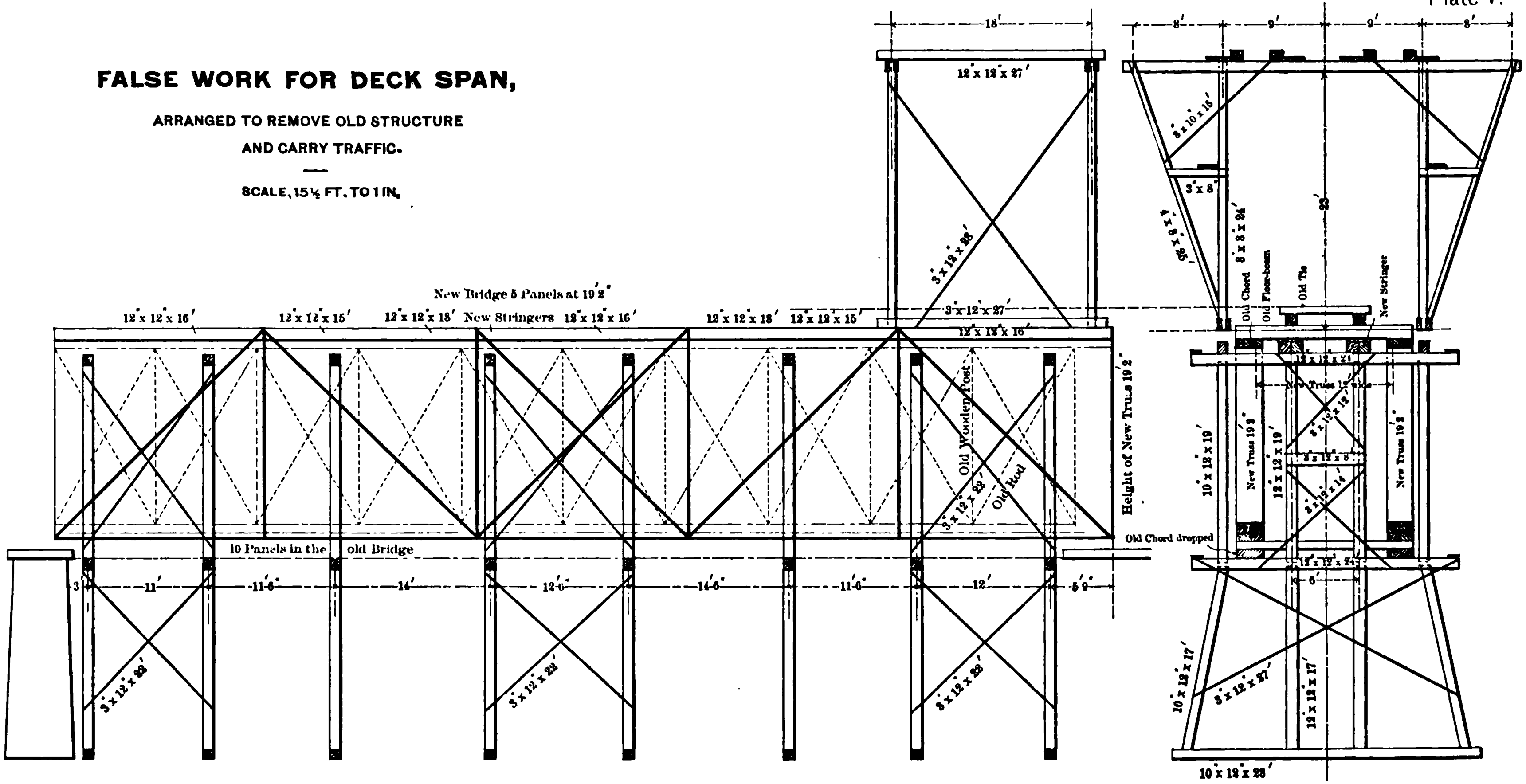
SCALE, 15½ FT. TO 1 IN.

MATERIAL FOR TRAVELLER.

4 Caps,	3 x 12 in.	36 ft.
4 Legs,	8 x 8 "	24 "
4 Braces,	4 x 8 "	25 "
4 "	3 x 10 "	15 "
4 "	3 x 8 "	7 "
4 Sills,	3 x 12 "	21 "
4 "	12 x 12 "	21 "
12 G Planks,	3 x 10 "	21 "
12 Bolts,	¾ in. diam.	13 in.
24 "	" "	14 "
16 "	" "	15 "
4 "	" "	18 "
112 Washers.		

FALSEWORK.

16 Caps,	12 x 12 in.	24 ft.
16 Legs,	" "	17 "
16 "	" "	19 "
16 "	10 x 12 "	19 "
16 "	" "	17 "
12 Stringers,	12 x 12 "	16 "
8 "	" "	15 "
8 "	" "	18 "
6 "	10 x 12 "	16 "
4 "	" "	15 "
4 "	" "	18 "
16 Braces,	3 x 12 "	12 "
16 "	" "	14 "
16 "	" "	27 "
16 "	" "	22 "
8 "	" "	8 "
40 "	" "	18 "
30 Floor Plank,		14 "
40 Blocking,		8 "



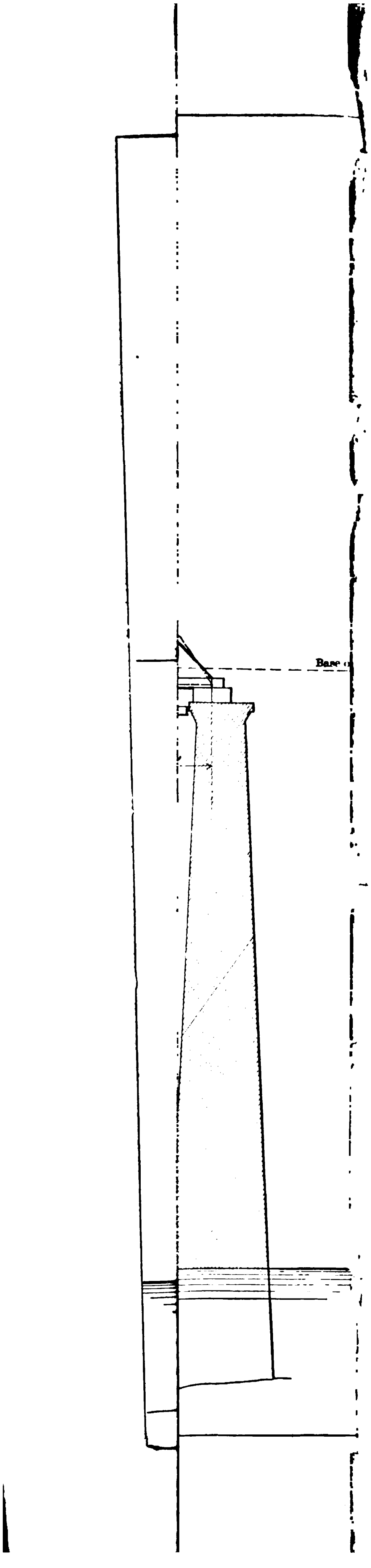
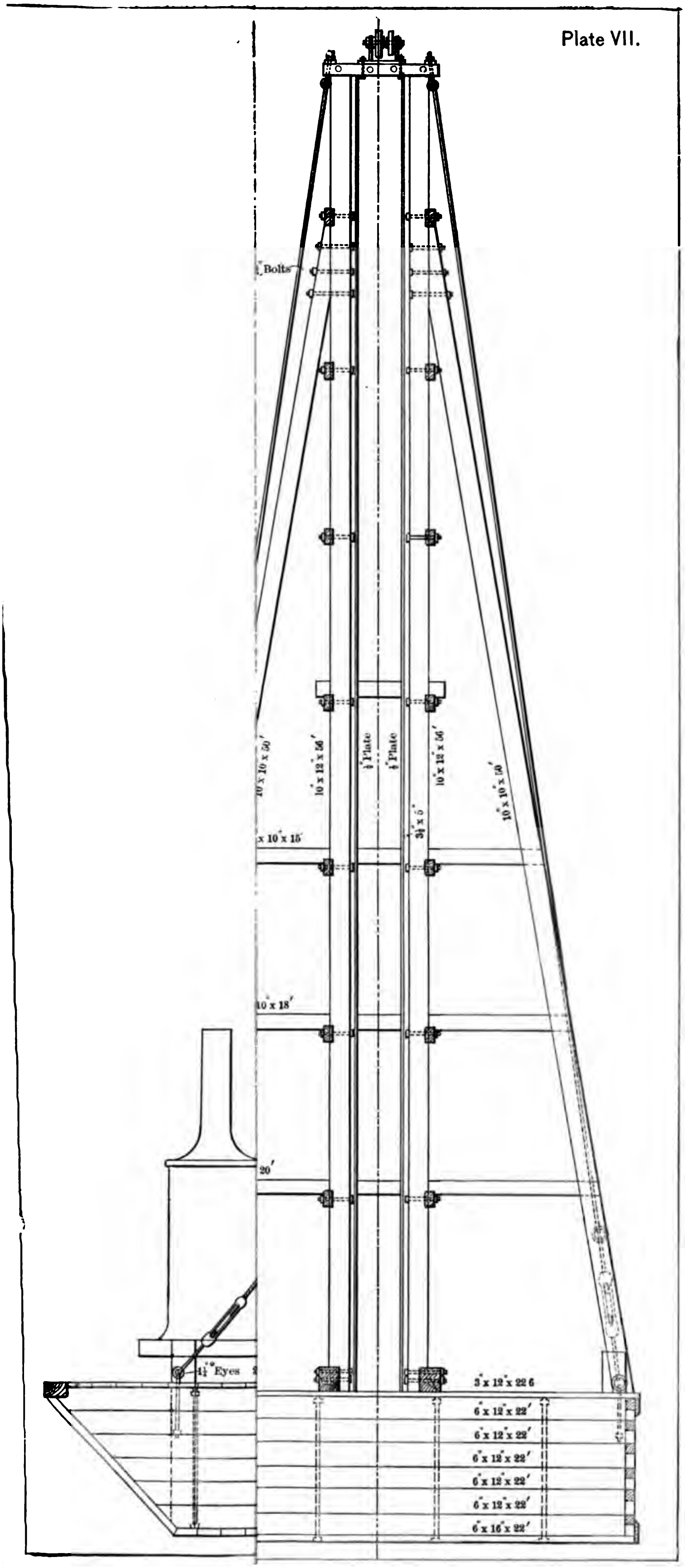
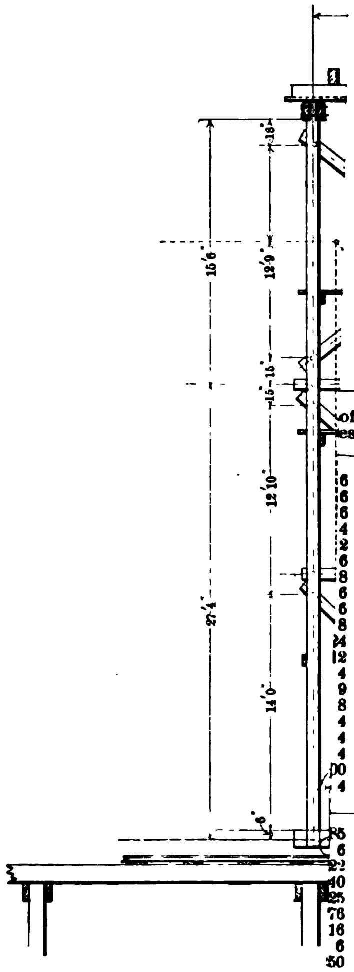

Base

Plate VII.



Bolts
10"x 10"x 50'
10"x 12"x 56'
½" Plate
½" Plate
10"x 12"x 56'
10"x 10"x 50'
3½"x 5'
x 10"x 15'
10"x 18'
20'
1½"φ Eyes
3"x 12"x 22 6
6"x 12"x 22'
6"x 12"x 22'
6"x 12"x 22'
6"x 12"x 22'
6"x 12"x 22'
6"x 16"x 22'

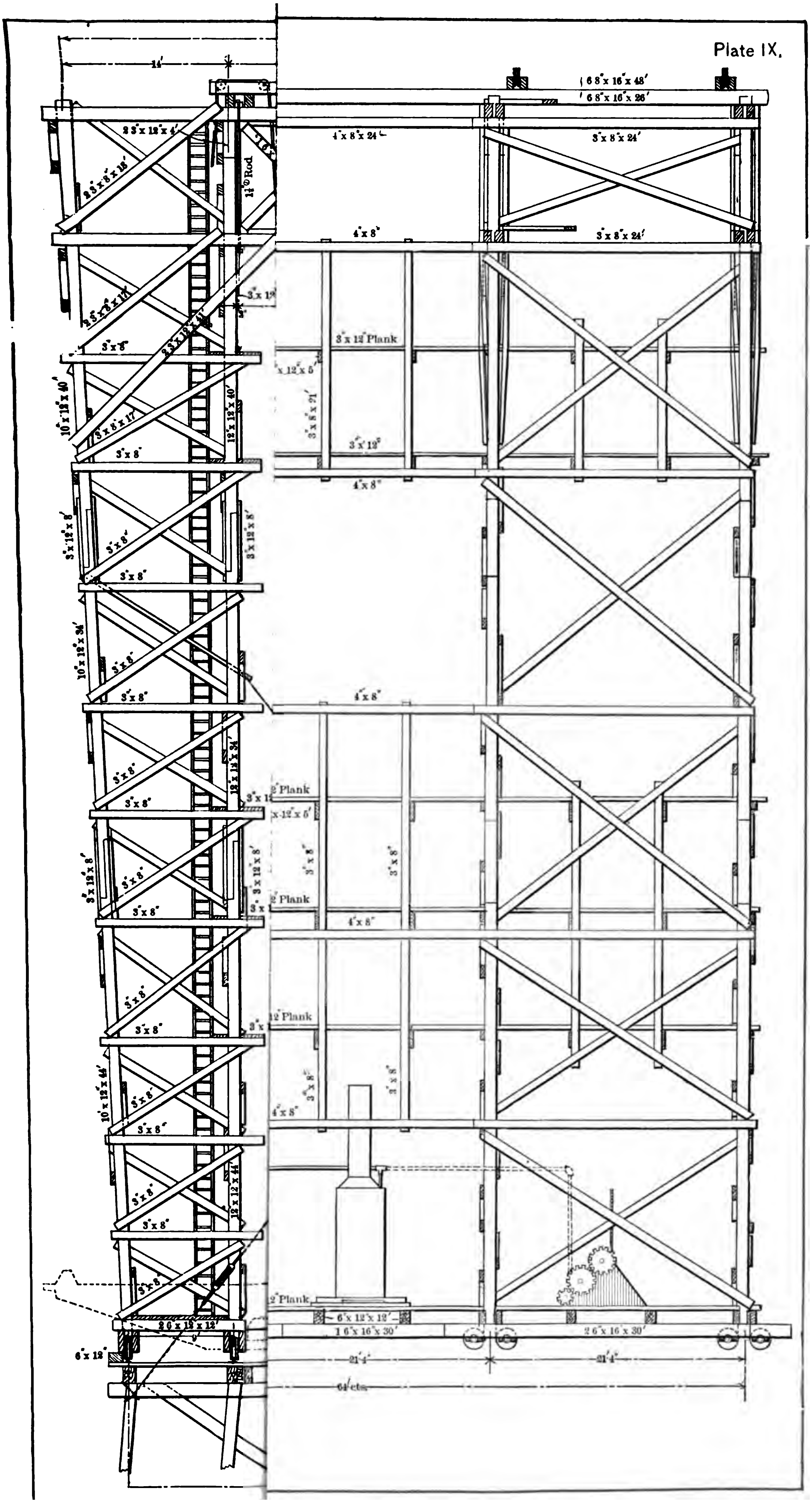


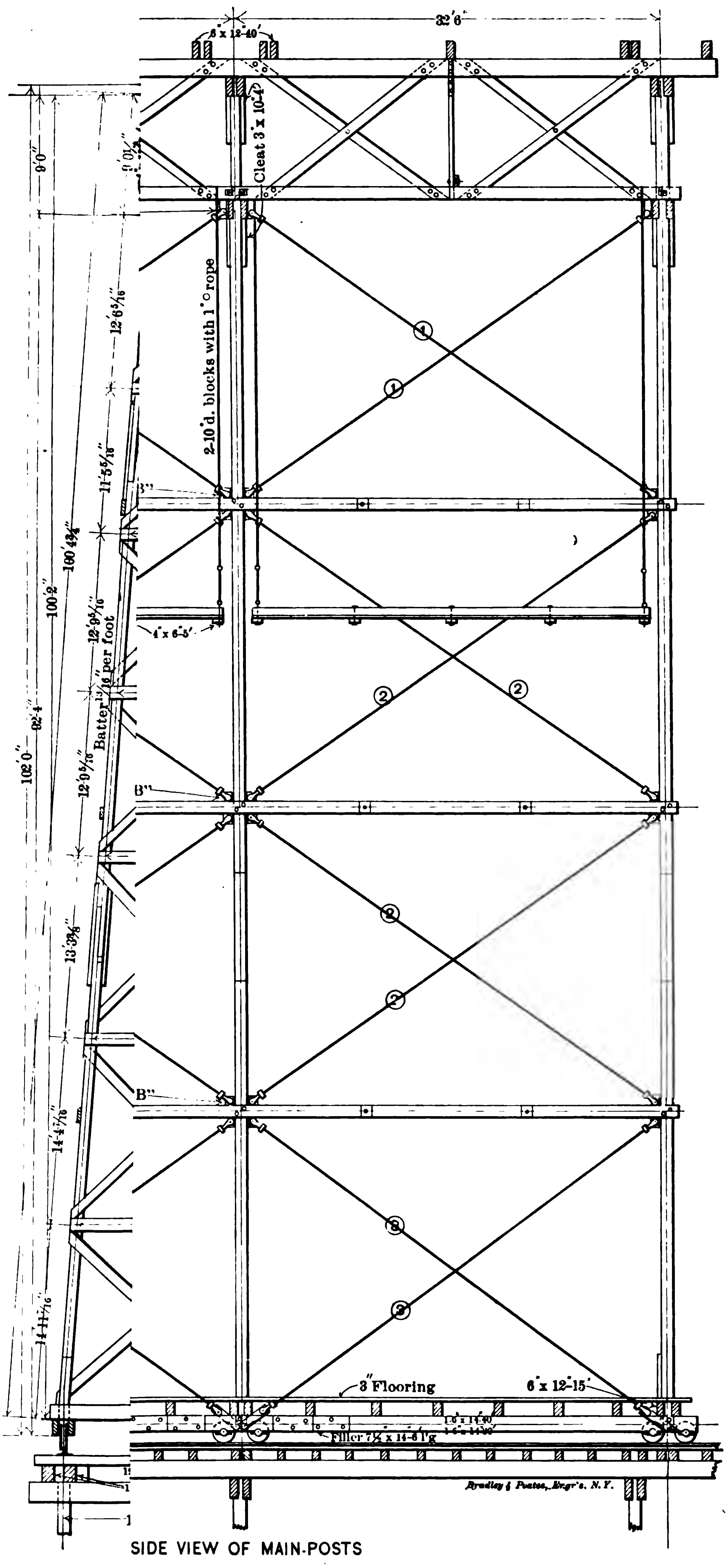
BILL OF MATERIAL.

No. of Pces.	Description.	Size.	Length.
6	Legs	8″ × 8″	45′ 0″
6	Braces	4 × 8	45 0
6	Cap	4 × 14	38 6
4	Sill	3 × 12	35 0
2	Braces	3 × 8	18 6
6	"	3 × 8	14 0
8	"	3 × 8	8 0
6	"	3 × 8	9 0
6	"	3 × 8	10 0
8	"	3 × 8	6 0
24	"	2 × 6	8 0
12	"	2 × 6	5 0
4	Stringer	6 × 12	36 0
9	Staging plank	3 × 12	36 0
8	Brace	3 × 8	22 0
4	"	3 × 8	34 0
4	"	4 × 8	42 0
4	"	3 × 8	23 0
00	Lin. ft.	1½ × 3	Ladder pcs.
4	Ladder-pcs.	2 × 6	23′ 0″
	Total, 8,547 ft. B.M.		
85	Bolt	¾″ dia.	12½″
6	"	" "	15½
22	"	" "	16½
40	"	" "	18½
25	"	" "	14½
76	"	" "	11½
16	"	⅝″ "	11½
6	"	⅝″ "	27½
50	Washer	For ¾″	Bolt
4	Traveler wheels and boxes		

2 Pieces 6″ × 12″ — 28′ 0″
4 " 3″ × 8″ — 18′ 0″

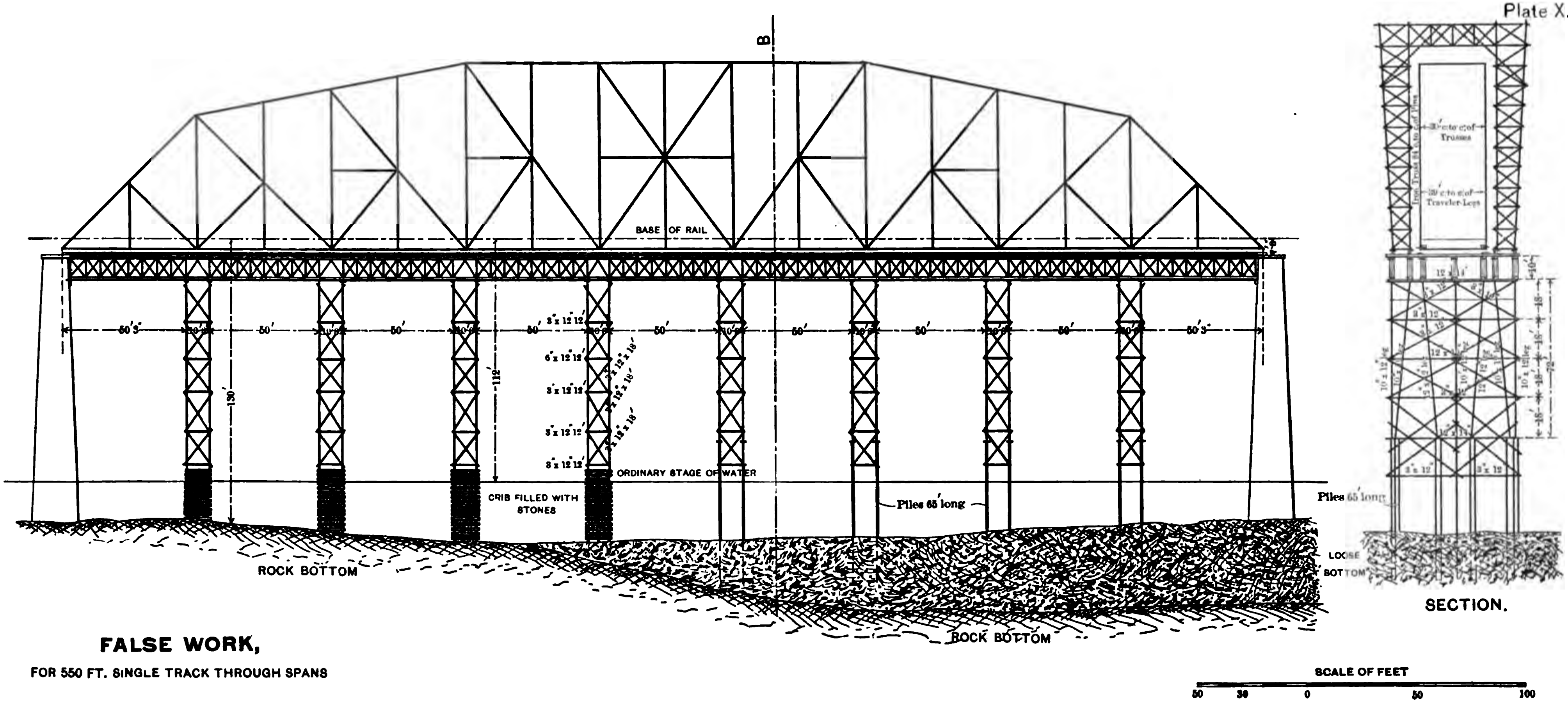
Plate IX.





SIDE VIEW OF MAIN-POSTS

Plate X.



FALSE WORK,

FOR 550 FT. SINGLE TRACK THROUGH SPANS

Plate XI.

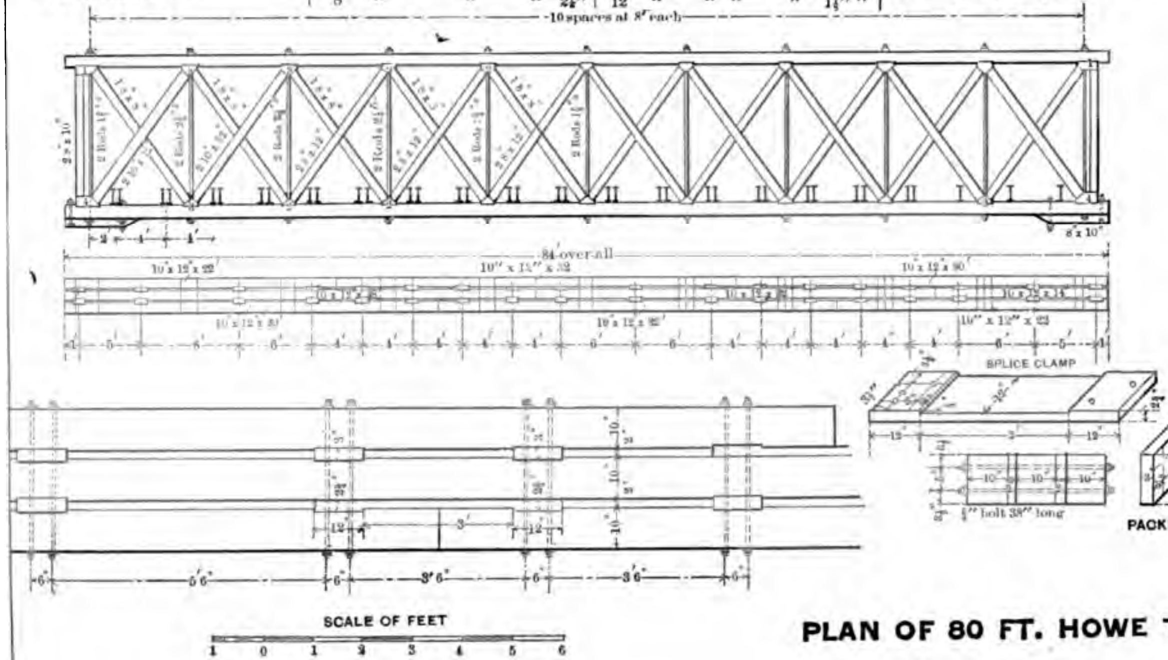
BILL OF MATERIAL.

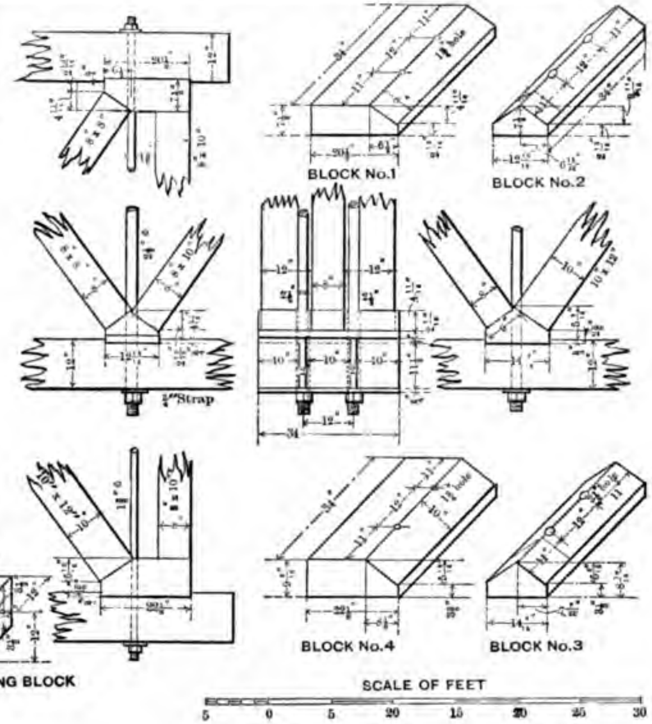
4 pcs.	10 x 12″	38′	
12 "	"	32′	
8 "	"	30′	
8 "	"	22′	
4 "	"	14′	
12 "	8 x 10″	6′	
24 Splice Clamps.			
96 Packing Blocks.			

16 Braces,	10 x 12″		13′
24 "	8 x 12″		13′
20 "	8 x 8″		13′
8 "	8 x 10″		10′
4 Blocks,	No. 1,	holes	1¼″
4 "	No. 4,	"	"
12 "	No. 3,	"	3¼″
4 "	No. 2,	"	3″
8 "	"	"	2¾″
8 "	"	"	2½″

4 Blocks,	No. 2,	holes	1¼″
8 Upset Rods,	1½″ di.		13′ 6″
4 " "	" "		13′ 8″
8 " "	1¾″ "		13′ 8″
8 " "	2¼″ "		13′ 9″
8 " "	2½″ "		13′ 10″
8 " "	2¾″ "		13′ 10″
16 Straps, 6 x ¾ x 34″	holes		3¼″ di.
16 " " "	"		2¾″ "
12 " " "	"		1¾″ "

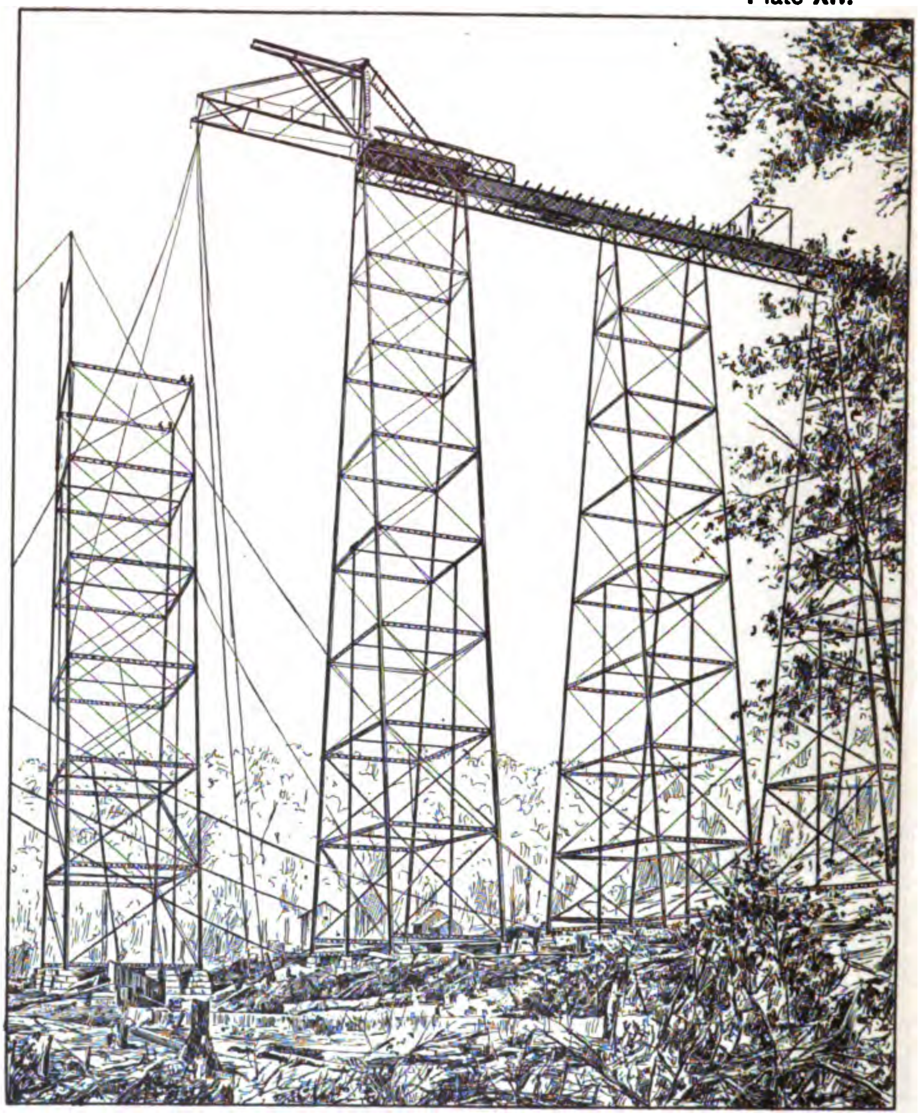
150 Bolts,	¾″ diam.	38″
24 "	"	24″
20 "	"	36″
390 Washers,	⅞″ hole,	2½ x ⅜″
34 Steel I-beams,	15″-150	44′
6 " "	"	"
FLOORBEAM SUSPENDERS.		
12 Rods,	1¼″ diam.	3′ 4″
24 Washers,	6″ square.	





PLAN OF 80 FT. HOWE TRUSS.

Plate XII.



KINZUA VIADUCT, PENNSYLVANIA.
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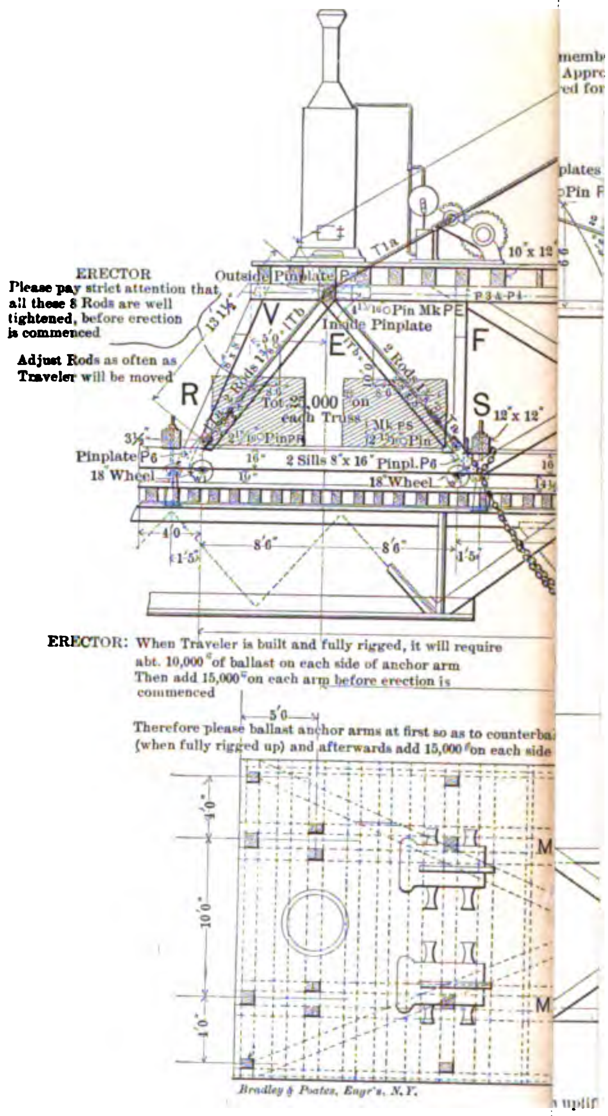

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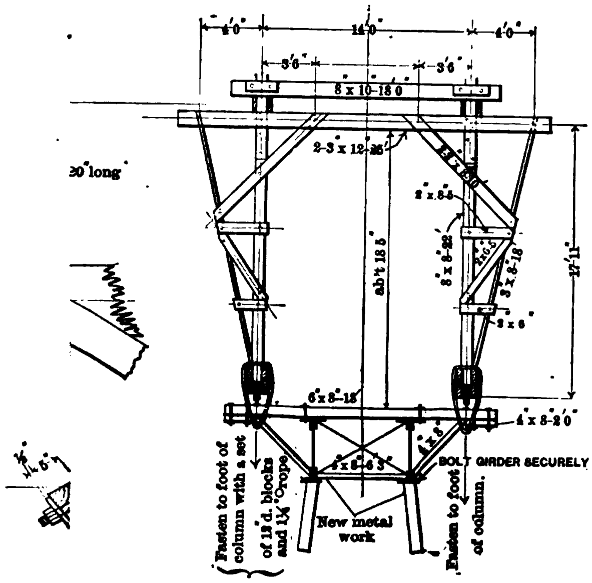


CROSS-SECTION OF UPRIGHT BENT

TOP TRAVELER FOR SMALL VIADUCTS

PLATE XII-B.

BILL OF MATERIAL.

No. of Pieces.	Description.	Size.	Length.	
4	Top chord pcs.	4″ × 14″	31′ 0″	
4	" " "	4 × 14	38 0	
4	Caps	3 × 12	24 0	
4	Posts	8 × 8	21 0	
2	Fillers	6 × 14	12 0	
2	Booms	8 × 10	41 0	
4	Braces	3 × 8	18 0	
1	Beam	8 × 10	18 0	
4	"	4 × 14	18 0	
4	Sills	5 × 12	35 0	
4	"	5 × 12	7 0	
2	Laterals	4 × 8	38 0	
2	Braces	6 × 8	34 0	
1	Beam	6 × 8	36 0	
11	Braces	2 × 8	10 0	
4	"	2 × 8	20 0	
4	"	2 × 6	5 0	
4	"	4 × 8	10 0	
30	Ties	6 × 8	18 0	3,360 ft. B.M.
50	Braces	4 × 8	9 0	
	Total ft. B. M. 8,800			
8	Bolts	¾″ dia.	30″	
6	"	" "	26	
6	"	" "	24	
12	"	" "	20	
38	"	" "	18	
104	"	" "	16	
22	"	" "	12	
50	"	" "	14	
2	"	" "	8	
500	Washers			
4	Rods	1¼″ "	38′ 6″	
2	Washers	¾ × 8″	24″	
2	"	¾ × 8″	20″	
6	Traveler wheels and bearings			
50	Hook bolts	¾″ dia.	10″ to 12″	

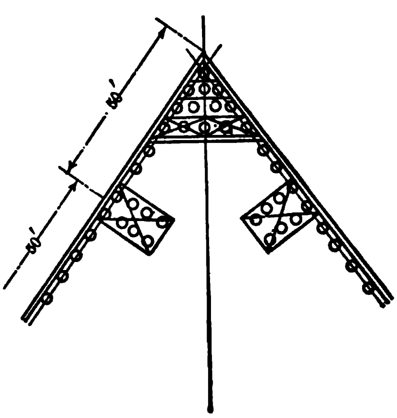
to raise and handle these long spans; the erection of the spans themselves does not differ, in plan, from that described under the head of smaller spans; erection being begun at the centre panel and working first toward the "fixed" end and finishing with the "roller" end.

SPANS, 300' TO 600' LONG.

For the very longest spans, where the trusses run up to 85' and more in depth, the traveller becomes a very important and expensive part of the erection plant. Usually it is of four bents, with its upper bracing a Howe truss 11' deep (see Plate IX.). Such a traveller, when fully rigged for work, has four hoisting engines, of the type shown in Sketch 4, with two boilers; lines from these engines run to snatch blocks fastened to the lower platforms, and from there to the top of the traveller, passing through sheaves in twin beams resting on the stringers which bear directly on the Howe trusses, and directly over the new trusses. Fifty sets of blocks are required, varying in size from the heaviest 16" triple to 8" single blocks; some 35 coils of rope are needed, varying from $1\frac{1}{4}''$ to $\frac{3}{4}''$. Such a traveller contains 75,000 feet B. M. of lumber, and requires twelve days to frame and erect, and three days to rig complete. In cases where long spans are to be erected on very high false works, in localities subject to high winds, it is best to give more width at the top of false work and widen the traveller correspondingly; having the narrowest part of traveller at the top instead of at the bottom, as is usually the case. (See Plate IX*a*.) It is unnecessary to point out the great increased stability of both false work and traveller in this design. However, only in special cases mentioned would the extra expense involved be warranted.

The false work for these extremely long spans is similar to those already described, where the same height and character of bottom is found; these spans, however, are generally designed for the crossing of important streams, often those subject to sudden and heavy rises, and for such cases it is advisable to still further increase the unbraced open spans, and keep the braced towers comparatively narrow. A usual division is to make towers of about 11' and adjoining openings 50'. (See Plate X.)

These towers are formed first of piles, driven as previously described, and capped and braced; upon these the bents of the false work are erected and braced, and on top of the caps the necessary trusses or stringers, the long spans being Howe trusses about 14' deep. (See Plate XI., showing a design of heavy Howe truss.) These trusses for false work weigh probably six tons, and can be framed and connected together and launched into position by an "upper" traveller with overhanging boom rigged for the purpose. This traveller should also be provided with a boom sufficiently long to reach over the tower in advance, so that it can be erected by hoisting directly from the traveller. In case there is not sufficient material over the rock in which piles may be driven, it is necessary to build a crib of rough timbers and fill the same with stones until it rests firmly on the bottom; and on this temporary pier erect the towers to carry spans. (See also Plate X.) In those localities where the stream is not only subject to sudden rises, but also to a heavy run of drift, the false work is not only liable to be carried away by the heavy pressure against it, but is subject to the greater danger of being scoured out, or literally washed out, by having drift accumulate against it; and after packing nearly to the bottom of the stream, the rush of water underneath scours out the false work until it is so undermined and weakened that it fails.



To prevent this, a "protection" or boom should be placed above the false work, of V shape, and having the point of the V at least the full length of the span above the centre of the opening, giving the sides thereby a good slope. This protection is best made of piles, well

driven, with centres about 4′ apart, and of sufficient length to be above water during a heavy rise; these piles are connected on the outside by horizontal pieces of 6″ × 8″, with spaces between each row of about 6 inches. At points about 50′ apart a group of 8 or 10 piles should extend on the inside of the V and be thoroughly braced together, giving great strength to the protection to resist the pressure of water and drift. Such a protection, in plan, would appear like the sketch on preceeding page.

The false work of the centre span of the Ohio River bridge at Cincinnati, built in 1888, and erected on piles driven into the bottom 18′ to 20′, and the whole work being of the most substantial character, sustained successfully a flood and depth of water of 45′, and a most unusual run of heavy drift lasting for three days, the drift having accumulated during this time in a solid triangle above and against the false work, extending to over 500′ above the bridge. This drift was nearly solid to the bottom of river. Even with this tremendous pressure the false work showed no signs of yielding, and not until the upper line of piles was completely undermined by scouring out, as was afterwards plainly shown, to a depth of 12′, did the false work yield and collapse, as it did on August 26, 1888, falling *up stream.* When the false work was again put up, as was done immediately, it was protected in the manner above described, and this protection withstood without sign of weakness two floods of 48′ of water, and while the run of drift was not very heavy, it is believed, however, it would have stood equally well the heaviest drift, as the sloping sides would not allow it to accumulate and start scouring. It is not necessary to further discuss the erection of the iron-work of these long spans; the plan pursued is precisely similar to that described for smaller spans, as given earlier in this chapter; the long spans simply demanding extra-heavy rigging, and care in the handling of the enormous pieces required in their construction; some of the members weighing 40,000 pounds.

DRAW SPANS.

The erection of draw spans presents no new problems, as far as the false work and travellers are concerned, the same conditions of height, length of span, etc., calling for the same method of erection.

Still greater accuracy, however, is demanded in the masonry adjustment and alignment of the span. The upper surface of the lower track, upon which the draw revolves, should be set perfectly level. This is usually secured by imbedding track segments in a composition of iron filings and sal-ammoniac (100 parts filings, 1 part sal-ammoniac), which composition soon sets into a very hard and compact mass. With this track set perfectly level and at the exact elevation below grade, the further erection of the turn-table should give no trouble, as it is all machine work and " iron to iron." Especial care must be taken to set the pinion so that it does not gear too deeply into the gear segments on the track, or hard turning, caused by binding, will be the result. Most of the draws are now designed to carry all the dead load to the centre; therefore, when simply carrying its own weight, the rollers or wedges under the ends of the draw should be so adjusted that they simply come to a bearing. The " locking gear " and shaft operating these rollers or wedges also demands the most careful attention, to see that it is in perfect alignment and adjustment, that there may be no binding. Draws are usually erected upon the " rest " piers, that is, open; the iron-work at the centre is raised first, and from there each way to the ends. All pin-truss draws have an " open joint " in one of the lower chord panels near and on each side of the centre; shimming plates varying in thickness from $\frac{1}{8}''$ to $\frac{3}{4}''$ are provided for insertion in this joint, as the case demands. It is impossible to estimate exactly the deflection of the ends of draw trusses, owing to the inaccuracy of workmanship and other causes, and this deflection is found exactly during the erection, and just sufficient plates

put in this "open joint" to raise or lower the end to its proper elevation. In the case of lattice girder draws or plate girders, where there is no open joint, and the girders are shipped in pieces, the ends should be blocked up *above* their final position from 1″ to 2″, according to length of span, before riveting is commenced, to allow for the deflection when the temporary support is removed.

VIADUCTS.

Under the head of Viaducts we class those structures composed of short spans resting upon bents or towers. These towers are erected by two principal methods: first, by means of gin-poles; second, by a traveller on top, with long, projecting boom. Following the first plan, a gin-pole, about 10 feet longer than the height of each story of the tower, is placed at each corner, and near the position of a vertical column.

These gin-poles are thoroughly guyed, and have a set of blocks lashed to the top. The columns of the tower are hooked to this set of " falls," and hoisted into position; the transverse and longitudinal bracing is put in place between the columns, and the first story is complete.

The next proceeding is to raise all the gin-poles sufficiently, so that the second-story columns and bracing can be hoisted and placed in position; this is done by raising poles to a sufficient height, and clamping them thoroughly to the columns of the story last finished; the second story is then erected, and so on to the top. This plan was used very successfully on the famous Kinzua Ravine Viaduct, near Bradford, Pa., at the time erected the highest in the world, being 306 feet from the bed of the small stream to the base of rail; this viaduct was designed and erected by Clarke, Reeves & Co., now The Phœnix Bridge Company. The viaduct, as before stated, is 306 feet high, 2,052 feet long, and is composed of spans of 60 and 40 feet, each tower containing 4 columns. The structure in course of erection is shown in Plate XII.

If it is proposed to use the *second* plan, by using the upper traveller, both for setting the girders and raising the towers, the traveller must be designed with a horizontal stiff boom sufficiently long to reach from the completed portion of the structure directly over the tower to be raised. (See Plates XII*a.* and XII*b.*) This second plan is probably the cheapest and best plan for most cases. It was used in the erection of the Pecos Viaduct on line of Southern Pacific Railway in Texas. (See Plate XII*a.*) This viaduct is 326 feet high. The traveller used is shown on Plate XII*a*. The traveller shown on Plate XII*b.* is so arranged as to permit traffic to be undisturbed during erection of new structure; the support for extra width required being obtained by using extra length ties, with temporary inclined struts running from lower flange of new girder to end of tie. The only special care to be exercised in the erection of the towers is in the adjustment of the bracing, to see that all columns are carried up plumb and square; this requires the aid of the transit, for high towers, to see that it is accurately done. All the adjustment should be complete, before the final riveting of the bracing or joints is done. After the towers are raised, the girders supporting the track are usually brought in from one end on a " dolly " car, and run out to the traveller at the end of the structure just finished, where they are taken hold of by a set of falls, fastened to the upper overhanging boom of the traveller, and lowered into position.

In some rare cases it may be better to hoist the girders from the bottom of the structure to the top of the tower, and place them on their seats on the columns.

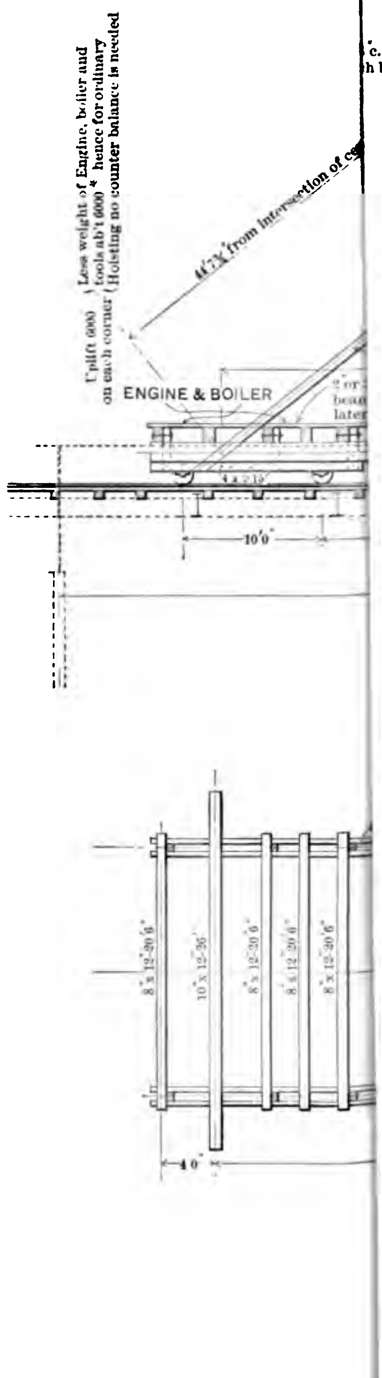
ELEVATED RAILWAYS.

Elevated roads similar to those in New York and Brooklyn come practically under the head of viaducts, as just discussed; but as most of these roads traverse crowded thoroughfares, it is absolutely essential that the streets be obstructed as little as possible; the traveller, therefore, is placed on top, and arranged with booms sufficiently long to reach out and set the columns of the bent ahead, also the transverse girders and bracing, and then hoist and place the longitudinal girders in place. If, however, the streets are not so important, and can be more or less blocked, a traveller designed to run on sills on the ground and spanning the structure is probably the more economical plan to pursue, and no doubt the iron can be thus placed in position more rapidly. The traveller, with its engine, boiler, and rigging complete, is necessarily an expensive piece of machinery, and it should only be employed in raising and placing the larger and more important members of the structure, only sufficient bracing being put in to enable the traveller to be run out safely on the completed structure, or permit it to move ahead, if it is designed to run on the ground. (See Plate XIII. for ground traveller.) The remaining part of the work, such as the bracing, ties, guards, and rails for the track, can be raised with a much simpler arrangement; even a common gin-pole, with a set of blocks lashed to the top, answering the purpose. At least 150 feet of elevated railway structure should be erected complete each day of ten hours, raising on the above plans, where the spans are about 40 feet, and the columns not over one story, or 20 feet high. (See Plate XIV. and XIV*b*. for viaduct top traveller.)

TRAIN SHEDS, ROOFS, ETC.

The erection of ordinary roof trusses is a much more simple problem than that of railroad structures, owing principally to the fact that they contain very much lighter pieces to handle and connect. For roofs up to 50-feet span the trusses are usually riveted together on the ground, and after the columns supporting the same have been placed in position at the proper points, the truss is hoisted bodily, and placed on columns by means of a pole placed on the centre line of the building, and extending 10 or 15 feet above the highest point of the truss; the top of the pole being well guyed in four directions, and having a set of falls fastened to it.

As the trusses increase in length they soon become so heavy that it is best to use two poles, one for each side. Only assemble one-half of each truss on the ground, hoist the same into position, supporting each half with poles until the centre connections are made, thus forming one complete truss. When the span reaches 100', or more, it is generally found better to design the roof trusses with pin-connections, in which case it is necessary to put a couple of bents of false work in, to temporarily support the trusses while connections are being made. For train sheds of extra long span, say 250 ft. to 300 ft., two- and three-hinged arches are generally used, and in this case more elaborate erection plant is necessary. First, the arch must be supported until finally connected, and movable traveller must be so placed as to command the arch throughout its entire length. The plant used in erecting the trusses of the Reading Terminal Train Shed in Philadelphia is shown on Plates XV. and XV*a*. It will be noticed, false work is placed on sills supported by wheels having free movement longitudinally, so that it can be transferred easily from arch to arch as fast as erected; while the movable traveller rests on lower sills of this false work, and is arranged to move transversely, to pick up and put in place material at any point of the arch.

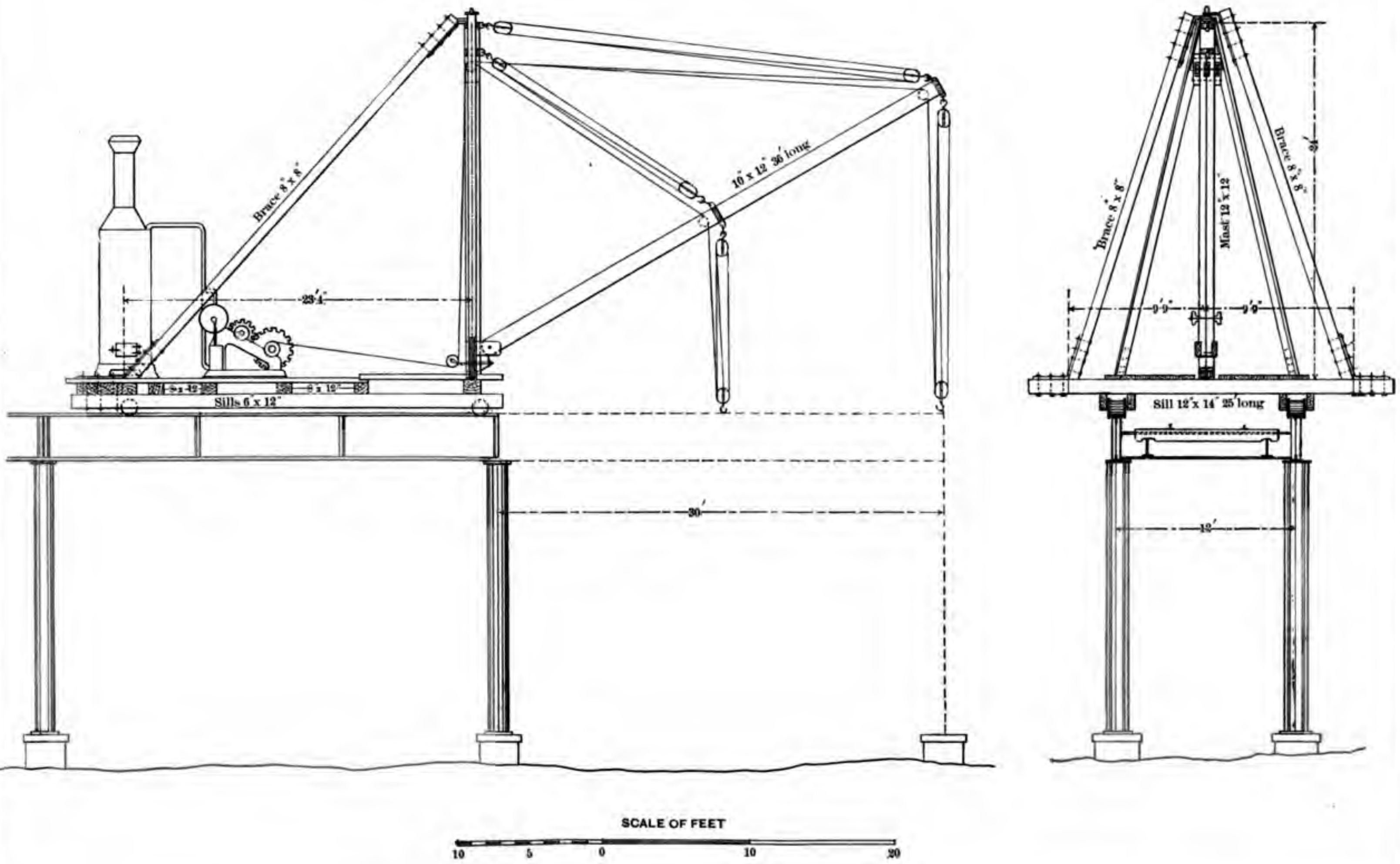


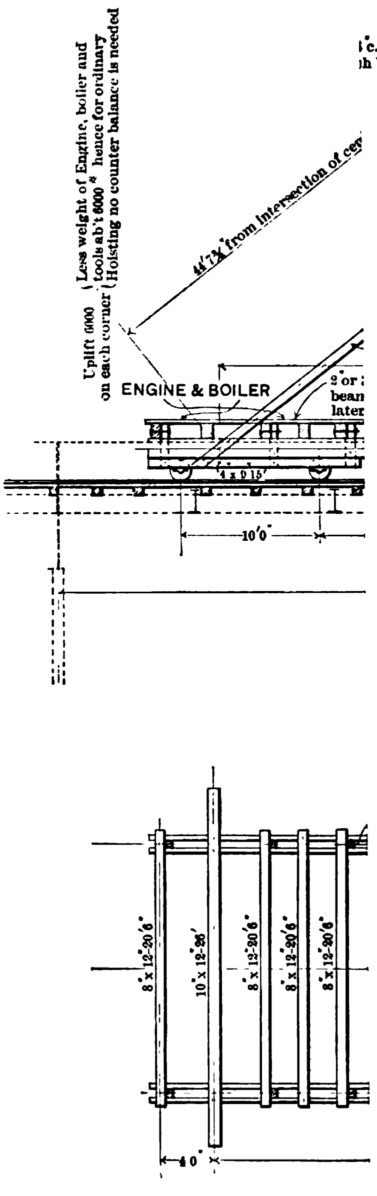
BILL OF MATERIAL.

No. of Pieces.	Description.	Size.	Length.
2	Masts	12″ × 12″	24′ 0″
4	Boom-pieces	4 × 14	55 0
2	Fillers	2 × 14	5 0
2	Posts	8 × 10	25 6
1	Cap	10 × 12	28 0
2	Sills	8 × 12	22 0
6	Sills	8 × 12	20 6
1	Sill	10 × 12	26 0
8	Braces	3 × 10	20 0
2	Braces	8 × 10	16 6
2	Stiff legs	8 × 8	46 0
2	Braces	4 × 8	18 0
1	Brace	4 × 6	19 0
4	Sills	4 × 16	43 0
4	Sill-pieces	4 × 9	15 0
4	Sill-pieces	4 × 9	5 0
800	Lin. ft. plank	3 × 12	
1	Cross-beam	8 × 8	22 0
8	Posts	6 × 8	4 0
6	Braces	4 × 8	8 0
10	Fillers	6 × 8	2 6
	About 9000 Ft. B.M.		
50	Bolts	¾″ dia.	10½″
40	"	" "	12
40	"	" "	14
15	"	" "	16
75	"	" "	17
40	"	" "	18
10	"	" "	20
10	"	" "	22
10	"	" "	23
10	"	" "	26
10	"	" "	29
10	"	" "	30
16	"	" "	5½
8	"	⅞″ "	13
8	"	1″ "	14½
4	Eye-bolts	1″ "	16
2	Bolts	⅞″ "	18
2	Pins	1″ "	21
2	"	" "	17½
2	"	" "	16½
2	"	" "	9½
2	"	" "	18
2	"	2″ "	19
2	Mast-pins	2½″ "	3′ 6″
2	Mast-rings	4″ × ⅝″	1″ link
2	" "	3 × ½	
2	Stiff-leg iron	1 × 7	45½″
2	Plates	1 × 7	15
4	Cap-plates	½ × 10	20
2	Boom-rings	¾ × 4	2 1″ links
4	Boom-plates	½ × 14	15″
4	" "	¼ × 14	12
2	1¼″ links	11″ dia.	
2	Clevises	1¼″ "	10″
2	Cast derrick	Bases	
10	Cast sheaves	9″ dia.	
2	" "	6″ "	
20	Sheave plates	½″ × 3″	20″
6	Traveler wheels 18″ diam.		
12	Boxes for these wheels		

TOP TRAVELER,

FOR VIADUCTS AND ELEVATED RAILWAYS





BILL OF MATERIAL.

No. of Pieces.	Description.	Size.	Length.
2	Masts	12″ × 12″	24′ 0″
4	Boom-pieces	4 × 14	55 0
2	Fillers	2 × 14	5 0
2	Posts	8 × 10	25 6
1	Cap	10 × 12	28 0
2	Sills	8 × 12	22 0
6	Sills	8 × 12	20 6
1	Sill	10 × 12	26 0
8	Braces	3 × 10	20 0
2	Braces	3 × 10	16 6
2	Stiff legs	8 × 8	46 0
2	Braces	4 × 8	18 0
1	Brace	4 × 6	19 0
4	Sills	4 × 16	43 0
4	Sill-pieces	4 × 9	15 0
4	Sill-pieces	4 × 9	5 0
800	Lin. ft. plank	3 × 12	
1	Cross-beam	8 × 8	22 0
3	Posts	6 × 8	4 0
6	Braces	4 × 8	8 0
10	Fillers	6 × 8	2 6
	About 9000 Ft. B.M.		
50	Bolts	¾″ dia.	10½″
40	"	" "	12
40	"	" "	14
15	"	" "	16
75	"	" "	17
40	"	" "	18
10	"	" "	20
10	"	" "	22
10	"	" "	23
10	"	" "	26
10	"	" "	29
10	"	" "	30
16	"	" "	5½
8	"	⅞″ "	13
8	"	1″ "	14½
4	Eye-bolts	1″ "	16
2	Bolts	⅞″ "	18
2	Pins	1″ "	21
2	"	" "	17½
2	"	" "	16½
2	"	" "	9½
2	"	" "	18
2	"	2″ "	19
2	Mast-pins	2½″ "	3′ 6″
2	Mast-rings	4″ × ¾″	1″ link
2	" "	3 × ½	
2	Stiff-leg iron	1 × 7	45½″
2	Plates	1 × 7	15
4	Cap-plates	½ × 10	20
2	Boom-rings	¾ × 4	2 1″ links
4	Boom-plates	½ × 14	15″
4	" "	½ × 14	12
2	1¼″ links	11″ dia.	
2	Clevises	1¼″ "	10″
2	Cast derrick	Bases	
10	Cast sheaves	9″ dia.	
2	" "	6″ "	
20	Sheave plates	½″ × 3″	20″
6	Traveler wheels 18″ diam.		
12	Boxes for these wheels		

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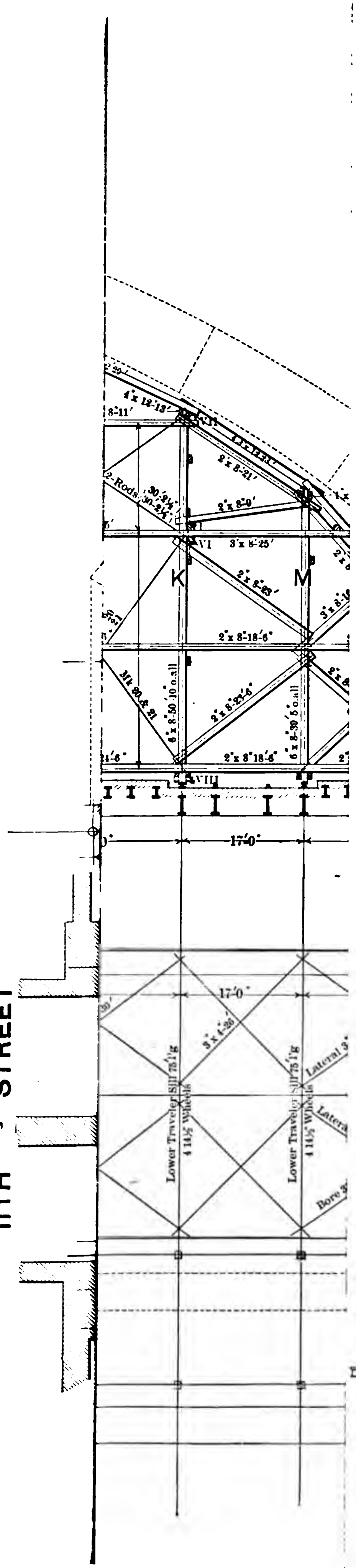
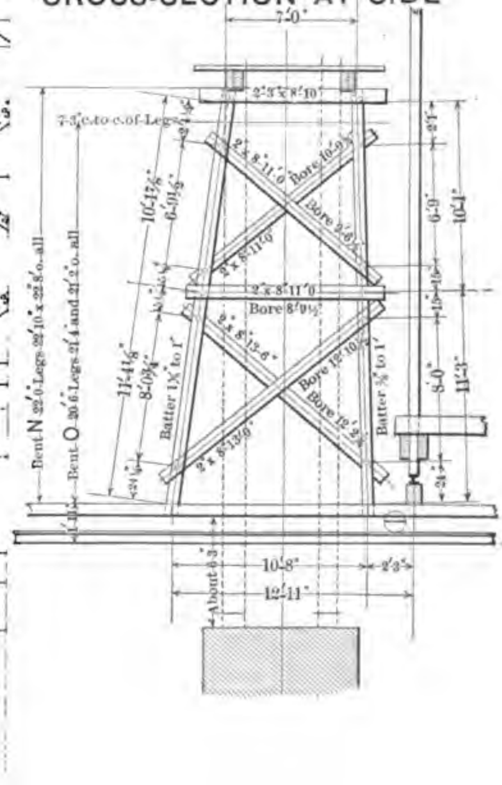
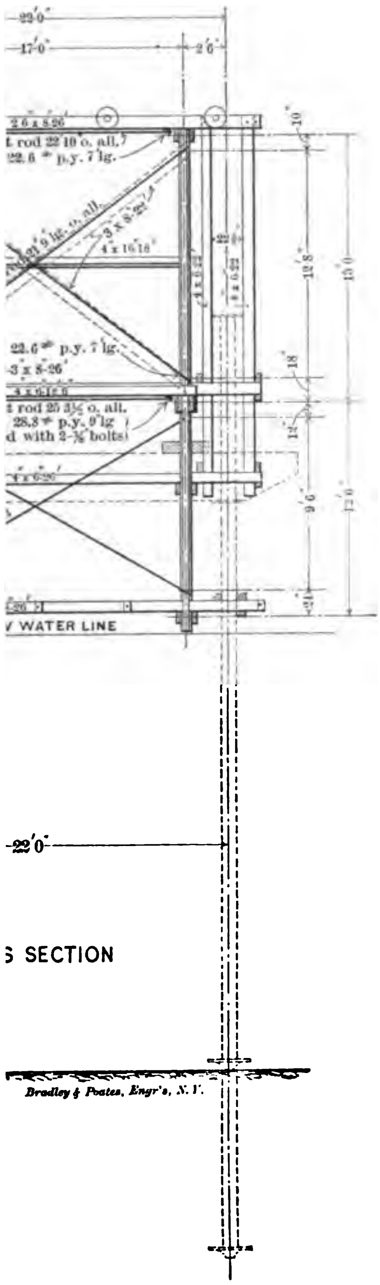
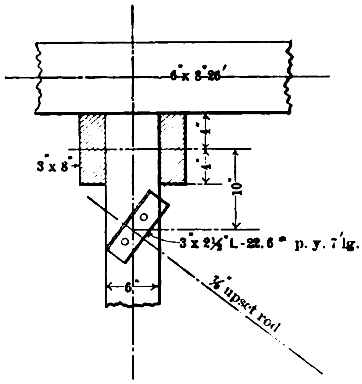


CROSS-SECTION AT SIDE







BILL OF MATERIAL.

4 Hammer-guides 4″ × 6″—22′.

2	Posts	6″ × 8″	17′ 0″	2	Rods	1″ dia. upset	33′ 2″
2	Braces	3 × 8	22 0	2	"	" " "	25 8¼
4	Sills	4 × 10	33 0	2	"	⅞″ " "	21 9
4	"	3 × 10	24 0	2	"	" " "	20 0
4	Upper chords	3 × 8	23 0	2	"	" " "	22 10
2	Stiff legs	6 × 6	32 0	4	Washers	4″ square	¼″ thick
2	Cross-pieces	4 × 6	18 0	4	∠s 3″ × 3″	28.8 lbs. p. yd.	9″
1	"	4 × 6	18 6	12	∠s 3″ × 2½″	22.6 " "	7
2	Braces	4 × 6	9 0	4	Sheaves	12″ diam.	
2	"	4 × 6	16 0	6	Traveler wheels		
2	"	4 × 6	17 0	10	Bolts	¾″ diam.	11″
4	"	4 × 6	26 0	20	"	" "	12
2	"	6 × 8	25 0	10	"	" "	13
2	Cross-beams	6 × 8	26 0	100	"	" "	14
4	Braces	2 × 8	24 0	40	"	" "	16
4	Cross-pieces	3 × 8	26 0	10	"	" "	18
2	Posts	6 × 6	29 0	10	"	" "	20
4	Braces	3 × 8	9 0	10	"	" "	25
4	"	3 × 8	5 0	400	Washers		
2	Fillers	6 × 10	4 0	4	Bolts	⅞″ "	12
4	Cross-beams	8 × 10	19 0	4	"	" "	10
1	"	6 × 8	19 0	4	"	1″ "	19
8	Cross-pieces	4 × 6	4 6	2	Pile-driver hammers about 1800 lbs. each.		
10	"	3 × 4	3 6				

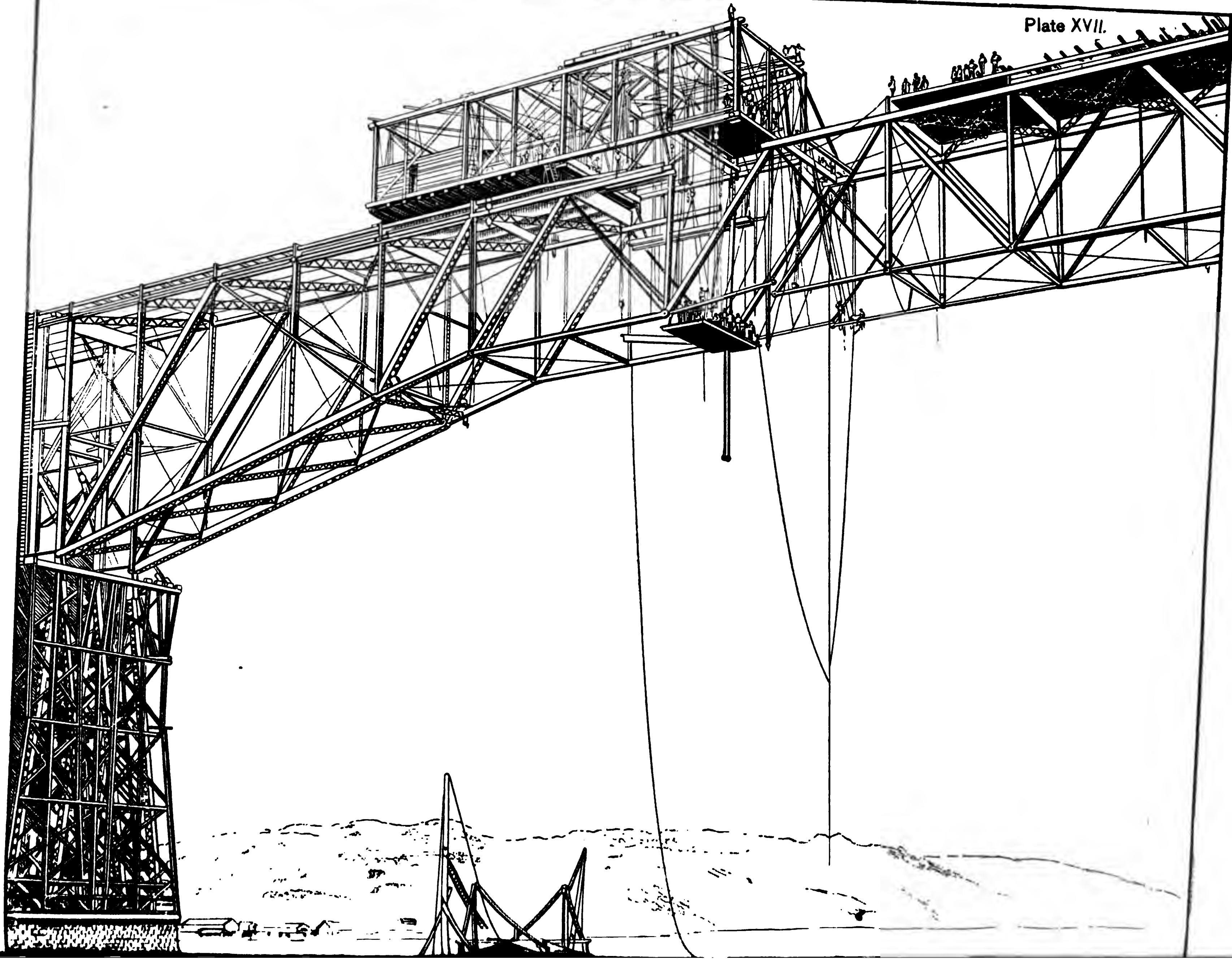
TRAVELER

ATE XVI

the upper and lower systems of lateral bracing; second, "Deck" truss cantilevers, or those in which the live load is applied to the floor above or in the same plane as the upper lateral bracing. As notable examples of the latter may be mentioned the Niagara bridge of the Michigan Central Railway, and the very fine structure just finished spanning the Hudson River at Poughkeepsie, N. Y. As examples of the former may be mentioned the bridge over the Ohio River between Louisville and New Albany, Ind., and the longest span in this country, now in course of erection (1890) at "Red Rock," California, over the Colorado River, on the line of the Atlantic and Pacific Railway, it being 660 feet centre to centre of piers. The renowned "Forth Bridge" in Scotland is also a cantilever structure of this style, but the magnitude of the work (span being over 1,700 feet) demanded such treatment, being literally built piece by piece in place, that it cannot be cited or described as illustrating any general or practical mode of erection.

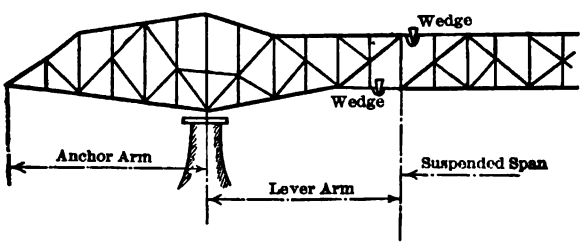
The "Deck" cantilever structures, presenting as they do the least difficulties in erection, will first be considered. The false work to temporarily support the anchor arm of the structure is first put in place, using the same plan and methods, according to the various conditions of height, water, and character of bottom, as have been given under the head of ordinary truss spans, previously considered. The pedestals and feet are then set on the pier with the greatest care, both as regards elevation and lateral position, all being done under the immediate supervision of the engineer, and set to his marks and centres. From the centre of the pin in this foot as a starting-point, the lower chord is lined out and connected to bars in the anchor pier; it is assumed that the anchorage, including the necessary eye-bars to connect with the trusses, has been put in place during the construction of the pier. The traveller to erect the trusses is of a pattern described earlier in this chapter, runs on the regular track, and is so designed that the overhanging portion projects nearly two panels ahead of that part of the structure connected. After the chord has been lined out from the pier to the anchorage, the pieces joined together by the end anchorage pin are then hoisted and held in place from the traveller while the pin is driven; the bars and members thus connected are hung to the traveller, while the first panel of the upper chord is lowered into position over the posts, and the upper ends of the bars just connected in the lower chord are hoisted to their proper upper panel point and the pins driven. This completes one panel of the truss. The floor beams, usually set directly upon the chord at the panel points, are then put on and the first panel of the stringers placed in position, the horizontal and transverse laterals connected, sufficient wooden ties and rails put on the stringers, and the traveller can then be run ahead one panel and the erection proceeded with in exactly the same manner, panel by panel, to the pier. It may be necessary to support the loose ends of partly connected members to the traveller while it is moved ahead; this can be done by a proper arrangement of the supports. It will also be noticed that it is necessary to design the details so that the chord splices occur near the panel points, but on the side away from the traveller; otherwise the traveller would have to reach out nearly three panels. After the anchor arm is complete the erection of the lever arm continues, panel by panel, in the same manner, only there is no false work under it, none being necessary, as it, with the suspended span, is held up in position by the dead weight of the anchor arm and the masonry of the anchorage. The erection of this lever arm presents no new problems or difficulties until we reach the panel between the lever arm and the suspended span. In this panel large and powerful vertical wedges are placed on line of both upper and lower chords. These wedges are for two purposes: first, to raise or lower the centre of the suspended span to facilitate the final connections; second, to shorten or lengthen the distance between the centre of the suspended span and the end pin of the lever arm, or, in other words, to increase or to

Plate XVII.



diminish the length of iron-work between piers. This is found to be absolutely necessary, for, even after exercising the greatest care in the triangulation of the span lengths and in the placing of the pedestals and shoes on the piers, the distance c. to c. is liable to vary considerably. It will also be seen upon reflection how necessary it is to have the adjustment vertically, for with the traveller, tools, and men on the centre of the suspended span, it would be impossible to figure the deflection to the nicety required to make the final connections. These wedges work against frames built in the members composing the upper and lower chords, and are worked by means of heavy, powerful screws, passing directly through the wedges vertically, the nuts of which screws bear on an independent frame. The pins connecting the sections of these chord panels at the wedges pass through oblong holes in the main members, permitting the lengthening or shortening of the panel.

Upon examination of the diagram it will be seen that the province of the upper wedge is simply to raise or lower the centre of the span, as it cannot possibly lengthen or shorten the distance between the end pins; while the movement of the wedge in the lower chord panels affects both the elevation of the centre and the length between the end pins. As previously stated, the erection of the lever arm is proceeded with precisely as for the anchor arm, each panel being supported by that portion of the structure previously erected. This also applies to the panels connecting the lever arm and the suspended span and the suspended span itself, and with the elevation and length of span in our control, to cover any discrepancy in the figured length or deflection, no difficulty should be experienced in the final connection. After this connection is made, the wedges should be removed, allowing the suspended span to hang by the vertical bars to the lever arm, and free to lengthen or shorten, by means of the oblong holes around the pins, for variations of temperature and loading. Before the erection of the suspended span is begun, it is advisable to so adjust the wedges that no raising of the centre will be necessary, as it is a much easier matter to lower than to raise.



There are no peculiar points to watch during the erection, not already covered in this discussion; of course, the foreman will see that his traveller is well anchored down when lifting heavy pieces, or when it has great weight hanging to it; he will also watch particularly the alignment of his trusses as the erection proceeds, and have ready means at hand for lashing the traveller down in case of high winds, as it is in a peculiarly exposed position, and without the convenience of false work to guy to. If the structure is so situated, it is better, and at times cheaper, to hoist the iron directly from boats below and place it in position; if this is not possible, it will be run out, on top, from the bank on a separate car, to the traveller, where it can be reached with a set of "falls" and lowered into position. Swinging platforms are hung from the traveller at convenient points for the men to work at the connections, driving pins, etc. Plate XVII. shows the Poughkeepsie Bridge in course of erection, including the traveller, wedges, etc., and when ready to connect the centre and last panel. It is advisable to place the hoisting engine and boiler directly on the traveller, near the rear end, as it is convenient for hoisting material, and the weight of machinery, etc., is just what is needed to help counterbalance the loaded overhanging portion of the traveller.

THROUGH CANTILEVERS.

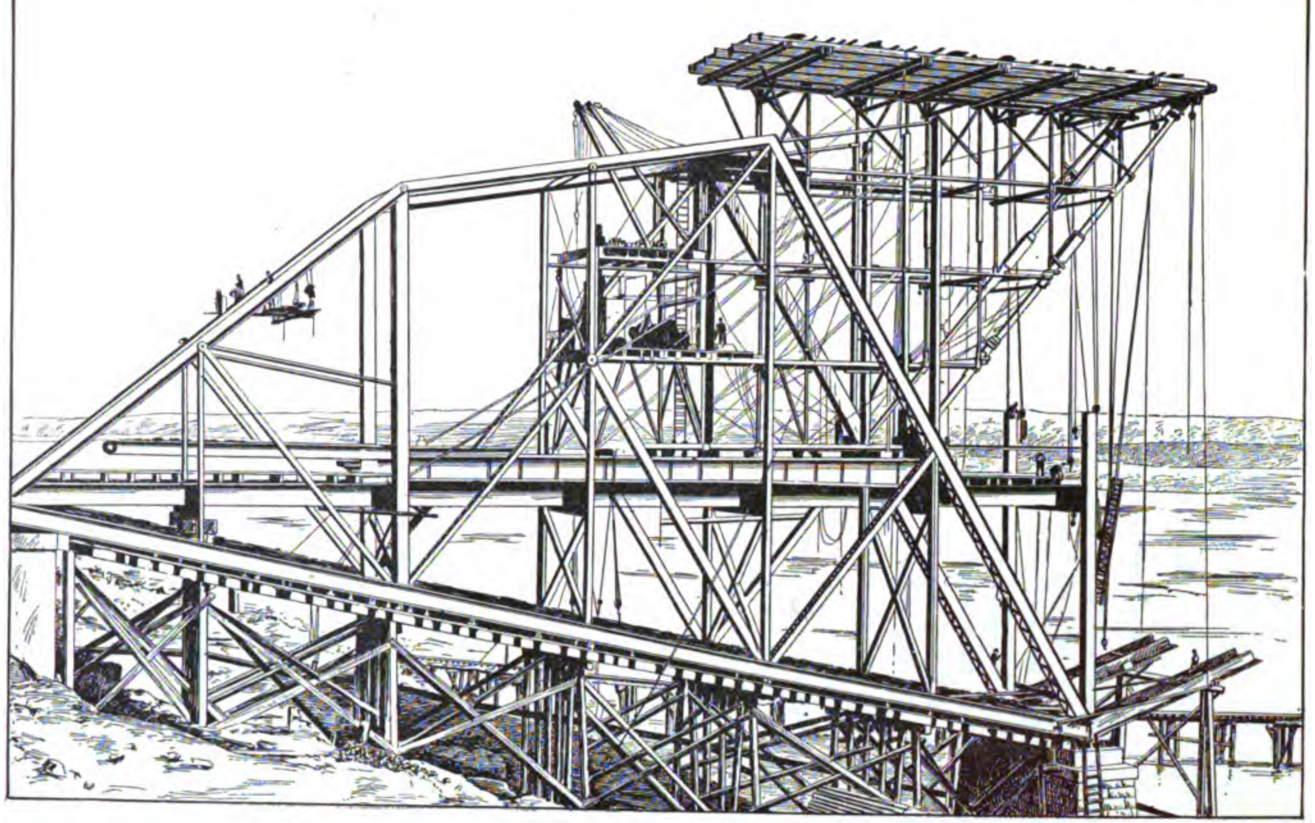
As has been stated, "Through Cantilevers" present more difficulties in erection than "Deck" structures, necessitating greater care and attention. First, the traveller must run

on the track and on the inside of the trusses, as there is no means of support outside; this necessitates a very narrow traveller. Second, the traveller being on the inside and extending above the trusses, it is not possible to put in the transverse and upper lateral bracing until the whole traveller has passed the panel point; this necessitates the omission of bracing, above the floor, at two panel points back of the panel being erected, and demands the closest attention of the foreman to see that the bracing is put in at the earliest possible moment, and that he is not caught napping by high winds. This design of structure also demands the extra handling of iron; part, as will be seen upon examination of Plate XVI*a*. must be hoisted and placed in position above, part must be lowered into place below; while all must be brought out over the track to the traveller and swung out from the centre until it clears, leaving the overhanging boom, where it is taken hold of by other sets of "falls" and passed back alongside of the traveller, and directly over the centre of the trusses to the proper point, and lowered or raised to its position. The erection of "Through Cantilevers" is begun and proceeded with in the same manner as described under the head of "Deck Cantilevers;" the wedges are provided in the panels connecting the lever arm and the suspended span, and operate in the same manner.

The Plate XVI*a*. shows "Through Cantilever" traveller in position to raise the lever arm, and by studying it closely the above description can be more intelligently followed. If the permanent structure details will permit it, outside temporary brackets at the panel points, with stringers to carry the traveller, could be provided, greatly simplifying the erection and reducing danger, as the traveller would then be outside of all, and the bracing could be put in immediately. Of course, it would only be necessary to provide brackets and stringers for the number of panels covered by the traveller, as they can be moved ahead as the erection proceeds.

Plate XVIII. shows the traveller, shown in detail in Plate XVI*a*., raising the Red Rock Cantilever.

Plate XVIII.



CHAPTER XIV.

MODERN HIGH BUILDINGS.

By William W. Crehore, Assoc. M. Am. Soc. C. E.

1. History.— Immense structures involving important engineering problems have occasionally been built ever since the days of the Pyramids; but the construction of very high buildings, for commercial purposes primarily and for architectural effect secondarily, is so distinctly modern that we have no records or experience to guide us in determining with any degree of certainty whether our methods will produce permanent or temporary structures as compared with the world-renowned architectural landmarks of Europe, or even with some less aged and less renowned in our own country. A considerable part of the so-called fire-proof construction going on to-day is recognized to be temporary, and is expected to deteriorate rapidly in a few years. The larger part, however, is designed with care and intelligence, and is expected to remain—how long? The gradual introduction of methods of fire-proofing has helped the development of the high building by opening markets for the very materials which have become indispensable in high-building construction. Improvement in the quality of cement has been an important factor in this development—not only by increasing the strength and capacity of masonry walls, but more especially in the increased value of cement concrete for heavy foundations. Many early modes of fire-proof floor arching appeared and disappeared after a short existence; but the hollow tile introduced in this country about 1871 has survived all competitors to the present day. Iron for columns and floor beams was originally used as much on account of its fire-proof properties as because of its superior strength.

Previous to 1885 a building of eight or ten stories was very close to the practical limit of height. Its walls were built heavy enough to carry their share of the floor loads, and the floor beams and girders rested on them. Interior columns were usually round cast-iron or Phœnix columns, the latter often having separate cast-iron caps at each floor for the reception of the floor beams. The floor arches, if fire-proof, were usually segmental brick or hollow tile arches, filled in above to the finished level. The whole construction was heavy, clumsy, and lacking in economy, according to our more modern view; but so long as the ground was not overloaded and so long as less wasteful methods of design were unknown, such a building was the standard by which the price of land was measured.

In the year 1885 an eleven-story building was completed in Chicago, in which iron columns built into the exterior walls received the floor loads from the beams and girders, thus relieving the walls from a duty they had previously been accustomed to perform. These walls were self-sustaining only, and the frame-work of the structure was erected entirely independent of them. This was a distinct step in advance, and was indicative of future possibilities. Other buildings constructed on the same general principle soon followed, each improving on its predecessors in important matters of detail; until within two years from that time the first real skeleton construction appeared, in which the exterior walls as well as all other loads were carried by the columns.

On account of the subsoil of clay which underlies Chicago, any attempt to increase

concentration of loading had to be met by adequate means of redistributing these loads upon the available ground area, without going down so deep that the layer of underlying clay was seriously diminished in thickness. The first effective solution of this problem was to use two or three courses of steel rails laid alternately at 90 degrees to each other and bedded in Portland cement concrete, to form a solid and rigid bed which would resist bending at any point. With the adoption of this system it was found that, if the columns were properly arranged, the weight of the structure (and consequently its height) might go on increasing until the footings covered the whole lot within the required limit of bearing per square foot. New buildings were made higher and gradually higher as better and lighter building material became known; and by the year 1890 the first twenty-story building in Chicago and in the country had been erected. The use of steel rails in the column footings was soon discontinued, as steel I-beams were found to be more suitable, and more economical after the collapse of the Steel-Beam Trust in 1890 and the consequent decline of 30 to 40 per cent. in prices. Spread-out footings are now made with one course of steel beams imbedded in concrete, the column loads being distributed on this bed by plate or box girders or other deep beams properly arranged.

Other cities have had the same or different problems to meet in the design of high buildings; but the history of this subject began in Chicago, where the most important problems were first met and solved, and even to-day one occasionally hears the method of skeleton construction referred to as the "Chicago Style." New York, Philadelphia, Boston, and other large cities have been applying the new principles of construction during the last six years with very substantial success. With the multiplication of rental space the price of land has increased, and with the improvements in fire-proofing insurance rates have decreased. The impetus thus given to building construction in New York City is likely to be felt until most of the downtown business property of twenty years' standing or more has been rebuilt. The realization of this fact is forcing itself more and more strongly upon the owners of the old buildings as each spring finds them with expired leases not renewed. The new buildings are more attractive, more convenient, and cleaner; the elevator service is more efficient, and all the details are more complete and more comfortable. A new era has begun.

2. MODERN STEEL SKELETON CONSTRUCTION.—Steel skeleton construction is what has made tall buildings a commercial possibility. To support any weight, or even to stand alone, a brick wall of very great height must broaden out towards the base, and would thus occupy considerable very valuable space in the lower stories of a tall building. Where space is as desirable as it now is in the business centres of our large cities, the building of very high self-supporting walls is precluded. The theory of modern skeleton construction is that the steel framework shall be complete in itself, furnishing the strength and rigidity; and all other portions of the structure—both inside and out,—live load and dead—shall be carried by it. According to this theory the walls of a building, being supported at intervals, need be no thicker at the bottom than they are at the top, provided the intervals are not too great. Such walls are known as curtain walls, as they are theoretically merely a covering or protection from the weather. Practically, however, curtain walls afford very great rigidity to the structure, and thus aid as well as protect the steel skeleton in the performance of its work.

The increasing use of steel construction has stimulated the ingenuity of designers, and called forth many schemes for reducing the interior dead weight of the building. Thus instead of brick arches to support the floor between beams, there are now a dozen or more floor systems in use which weigh less per square foot. Some of these floor systems, however, have very little merit from an engineering standpoint, and it is well to be on guard in selecting one. A few cardinal principles should be borne in mind. Systems requiring

close spacing of the floor beams for ordinary loading should be avoided, since the beams if designed economically will not be deep enough to give rigidity. Systems making use of rolled iron or steel sections whose shape differs radically from that of the I-beam (whether large or small sections) should be avoided, since the I-shaped section is the best and most economical section known for beam work. Systems relying to any extent on the tensile strength of concrete should be avoided.

The Flat Hollow Tile Arch, either side or end construction, is at present more universally used than any other system. (See Fig. 1.) The terra-cotta blocks are made 8″, 10″,

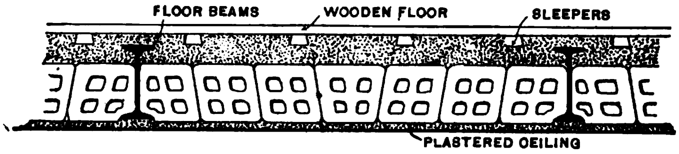


Fig. 1.—Hollow-tile System of Arching.—Side Construction.

or 12″ deep, and are used on spans from 6 to 6½ times their depth. A filling of cinder concrete is spread over the top, covering the arches and floor beams to a depth of 2 or 3 inches. In this filling the floor sleepers are imbedded, and the finished floor is then nailed on. Other systems of arching have gained ground very rapidly within two or three years, notably the Metropolitan, the Roebling, the Melan, and the Expanded Metal systems. The *Metropolitan* system (Fig. 2) consists of a series of wire strands about 1½ or 2 inches

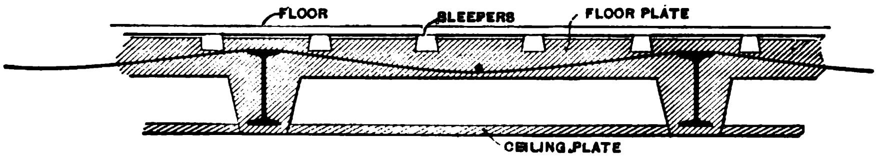


Fig. 2.—Metropolitan System.

apart laid over the floor beams and anchored at every fifth or sixth beam. Between the beams these strands are held down by a ⅜″ diam. rod to a centre deflection of 5 or 6 inches. A mixture of plaster of Paris and sawdust is then poured on covering the strands and encasing the floor beams. As this mixture is in a semi-liquid state, false centres are required for keeping it in position until it hardens or sets. So rapidly does this setting take place that the centres can be removed within half or three-quarters of an hour. The wooden sleepers are sometimes set in place before the mixture is poured on, in which case they are firmly imbedded in the floor plate when it sets, and sometimes they are laid on top of the floor plate after it has set.* The *Roebling* system (Fig. 3) is theoretically an arch of con-

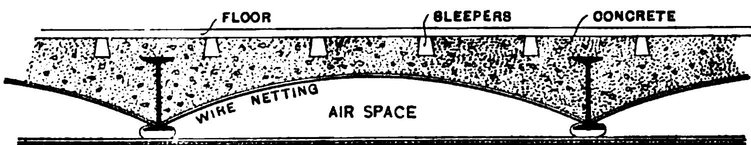


Fig. 3.—Roebling System.

crete between the floor beams. Permanent centres, consisting of wire netting stiffened by $\frac{7}{16}$″ iron rods every foot or two, are arched to a central height of about ⅛ of the span. Con-

* The Metropolitan Fire-proofing Co. is at present just introducing a modification of their system in that the ceiling is supported by 1″ × ⅛″ steel bars set edgewise about 18 inches apart. These bars are clamped to the under side of the I-beams, and to them in turn is secured the wire lathing which carries the plastered ceiling. This construction leaves the floor plate entirely separate. The sides and soffits of the beams are covered with small blocks of the material securely clamped on.

crete is then spread on these arches and levelled off to a depth of $1\frac{1}{2}$ or 2 inches at the crown of the arch. The sleepers and wooden flooring are then laid in the usual manner. The strength of the *Melan* system of arching (Fig. 4) depends largely upon the use of steel ribs (usually the small sizes of I-beams) bent to the shape of the arch, having a central height of $\frac{1}{10}$ to $\frac{1}{12}$ the span, and imbedded in the concrete, which is levelled off on top and completes the arch. This system is used on spans of 12 to 16 feet, where the other systems mentioned are used up to 6- or 7-foot spans only. The steel ribs are spaced from 3 to 5 feet apart, according to the strength of floor required. Tie-rods are placed one under each rib to take up the thrust. Many engineers criticise the Melan system severely on account of its use of concrete to do beam work and to act as an arch at the same time. This objection becomes less important the closer the ribs are spaced. The owners of the *Expanded Metal* system claim a great variety of uses for it. On wide spans of 8 to 16 feet they make use of arched channels spaced every 4 or 5 feet to re-enforce and stiffen the floor

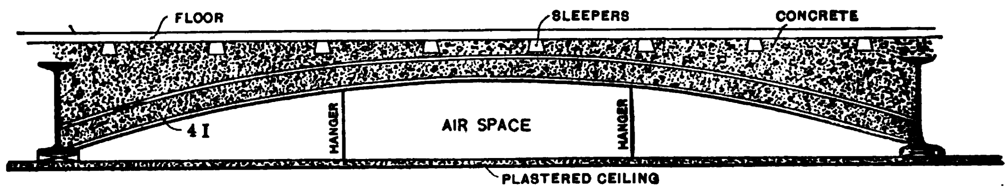


Fig. 4.—Melan System.

plate, which is itself composed of sheets of expanded metal laid horizontally over the tops of the floor beams and imbedded in a layer of concrete 3 or 4 inches thick. On this the sleepers and finished floor are laid as in other systems. When the spans are less than 8 feet the arched channels are dispensed with, and the system then depends upon the expanded metal for its strength. Sometimes flat bars used in suspension and hooked or strapped over the floor beams take the place of the arched channels as re-enforcement pieces on the larger spans. Another method is to use sheets of expanded metal for a permanent arch centre and build the concrete arch on it. This should be done only where the span is small enough to allow a central height of $\frac{1}{4}$ to $\frac{1}{8}$ the span, and corresponds in theory to the Roebling system, which uses a wire netting in place of the expanded metal. Where a flat ceiling is desired with any of these patent systems a separate ceiling plate is constructed of the same material as the floor plate, and is hung beneath the floor beams. The hollow-tile flat arch, however, requires no ceiling plate, as it fills up the whole space to the lower flange of the floor beams. As long ago as 1889 a system of concrete and iron flooring was used by a well-known firm of Philadelphia architects, who held no patents and made no particular claim to originality. The span between floor beams being anywhere from 10 to 18 feet, flat iron straps were suspended at intervals of one to two feet, the ends of each strap being bent or hooked over the top flanges of the beams, and the straps being curved down, so that midway between beams they hung close to the ceiling line of the story below. A concrete floor plate was then built in, completely encasing the straps and the floor beams, being levelled off on top and bottom to prepare for the usual floor and ceiling finish. The strength of such a system depends largely upon the size and spacing of the straps. It would be classed as more or less obsolete to-day, except in the rare instances where a heavy floor is desirable; but as such does not possess the rigidity of the segmental brick arch.

The Guastavino system of fire-proof flooring was about the first to appear whose strength on long spans and whose relative weight made it an economical system to use. Courses or layers of hard, well-burned clay tile weighing 100 lbs. per cubic foot are laid flat, either in the usual segmental form or dome-shaped, the central rise being about one tenth of the span. These tile blocks are 1 inch thick, 6 inches wide, and 12 inches long,

On spans up to 12 feet three courses are used; up to 16 feet, four courses; and up to 20 feet, five courses. This arch when not accompanied by a ceiling plate leaves the tie-rods exposed to view from below—which is a serious disadvantage; but it has been used with great effect in ornamental ceilings for theatres, churches, corridors, etc.

It ought also to be mentioned that the terra-cotta hollow-tile segmental arch has been occasionally used on very long spans with great success: for instance, one of fifteen (15) feet. with rise of one twelfth. There are a few buildings in New York City containing this construction, and the tests made on these floors have been in every way satisfactory.

Tie-rods spaced at intervals of eight times the depth of the floor beam to take up the thrust of the arch are required by the Building Department in New York City in all systems of floor arching. Their use began with the brick and tile segmental arches and continued with the hollow tile flat arch, but with some of the new systems of flooring they are of doubtful necessity.

Occasionally a very heavy floor is desirable, viz., in cases where the live load is to be applied in the form of shock or sudden vibration. The segmental double brick arch (Fig. 5), having a central rise of one sixth to one eighth of the span, will prove to be a rigid and

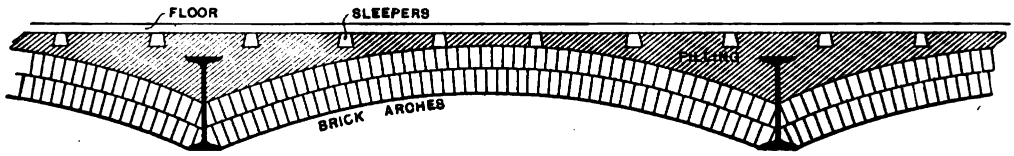


Fig. 5.—Double-brick Segmental Arch.

economical mode of construction for this purpose. It is of the utmost importance, in using any of the systems of floor arching, to have the material properly mixed and set with great care. Each manufacturer publishes a little catalogue (which may be obtained on application) describing and illustrating his own system in detail. It must not be forgotten that the tests recorded in these catalogues were made on sections of flooring specially prepared by skilled workmen and under careful supervision. To produce the same results in practice something like similar conditions must exist.

Fire-proof Partitions formerly were an important part of the interior dead weight of a building. These are at the present time most often built of 2 or 3 inch terra-cotta blocks, stiffened at intervals with light angle-iron "furring" (as it is called) and covered with wire lath and plaster. Each of the patent floor systems, however, is accompanied by a corresponding system of partitioning made similarly and with the same materials. The result is that the interior partitions of skeleton-constructed buildings have come to be regarded of very little consequence as dead load, and are placed anywhere on the floor regardless of the positions of the floor beams—rather, the floor beams are placed regardless of the locations of the partitions. There is the additional advantage in this that the partitions may be altered, removed, or torn down at any time without affecting the floor construction. In calculating the column loads their weight is usually considered as part of the assumed distributed total load per square foot of floor area.

The Floor Beams receive the floor loads directly from the arches or the fire proof floor plates and transmit them to the girders. Rolled steel I-beams are invariably employed for this purpose. As previously stated, the spacing of the floor beams depends somewhat upon the system of fire-proofing or arching to be used. It also depends upon the spacing of the columns in the building when their position has to be fixed by other than engineering considerations. A floor beam should be placed opposite each column to give stability and to aid in erection, and the space between columns should be divided into an odd number of bays, when feasible, so that the girders may not be loaded at their centres. The principle

is the same as that which makes a truss having an odd number of panels more economical than one having an even number, the span and load being the same for each. Another consideration which should have weight in spacing the floor beams is that of two beams which will do the same work the deeper beam is stiffer and lighter. When possible, therefore, the spacing should be so arranged as to use the lightest weight of a given-size beam up to the allowed limit of the specifications.

The Floor Girders receive the floor loads directly from the floor beams and carry these loads to the columns. A girder may be simply an I-beam when the conditions permit the use of one; or it may be composed of two I-beams when the use of one alone is prohibited by the depth allowed or the load to be imposed; or it may be a built plate or box girder when required by the loading. Girders carrying walls are usually composed of two or more I-beams (even when one beam could be found strong enough to do the work), so that there will be sufficient horizontal surface for the wall to bear on. The same result is sometimes accomplished by riveting a plate of sufficient width on the top or bottom flange of the single beam, but usually this method is found less economical than the other, to say nothing of the practice of using rivets in tension. When two or more I-beams are used as one girder they are bound together by bolts passing through their webs and through cast-iron separators placed between them at intervals of six feet, more or less.

The Columns receive their principal loads from the girders in one direction and partial loads from the floor beams in the other direction, and carry these loads down to the foundations. Their construction is a very important part of the work in a fire-proof building. The first question which always arises is whether built-up rolled-steel columns or cast-iron columns shall be used. In a majority of cases in buildings over eight or ten stories high built columns are selected without discussion. There are some buildings twelve or fourteen stories high, however, which are carried by cast iron columns. At the present time the difference in price is not so much in favor of cast-iron colnmns as it used to be. The main objection to their use in very high buildings is the impossibility of using riveted connections which are a great aid to lateral stiffness. In tall, narrow buildings cast-iron columns are practically prohibited on account of the importance of the wind strains. It may be said in favor of cast-iron columns that they are not as likely to warp and buckle in case of fire as the built columns are. There are very few cases of fire on record, however, where any serious damage has resulted from the collapse of the columns, whether steel or cast iron. When fire attains a degree of heat sufficient to affect the skeleton structure, the beams and girders give way first, owing to their transverse loading. The uncertainty as to invisible defects, the difficulty of inspection, the greater dead weight to handle during shipment and erection, and the necessity of using bolts for making connections are some of the points which operate against the use of cast-iron columns in the construction of a first-class

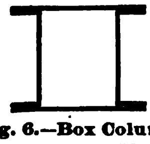


Fig. 6.—Box Column. Channels and Plates.

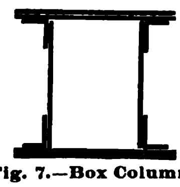


Fig. 7.—Box Column. Angles and Plates.

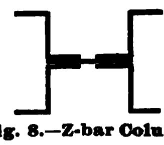


Fig. 8.—Z-bar Column.

building. Cast columns are made square, round, or rectangular. The round columns are the most economical, but the square and rectangular are used for wall columns, because it is easier to build brickwork and masonry around them.

There are several kinds of built steel columns in use to-day. Probably the one most commonly used is the box column built of channels and plates or angles and plates, as the

case may require. (See Figs. 6 and 7.) The Z-bar column (Fig 8) and the Phœnix column (Fig. 9) are largely used but have been proved less economical than the box column for very heavy work For lighter loads we find columns composed of two channels placed as in Fig. 6, but bound together by lattice bars instead of plates; also columns composed of four angles and one plate, as in Fig. 10. A style of column lately introduced, and used

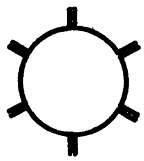


Fig. 9.—Phœnix Column.

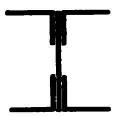


Fig. 10.—Four Angles and one Plate.

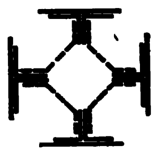


Fig. 11.—Gray Column.

quite frequently in Chicago, Philadelphia, and Buffalo, is the Gray column (Fig. 11). The working section of this column is contained in the angles and the outside plates. The dotted lines represent bent plates 8 or 9 inches wide, spaced two and a half feet apart vertically. These bars are intended to bind the members of the column together to make them act as a unit. Upon this point some eminent engineers have criticised this style of column adversely. The published tests, however, give good average results.

There are advantages and disadvantages to be found in the use of any kind of column, and each case ought to be studied by itself with reference to the kind of loading, the range of loading (i.e., the difference between the average load on top-story columns and that on basement columns), the facility for connecting the beams and girders to the columns, and provision for taking up wind strains. The greater the least radius of gyration for a given load and unsupported length, the smaller will be the amount of metal required in the column; similarly, the greater the load for a given unsupported length and area of section, the greater the least radius of gyration must be. A column whose working members are situated as far as practicable from the centre of gravity of its section, having the greater least radius of gyration, will consequently be more efficient than a column otherwise constructed, other things being equal. The Phœnix column is ideal in this respect, but does not compare favorably with the box column in mill or shop construction or in facility of making field connections. The box column is particularly advantageous in building construction, because it presents a square surface for beam and girder connections, and is easily built into the wall. The same may be said of the Gray column, but a comparison between the Gray and the box columns shows that there is considerable superfluous metal (the straps shown in Fig. 11 by dotted lines) in the former which cannot be counted in the sectional area of the column, whereas in the latter all the sectional area is available. For buildings of moderate height and moderate loading the Z-bar column is used very often, and is found economical because the saving in shop expense by reason of having to drive only two lines of rivets and the facility in making beam and girder connections outweigh the advantages possessed by any other kind of column. When the required area of section is greater than can be made up of Z-bars without the use of outside plates the box column begins to compare favorably with it. It is quite common to find two or three styles of built columns in the same building, especially when the range of loading is wide.

Bases.—It is necessary to use a shoe or distributing base underneath all basement columns to apportion the load properly upon the foundation. The kind most frequently used is the cast-iron shoe, such as that shown in Fig. 12*b*. This shoe is made separate from the column, and can be very accurately set on the masonry or steel-beam footings, and well grouted. Cast-iron shoes are often found under built columns, although the style of shoe most commonly used with the built column is shown in Fig. 13. It is riveted fast to the

column itself, and made up of angles and plates. It is somewhat more difficult to set the basement column with the shoe attached than it is to set the shoe separately; but there is this advantage in the riveted shoe, that by means of it the column's load can be more efficiently distributed over a rectangular base plate than by means of a cast-iron shoe. It

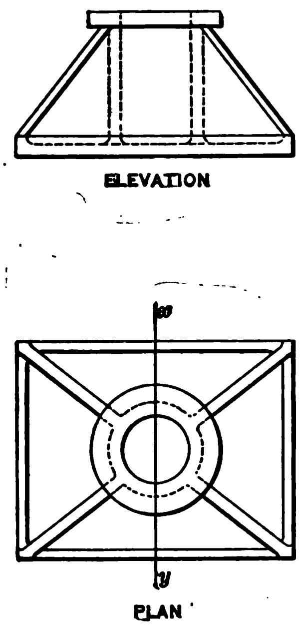


Fig. 12a.

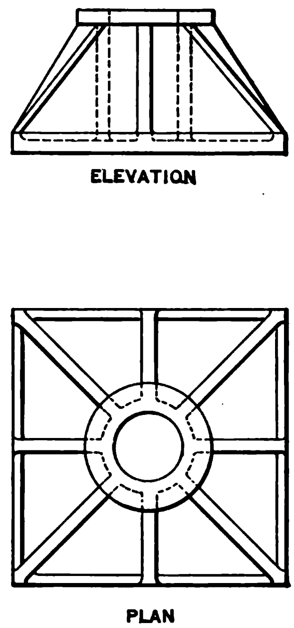


Fig. 12b.

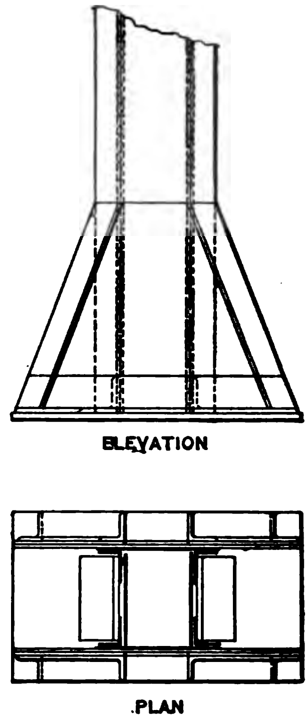


Fig. 13.

is sometimes better, in covering a great many beams in the upper course of the grillage work, to have the base plate of the shoe longer than it is broad. A cast-iron shoe can, of course, be made to cover a rectangular area, but it must be very carefully designed. The shoe shown in Fig. 12*a* has a rectangular base, and only four ribs—one on each corner. Under great pressure, and especially if there was any unevenness in the grouting, shoes like this one have been known to crack and give way on the line *x*—*y*. To be really efficient, a shoe of that size should have four intermediate ribs besides the ones at the corners, as shown in Fig. 12*b*. Occasionally we find built shoes made up of angles and plates, but separate from the column, like cast-iron shoes.

In order to protect the metal in the columns in case of fire some method of fireproofing them is always provided. Frequently the same material is used for fireproofing the column as is used in floor plates for fireproofing the beams; and most of the patent systems of fire-proof flooring include systems of fireproofing the columns. It makes very little difference what the original shape of the column was, as it can be filled out with fire-proof material to form any conceivable shape which the architect may desire. There are numerous ways of covering the columns. Occasionally, and quite frequently when cast-iron columns were in the ascendant, a ¾-inch cast-iron shell was used, completely covering the column, leaving an air space of one and one-half to two inches between the shell and the column; this, with an ornamental cap and base, was quite sufficient decoration as well as protection for an interior column. The use of these shells is somewhat obsolete at the present time, owing to the advantages and increased popularity of other methods of fireproofing.

Wind Bracing.—This subject received very little attention so long as the walls of a building were relied upon to carry the loads, but as soon as the custom of carrying the walls themselves came in vogue some method of stiffening the structure laterally had to be adopted. The necessity for wind bracing increases with the ratio of the height to the least dimension of the building. When this ratio is very great, as for instance in a ten-story building on a 25-foot city lot, transverse bracing must be located at intervals in the length of the building throughout its whole height.* The system of bracketing shown in Fig. 14 is used frequently, but is less efficient than any of the other systems here shown. Fig. 15 represents a system of diagonal rod bracing. Since diagonal bracing is always most efficient when placed at an angle of 45 degrees, it sometimes happens that the distance between columns is great enough to require one panel of bracing to extend two stories in height, instead of one, as shown by the dotted lines. Rolled angle bars are sometimes used instead of rods, the connections in such a case being riveted. Fig. 16 represents the most common form of wind bracing in

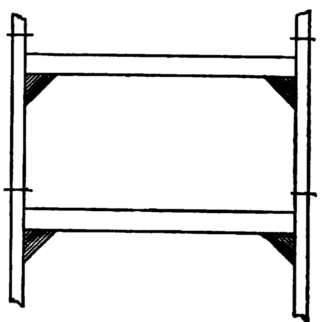


Fig. 14.—Wind Bracing by means of Brackets.

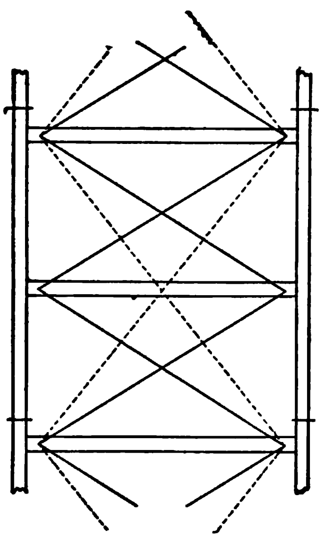


Fig. 15.—Wind Bracing by means of Rods.

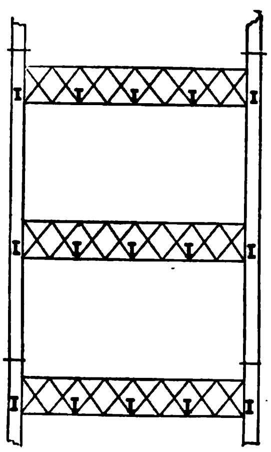


Fig. 16.—Wind Bracing by means of Lattice Girders.

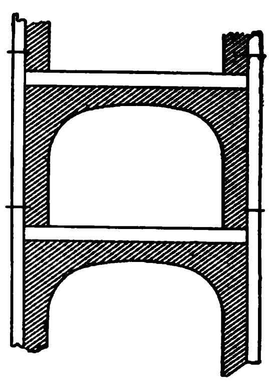


Fig. 17.—Wind Bracing by means of Portals.

use to-day, i.e., a system of lattice girders. They are usually situated in a wall or partition in order not to interfere with the architectural effect, and are made the full depth available between the window lintels of one story and the sills of the next. Being usually located in a curtain wall, these lattice girders do double duty in carrying the loads and acting as struts at the same time. The ends of the floor beams are connected to them wherever they happen to come, but preferably at a panel point. The system shown in Fig. 17 is an efficient but expensive one. The shaded portions represent solid plates, which are spliced at convenient intervals and whose thickness is determined by the shearing stresses due to the wind forces. This system has been used more in Chicago than elsewhere.

The designer is usually limited in his choice of wind bracing by other than engineering considerations, and the greatest problem is how to adapt the design to the existing conditions and preserve its efficiency. That this problem is too often neglected is partly because

* An eighteen-story building is now being erected in New York City on a 25-foot lot.

of its difficulty, partly because no serious accidents have as yet been recorded where tall buildings have failed for lack of wind bracing, and partly because much reliance is placed upon the stiffening effect of the curtain walls. The walls do provide great stiffness,—just how much it is difficult to say,—but it should not be relied upon.

For rapidity and economy in erection riveted steel columns are now generally made in one length for two stories. The lattice girders used for wind bracing are often omitted in such cases on every alternate floor and are placed at the tops of the columns, their place being supplied by beams or beam girders at the intermediate floor. This omission should not be made in narrow buildings where the wind bracing is important.

Connections are made either by bolting or field riveting. The former method is cheaper and quicker, but when a structure has a comparatively small base in proportion to its height and great rigidity is required, bolted connections are used with considerable risk to the owners and occupants of the building. With cast-iron columns bolted connections are necessary, since rivets cannot be driven without injury to the castings. This fact effectually precludes the use of cast columns in buildings where wind bracing plays an important part in the design. In connecting a riveted steel column to the one above it vertical splice plates are usually placed on opposite sides, extending a foot and a half or two feet both above and below the joint. The end sought by this means is to render the shaft continuous throughout the whole height of the building and to assist in overcoming any outside influence to torsion or tension.

The connections of all girders and beams to the columns are important, since the stiffness of the structure depends upon the transmission of the lateral stresses through these connections and into the columns. Specifications now commonly require these connections to be riveted, whereas connections of beams to beams or beams to girders are allowed to be bolted. If excessive eccentric loading of columns cannot be avoided, such loads are provided for by increasing the column's sectional area for bending and not by designing the column unsymmetrically. Our conception of a column should be that of a continuous shaft tapering from the bottom to the top of the building, and being loaded very irregularly at best by a multitude of comparatively small loads. The building laws in most of the large cities require such factors of safety, and the single loads usually constitute such a small percentage of the total load on a column, that any great refinement in the treatment of eccentric loads is unnecessary. It is really more important that the brackets or seats which transmit the girder loads to a column should be designed so as to bring these loads as soon as possible to the column's centre of gravity.

Special Features.—The necessity for special features continually arises, and it is in devising methods for producing the desired result under the existing conditions and restrictions that the engineer finds a large field for his ingenuity. A few of these features are briefly described and illustrated below.

Fig. 18 illustrates a truss set in a partition longitudinally through the centre of a building. On its lower chord rest the ends of the fifth-floor beams, on its upper chord rest the sixth-floor beams, the truss being the full height of the fifth story; the fourth floor beams are hung from the panel points of the truss. Some construction had to be made to avoid the use of columns in the second and third stories directly underneath, and this was adopted as being the best way out of the difficulty. It also happened that the sixth story was to be kept clear of columns, so that there were no concentrated loads on this truss from above the sixth floor. The diagonal members of this truss had to be arranged to permit door openings at fixed points, but the truss itself was entirely enclosed in a partition.

Fig. 19 represents a somewhat similar problem, but on a larger scale, the truss being two stories in height instead of one, the columns themselves being utilized as compression members in the truss. The problem was to keep the ground floor clear of columns, yet

make the building a great many stories high. It will be noticed that in both of these figures (18 and 19) very heavy concentrated loads have to be taken through the columns at the ends of these trusses and down to the foundations.

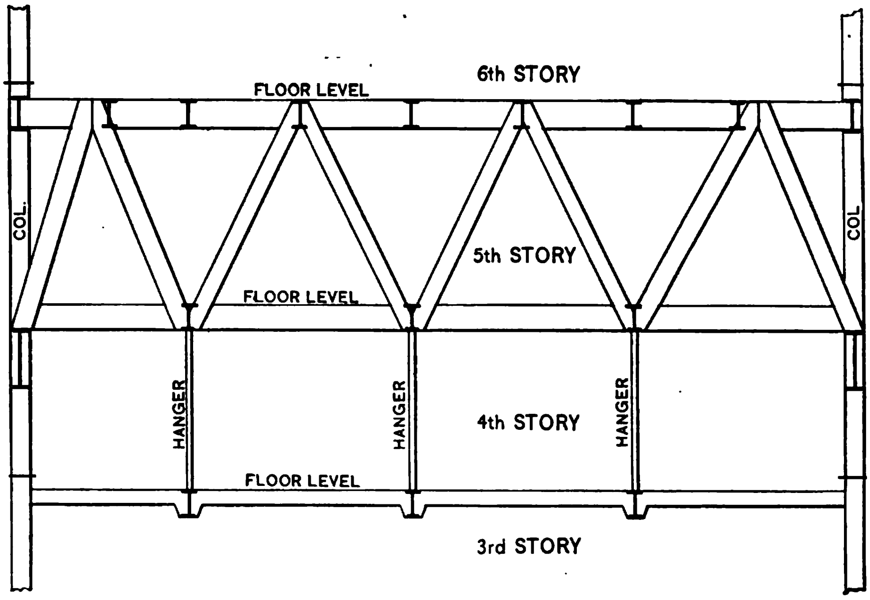


Fig. 18.

Fig. 20 shows a common form of cantilever girder used in foundation work. As is always the case when the building has to cover every square inch of the lot on which it rests, some means must be contrived for keeping the foundations also wholly within the lot.

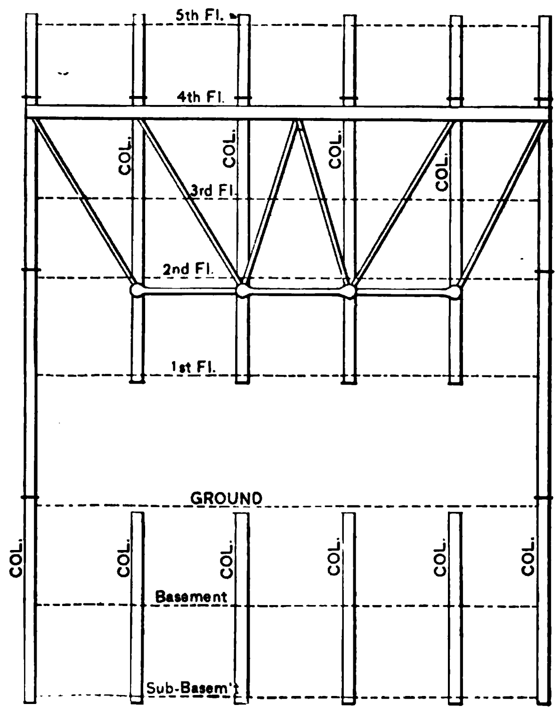


Fig. 19.

Before these very heavy buildings were the fashion there was seldom any difficulty in providing a broad footing underneath the side walls so that the line of thrust should fall within

the middle third of the foundation course; but with the introduction of tall buildings exceedingly heavy loads had to be placed upon single footings or groups of footings, and, as the columns which carried these loads down through the building were situated in the side walls themselves, it was found impossible to bring the centre of gravity of a column

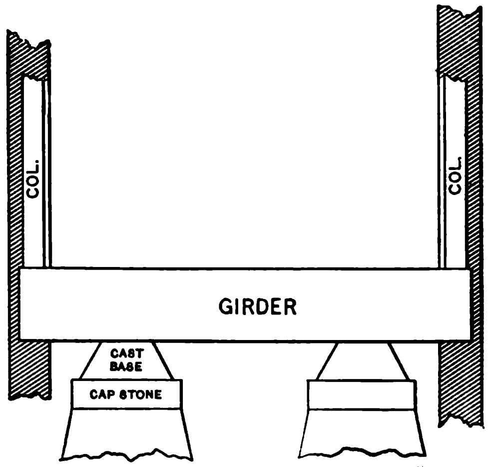


Fig. 20.

over the centre of its footing by any of the old methods. The commonest way of overcoming this difficulty is by setting the footing pier back from the property line a sufficient distance to spread it equally in both directions from the centre of pressure, and by the use

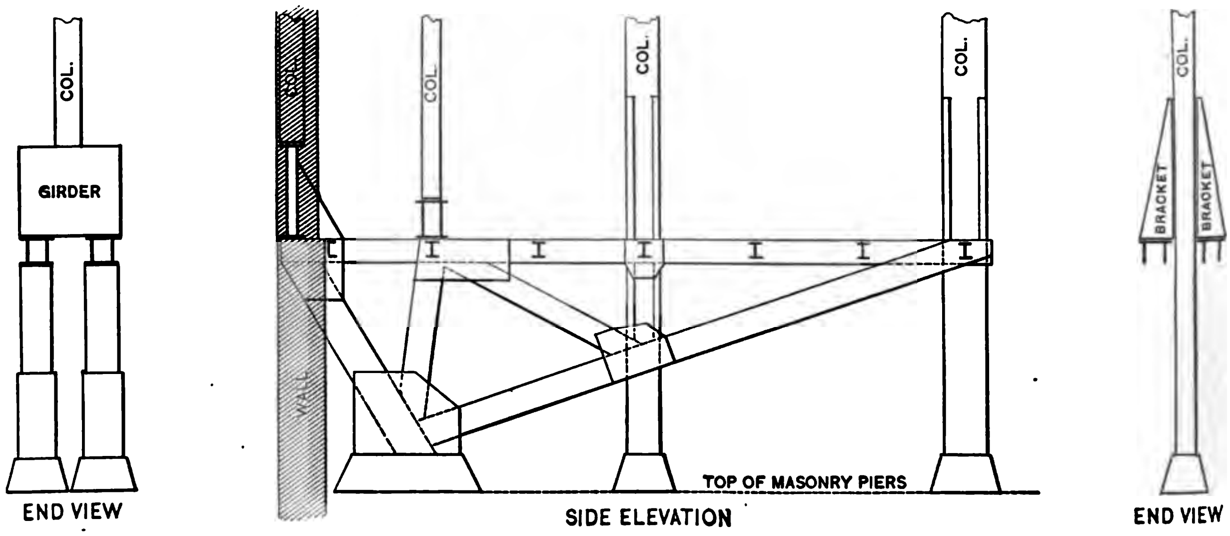


Fig. 21.

of steel girders (as shown in the figure), arranged in the form of a cantilever, to carry the column on the projecting end.

Fig. 21 represents a form of cantilever truss. The object of using the triangular truss instead of the usual riveted girder was in this case twofold, namely, to save metal and to give extra room in the sub-cellar without taking away materially from the head room. It will be seen that the long arm of the cantilever includes two columns, both of which rest upon the ground. These two columns were required as anchorage since there was not sufficient dead load in one of them to serve the purpose. The beams of the basement floor rest directly upon the horizontal chord of the truss. The triangular trusses were

made in pairs, one on either side of the anchorage columns, and firmly secured to them by large inverted brackets. The load on the short projecting arm is brought from the wall column to the trusses by short plate girders. This construction was thought to be eco-

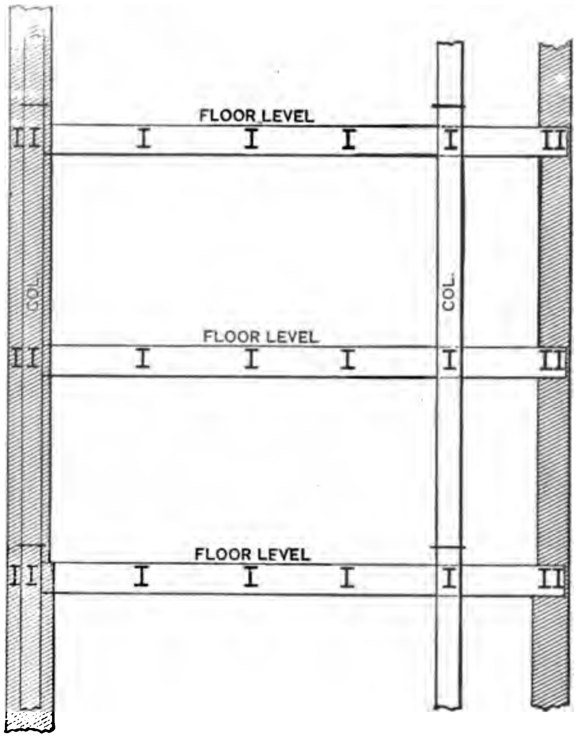


Fig. 22.

nomical on account of the gain in depth over the ordinary foundation cantilever made of riveted girders.

Rarely we find the form of construction shown in Fig. 22, where the side wall is

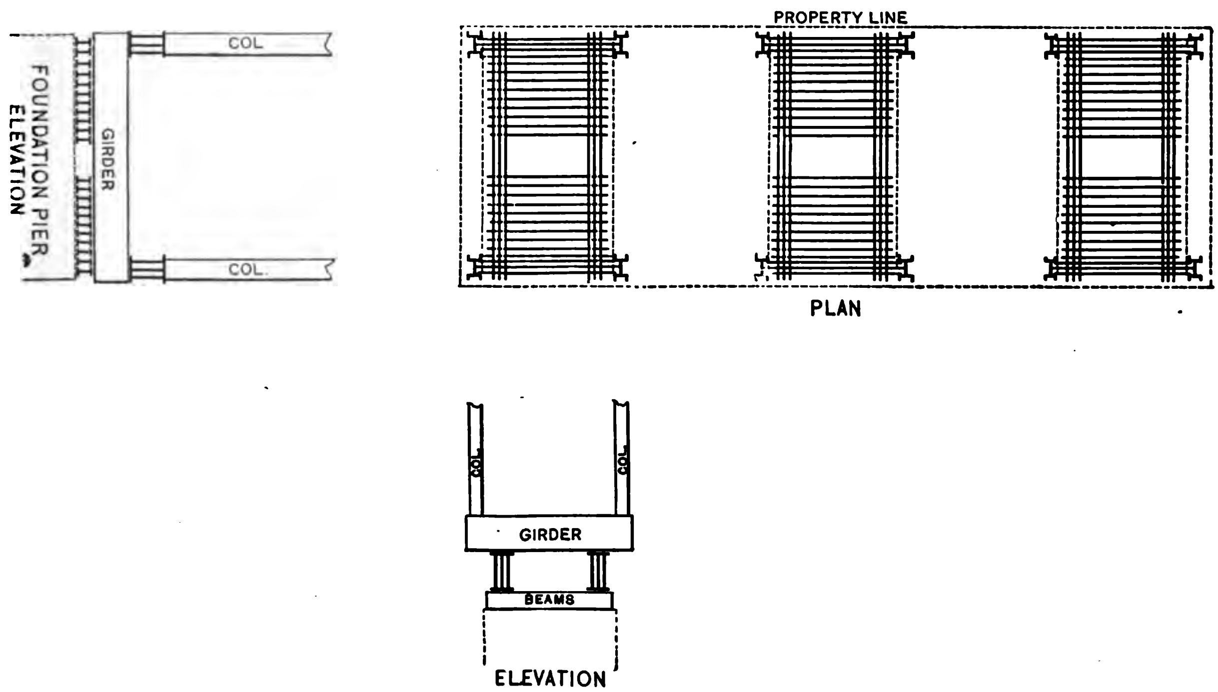


Fig. 23.

carried by cantilever beams and girders at each floor level; the projecting arm being long enough to span a hallway in the building. This construction is not usually an economical one if the building is very high, as it requires more metal in the girders than one single cantilever girder in the foundations would.

In Fig. 23 we find rather a novel arrangement. The building (now in process of construction) is to be eighteen stories high, and stands on a very narrow lot. The twelve columns are all wall columns, and the loads from them are transferred by means of two sets of cantilever girders into three foundation piers, so that each of these piers will finally receive the load from four columns. The manner of transferring these loads through the two sets of cantilever girders can be seen from the figure.

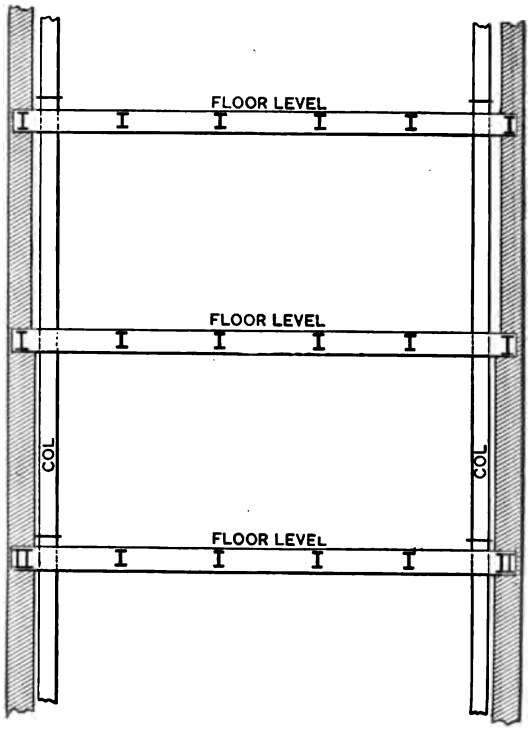


Fig. 24.

Now and then an architect desires that no columns shall be built in the walls, but that they shall be fireproofed inside of the building itself, so that they may be easily gotten at and examined. Fig. 24 shows such a system. The tendency to eccentric loading is relieved by using double beams or channels for the floor girders and allowing them to project by the columns on either side, forming cantilevers whose short arms carry the ends of the wall girders.

There are other features of modern building to be found in the construction of theatres, armories, train-sheds, factories, etc., etc., which are not properly classified under skeleton construction, and are consequently outside the province of this chapter.

3. METHOD OF DESIGN.—The work of designing naturally proceeds in the following order: Select the fire-proof floor arch; arrange the spacing of beams, girders, and columns, determine the wall sections and method of supporting the walls; make schedule of loads on columns and foundations; design the foundations; calculate sizes of beams and girders; calculate the columns; calculate wind bracing.

Before proceeding with the design an assumption for the live loads must be made. Generally accepted practice calls for 40 to 75 lbs. per square foot on floors used for office

purposes, hotels, or dwellings; 100 to 120 lbs. per square foot on floors used for stores, ball-rooms, theatres, or places of public assembly; 150 lbs. per square foot and upwards for factory or warehouse floors or floors subject to vibration or shock.

The New York City building law at present requires the following assumptions of live load to be made: 70 lbs. per square foot for floors in hotels and dwelling-houses; 100 lbs. for floors in office buildings; 120 lbs. for floors in all places of public assembly; 150 lbs. and upwards for floors in factories, stores, warehouses, etc. This is a heavier requirement than municipal governments usually enforce; but considering that it includes the weights of partitions, stationary and movable furniture (including small safes) in the general requirement for live load, and that the law itself makes no provision for increasing the loads on the columns affected by wind forces, the general result produced by its enforcement is not far removed from good practice.

The choice of fire-proof arch depends somewhat upon the purpose for which the floor is to be used, but of the several efficient systems that which weighs least will ordinarily be cheapest, as thereby the amount of metal in the beams and columns will be slightly less. Because one floor system weighs less than another in a given case, it should not be concluded that it will in another case using a different size of beams differently spaced. As conditions vary greatly, each case requires special study, and some consideration should be given at the same time to the beam and column spacing, since restrictions here often help in determining the selection of the floor arch. When the selection has been made the total dead weight of the floor per square foot should be carefully calculated from the actual material composing it.

In designing the floor beams use the formula

$$R = \frac{3myl^2}{2T}, \quad \ldots \ldots \ldots \ldots \quad (1)$$

where m = total load in load in lbs. per square foot,
y = distance between beams in feet,
l = span of beam in feet,
T = allowed fibre stress in lbs. per square inch,
and R = the section modulus in inch units,* which for a plate girder is equal to the effective depth in inches multiplied by the area of one flange in square inches;

or the formula

$$R = \frac{3Wl}{2T}, \quad \ldots \ldots \ldots \ldots \quad (2)$$

where W = the total uniformly distributed load on the whole beam, the other quantities being as denoted above.†

Having found from equation (1) or (2) what the section modulus should be, the proper I-beam can be selected from any of the mill hand-books which give as beam properties either the section modulus or the moment of inertia,—the former being the quotient obtained by dividing the latter by one half the depth of the beam. Tables based on equations (1) and (2) are more convenient in practice than those giving safe loads which are commonly found in the mill hand-books (although the latter are more easily comprehended by the layman), because R represents a characteristic property of the beam which might be termed its strength, and its value may be very readily compared with the weight of the

* Until recently improperly called the moment of resistance.

† It is assumed that the student can derive these simple equations, and others which are to follow, by the elementary principles learned in the early part of his work.

beam per linear foot, both of which quantities are constant for all spans and conditions of loading.

The lightest beam which will do the work (i.e., whose value of R is sufficiently large) should be selected. Adherence to this rule will invariably secure the deeper of two beams having their section moduli alike, a fact which confirms the economy of the deeper beam. If, now, no beam can be found whose section modulus is close to the required value of R, it may be feasible to rearrange the spacing of the beams to suit some one particular size of beam. For this purpose solve equation (1) for y, substituting the value of R desired and the other quantities as before. This will give the maximum spacing allowed for the given I-beam.

If, however, no change in the spacing is feasible, perhaps a rearrangement of the columns might be made so as to change the span of the beams. To find the maximum span on which a particular size of beam might be used solve equation (1) for l, substituting all the other quantities as before. Nearly always the designer will find the column spacing fixed by conditions beyond his control, and quite as often the direction in which the girders must lie is indicated by some restriction or other, so that he is finally reduced to "Hobson's choice" in picking out the most economical size of beam.

Should there be a restriction on the depth of floor beams, and should the economical size of beam exceed the allowed depth, a heavier and shallower beam of equal or sufficient strength must be chosen. In selecting a shallow beam to do the work required of a deeper one, the limiting span should not be exceeded. The limiting span is determined by the amount of centre deflection allowed by the specifications, which in New York must not exceed $\frac{1}{360}$ of the span when the beam is fully loaded. The following formula gives the limiting span, the centre deflection being $\left(\frac{1}{c}\right)th$ of it:

$$l = \frac{2Eh}{5Tc}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where E = modulus of elasticity in same unit as T,
h = depth of beam in inches, and
T = allowed fibre stress.

Having determined the beam and girder spacing for the typical size of beam in the job, calculation of the other floor beams may be wisely postponed until some attention has been given to the foundations, especially if the structure is to rest on yielding soil. If the building is to be very high and very heavy, the problem of arranging the foundations properly for economy as well as safety is the chief part of the work, and in solving it if it should be found necessary to shift the position of a column here and there, as frequently occurs, the beams or girders adjacent to that column throughout the building would have to be recalculated.

The wall sections should now be determined, and the question at what floors to carry the walls settled. In New York City the law requires that a 12-inch curtain wall shall be carried at every floor, but permits a 16-inch curtain wall to be two stories high without support. The same law requires curtain walls to be 12 inches thick in the four top stories (i.e., from the roof down about 50 feet), and every lower section of 50 feet shall have a thickness of four inches more than is required for the section next above it, down to the tier of beams nearest to the curb level. The absurdity of this requirement (passed in 1892) became so manifest in the planning and erection of very high buildings, that the Board of Examiners (a body empowered to modify the Building Law within certain limits) has frequently allowed concessions on this point, although it has never felt at liberty to take so radical a

step as to permit the use of a 12-inch curtain wall throughout the full height of a building 12 or more stories high.

If any of the ornamental front work is heavy or of peculiar construction, or if any special beams are needed to carry bay-windows or spandrel sections, arrangements should be made at this point for all of these features, and the system of wind bracing should be decided upon enough in detail to show how much additional load the columns must sustain on this account.

Having provided for the distribution of all special loads, and having located the principal beams and girders, the next work is to calculate all the column loads down through the building. If the building is to rest on yielding soil, the live and dead loads should be kept separate in these calculations. Most authorities agree that the floor beams should be calculated to sustain all the assumed live load in addition to the actual dead load; very many, however, maintain that the total live load on a floor never reaches the girders, and that still less of it ever reaches the columns. A great many theories have been advanced as to what portions of the live load are actually carried by the girders and columns. The present building law in Chicago requires that the girders shall be calculated to sustain eight tenths of the assumed live load in addition to the dead load, and that the columns be calculated to sustain six tenths of the assumed live load in addition to the dead load. In New York City the law requires that the girders and the columns shall be calculated to sustain the total live load assumed for each floor in addition to the actual dead load, and that this total load be assumed to rest upon the foundations. This rather rigid requirement is in excess of the best practice among engineers who are not restricted in any way in their apportionments of the loads; and, were it not for the exceptional character of the ground underlying New York City, such a requirement would be a great hardship in the construction of very tall buildings. The law, however, is somewhat compensatory in permitting a pressure of four tons per square foot on "good earth," without in any way defining the character of the soil referred to, and leaving it discretionary with the Superintendent of Buildings as to whether or not this requirement should be less. Previous to the enactment of the Chicago law referred to, it was customary among engineers and architects to assume that a certain percentage of the sum of the live loads of all the stories above it was carried by the column in a given story, and that this percentage diminished uniformly from the top to the bottom of the building. It was also considered good practice to ignore the live load entirely in proportioning the foundations; in so doing, however, the unit of bearing area upon the ground was made low.

The writer believes that, theoretically and practically, it is right to proportion the foundations to the dead loads only (meaning by dead loads all the permanent loads), even admitting that all of the live load eventually reaches them. Under the present New York City requirements this can be done as follows: Having scheduled the live and dead loads separately, the total dead load and the total live load on each footing are known; the sum of these two in each case gives the total load on the footing, which by law is required to cover such an area of ground that not more than four tons shall bear upon each square foot; the ratio of the dead load, therefore, to the total load in each case must equal the ratio of that portion of the allowed unit, which is used up by the dead load only, to the unit itself, namely, four tons. The third term of this proportion gives the unit due to the dead load only, which can easily be fixed low enough, by observing a sufficient number of cases, so that finally in no case shall the total load per square foot of any footing exceed the required unit, namely, four tons. The writer has used this method frequently, under the jurisdiction of the New York City law, where an equitable distribution of the permanent load was important. The method is rather rigid and, perhaps, wasteful in giving larger footings under interior columns where the dead load is much less than it is in the

wall columns; but if the soil is of a yielding nature the principle of proportioning the footings according to the dead load should be closely adhered to.

A convenient form for load schedule is the following:

LOAD SCHEDULE. TONS.

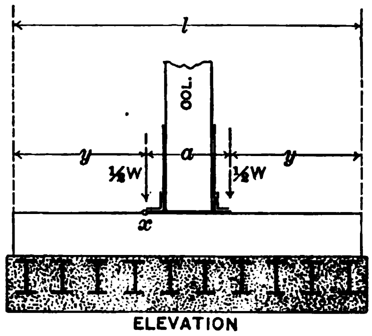
	Col. 1.			Col. 2.			Col. 3.			Etc.		
	Dead.	Live.	Total on Col.	D.	L.	Total.	D.	L.	Total.	D.	L.	Etc.
Roof..........	4	3		4	3		12	12				
Wall...........				11			8					
Special.........	13	4		3	4		6	7				
12th story. Floor..........	4	6	24	10	17	25	8	12	45			
Wall...........	27						20					
Special........												
11th story. Floor..........	3	4	61	11	16	52	6	9	85			
Wall...........	16						10					
Special........				43	66							
. . . .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
1st story. Floor..........	3	4	319	11	16	433	6	9	350			
Wall...........	22						20					
Special........							9					
On Bmt. Col..............			348			460			394			
On Foundation..............	267	81		193	267		255	139				

Each column is usually numbered on the plan; the different stories of the building being represented by the letters of the alphabet, "A" being the first story, "B" the second etc., so that column B6 would mean column number six in the second story. The division of the loads, in the left-hand column, into three parts, viz., floor loads, wall loads, and special loads, is usually sufficient for all practical purposes. The floor loads are understood to include an allowance for the fire-proof partitions, the beams, the girders, the weight of interior columns and the fireproofing for them, and for the plumbing and heating fixtures etc., etc., all of which items should be calculated and reduced to a uniform amount per square foot of the floor area, and added to the dead load of the floor itself. The wall loads include, besides the weight of the walls, the wall columns, the windows, and everything in the walls themselves. It is customary in New York to deduct for all window openings one half the weight of wall which would otherwise fill the opening, adding nothing for the windows themselves. If the walls are very irregular and contain many projections, as might be the case with ornamental front walls, these irregularities should be taken into account and lumped as wall loads. Under the head of special loads should be placed the allowance for wind (a live load), the weights of tanks, vaults, safes, elevators, and all permanent machinery; these loads should be treated as concentrated loads. Although not necessary to the subsequent work of designing, still the division of the loading as here given is convenient and desirable, because in looking over the work to detect any omission a certain amount of detail is advantageous; and further, if any changes are desired in the interior arrangement, it is very easy to rearrange the loads for such changes when they have been kept separate from the beginning. The total load which each column must sustain is put in the third vertical column of the load schedule opposite the proper story just

under the horizontal line dividing the stories, and is the sum of all the partial loads above that line. At the bottom of the load schedule the final total gives the load on the foundations in two parts, dead and live, the sum of which must equal the total on the basement column; to find the pressure on the ground the actual weight of the footings themselves should be added to this.

Grillage. — The distribution of the column loads on the ground by means of I-beams or girders and concrete beds is a part of the work so closely allied to the designing of the superstructure itself, and is so important a problem on account of the excessive concentration of loads in the skeleton-constructed building, that it ought to receive attention here.

CASE I. *When the concrete bed is symmetrical and receives the load from one column situated at its centre.*—The column's shoe must of course be large enough to bear across all the grillage beams or girders in the upper course. The shoe, whether cast-iron or riveted steel, is strong enough to take all the column's load on its perimeter; otherwise the slightest deflection in the upper course of grillage beams would crack or bend it. If, then, a is the width of the shoe's base plate (Fig. 25), y is the projection of the beams beyond the shoe in feet, l the length of the beams in feet, and W the load transmitted through the column, the moment at the point x will be



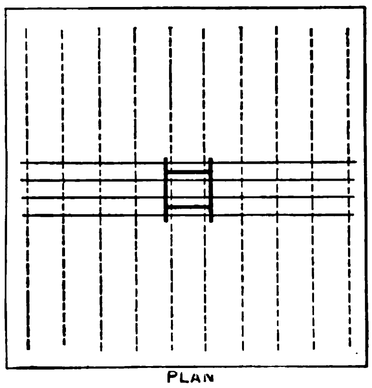


Fig. 25.

$$M=\frac{y}{2}\times\frac{y}{l}\times W=\frac{y^2W}{2l}. \quad (4)$$

The section modulus required to withstand this external moment would be

$$R\left(=\frac{12M}{T}\right)=\frac{6y^2W}{Tl}. \quad (5)$$

To find the required value of R for one beam this value is then divided by the number of beams or girders used. The beams in the lower course are calculated in the same way, the point of moments, x, being situated on the edge of the outside beam of the course above.

CASE II. *When the concrete bed is symmetrical and receives the load from two columns, one situated at each end.*—This case is rarely met with in practice, since if the two columns are not equally loaded the concrete bed must be trapezoidal in form (see Case IV.), or else must extend some distance beyond the more heavily loaded column (see Case III.), in order that the centre of gravity of all the imposed loads may coincide with the centre of pressure of the ground area covered. In this case (see Fig. 26), the columns being loaded equally

the problem is exactly similar to that of a uniformly loaded beam supported at each end (invert Fig. 26) and the greatest moment is

$$M = 2W \times \frac{l}{8} = \frac{Wl}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence the required section modulus is

$$R = \frac{12M}{T} = \frac{3Wl}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The beams of the lower course are calculated by equation (5).

CASE III. *When the concrete bed is symmetrical, and receives the load from two unequally loaded columns situated, one at one end, and the other at a distance y from the other end.*—The distance $l-y$ (Fig. 27) and the load on each column being known, the centre of

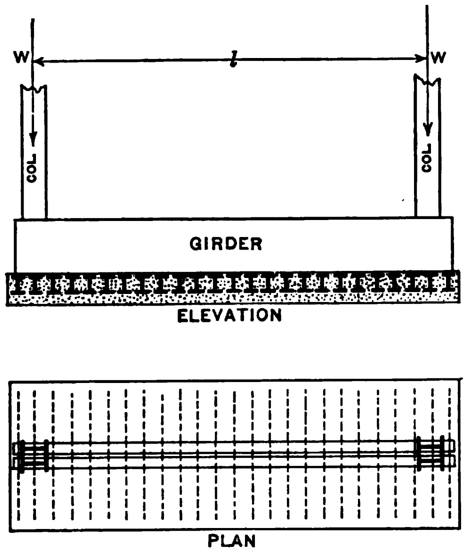


Fig. 26.

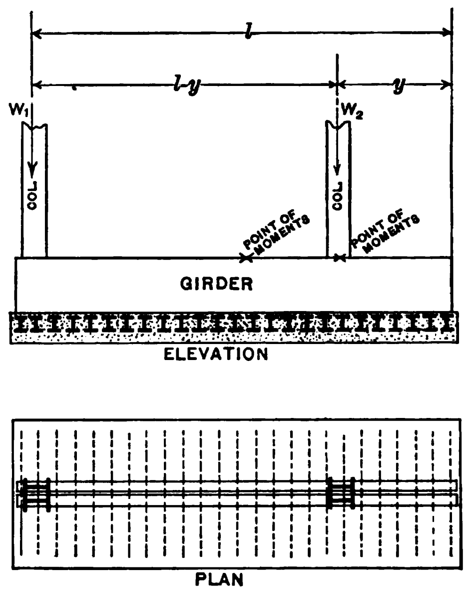


Fig. 27.

gravity of the two loads W_1 and W_2 is known to be at a point distant $\frac{W_2(l-y)}{W_1+W_2}$ from the load W_1. The length of the bed should be twice this distance in order that its centre of pressure and this centre of gravity may be coincident. Therefore

$$l = \frac{2W_2(l-y)}{W_1+W_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Assuming that no deflection occurs in the upper grillage beams, the pressure on the lower course beams is now uniform from end to end of the bed and, per linear foot, is equal to

$$\frac{W_1+W_2}{l} = p \text{ (say)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Next, the points of no shear must be determined in order to calculate the moments at these points and find the greatest. From the load W_1 moving to the right the first point of no shear is $\frac{W_1}{p}$ feet distant. The second point of no shear is under the column at W_2, not exactly at its centre, but located so that enough of W_2 bears to the left of it to be

added to W_1 and equal $p(l - y)$. As the column's shoe in the direction indicated is usually not more than 20 to 24 inches wide (see Fig. 25), it is sufficiently accurate to use the centre of the column for this point of moments—just as the centre of the column at W_1 is used for the end of the span.

Considering all the forces left of the first point of no shear, the moment at that point is

$$M = \left(W_1 \times \frac{W_1}{p}\right) - \left(p\frac{W_1}{p} \times \frac{1}{2}\frac{W_1}{p}\right) = \frac{W_1^2}{2p}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

or, substituting the value of p from equation (9), we have

$$M = \frac{W_1^2 l}{2(W_1 + W_2)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Coming to the point under the column at W_2 and considering the forces to the right, the moment there is

$$M = \left(py \times \frac{y}{2}\right) - \left(\frac{W_2}{2} \times \frac{a}{4}\right) = \frac{py^2}{2} - \frac{W_2 a}{8}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

in which a is the width of the column's shoe in feet; and substituting for p as before, we obtain

$$M = \frac{(W_1 + W_2)y^2}{2l} - \frac{W_2 a}{8}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The greater of the two moments obtained from equations (11) and (13) is then substituted for M in equation (7), and the required value of R is found.

The beams of the lower course are again calculated as before by equation (5).

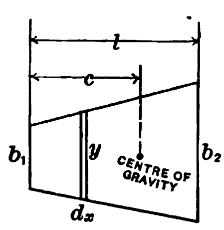
CASE IV. *When the concrete bed forms a trapezoid, and receives its load from two unequally loaded columns, one situated at each end.*—These trapezoidal beds are necessary when the space beyond the more heavily loaded column cannot be utilized, as, for instance, on the edge of the property. The centre of gravity of the loads and the centre of pressure of the ground area must be coincident, as before. If c denote the distance of the centre of gravity from the lighter load, W_1, then

$$c = \frac{W_2 l}{W_1 + W_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

but this distance should be the same as the perpendicular distance from the centre of gravity of the trapezoid to the shorter of its parallel sides, which by the calculus is

$$c = \frac{2b_2 + b_1}{3(b_2 + b_1)} \cdot l,* \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

the notation being as in Fig. 28.



* This equation is deduced as follows:

A being the area,

$$dA = ydx,$$

and

$$y = b_1 + \frac{x}{l}(b_2 - b_1);$$

hence,

$$dA = b_1 dx + \frac{b_2 - b_1}{l} . xdx,$$

and

$$xdA = b_1 xdx + \frac{b_2 - b_1}{l} . x^2 dx.$$

But

$$\int_0^l xdA = Ac,$$

The area of the trapezoid is equal to

$$A = \frac{b_2 + b_1}{2} \cdot l. \quad \ldots \ldots \ldots \ldots \quad (16)$$

This area is of course known by dividing the sum of the two loads, W_1 and W_2, by the

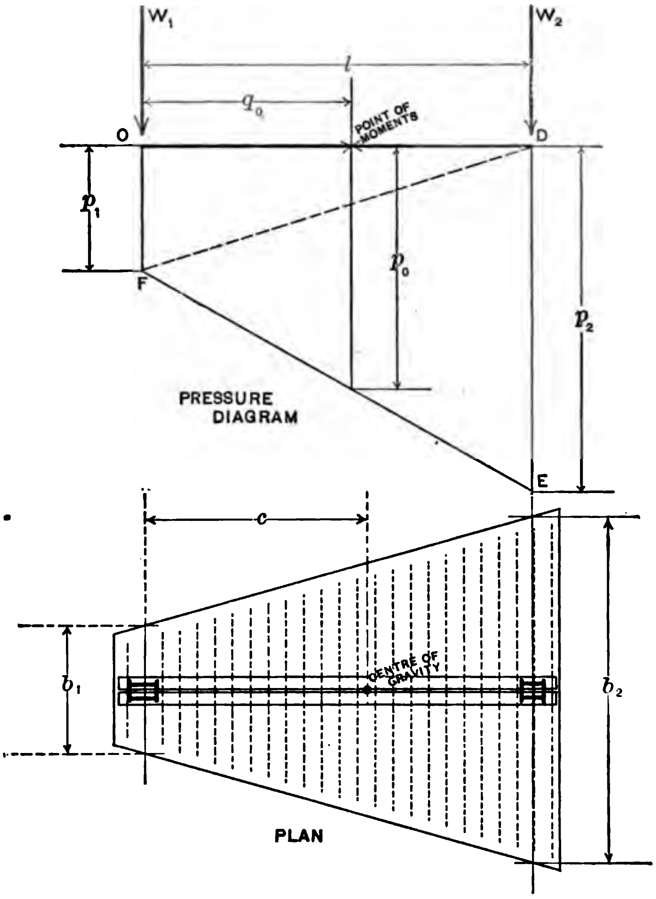


Fig. 28.

allowed bearing pressure per unit of area. Consequently, we have two equations, (15) and (16), containing but two unknown quantities, b_1 and b_2.

Solving for these, we obtain

$$b_1 = \frac{2A}{l^2}(2l - 3c), \quad \ldots \ldots \ldots \ldots \quad (17)$$

---

hence

$$Ac = b_1 \int_0^l x dx + \frac{b_2 - b_1}{l} \int_0^l x^2 dx + \text{constant},$$

the constant being zero, because $x = 0$ when $A = 0$. Therefore

$$Ac = \frac{b_1 l^2}{2} + \frac{(b_2 - b_1)l^2}{3} = l^2\left(\frac{b_2}{3} + \frac{b_1}{6}\right).$$

Since

$$A = \frac{b_1 + b_2}{2} . l.$$

we have

$$c = \frac{2b_2 + b_1}{3(b_2 + b_1)} . l. \quad \text{Q. E. D.}$$

and

$$b_2 = \frac{2A}{l^2}(3c - l).^* \quad \dots \dots \dots \dots \quad (18)$$

Substituting for c in each of these equations its value in equation (14), we have

$$b_1 = \frac{2A(2W_1 - W_2)}{l(W_1 + W_2)}, \quad \dots \dots \dots \dots \quad (19)$$

and

$$b_2 = \frac{2A(2W_2 - W_1)}{l(W_1 + W_2)}. \quad \dots \dots \dots \dots \quad (20)$$

It will be observed that the trapezoid becomes a triangle when $b_1 = 0$; which, from equation (19), means when $W_2 = 2W_1$. The interpretation of this is that Case IV. does not apply when either load is less than half the other.

Considering a longitudinal section through this bearing area, as shown in elevation (Fig. 28), the pressure per linear unit is of course greater under the heavier load and gradually diminishes to the other end. Denoting these linear unit pressures by p_1 and p_2 under the lighter and heavier loads respectively, and knowing these pressures to be made up of as many square unit pressures as the trapezoid contains widthwise at any given point we have the equations

$$p_1 = b_1 U, \quad \dots \dots \dots \dots \quad (21)$$

and

$$p_2 = b_2 U, \quad \dots \dots \dots \dots \quad (22)$$

where U is the assumed unit pressure per square foot on the ground area. The unit pressure due to either load is zero at the other end and increases uniformly to the load itself, so that it may be represented at any point by the ordinate to the straight line which completes the triangle whose base is the unit pressure and whose altitude is the distance between the loads. Putting the two triangles (*OFD* and *DFE*) together forms the trapezoid whose parallel sides are p_1 and p_2 (Fig. 28), from which the linear unit pressure due to both loads can be read at any point along the upper grillage beams or girders.

It is easily proved that at the point of no shear this ordinate is

$$p_0 = \sqrt{\frac{2W_1}{l}(p_2 - p_1) + p_1^2}.\dagger \quad \dots \dots \dots \dots \quad (23)$$

* Equations (17) and (18) may also be deduced geometrically.

† If x represents the abscissa and y the ordinate of any point on the line *FE* (Fig. 28), then the shear at any point on the beam *OD* is

$$h = W_1 - \frac{p_1 + y}{2} \cdot x,$$

But

$$y = p_1 + \frac{x}{l}(p_2 - p_1);$$

therefore, by substitution, we have

$$W_1 - p_1 x - \frac{x^2}{2l}(p_2 - p_1) = h,$$

an equation showing that the line in which the ordinates representing shear terminate is the curve of the parabola. When the shear is zero, $h = 0$, and

$$p_1 x + \frac{x^2}{2l}(p_2 - p_1) = W_1.$$

Solving this for x, we find

$$x = \frac{\pm\sqrt{2W_1 l(p_2 - p_1) + p_1^2 l^2} - p_1 l}{p_2 - p_1},$$

and, substituting this value of x in the above equation for y, remembering that $y = p_0$ at the point of no shear, we have

$$p_0 = \sqrt{\frac{2W_1}{l}(p_2 - p_1) + p_1^2}. \qquad \text{Q. E. D.}$$

If q_0 represents the distance of this point from the load W_1, then from the similarity of triangles

$$q_0 = \frac{p_0 - p_1}{p_2 - p_1} \cdot l. \qquad \qquad (24)$$

With this as a point of moments, considering all the forces to the left of it and remembering that the sum of all the unit pressures between the point of no shear and the load W_1 is equal to W_1 itself, we have

$$M = W_1 q_0 - W_1(q_0 - c_1) = W_1 c_1, \qquad \qquad (25)$$

where c_1 represents the distance from the load W_1 to the centre of gravity of the said sum of unit pressures, or to the centre of gravity of the trapezoid whose parallel sides are p_0 and p_1. Changing the notation in equation (15) to suit this trapezoid, we obtain

$$c_1 = \frac{2p_0 + p_1}{3(p_0 + p_1)} \cdot q_0. \qquad \qquad (26)$$

Having found the moment from equation (25), the required value of R may be obtained from equation (7) as formerly.

The lower course grillage beams, being of different lengths and distributing different pressures, should be computed singly. Knowing how far apart these beams will be located, we may represent their positions along the line OD (Fig. 28) by points, at each of which ordinates drawn to the line FE will measure the respective unit pressures. Therefore each ordinate multiplied by the distance between two successive ordinates (or two adjoining beams) will give the load W which is to be substituted in equation (5) to determine R or the size of the beam.

CASE V. *When the concrete bed is rectangular and receives its load from three or more*

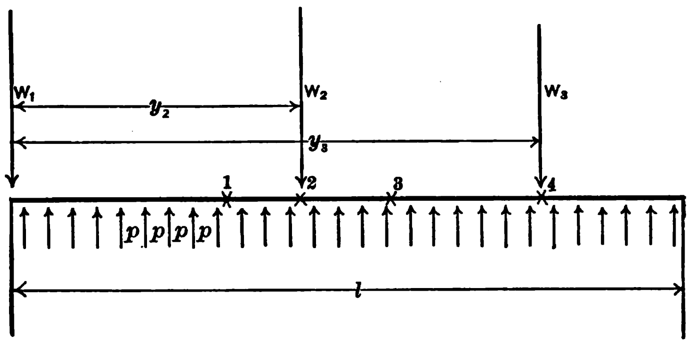


FIG. 29.

unequally loaded columns.—The centre of gravity of the three loads (Fig. 29) is distant $\frac{W_2 y_2 + W_3 y_3}{W_1 + W_2 + W_3}$ from W_1, and as the bed must extend an equal distance each side of this point, the length

$$l = \frac{2(W_2 y_2 + W_3 y_3)}{W_1 + W_2 + W_3}, \qquad \qquad (27)$$

supposing the centre of gravity to be nearer W_3 than W_1. As in Case III., the pressure on the lower course beams is now uniform, and, per linear foot, is

$$p = \frac{W_1 + W_2 + W_3}{l}. \qquad \qquad (28)$$

At the several points of no shear the equations of moment are made up as follows:

$$\text{at 1,}\quad M=\left(W_1\times\frac{W_1}{p}\right)-\left(W_1\times\frac{W_1}{2p}\right)=\frac{W_1^2}{2p}; \quad\ldots\ldots \quad (29)$$

$$\text{at 2,}\quad M=(W_1\times y_2)-\left(py_2\times\frac{y_2}{2}\right)+\frac{W_2}{2}\times\frac{a}{4}=W_1y_2-\frac{py_2^2}{2}+\frac{W_2a}{8}; \quad\ldots\ldots \quad (30)$$

$$\text{at 3,}\quad M=\left(W_1\times\frac{W_1+W_2}{p}\right)+\left[W_2\times\left(\frac{W_1+W_2}{p}-y_2\right)\right]-(W_1+W_2)\times\frac{W_1+W_2}{2p}$$

$$=\frac{(W_1+W_2)^2}{2p}-W_2y_2; \quad\ldots\ldots \quad (31)$$

$$\text{at 4,}\quad M=W_1\times y_4+W_2(y_4-y_2)-\left(py_4\times\frac{y_4}{2}\right)+\frac{W_3}{2}\times\frac{a}{4}$$

$$=(W_1+W_2)y_4-W_2y_2-\frac{py_4^2}{2}+\frac{W_3a}{8}, \quad\ldots\ldots \quad (32)$$

a being the width of the column's base plate, as before.

Equations giving moments at each of these points of all the forces to the right can be formed in a similar manner and used for checking.

The greatest bending moment on the upper course of grillage beams is given by one of the four equations (29), (30), (31), and (32). When found this moment is substituted in equation (7) to obtain R, as before.

The concrete bed being rectangular, the beams of the lower course may be calculated by equation (5).

CASE VI. *When the concrete bed is trapezoidal in form and receives its load from three or more unequally loaded columns.*—Having found the centre of gravity of the three or more loads, the parallel sides of the trapezoid can be computed from equations (17) and (18), remembering that c is measured from the short end of it and that A is the sum of *all* the imposed loads divided by the allowed pressure per unit of area. If Fig. 30 represents a longitudinal section through such a system, we may lay off p_1 and p_2, as in Case IV., by equations (21) and (22), and draw the line FE, to which an ordinate from OD at any point represents the pressure per linear unit at that point. The points of no shear, 1 and 3, are readily obtained by adapting equations (23) and (24) as follows:

$$p_3=\sqrt{\frac{2W_1}{l}(p_2-p_1)+p_1^2}; \quad\ldots\ldots \quad (33)$$

$$p_4=\sqrt{\frac{2(W_1+W_2)}{l}(p_2-p_1)+p_1^2}; \quad\ldots\ldots \quad (34)$$

$$q_3=\frac{p_3-p_1}{p_2-p_1}.l; \quad\ldots\ldots \quad (35)$$

$$q_4=\frac{p_4-p_1}{p_2-p_1}.l. \quad\ldots\ldots \quad (36)$$

Another point of no shear will occur under the load W_2, which may be considered to be at the centre of the column, although this is not exact, as previously explained. The

distance from the short side to the centre of gravity of any of these trapezoids may now be found by adapting equation (15), and hence the moments at the points 1, 2, and 3 can be computed in the usual manner. The greatest moment should then be selected, and equation (7) solved for *R*.

The lower course grillage beams are calculated as in Case IV.

At this point it is usual to go back to the floor plans and calculate the sizes of all the beams and girders not yet known. In doing this it is well to keep as close to the typical size of beam as practicable, in order that there may not be too many sizes in the job; the extra cost of a slight excess in material, by using the typical beam when a lighter beam would be sufficient, is often more than balanced by the fact that unless many of the latter were needed in the job, there would probably be delay in procuring them from the mills.

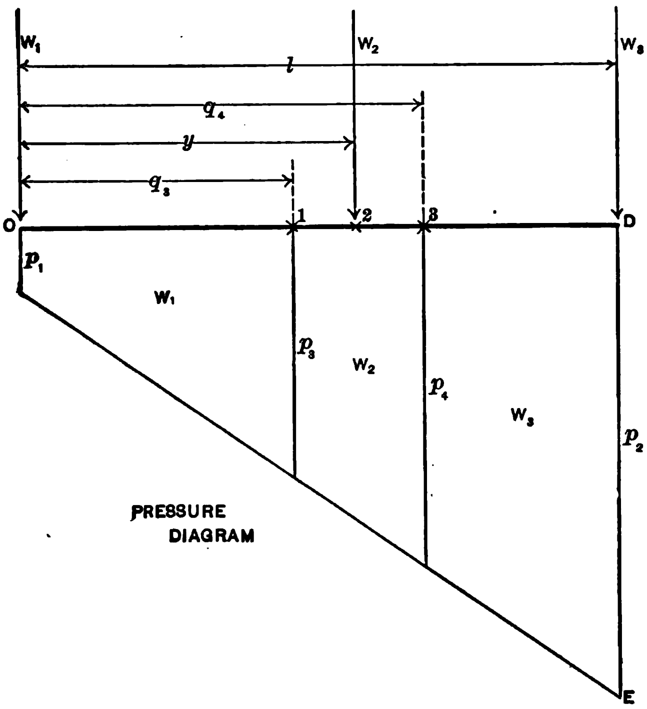


Fig. 30.

Girders should be calculated for the series of concentrated loads brought to them by the beams which frame into them, since this method usually gives a smaller moment than to assume the girders loaded uniformly. No beam should frame into another one of less depth than itself, even though the respective loading may suggest it, as thereby not only is the labor in the shop and on erection increased, but there is difficulty in designing a sufficient connection. Beams should connect to the webs of the girders in preference to resting on the girders; although the latter is a cheaper mode of construction, it lacks stiffness and rigidity.

The permissible fibre stress, *T*, [see equations (1) to (7)] for steel floor beams and girders is usually taken at 16,000 lbs. per square inch unless otherwise specified. The New York law limits its value to 15,000 lbs. For grillage beams, however, *T* may be taken as high as 18,000 or 20,000 lbs. within the limits of good practice; first, because the load upon them can never be applied suddenly or with a shock; and second, because the concrete in which they are imbedded aids materially in the performance of their work.

Columns.—In calculating the sizes of the columns the load which each column has to carry may be taken directly from the load schedule already prepared; a schedule for each column is usually made out somewhat as follows:

COLUMNS NO. 8 AND 14.

Story.	Required Load. Tons.	Composition.	Area. □″	No. of Sketch on Sheet 16.	$\frac{l}{r}$.	Unit Stress. Tons.	Safe Load. Tons.	Wt. pr. foot. lbs.
12	32	4 ∠s 3″ × 3″ × ¼″ – 1 pl. 8″ × ¼″	7.76	1	$\frac{12.0'}{1.21''}$	4.12	32	26
11	62	4 ∠s 3½″ × 3″ × 7/16′ – 1 pl. 8″ × ¼″	12.60	1	$\frac{10.0'}{1.48''}$	5.07	64	43
10	90	2 [s 10″ × 16½ lbs. – 2 pls. 14″ × 5/16″	18.45	3	$\frac{9.5'}{4.5''}$	5.86	108	63
9	119	2 [s 10″ × 20 lbs. – 2 pls. 14″ × 5/16″	20.50	3	$\frac{9.5'}{4.36''}$	5.86	120	70
8	152	2 [s 12″ × 20 lbs. – 2 pls. 16″ × 7/16″	25.75	4	$\frac{9.5'}{5.51''}$	5.91	152	87
etc.	etc.	etc. etc.						

The properties of each column when calculated are set down in their proper places. The safe load is of course the product of the unit stress by the area. The weight per foot here given does not include an allowance for splice-plates, connections, etc. It will be observed that the columns are designed to be continuous through two stories,—the ninth and tenth story column having 14″ × 5/16″ plates running full length, while the channels are spliced at the tenth-floor level,—the eleventh and twelfth story column having one 8″ × ¼″ web plate extending full length, while the angles are spliced at twelfth floor. Columns in the eighth, ninth, and tenth stories are in style like Fig. 6; those in the eleventh and twelfth, like Fig. 10. As a mere matter of economy it can be readily figured out whether any attempt to reduce the column's section in the upper story will be more or less costly than to continue the section of the lower story through the upper one also. Where the difference is slight no change of section should be made.

In the above schedule the columns were calculated by the Gordon-Rankine formula,

$$p = \frac{f}{1 + \frac{144 l^2}{a r^2}},$$

l being in feet and r in inches. The constant f used was 12,000 lbs., and the constant a was 36,000. The New York law requires the use of this formula, and specifies that f shall equal 12,000 for steel and 10,000 for wrought iron, but makes no mention of the constant a. Many engineers prefer a "straight-line" formula, and the Chicago building law provides one for steel columns as follows: $p = 17000 - \frac{60l}{r}$ for columns more than 60 radii in length, and $p = 13500$ for columns under 60 radii,—l and r both in inches. Good practice restricts the length of built columns in this class of work to thirty times the least dimension or about 120 radii of gyration, and that of cast-iron columns to twenty times the least dimension. The subject of column formulas, including provision for eccentric loading, etc., etc., is fully discussed in the chapter on "Theory of Flexure."

Wind Bracing.—Of the various methods of stiffening or wind bracing, those shown in Figs. 14 and 16 are in general use,—that of Fig. 15 being practicable only where windows or doors can be omitted, and that of Fig. 17 being expensive. The use of lattice girders

for this purpose (Fig. 16) is becoming so general as to justify a little special discussion at this point. Referring to Fig. 31 for notation, we observe that the external force of the wind is F, applied outside the column, one half in a line with each chord of the girder. These forces are each made up of the wind pressure upon a superficial area of the building extending half-way to the next force in either direction, horizontally or vertically. This pressure is commonly assumed to be from 30 to 40 lbs. per square foot. The horizontal shear due to the force F must be resisted by the two columns at any point between C and the next girder below, and hence at the bottom of the column. If the total horizon-

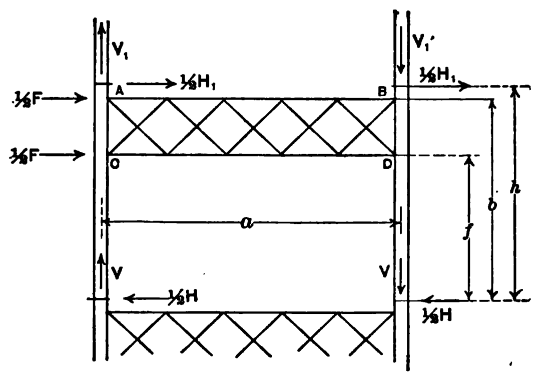


Fig. 31.

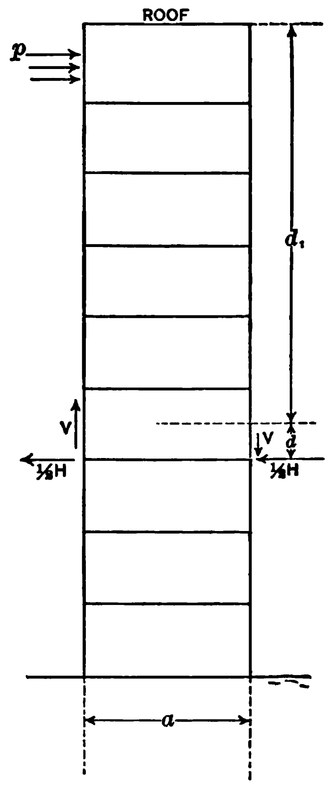


Fig. 32.

tal shear from all the stories above is H_1, one half resisted by each column, then the total shear at the bottom of the story is

$$H = H_1 + F. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Representing the compression in the leeward column due to the wind by V, and that from all the stories above by V_1, we have

$$V = V_1 + \frac{H_1 h + \frac{1}{2}F(b+f)}{a}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

which is also the tendency to tension in the windward column. This value of V is a live load on the leeward column, and should be added to all the other column loads.

It may be obtained more simply from the following equation, based on notation in Fig. 32, and its value here should agree with that given by equation (38):

$$V = pd_1 \times \left(\frac{d_1}{2} + d\right) \times \frac{1}{a} = \frac{pd_1(d_1 + 2d)}{2a}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

where p is the wind pressure per linear foot of height, and d_1 represents the distance from the roof to a point midway between the lattice girders next above and below the story in question. In closely built up cities the wind pressure is usually assumed to act above the fourth or fifth story only, i.e., above the average height of surrounding buildings.

The stress in the upper flange AB (Fig. 31) is

$$AB = \frac{\frac{1}{2}H_1(h-f) + \frac{1}{2}Hf}{b-f} + \frac{1}{2}F, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

which is compression; the stress in the lower flange CD is

$$CD = \frac{\frac{1}{2}H_1(h-b) + \frac{1}{2}Hb}{b-f} - \frac{1}{2}F; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

also compression. The moment on the column at the point C, considering both ends fixed, is

$$M = \frac{1}{2} \times \frac{H}{2} \times f = \frac{Hf}{4}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

and the column should be so designed that it can resist this bending moment in addition to its other work. In tall, narrow buildings the columns are made much wider crosswise than lengthwise of the building for the purpose of taking up this moment.

Since these lattice girders usually carry wall and floor loads they must be designed for these first in the usual way. The chord sections should then be increased for the wind strains as here shown, and the diagonals may be slightly increased for the extra work of providing rigidity between the chords.

For a thorough discussion of overhead and portal sway bracing to which the above system is analogous, and for the treatment of knee-bracing or the system of brackets in Fig. 14, the reader is referred to the chapter on "Wind-bracing." Diagonal-rod bracing (Fig. 15) should be treated in a similar manner to the lattice-girder bracing, the differences being that the former puts no bending moment in the column and that the forces F and H_1 (Fig. 31) all act at one point, viz., the floor level. The stress in the diagonal rod is the product of the stress in the horizontal strut above it and the secant of the angle between them. The wind load on the leeward column is given by

$$V = V_1 + \frac{(F + H_1)h}{a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

Unless the rods are attached at the centre of the columns, this load on the columns will be eccentric, and to this extent does cause a bending moment.

In the system of portals (Fig. 17) there should be thickness of plate enough on any horizontal or vertical section to take up the shear in that direction, and the flanges should be proportioned to resist the greatest moment at any point on the curve.

4. DETAILS.—Like all other structural steel work, it is very important that the details should be carefully designed for economy, strength, and facility of erection; much can be wasted or saved in the designing of the details, and a large share of the strength of the structure is either present or lacking in them. The principles of detailing are about the same as in other branches of structural steel work, and they should be applied. In connecting beams or girders to columns web connections are preferable, since they are more efficient in providing lateral stiffness; if, however, it is not always possible to make the web connections strong enough for the purpose, a seat re-enforced by stiffening angles is placed under the end of the beam to help support it. If cast-iron columns are used this seat

should be supported by a bracket directly underneath the web of the beam or girder resting upon it; and, if there are two beams, there should be two brackets. If the column is a built column the seat is usually made the horizontal leg of an angle, and stiffeners for supporting it are placed directly under the web of each beam or girder.

An allowance of three eighths of an inch should be made at each end of each beam or girder which comes against a cast-iron column; with steel columns, this allowance is usually made one fourth of an inch. Where a beam frames in between two header beams or girders an allowance of three sixteenths of an inch on the total actual distance between the webs of the girders should be made. These allowances are made to facilitate erection and to overcome any unevenness there may be in the material, as it is found that when material is cut to exact lengths there is always a cumulative error in some one direction which forces the total over-all dimensions considerably out. The above allowances should be made from the last holes to the end of the beam; that is, the distance between the holes in the beam itself should be exact. All dimensions should be given on the drawings in exact figures; distances from centre to centre of beams and girders should be distinctly marked; and, unless the connection is a standard one, and can be readily referred to as such, it should be detailed and every hole accurately dimensioned. The cost of erecting the building is materially affected by the way in which the details are made; time can be saved and thus gained for the capital invested if they are made in an advantageous manner. There is almost no other branch of engineering where so much can be done, by way of careful preparation, to facilitate the speed as well as the accuracy of the work.

5. ERECTION.—Much depends upon the accurate alignment of the base plates or shoes for the basement columns; they should be set exactly, both for line and for level, and securely grouted or bolted in their places on the foundation. Built-steel columns are usually erected in two-story lengths, occasionally in three-story lengths—a practice which results in much saving of time and some expense; but it is not necessary that the column whose length is two stories contain the same section throughout its length, since outside plates may be riveted on through the lower story only, or part of the section may be spliced at the intermediate floor the remainder running full length. The beams and girders are first bolted temporarily in their places, about one third of the bolt-holes being filled; if any of the connections are to be riveted, a riveting gang follows closely behind the erectors; tie-rods should be drawn just taut, so that the beams remain in perfect alignment and no initial stress is put in the rods. Columns should be jointed just above a tier of beams so that the beams frame near the top of the columns; when columns are jointed just below a tier of beams and the beams frame at the bottoms of the columns, erection is much more difficult and not as safe. The rapidity of erection is not determined so much by the cubic contents of the building as by its linear height, the rate of putting the work together being usually about two tiers of beams per week without regard to the size of the building. Of course when the building covers a *very* extensive ground area the above statement should be modified somewhat, since material cannot then be picked up directly from the street and landed at its destination with one sweep of the boom.

6. SPECIFICATIONS.—In writing the specifications to accompany a set of plans it is well to avoid useless repetitions of the ordinary facts plainly set forth on the plans, as thereby conflicting statements between the plans and specifications often result; if there are any very unusual features in the work, which affect the cost, these should be carefully described in the specifications, and the limitations of the work should be defined with accuracy so that the iron contractor will know exactly where he is to stop and the next contractor to begin, or *vice versa*. The quality of the steel or iron ought to be distinctly defined—broadly enough so as not to restrict the market too much and yet to be on the side of safety; most of the mills now furnish material under specifications of their own, and want an extra price

or extra length of time for work under more rigid specifications. This action on their part is the outcome of the large demand for steel during the last four or five years by people who do not know whether they are getting good or bad material and never hire anybody to tell them. Formerly, before the days of skeleton construction, the larger portion of the demand for structural iron and steel was from railroads, municipalities, and others who defined what they wanted and enforced their requirements by rigid inspection. Of late years the demand for steel in building construction has been so enormous in comparison with the other that it has regulated the market and has enabled the mills to get together and practically to dictate the specifications. The New York market receives and uses about every grade of steel rolled without much investigation as to its quality. Many architects argue that the building law requires such an excessive assumption for live load, besides a large factor of safety, that time or money spent in enforcing a requirement for any particular grade of steel is wasted. The consequence is that material rejected on work where inspectors have been employed easily finds its way into buildings on which there is no shop or mill inspection. It stands to reason, however, that the life of a building which is constructed with high-grade material, well designed and well put together, will be much greater than the life of one otherwise constructed, although to the unskilled eye the difference would not be apparent during the first few years of their existence.

The following requirements will produce a grade of steel which will give thoroughly satisfactory results in this class of work, and come well within the present practice of any first-class rolling-mill:

All material shall be open-hearth steel.

Ultimate tensile strength shall be between 58,000 and 68,000 lbs. per square inch.

Elastic limit shall be at least one half the ultimate strength.

Elongation shall be at least 24 per cent in 8 inches.

Reduction of area shall be at least 48 per cent.

The specimen shall bend cold, or quenched from a dark cherry-red heat in water of 80 degrees Fahr., 180° around a diameter equal to thickness of the piece bent without sign of rupture on the convex side; and shall bend 180° flat at any heat from a dark red to a light yellow without sign of rupture on the convex side.

Chemical analysis shall show not more than .10 of one per cent phosphorus for acid steel, and not more than .05 of one per cent phosphorus for basic steel.

If the reader desires thorough information on the subject of structural steel, he is referred to "A Manual for Steel Users," by Wm. Metcalf, published by John Wiley & Sons, 1896; and "Manufacture and Properties of Structural Steel," by H. H. Campbell, published by the Scientific Publishing Co., 1896.

Specifications for the shop work are not as rigid in this class of work as in railroad bridge work. Drilling rivet-holes is almost never required, and a moderate use of the drift-pin is allowed both in the shop and field. Any considerable degree of mismatching should be corrected by reaming. The ends of all columns should be milled to a true surface exactly normal to the column's axis, leaving the finished length of the column absolutely accurate as per plans. Requirements for rivet spacing and driving are the same as for other classes of work.

7. SUPERVISION AND INSPECTION.—Until recently, almost the only examination or inspection which building material was subjected to occurred after it had been delivered on the job, but since these very high buildings have come in vogue a few of the persons charged with the responsibility of making them safe feel that every known precaution to make them so ought to be taken and, consequently, have resorted to the method of shop and mill inspection so long customary in bridge work. If the shop inspection is done with

care mistakes are frequently found before the work is shipped and thus important delays are avoided. The mill inspection is valuable or otherwise depending upon its rigidity.

In supervising the erection there are many little points to look out for which can only be learned by experience in this line of work; a man with a practised eye will very soon discover the weaknesses of any particular contractor, and be on the lookout for these particular points. If the drawings have been accurately made, a close adherence to them should be compelled, as any deviation from the plans at the beginning will affect the work later on.

Where connections are bolted the nuts should be left tight, and bolts should be long enough to fill the nut. Field rivets should be driven absolutely tight, the button head being as near the centre as possible; rivets with cracked or eccentric heads should be cut out and replaced. Assuming that the connections have been designed with the usual ten per cent excess over the required amount of bolts or rivets, a single rivet here and there which did not come up to the standard of workmanship might be passed rather than that the work be delayed by having it replaced. In general it is just as easy to do the field work right as wrong, and the mere presence of a competent inspector or supervisor always has a wholesome effect.

8. TABLES AND DIAGRAMS.—The work of calculating the sizes of columns, beams, girders, or trusses has of late years been greatly simplified and much of it eliminated by the use of tables and diagrams based on fundamental formulæ, both rational and empirical. Almost every engineer has his own peculiar methods of working, and consequently uses tables or diagrams adapted to them. A set of tables and diagrams on the loading and spacing of beams and columns, and the calculation of plate girders for this class of work, was published in 1894 by the writer, and copies may be procured of the Engineering News Publishing Co., New York. Each table or diagram is accompanied by an explanation and suitable examples.

9. REFERENCES.—Periodical literature on the subject of skeleton-constructed buildings and allied topics is, of course, comprised within the last ten or twelve years, and even down to five years ago is decidedly meagre. The following list of references has been compiled from four well-known sources, viz.: (1) *Transactions of the American Society of Civil Engineers;* (2) *The Engineering News;* (3) *The Engineering Record* (from December 1887 to December 1890 known as *The Engineering and Building Record,* from June to December 1887 known as *The Sanitary Engineer and Construction Record,* and previous to June 1887 known as *The Sanitary Engineer*); and (4) *The Railroad Gazette.* The classification of subjects is made as brief as possible, and only those special headings are used which seem to be most important; the general heading covers references to those articles only which treat no particular branch of the subject, and could therefore be otherwise classified only by endless duplication. There is no duplication in the list as here given; each reference is given once only, except in rare instances where two particular subjects are treated in one article.

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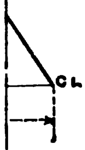
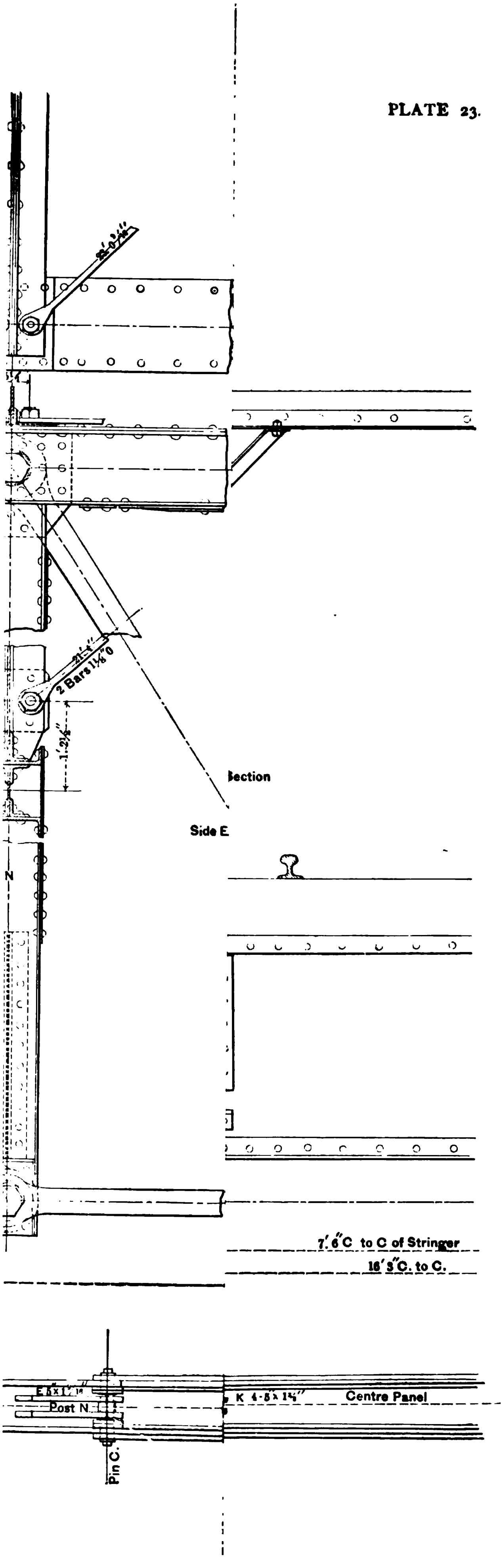
PLATE 22.

STRAIN SHEET.

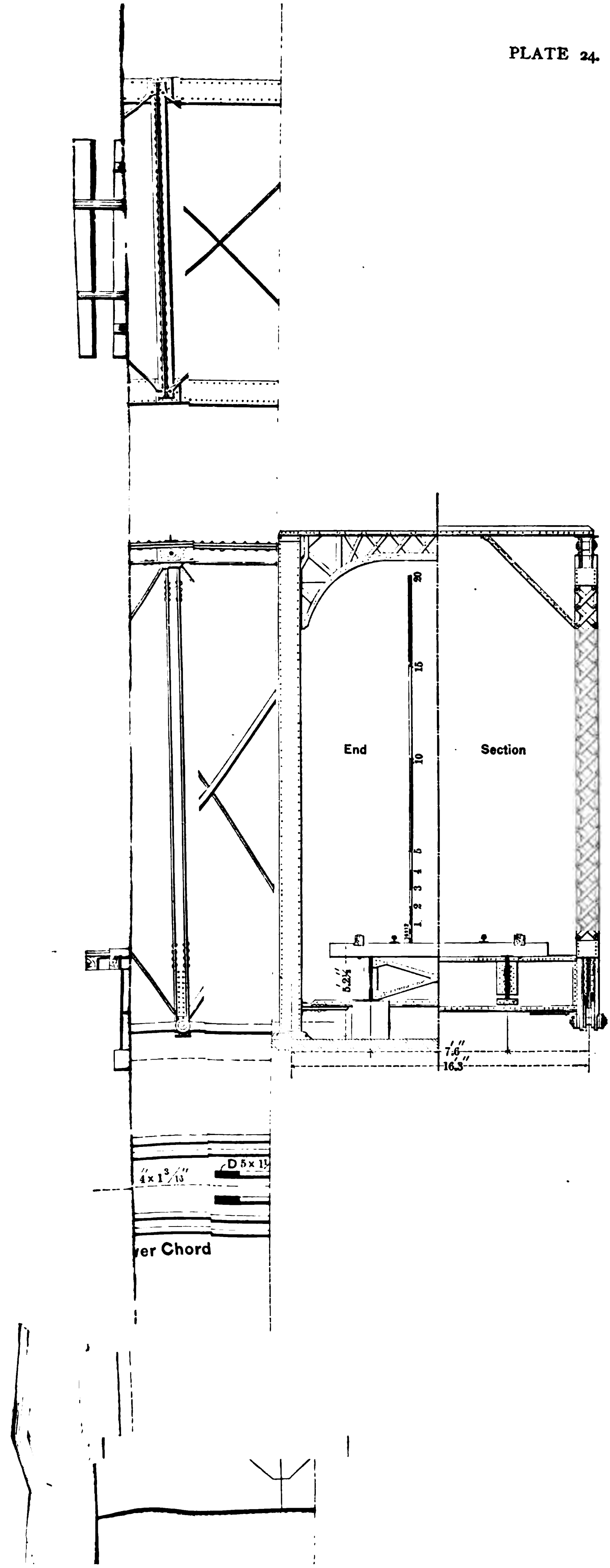


		Total Cross Section ▭″.	Max. strain per ▭″ Lbs.
Strain on A	....	.785	5,642
B	....	4.810	7,670
C	....	7.5	8,826
D	....	11.25	9,124
E	....	15.625	9,085
F	...........net.	5.469	7,209
G	....	11.5	8,751
H	....	19.	9,090
I	....	25.	8,739
K	....	27.5	8,875
L	tes 12″ × ¼″....	6.	5,663
M	ars 2 × ¼″ double.	9.6	6,093
N		13.2	6,742
O	....	26.62	6,907
P	⅜″	24.75	6,978
R	....	29.05	7,530
S	....	31.5	7,748
T	....	31.5	7,748
Floor Beams	....	....	......
Max. Load o	a 8″.29 net...	T. Gross 9.27 B. Net 8.29	7,153 7,999
Centre	....	....	......
	....	....	......
Max. Load o	Area, 7″.0 net	T. Gross 7.79 B. Net 7.	6,431 7,157
Centre	′ long....	....	......
	plates $\frac{5}{16}$″ thick.	....	......
Tr	½″	....	......
Ma	d plates 1″–0....	....	......
Bo	....	....	......
	....	....	......
	....	....	......
	....	....	......
	....	....	......
Ar	....	....	......
To	....	....	......
Fo	....	....	......
Di	....	....	......
	....	....	......
	....	....	......
	....	....	......
	....	....	......
	....	....	......

PLATE 23.



2 Bars 1⅛" ⌀
Section
Side E
7' 6" C to C of Stringer
18' 3" C. to C.
Post N.
K 4-5 x 1¼"
Centre Panel
Pin C.


End
Section
7.6"
16.3"
5.2½"
D 5 x 1½
4 x 1 3/13"
ver Chord

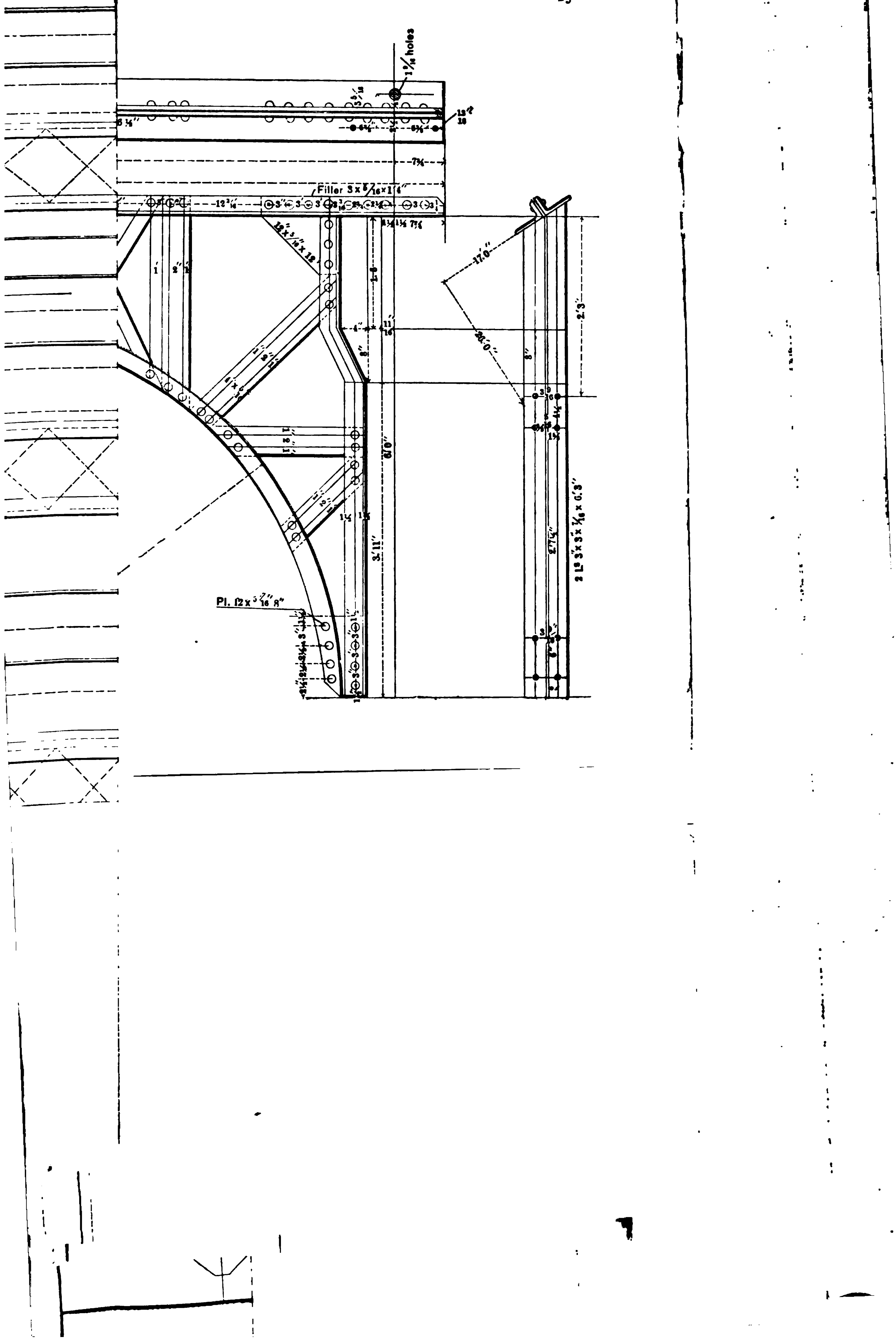

1 5/16 holes
Filler 3 x 5/16 x 1' 4"
Pl. 12 x 5/16" 8"
2 Ls 3" x 3" x 7/16 x 6' 3"
17' 0"
20' 0"
6' 0"
3' 11"
2' 3"

PLATE 26.

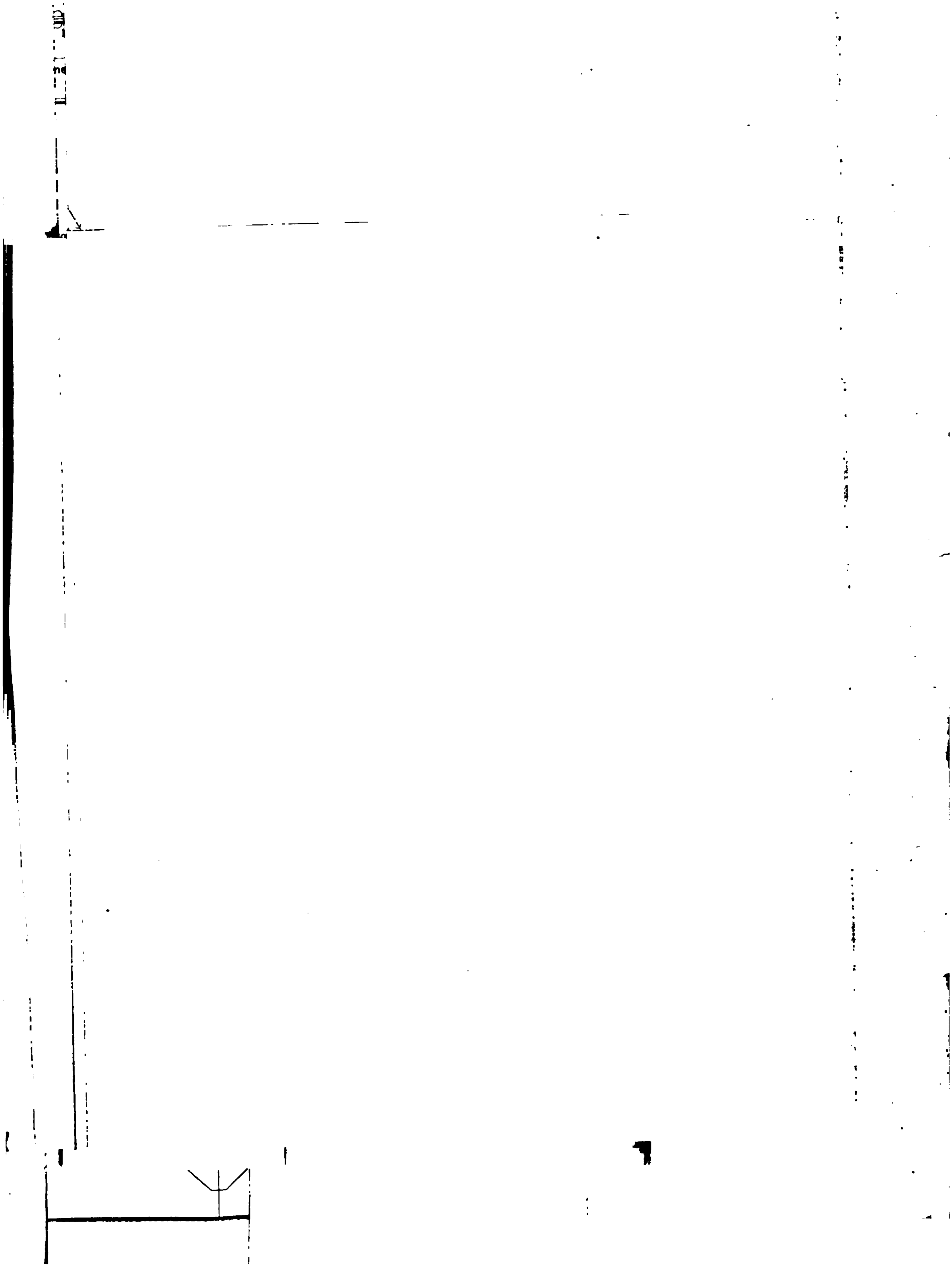


PLATE 27.

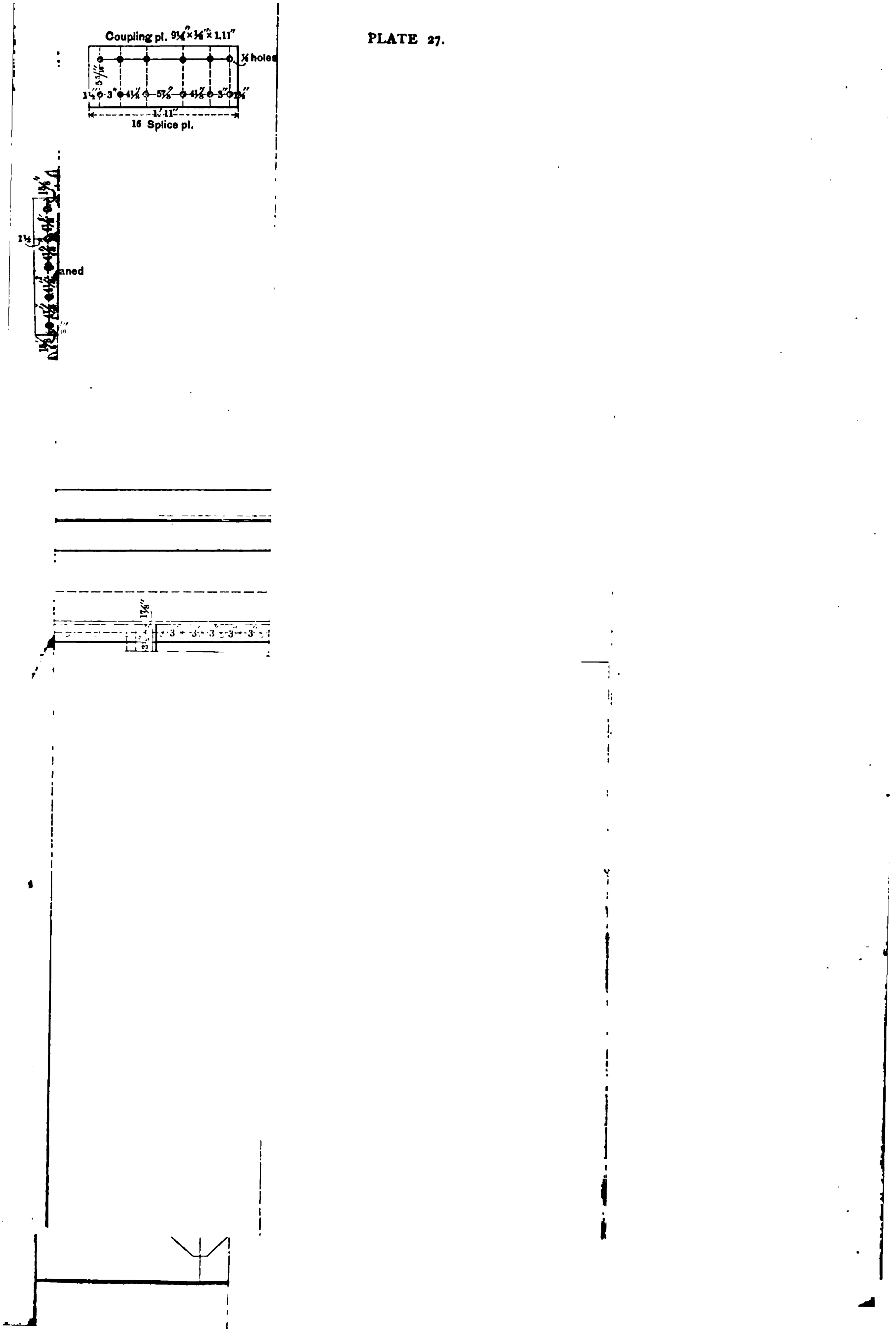
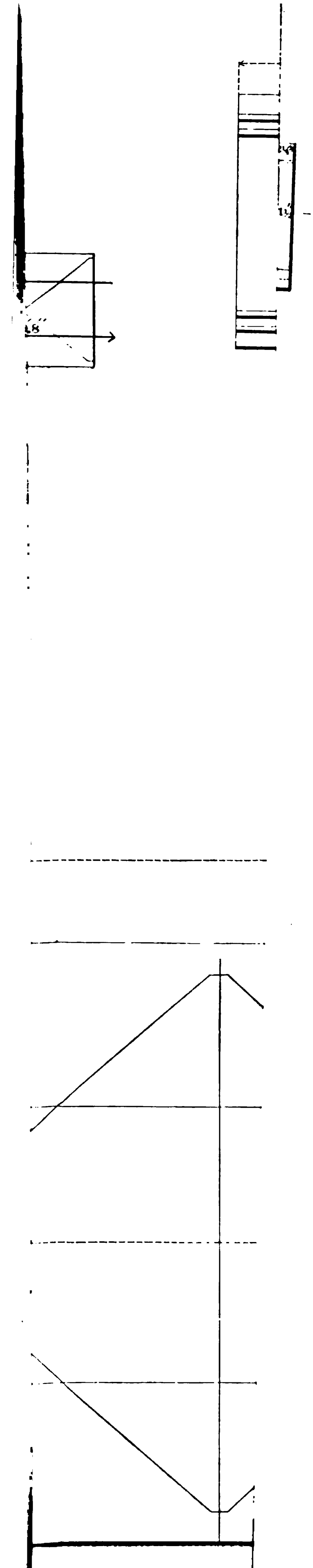


PLATE 28.



M. P. R. R.

Bridge No.44.

SCALE OF FEET

0 1 2

KELLOGG & MAURICE,

Athens, Pa.